SPECIFICATION OF FIRM LEVEL RISK BEHAVIOR MODELS:

ANOTHER LOOK AT THE ALTERNATIVES

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Members of this committee will recall the report presented by Douglas Young at our last meeting (Young, et. al.). The papers that followed and the subsequent discussion suggested a number of topics that could be usefully considered in more detail. This paper will discuss several questions that frequently must be answered when developing an approach to risk in firm level research and extension applications. An attempt is made to identify and evaluate the alternatives based upon the current state of knowledge. This discussion will be limited to the static models of portfolio selection.

It will be useful to restate the traditional decision model. Let A_j represent the jth course of action and S_i the occurrence of the ith state. Let C_{ij} represent the consequence when selection of the jth action is followed by the occurrence of the ith state. This model is represented in matrix form in Figure 1.

		Actions			
		A_1	A_2	A_j	A_J
	S_1	C_{11}	C_{12}	C_{1j}	C_{1J}
	S_2	C_{21}	C_{22}	C_{2j}	C_{2J}
States	S_i	C_{i1}	C_{i2}	C_{ij}	C_{iJ}
	S_I	C_{I1}	C_{I2}	C_{Ij}	C_{IJ}

Figure 1 The traditional decision model where the C_{ij}'s are the consequences.

This decision model can be used as a starting point a) in studying actual behavior, b) in determining how a decision maker ought to behave, c) in helping a decision maker select among possible actions or d) in predicting behavior. However, utilization of the decision model framework requires answers to most if not all of the following:

1. What criteria are to be assumed in evaluating actions?
2. What is the action set?
3. How is the action set reduced to a manageable set?
4. How are the possible states and consequences determined?
5. How is a preferred action selected?

Particular focus will be given here to the first three of these questions.

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The Criterion Function

The maximum expected utility criterion is widely accepted as a normative basis for decision making (Whitmore and Findley, p. 14). However, alternatives have been proposed that consider, for example, a trade off between mean and variance of returns without being particularly concerned about motivating the preference ordering from expected utility theory. In addition there are various safety-first and related chance constrained formulations of the problem that have been proposed. See for example the works reproduced in Ziemba and Vickson, pp. 203-330.

The expected utility hypothesis assumes the criterion function:

$$(1)\quad \max_j \; E(U[C_{ij}]) = \textstyle\sum_i Pr(S_i)U[C_{ij}]$$

where

$Pr(S_i)$ = probability of the ith state, S_i, and

$U[C_{ij}]$ = utility of the consequence C_{ij}.

The mean-variance, E-V, approach on the other hand assumes that the consequences of an action are described sufficiently by knowing the mean and the variance of those consequences. As a result, the decision maker can limit consideration to those actions that are E-V efficient, e.g., those actions where no other action exists that has the same mean and a lower variance. It has been shown that when U is quadratic the expected utility criterion results in selecting from the E-V efficiency set (Lintner).

The E-V approach results in positive and negative deviations about the mean being equally weighted. If the decision maker is more sensitive to deviations below the mean, a semivariance approach which gives weight only to negative deviations about the mean may be desirable. The mean-semivariance, E-SV, approach when semivariance is calculated around a fixed point has been shown to result in an efficiency set that is a subset set of the second degree stochastic dominance efficiency set (Porter)[1]. This same result can not be shown for the E-V efficiency set unless the consequences are normally distributed, in fact, Porter and Carey found in a study of common stocks that the E-V efficiency set included a stock that was inefficient based upon first order stochastic dominance.[2] This result is expected in theory since a quadratic utility function is not monotone increasing.

The safety-first models have also been motivated from a belief that decision makers are more sensitive to deviations below the mean. Those models have been reviewed in detail by Pyle and Turnovsky. The Telser version of the safety-first principle, for example, assumes a criterion function where the objective is:

$$(2)\quad \max_j \; E\,(C_{ij}) \text{ subject to } Pr\,(C_{ij} \leq d) \leq \alpha$$

where

$$E\,(C_{ij}) = \textstyle\sum_i Pr(S_i)\, C_{ij}$$

The probability α that the consequences of selecting action A_j are less than or equal to the disaster level d are assumed to be set a priori by the decision maker. Pyle and Turnovsky show that the safety-first criteria result in linear indifference curves in mean-standard deviation space. Therefore, if the mean-standard deviation efficiency set is strictly convex there is a unique safety-first indifference curve that is tangent to the efficiency set. Hence the same results can be obtained from applying the safety-first criterion and the E-V approach where the efficiency set is a strictly convex E-V efficiency set. This correspondence breaks down, however, when there is a risk free asset. It remains to determine which approach is to be faulted for this lack of correspondence, however. Also the performance of the safety-first criterion where distributions contain more than two parameters has yet to be investigated.

