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Risk Modeling in Agriculture: Retrospective and Prospective

Program proceedings for the annual meeting of the Technical Committee of S-232, held March 24-26, 1994, Gulf Shores State Park, Alabama.

Iowa State University
Ames, Iowa 50011-1070
August 1994

ALTERNATIVE FORMULATIONS OF RISK PREFERENCES IN DYNAMIC INVESTMENT MODELS*

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A number of authors have noted that off-farm investments have the potential to reduce risks faced by agricultural firms. For example, Young and Barry found that Illinois cash grain farms can reduce risk by holding a portfolio of farm assets, Standard and Poors 500 stocks, and passbook savings. In somewhat the same vein, Moss et al. found that farm assets enter into expected return—variance efficient portfolios containing off-farm assets.

These studies were static in that they do not consider firm growth. However, previous studies examining firm growth do not consider the possibility of off-farm investments (e.g., Gwinn, Barry, and Ellinger; Larson, Stauber, and Burt). Given that growth is a relatively risky process and that previous studies indicate that off-farm investments reduce risk, it is reasonable to expect that including off-farm assets in growth plans will reduce risk. In this paper, we examine risk reductions possible by including off-farm assets with farm assets in a firm growth context.

The investment model we use is caste in terms of a stochastic dynamic programming problem. It follows the theoretical development of a financial model that maximizes the expected utility of terminal wealth (Elton and Gruber). We use dynamic programming because it (1) allows investments to be lumpy and irreversible, characteristics common to many agricultural investments, (2) allows incorporating dynamic relationships across asset returns, and (3) allows optimal decisions to be related to the current financial structure of the firm.

In our analysis, we examine alternative objective functions. These objective functions embody different preferences for risk that change as wealth level changes. Similar to Featherstone et al., we focus attention on utility functions that exhibit constant absolute risk aversion and constant relative risk aversion. Unlike most previous approaches, however, we specifically allow for bankruptcy. Including bankruptcy allows incorporation of preferences for bankruptcy avoidance, a goal that seems important to many firm managers (Patrick and Brake). This examination will provide guidance for specifying an objective function for use in dynamic investment models.

Organization of the remainder of this paper is as follows: First, we detail considerations in specifying objective functions for a dynamic, investment model. Then, a conceptual model for examining dynamic investment decisions is given and numerically solved for an Albertan

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* A paper presented at S-232, Regional Risk Management meeting, Gulf Shores State Park, Alabama, March 24-26,1994.

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hog finisher. Parameters and numerically results are respectively presented in the third and fourth sections of this paper. We follow by giving conclusions and suggestions for future research.

Objective Functions

Financial theory suggests two alternatives for determining optimal investment decisions over time. One approach uses a multiperiod consumption-investment model to jointly find consumption and investment decisions that maximize the discounted, expected utility of consumption over time (see Fama, Merton, Samuelson, and Elton and Gruber). This model requires an objective function that incorporates both risk and time preferences for consumption. Alternatively, a terminal wealth model determines investment decisions that maximize a function of wealth in the terminal time period (see Elton and Gruber). In the terminal wealth model, the objective function incorporates only risk preferences.

We use the terminal wealth model in this paper. By using the terminal wealth model, time preferences for consumption are not specified. Therefore, focus is given to the impacts of risk preferences on investment decisions. We justify this choice by noting that consumption withdrawals often are relatively small cash flows on mid- to large-sized farms. Moreover, changes in farm income have relatively small impacts on consumption (Langemeir and Patrick). These factors suggest that changes in consumption are likely to have little impact on investment decisions.

We use a terminal wealth utility function adapted for use in cases on limited liability (Robison and Barry; Robison and Lev; Collins and Gbur):

$$U(w_T) = \begin{cases} u_1(0), & w_T \leq 0 \\ u_2(w_T), & w_T > 0 \end{cases} \tag{1}$$

where w_T is terminal wealth $u_i(\cdot)$ are functions. This utility function has two parts: one for cases when terminal wealth is greater than zero (i.e., $u_2(w_T)$) and the other when wealth is less than zero (i.e., $u_1(0)$). The $u_2(w_T)$ function is continuous and concave, representing the typical function for ordering risky choices. The $u_1(0)$ accounts for truncations to the wealth distribution. In our model, truncations occur because of bankruptcy. We define bankruptcy as occurring whenever a negative wealth level arises. When bankruptcy occurs, we presume that investment decisions are no longer made and zero terminal wealth results. Since bankruptcy truncates the wealth distribution, a discontinuity exists and a discrete probability mass may occur at the zero wealth level. Hence, finding the expected value of expression (1) with respect to investment decisions (θ) is equivalent to:

$$E[U(w_T)] = u_1(0)\,p(w_T \leq 0 \mid \theta) + \int_0^{\infty} u_2(w_T)\,g(w_T \mid \theta)\,dw_T \tag{2}$$

where $p(\cdot)$ is the probability of bankruptcy and $g(\cdot)$ is a probability density function for terminal wealth levels greater than zero (see Collins and Gbur for a more detailed discussion).

Of key importance in specifying a terminal wealth function is recognizing how risk preferences change with different wealth levels. If a utility function is continuous and concave, risk preferences for alternative wealth levels can be examined using Pratt-Arrow measures of:

1. absolute risk aversion — defined as $-u''(w)/u'(w)$, where u'' and u' denote the second and first derivative of the utility function, and

2. relative risk aversion — defined as $-w \cdot u''(w)/u'(w)$.

For both absolute risk aversion (ARA) and relative risk aversion (RRA) measures, a function is classified as exhibiting increasing, constant, or decreasing risk aversion, depending on whether the risk aversion measure increases, remains constant, or decreases as wealth level increases.

Investment properties for differing ARA and RRA classifications are known in the static, mean-variance portfolio case. While these properties will not necessarily translate to our dynamic model, they do provide insight into the types of investment decisions we will obtain. Under a constant ARA utility function, investments result in the same variance on terminal wealth regardless of the initial wealth level. If an individual has $1,000 of initial wealth and invests in a portfolio that yields a $10 variance, then that individual will select a portfolio that yields a $10 variance at wealth levels of $2,000, $5,000, etc. Hence, proportions of wealth held in assets with higher variances decline as wealth level increases. If the utility function exhibits decreasing (increasing) absolute risk aversion, variance on terminal wealth will increase (decrease) as initial wealth level increases.

Under a constant RRA utility function, proportions invested in assets will not vary with initial wealth level. If, for example, an individual has $1,000 of wealth and invests 50 percent of initial wealth in two assets, then that individual will invest 50 percent in each asset when initial wealth is $2,000, $5,000, etc. Elton and Gruber show that the constant proportions property extends to the multiperiod case when there is no probability of bankruptcy and asset investments are not lumpy. Their analytic results are not directly transferable to our model because we allow for bankruptcy and investment in agricultural assets is lumpy. However, their results are suggestive of the types of decisions we will obtain.

For our analysis, we examine two types of continuous, concave utility functions. The first is a constant relative risk aversion (CRRA) function of the following form:

$$u_2(w_T) = \frac{w_T^{(1-\alpha)}}{1-\alpha} \qquad (3)$$

where α represents a risk parameter equal to the Pratt-Arrow measure of relative risk aversion. We examine cases in which α is less than 1 and define $u_1(0)$ to be equal to expression (3) at a wealth level of zero (i.e., $u_1(0) = u_2(0) = 0$). The second is a constant absolute risk aversion (CARA) function of the following form:

$$u_2(w_T) = -e^{-\lambda w_T} \tag{4}$$

where λ is a risk parameter equal to the Pratt-Arrow measure of absolute risk aversion. We define $u_1(0)$ to be equal to expression (4) at a wealth level of zero (i.e., $u_1(0) = u_2(0) = -1$). Since the CRRA function exhibits decreasing ARA, we can examine investment decisions under decreasing and constant ARA. Moreover, we can examine investment decision under constant and increasing RRA because the CARA exhibits increasing RRA.

We also solve the investment model using two other utility functions:

1. Expected wealth maximization: In the expected wealth maximization case, risk preferences are neutral and the utility function has the following form: $u_1(0) = 0$ and $u_2(w_T) = w_T$.