It is of interest to note that the safety-first approaches have at times been characterized as departures from the expected utility framework (Pyle and Turnovsky, p. 75). The following utility function will be considered in more detail below:

$$(3)\quad U[C_{ij}] = C_{ij} + G[C_{ij},T]$$

where

T = target income,

$G[C_{ij},T] = 0$ if $C_{ij} < T$ and

$G[C_{ij},T] = \beta > 0$ if $C_{ij} \geq T$.

If expected utility is maximized for Eq.(3) the following criterion function results.

$$(4)\quad \max_j E(U[C_{ij}]) = E(C_{ij}) + \beta \Pr(C_{ij} \geq T)$$
$$= E(C_{ij}) + \beta(1-\alpha)$$

where

$\alpha = \Pr(C_{ij} < T)$

It can be shown that a parametric solution of the Telser problem, Eq.(2), where d = T results in the E-T (mean-target) efficiency set for the criterion function in Eq.(4) and that a unique solution will exist if the efficiency set is strictly concave. Hence under specific conditions, the safety-first framework can be generated from the expected utility hypothesis. The correspondence with stochastic dominance has yet to be explored.

Another alternative to the E-V approach is to consider additional moments. A Taylor series expansion of $U[C_{ij}]$ can be used. This alternative will be discussed further below.

The expected utility hypothesis is based upon a set of axioms that have been studied in considerable detail. There have been concerns reported in the literature that the independence axiom is frequently violated by decision makers. Machina has recently shown that the expected utility hypothesis is a specific case of a more general hypothesis, however, that does not require the independence axiom. Most of the results derived from expected utility maximization assuming concave utility functions, however, are still valid in the more general case.

A further comment on ranking alternatives according to risk is perhaps appropriate at this point. In an E-V framework, for example, the more risk averse decision makers will tend to select actions from the E-V efficiency frontier that have a lower variance. It may be useful to label as riskier those alternatives that would be selected by a less risk averse decision maker when maximizing expected utility. The use of the coefficient of variation is of particular interest here since it is a frequently used measure of dispersion and is often considered to be a measure of risk. To illustrate the problem involved in using the coefficient of variation as a measure of risk refer to Figure 2 below.

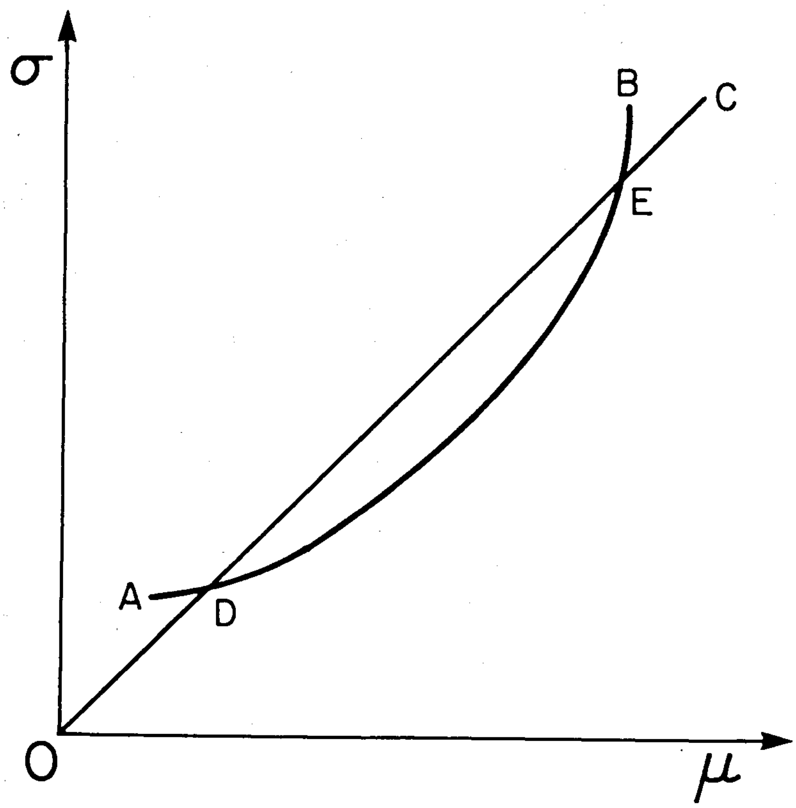

Figure 2 A mean-standard deviation efficiency frontier

The mean-variance efficiency frontier is represented in Figure 2 in mean-standard deviation space since the coefficient of variation is defined as the standard deviation divided by the mean, σ/μ. Let AB represent the μ-σ efficiency frontier. The coefficient of variation is constant along the ray OC hence the coefficient of variation is the same for an action characterized by point D and an action characterized by point E. Clearly points D and E are not equally risky, however, in that it would require a less risk averse decision maker to select E when maximizing expected utility. The coefficient of variation can be used in a mean-variance framework to rank actions that have the same mean. It cannot in general be used to rank alternatives according to risk that have different means.