2. Bankruptcy avoidance: Bankruptcy avoidance involves the following utility function: $u_1(0) = -1.0$ and $u_2(w_T) = 0.0$. The values of $u_1(\cdot)$ and $u_2(\cdot)$ result from a CARA function as the Pratt-Arrow measure of constant absolute risk approaches infinity (i.e., $u(w) = e^{-\lambda w}$ as $\lambda \rightarrow \infty$). In this case, investment decisions are made so that bankruptcy is avoided without regard to the ending value of terminal wealth. In essence, the bankruptcy avoidance case is the opposite extreme of the expected wealth maximization case. This case also represents a safety first objective function, when all weight is placed on "safety".

The Multiperiod Investment Model

We specify a multiperiod investment model in which investment decisions occur at the beginning of each month. Investments are made in three assets: hog finishing buildings, stocks, and financial holdings. The hog finishing building is a lumpy, irreversible investment, embodying the characteristics of many agricultural investments. The investment in a hog finishing building equals BNV. For this investment, a constant number of hogs (NH) are marketed each month, with returns from marketings equaling:

$$NH \cdot (H_t - FC) \tag{5}$$

where H_t equals per hog revenue minus variable costs, hereafter referred to as hog returns, and FC equals per hog fixed costs.

Stock holdings are invested in a market portfolio of publicly traded equity investments. This portfolio is equivalent to an unmanaged mutual fund in which investments are made in all traded stocks proportional to their relative values. Stock holdings generate returns (R_t) consisting of dividends and stock appreciation. We presume that stock investments are perfectly divisible. Moreover, the individual can invest and liquidate stock investments at no cost.

Financial holdings represent positive or negative holding of treasury bills. Positive holdings represent investments in treasury bills for which the agent receives a return equal to the interest rate (I_t). Negative holdings represent debt on which interest payments are made. The interest rate for determining interest payment equals the interest rate on treasury bills plus a borrowing differential (BD) that represents costs of financial intermediation.

Dynamic Returns and Investment Movements

Returns to the three assets are stochastic and are presumed to follow a first-order Markovian structure. Return movements can be modeled using the following general relationship:

$$f(H_{t+1}, R_{t+1}, I_{t+1} \mid H_t, R_t, I_t) \tag{6}$$

where f(·) is a probability density function. This function gives the distribution of next month's returns conditional on the realizations of current month's returns.

Each month, the agent makes two decisions:

1. Invest in another hog finishing barn. This decision is denoted as DB_t. DB_t may have two values: 0 = do not invest in another barn and 1 = invest in another barn.

2. Invest (Disinvest) in stocks. This decision is denoted as DS_t. Positive amounts indicate that additional funds are invested in stocks while negative numbers indicate that funds are withdrawn from stocks.

A decision variable is not required for financial holdings because investments in finishing barns and stocks uniquely determine financial holdings.

Based on these decisions, we can determine how holdings of the assets move over time. Investments in hog finishing barns are modeled by the number of hog finishing barns (B_t) according to the following relationship:

$$B_{t+1} = B_t + DB_t \tag{7}$$

which states that number of barns next month equals the number of barns in the current month plus barns acquired during the current month.

Stock holdings in the next month (SH_{t+1}) equal the stock holdings in the current month plus returns from stock holdings plus the decision to invest (disinvest) additional funds in stocks:

$$SH_{t+1} = SH_t \cdot (1 + R_t) + DS_t \tag{8}$$

For expository and solution reasons, stock holdings are restated as a proportion of funds invested in finishing barns. Specifically, we define S_t as stock holdings as a proportion of finishing barn investment (i.e., $S_t = SH_t / (B_t \cdot BNV)$). The primary advantage of this definition is that it allows us to more easily compare relative investments in finishing barns and stock holdings. Using the definition of S_t, we can restate equation (8) as:

$$S_{t+1} = \frac{(S_t \cdot B_t \cdot BNV) \cdot (1 + R_t) + DS_t}{B_{t+1} \cdot BNV} \tag{9}$$

Financial holdings in the next month (FH_{t+1}) equal the financial holdings in the current month (FH_t), plus returns (costs) of financial holdings ($FH_t \cdot IT(FH_t, I_t)$), plus returns from hog marketings ($B_t \cdot NH \cdot (H_t - FC)$), less investment in hog finishing barns ($DB_t \cdot BNV$), less investment (disinvestment) in stocks (DS_t):

$$FH_{t+1} = FH_t \cdot (1 + IT(FH_t, I_t)) + B_t \cdot NH \cdot (H_t - FC) - DB_t \cdot BNV - DS_t \tag{10}$$

where

$$IT(F_t, I_t) = \begin{cases} I_t & when\ F_t \geq 0 \\ I_t + BD & when\ F_t < 0 \end{cases}$$

adds the borrowing differential to the interest rate when financial holdings are negative. Similar to stock holdings, we redefine financial holdings for expository purposes. Specifically we define F_{t+1}, which states financial holdings as a proportion of investment in finishing barns and stocks:

$$F_{t+1} = \frac{FH_{t+1}}{B_{t+1} \cdot BNV + S_{t+1} \cdot B_{t+1} \cdot BNV} \tag{11}$$

When financial holdings are negative, F_t is interpreted as the negative of the debt-to-asset ratio. When financial holdings are positive, F_t gives financial holdings relative to holdings in other assets.

Terminal Wealth and Solution Procedures

Optimal investment decisions are found for each month by maximizing the expected utility of terminal wealth, where the general utility function is given in equation (1) and terminal wealth is defined as:

$$w_T = (B_T \cdot BNV + S_T \cdot B_T \cdot BNV) \cdot (1 + F_T) \tag{12}$$

This problem is solved recursively using Bellman's principle of optimality. The recursive objective function for each month's maximization is:

$$V_t(H_t, R_t, I_t, B_t, S_t, F_t) = \max_{DB_t, DS_t} E\left[V_{t+1}(H_{t+1}, R_{t+1}, I_{t+1}, B_{t+1}, S_{t+1}, F_{t+1})\right] \tag{13}$$

where $V_t(\cdot)$ is the recursive objective function that gives the expected utility of terminal wealth, given that optimal investment decisions are made from month t to the terminal month. This problem has six state variables: hog returns, stock returns, interest rates, number of finishing barns, stock holdings, and financial holdings. Transition equations for the state variables are given in equations (6), (7), (9), and (11). In addition to the state transition equations, the maximization in (13) is subject to the following bankruptcy condition:

$$V_t(\cdot) = u_1(0) \qquad \text{when } F_{t+1} < -1.0 \tag{14}$$

This condition states that when debt exceeds assets, as indicated by a F_{t+1} that is less than -1.0, the firm is bankrupt, terminal wealth becomes zero, and the expected utility of terminal wealth is given by the portion of the utility function associated with bankruptcy.

Numeric Specification of the Dynamic Investment Model

The multiperiod investment model was solved numerically for an Albertan hog finisher. For each finishing barn, 750 hogs per month are marketed, fixed costs per hog equal $5, and investment cost per barn is $290,000 (i.e., NH = 750, FC = 5, and BNV = 290,000). Stock holdings occur in a market portfolio traded on the Toronto Stock Exchange. Financial holdings occur in Canadian treasury bills.

Stochastic, Markovian relationships were estimated for the three assets using data from January, 1980 through December, 1989. A series of hog returns was constructed to be representative of an Albertan finishing operation. Stock returns were calculated as the natural logarithm of the monthly change in the Toronto Stock Exchange 300 total returns index. Interest rates represent market rates on treasury bills. Both monthly stock returns and interest rates were stated as a yearly return.

Time series methods were used to determine the Markovian structures of returns. Results and statistical tests indicated that a first-order, auto-regressive model adequately captures the dynamic movements of returns over time. The following estimates using ordinary least squares resulted:

$$\underset{}{H_t} = \underset{(3.20)}{1.800} + \underset{(9.70)}{.8350}\, H_{t-1} \qquad R^2 = .726 \qquad S_e = 10.8 \tag{15-a}$$

$$R_t = \underset{(3.78)}{.1689} + \underset{(.783)}{.0547}\, R_{t-1} \qquad R^2 = .271 \qquad S_e = .542 \tag{15-b}$$

$$I_t = \underset{(.810)}{.0015} + \underset{(60.8)}{.9890}\, I_{t-1} \qquad R^2 = .962 \qquad S_e = .0058 \tag{15-c}$$

where R^2 is the adjusted r-square, S_e is the standard error of estimate, and t-ratios are given below the parameter estimates.