<u>The Action Set</u>

The production, marketing, and financing alternatives to be incorporated in the action set are in part a function of the imagination of the researcher, the extension worker or the decision maker, but are ultimately limited by the principle of bounded rationality.

> The term <u>bounded rationality</u> is frequently used to describe the limited ability of a person to formulate decision problems in their totality. People have neither the time, the energy, nor the interest to search out all the possible courses of action on a decision problem. They consider the more obvious and potentially attractive alternatives and ignore the rest. (Whitmore and Findlay, p.6)

The various algorithms that have been applied in decision analysis have successfully expanded our ability to generate and evaluate actions. Although there is algorithm that will tell us that we should consider growing sunflowers, for example, if it hasn't already been provided as input to the algorithm, most of the algorithms can be used to generate combinations of production, marketing, and finance alternatives that might not otherwise have been identified. The algorithms that have been developed to apply the stochastic dominance, SD, criteria can not generate combinations of alternatives, however. Furthermore,

> There does not and may never exist an algorithm for developing weighted combinations of risky assets that are efficient by any of the stochastic dominance rules. (Richard Burgess in Whitmore and Findlay, p.165)

The approach that can be used in reducing the action set using SD is to develop an algorithm that will generate feasible actions and then apply the SD algorithms to reduce the action set to a manageable size. The algorithms developed by Porter, Wart and Ferguson can be used to apply the SD rules although some errors have been detected in the algorithms since their publication cf. Whitmore and Findlay, p. 79. An algorithm for SD is also given in Anderson, pp. 312-318.

Since the E-SV efficiency set for a fixed point is a subset of the second order SD efficiency set and Hogan and Warren have developed an efficient algorithm to find the E-SV efficiency set, for many purposes finding the E-SV efficiency set will be the approach to use. Further, Whitmore and Findlay (p. 367) have suggested that parametric solution on the fixed point for the E-SV efficiency set may result in the entire second order SD efficiency set, although this result has not yet been demonstrated.

The mean absolute deviation or MOTAD approach develops an efficiency set using linear programming, LP, that simultaneously approximates the E-V and the E-SV efficiency sets where the semivariance is calculated using the mean (Hazell). In contrast to calculating semivariance around a fixed point, however, calculating semivariance around the mean results in actions in the E-SV efficiency set that are second order SD inefficient (Porter). This suggests a need to revise the MOTAD models to calculate the semivariance around a fixed point.

Another alternative is to use a Taylor series expansion of a utility function to define a nonlinear programming problem. For example, a third order approximation results in a criterion function that has the mean, variance and the third moment about the mean as arguments. Nonlinear programming algorithms are not as generally available, however. Also the correspondence between the efficiency set derived from the use of higher moments and SD efficiency has yet to be investigated. In the limit, parametric solutions to a nonlinear programming problem using higher order moments in developing an efficiency set where appropriate constraints have been placed upon the parameters in the criterion function so that the utility function is everywhere increasing, for example, must result in a first order SD efficiency set. Rulon Pope is currently investigating the use of a nonlinear algorithm to derive an efficiency set where the third moment is included, but he has indicated having problems where the utility function is convex at low levels of income and concave at higher levels of income.

The use of chance constrained programming, CCP, models to solve safety first problems provides another alternative for generating and evaluating actions. Specifying a problem in terms of a discrete probability distribution makes it possible to solve CCP models using a LP algorithm. Consider, for example, the following safety first problem:

$$
\begin{aligned}
(5)\quad & \max_{X}\ E(C'X) \\
& \text{subject to} \\
& AX \leq b,\ X \geq 0 \\
& \text{and} \\
& \Pr(C'X < T,) \leq \alpha
\end{aligned}
$$

which for a discrete probability distribution can be represented as:

$$
\begin{aligned}
(6)\quad & \max_{k,X}\ E(C'X) \\
& \text{subject to} \\
& A\,X \leq b,\quad X \geq 0 \\
& \text{and} \\
& C_i'\,X \geq T,\ i \in k
\end{aligned}
$$