Realizations of next month's returns were calculated using normal distributions. Expected values for each of the returns are respectively given by the equations in (15). Standard deviations of next month's realizations were presumed to equal the respective standard error of estimates. When borrowing occurred, a .03 borrowing differential was added to the interest rate given in (15-c) to arrive at the interest rate for borrowing (i.e., BD = .03).

We limited the range of number of barns, stock holdings, and financial holdings. Number of barns ranged from 0 to 8, stock holdings as a proportion of the barn investment (S_t) ranged from 0 to 1, and financial holdings as a percent of finishing barn investment and stock holding (F_t) ranged from -1 to 1.

Results for Differing Objective Functions

Optimal decisions were found for the dynamic investment model given objective functions associated with (1) expected wealth maximization, (2) bankruptcy avoidance, (3) CRRA function having α parameters equal to .3, .6 and .9, and (4) CARA function having λ parameters of 1.0×10^{-6}, 5.0×10^{-6}, and 1.0×10^{-7}. Optimal investment decisions were recursive solved for 10 years and, in all cases, converged by the 5th year. In the next three sub-sections, we report the expected financial position over a five year period given that optimal investment decisions are made according to the converged decision rule. For all objective functions, the initial financial position is to own one finishing facility and have an additional $60,000 of financial holdings, giving an initial wealth level of $350,000.

Expected Wealth Maximization and Bankruptcy Avoidance Results

Results for the expected wealth maximizing and bankruptcy avoidance objective functions are reported in Table 1. Expected wealth at the end of year five is $879,903 for the expected wealth maximizing objective and $672,367 for the bankruptcy avoidance objective (see Table 1), suggesting that the expected wealth maximizing objective yields $207,536 more wealth than the bankruptcy avoidance objective. As one would expect, the variance of wealth and the probability of bankruptcy is higher under expected wealth maximization than under bankruptcy avoidance. By the end of the fifth year, there is a .162 cumulative probability of bankruptcy occurring between the initial and fifth year under expected wealth maximization; while the bankruptcy probability is .106 for the bankruptcy avoidance objective. For the bankruptcy avoidance objective, the bankruptcy probability may seem high; however, the high probability reflects the relatively high risks associated with finishing hogs.

Under the wealth maximizing objective, expected number of hog finishing barns increases each year. By the end of year 5, the expected number of barns is 2.84 when the firm is not bankrupt (Table 1). In addition, significant stock holdings occur. Expected stock holdings as a percent of investment in barns (S_t) equal .84 at the end of year 1, .78 in year 2, and .77 in year 3 through 5 (see Table 1). The financial holdings as a proportion of barn and stock investment (F_t) are always negative, indicating that debt is held. As stated previously, F_t can be interpreted as the negative of the debt-to-asset ratio when F_t is negative. The expected wealth maximizing objective results in an .50 expected debt-to-asset ratio in year 1 and a .45 expected debt-to-asset ratio in year 5 (see Table 1).

Table 1. Simulation Results Over Five Years, Beginning with One Barn and 350,000 Wealth, Expected Wealth Maximization and Bankruptcy Avoidance Objectives.

	Objective	
	Expected Wealth Maximization	Bankruptcy Avoidance
Expected wealth in year 5	$879,903	$672,367
Variance of wealth in year 5[1]	849,895	300,862
Probability of bankruptcy		
year 1	.000	.000
year 2	.028	.026
year 3	.075	.063
year 4	.119	.088
year 5	.160	.106
Expected number of barns[2]		
year 1	1.13	1.00
year 2	1.50	1.00
year 3	1.93	1.00
year 4	2.39	1.00
year 5	2.84	1.00
Expected stock holding (S_t)[2, 3]		
year 1	.84	.51
year 2	.78	.48
year 3	.77	.50
year 4	.77	.55
year 5	.77	.60
Expected financial holdings (F_t)[2,4]		
year 1	-.50	-.27
year 2	-.52	-.09
year 3	-.50	.09
year 4	-.49	.23
year 5	-.45	.36

[1]The variance is calculated for only positive wealth levels (i.e., it does not include bankruptcy wealth levels).

[2]Expectations are calculated conditional on the firm being not bankrupt.

[3]Stated as a proportion of investment in hog finishing facilities.

[4]Stated as a proportion of investment in hog finishing barns and stock holdings. Negative numbers are interpreted as debt-to-asset ratios.

Under the bankruptcy avoidance objective, investment in additional barns are not made, as indicated by expected number of barns equaling 1 in all years (see Table 1). However, stocks investments do occur. In our simulation results, expected S_t is .51, .48, .50, .55, and .60 in years 1 through 5, respectively (see Table 1). In early years of the simulation, stock investments are debt financed as indicated by expected F_t of -.27 at the end of year 1 and -.09 at the end of year 2 (see Table 1). Over time, debt is reduced and positive financial holdings occur: expected F_t equals .09 in year 3, .23 in year 4, and .36 in year 5. Use of debt may seem counter-intuitive for a bankruptcy avoidance objective. In our model, debt is used to purchase stocks which have zero correlation with hog returns. This diversification into stocks outweighs the risks associated with using debt capital. These results strongly support the idea that stock investments serve as an effective means of reducing risk for a hog finisher.

Constant Relative Risk Aversion (CRRA) Results

Results for CRRA functions are presented in Table 2. As one would expect, expected wealth decreases and variability of wealth decreases as risk aversion increases. For example, expected wealth equals $878,623 when α equals .3, $873,262 when α equals .6, and $684,426 when α equal .9 (see Table 2). Bankruptcy probabilities in year 5 equal .149, .145 and .106 respectively for α parameters of .3, .6 and .9.

Higher risk aversion levels lead to lower holdings of risky assets. For example, the expected number of barns in year 5 equals 2.76 and expected S_t in year 5 equals .75 for an α of .3 compared to an expected number of barns in year 5 of 2.71 and an expected S_t of .71 for an α of .6. When α equals .9, additional investments in hog finishing facilities do not occur.

Higher levels of risk aversion also lead to lower debt holdings. In year 5, expected F_t equals -.43 for α equal to .3, -.40 for α equal to .6, and -.04 for α equal to .9.

Constant Absolute Risk Aversion (CARA) Results

Results for CARA utility functions are presented in Table 3. Again, the usual results as risk aversion increases are exhibited:

1. Expected wealth decreases as risk aversion level increases. Expected wealth in year 5 equals $875,074, $844,081 and $716,587 for λ of 1.0×10^{-7}, 5.0×10^{-7} and 1.0×10^{-6}.

2. Variability of expected wealth decreases as risk aversion level increases. Variances of wealth in year 5 equals $841,057, $514,381, and $348,762 for λ of 1.0×10^{-7}, 5.0×10^{-7} and 1.0×10^{-6}.

3. Holdings of risky assets decline as risk aversion increases. Expected number of barns in year 5 equal 2.82, 2.25 and 1.16 for λ of 1.0×10^{-7}, 5.0×10^{-7} and 1.0×10^{-6}.

4. Use of debt declines as risk aversion increases. Expected F_t in year 5 equal -.45, -.34 and -.11 for λ of 1.0×10^{-7}, 5.0×10^{-7} and 1.0×10^{-6}.

Table 2. Simulation Results Over Five Years, Beginning with One Barn and 350,000 Wealth, Constant Relative Risk Aversion (CRRA) Utility Functions.

	α parameters		
	.3	.6	.9
Expected wealth in year 5	\$878,623	\$873,262	\$684,426
Variance of wealth in year 5[1]	781,844	762,771	300,862
Probability of bankruptcy			
year 1	.000	.000	.000
year 2	.027	.027	.026
year 3	.072	.070	.063
year 4	.113	.110	.089
year 5	.149	.145	.106
Expected number of barns[2]			
year 1	1.12	1.10	1.00
year 2	1.46	1.44	1.00
year 3	1.87	1.83	1.00
year 4	2.32	2.27	1.00
year 5	2.76	2.71	1.00
Expected stock holding (S_t)[2, 3]			
year 1	.80	.76	.66
year 2	.76	.71	.68
year 3	.75	.71	.71
year 4	.75	.71	.74
year 5	.75	.71	.76
Expected financial holdings (F_t)[2,4]			
year 1	-.48	-.46	-.37
year 2	-.49	-.46	-.25
year 3	-.47	-.45	-.15
year 4	-.46	-.43	-.08
year 5	-.43	-.40	-.04

[1]The variance is calculated for only positive wealth levels (i.e., it does not include bankruptcy wealth levels).