where

C_i = column vector of the net prices in the ith state

k = a subset of the I states where $\sum_{i \in k} \Pr(S_i) \geq 1-\alpha$

In other words, solving a LP problem where profits are maximized subject to the usual constraints plus a set of constraints that require profits to equal or exceed a target $(1 - \alpha)$ X 100% of the time results in one way in which the target can be achieved or exceeded $(1 - \alpha)$ X 100% of the time, i.e. for each k, the maximum profit level where $\Pr(C'X<T) \leq \alpha$ is the maximum profit achieved over all k. For any I, a small α requires that a relatively small number of LP problems be solved. For example, where 20 different states, I = 20, are used and $\Pr(S_i) = .05$ for all i=1, I, an $\alpha = .05$ would require 20 separate LP solutions.

Possible States and the Consequences

Douglas Young has addressed the problem of how to best use historical data to determine possible states and the consequences. In many cases, however, historical data may provide a poor representation of expectations. No attempt will be made here to review the literature concerning the formulation of expectations. Although formulating expected yields is not without its problems, the formulation of expected prices is considerably more complex. This very important problem is left for treatment elsewhere.

We have so far ignored the details of the mathematics of risk analysis, except for pointing out the limitations in the use of the coefficient of variation. However, since most of the models discussed involve the calculation of summary statistics, a comment is perhaps in order on how to calculate these summary statistics. This discussion is taken up here because the relationship between the possible states and the consequences determine the method required in calculating the summary statistics. This point will be illustrated for the calculation of expected profits.

Consider, for example, the production of a crop such as lettuce. Revenue is determined by the acres harvested, the crates harvested per acre and price per crate. Production costs depend upon the acres planted, the inputs required to grow the crop and the harvesting costs. Consider the following profit equation:

$$P = A_h\, Y\, P_y - A_p\, P'_x\, X - C[A_h, Y]$$

where

P = profits,

A_h = acres harvested,

Y = crates harvested per acre,

P_y = sales price per crate,

A_p = acres planted,

P_x = column vector of input prices,

X = column vector of inputs used to grow an acre and

$C[A_h, Y]$ = harvesting costs,

Assuming acres planted and growing costs are known, expected profits become:

$$E(P) = E(A_h\, Y\, P_y) - A_p\, P'_x\, X - E(C[A_h, Y])$$

It is often the case for lettuce that the acres harvested and the crates harvested per acre depend upon market conditions and the specific price received depends upon the quality of the crates which depends upon the intensity of harvesting and the number of crates harvested from each acre. Then expected revenues are an expectation of a product and $E(A_h\, Y\, P_y) \neq E(A_h)\, E(Y)\, E(P_y)$. Also since acres harvested and crates harvested per acre are stochastic, harvesting costs are stochastic and expected harvesting costs are likely to be the expectation of a nonlinear function of the stochastic variables so that $E(C[A_h, Y]) \neq C[E(A_h), E(Y)]$.

The lettuce example illustrates a situation where even the expected profit maximizer, the risk neutral decision maker, is sensitive to price and yield risk. Although this example is not typical, it does help emphasize the need to be clear about the assumptions that are needed to be able to calculate expected profits (the consequences) from expected yields and expected prices.

Selection of a Preferred Action

Selection of an action by a decision maker can be greatly facilitated by reducing the action set to a manageable set and clearly representing the relevant characteristics of those actions. Providing the decision maker with a systematic approach that is consistent with an agreed upon set of axioms (rules) will also be useful.

This group is familiar with the elicitation of utility functions. The work of Machina provides an approach that does not use the independence axiom so that models for eliciting or estimating preferences need not be restricted to the traditional expected utility maximization hypothesis. A potentially fruitful alternative approach to eliciting utility functions that has seldom been mentioned in the literature is what Whitmore and Findlay call the revealed preference method. Consider, for example, the use of the E-SV approach where sufficient information is collected to determine the expected value and semivariance for the action taken in a particular year and the relevant decision maker's characteristics are also collected for a sample of firms. With proper specification of a model, it would be possible to estimate a utility function that is dependent upon the characteristics of the decision maker. Pope has considered this approach where both the production function and the utility function are estimated. Jebuni has applied the revealed preference method by estimating a quadratic utility function using the first order conditions for maximizing expected utility. The revealed preference approach is ex post where the elicitation of utility functions is ex ante. On the other hand, the elicitation of utility functions is usually based upon hypothetical data where the revealed preference approach is not. A comparison of these approaches would be of considerable interest.