[2]Expectations are calculated conditional on the firm being not bankrupt.

[3]Stated as a proportion of investment in hog finishing facilities.

[4]Stated as a proportion of investment in hog finishing barns and stock holdings. Negative numbers are interpreted as debt-to-asset ratios.

Table 3. Simulation Results Over Five Years, Beginning with One Barn and 350,000 Wealth, Constant Absolute Risk Aversion (CARA) Utility Functions.

	λ parameters		
	1.0×10^{-7}	5.0×10^{-7}	1.0×10^{-6}
Expected wealth in year 5	$875,075	$844,081	$716,587
Variance of wealth in year 5[1]	841,057	514,381	348,762
Probability of bankruptcy			
year 1	.000	.000	.000
year 2	.025	.027	.026
year 3	.075	.070	.066
year 4	.118	.110	.092
year 5	.158	.145	.112
Expected number of barns[2]			
year 1	1.12	1.11	1.01
year 2	1.49	1.44	1.07
year 3	1.91	1.76	1.11
year 4	2.37	2.03	1.14
year 5	2.82	2.25	1.16
Expected stock holding (S_t)[2, 3]			
year 1	.80	.78	.79
year 2	.76	.74	.78
year 3	.75	.75	.80
year 4	.75	.75	.81
year 5	.76	.76	.81
Expected financial holdings (F_t)[2,4]			
year 1	-.49	-.47	-.45
year 2	-.50	-.48	-.32
year 3	-.49	-.45	-.23
year 4	-.48	-.40	-.15
year 5	-.45	-.34	-.11

[1]The variance is calculated for only positive wealth levels (i.e., it does not include bankruptcy wealth levels).

[2]Expectations are calculated conditional on the firm being not bankrupt.

[3]Stated as a proportion of investment in hog finishing facilities.

[4]Stated as a proportion of investment in hog finishing barns and stock holdings. Negative numbers are interpreted as debt-to-asset ratios.

Comparison of CRRA and CARA Utility Functions

Comparison of the CRRA and CARA of utility functions can best be made by examining optimal decisions for the two utility functions. CRRA utility functions tend towards stable values on S_t and F_t as illustrated in Figure 1. This figure shows B_t, S_t and F_t over time for a CRRA function with $\alpha = .6$ given that realizations of random returns occur at their expected values and that the initial financial position is to own one barn and have an additional $60,000 of wealth. Optimal decisions immediately lead to an S_t of .70 and a F_t of -.47. Over time, S_t increases to .90 and F_t increases to -.20. When these levels are reached, another barn is acquired and S_t and F_t fall. Over time, S_t again increases to .9 and F_t increases to -.20. When these levels are reached, another barn is purchased. Obviously, values of S_t and F_t will occur outside the ranges shown in Figure 1 because returns will not be realized at their expected values. However, optimal decisions tend to lead towards values within these ranges.