Finally a word about presenting an efficiency set as an aid to decision makers. A model can be quite complex for research purposes, but extension applications require some care in the selection of approaches. As an illustration of what can be done, consider the efficiency set from the E-T model discussed earlier. See Figure 3.

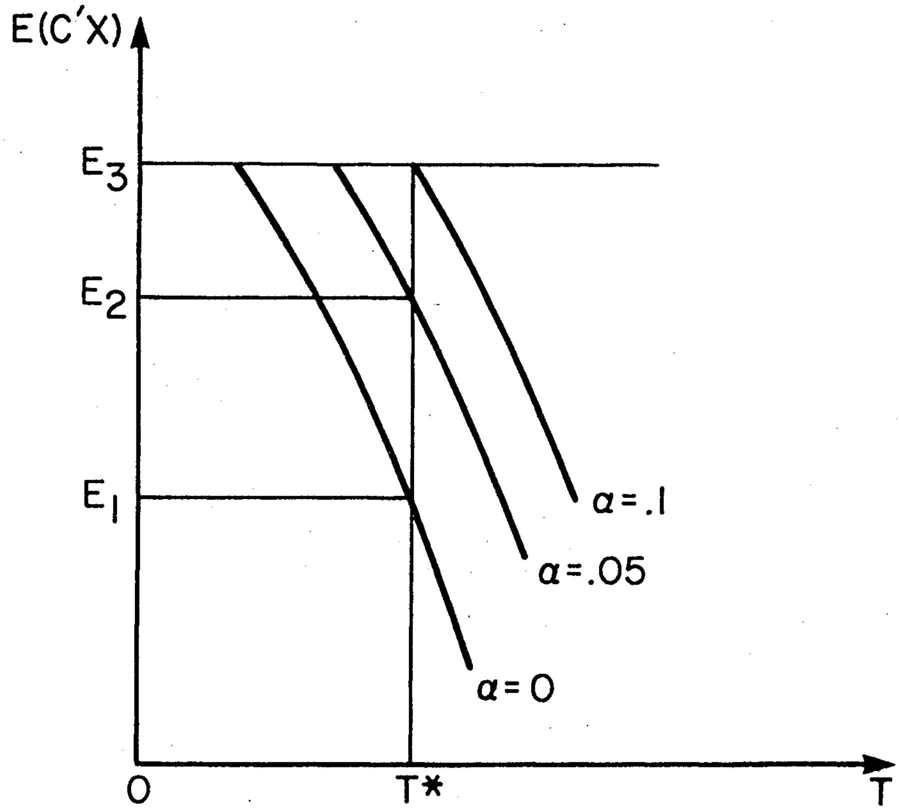

Figure 3 A representative efficiency set for the E-T

Referring to Figure 3, the target T* can be achieved ninety percent of the time, $\alpha = 0.1$ or $(1-\alpha) = 0.9$, and the mean profit would be E_3, the maximum possible profit. Alternatively, the same target, T*, could be achieved 95 percent of the time and mean profits would be E_2 or T* could be achieved 100 percent of the time and mean profits would be E_1. These results can likely be made understandable to a large audience of decision makers. The findings of this paper suggest that it would be desirable to be able to present the results of the E-SV model at least as effectively.

Conclusion

A review of the theoretical work on treatment of risk at the firm level suggests that a mean-semivariance approach is most consistent with current theory if the semivariance measure is calculated using a fixed point. For those that find a safety-first approach appealing, this mean-semivariance measure may be attractive where the fixed point is interpreted as a target or disaster level and the semivariance is the mean of the squared deviations below the target. A probability of achieving the target (or exceeding the disaster level) is not directly involved in the mean-semivariance approach, however. An algorithm is available for applying the mean-semivariance approach using a fixed point (Hogan and Warren). Also a MOTAD model revised to use a fixed point is possible.

Another possible approach that has been shown here to be consistent with the expected utility hypothesis is a modified version of the safety-first model. A linear programming approach to deriving an efficiency set has also been suggested. The correspondence of this model with stochastic dominance efficiency sets has yet to be investigated.

Footnotes

1. The second degree stochastic dominance efficiency set results from eliminating all actions that would be eliminated by any expected utility maximizer that has a utility function that is monotonically increasing and strictly concave i.e. with diminishing marginal utility to the consequences (returns). (Anderson pp. 284-288)

2. The first degree stochastic dominance efficiency set results from eliminating all actions that would be eliminated by any expected utility maximizer that has a utility function that is monotonically increasing, i.e. a comparatively weak condition. (Anderson, pp. 282-284)

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