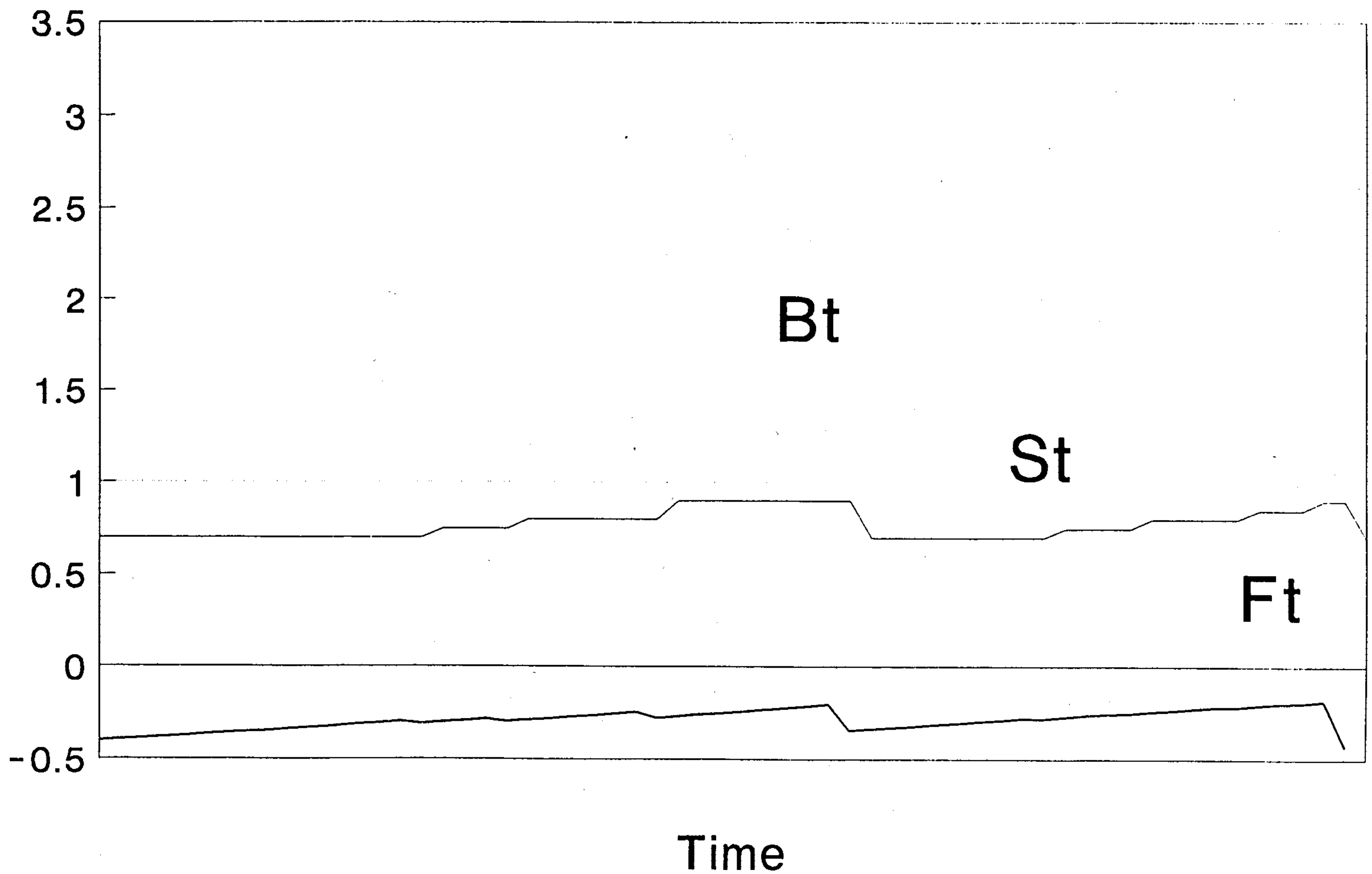
The CARA functions will not lead to similar results. CARA functions will reach a maximum B_t and a maximum S_t. This is easily illustrated by examining investments in hog finishing buildings. Total number of finishing barns will never exceed 2 when λ equal to 1.0×10^{-6} and will never exceed 4 when λ equal to 5.0×10^{-6}. Hence, CARA utility functions will limit firm size while CRRA functions will not.

Summary and Conclusions

Our results suggest that stock holdings can serve as an effective means of reducing risks during firm expansion. Under all alternative risk preferences we examined, stock holdings occur. Even under a bankruptcy avoidance objective, stock holdings occur when a hog finishing barn is already owned. We suggest that continued research concerning off-farm investments should be conducted. This research could compare the risk-reducing benefits of off-farm investments to other methods of reducing risks. Moreover, we suggest that outreach efforts directed at farm clientele concerning the benefits of off-farm investments should be conducted.

Our results also aid in selection of an objective function in a dynamic setting. Without empirical studies, we cannot say whether a CRRA or a CARA is the preferred approach. However, we can place the choice in a more concrete context. If a researcher believes that risk preferences limit firm size, a CARA (or, by extension of our results, an increasing ARA) function is the preferred choice. On the other hand, if a researcher does not believe that risk preferences limit firm size, a CRRA (or, be extension of our results, a decreasing ARA) function is the preferred choice.

Figure 1. Time Path of Optimal Decisions for .6 CRRA Utility Function (Return Realizations at Means)



References

Collins, R. A., and E. E. Gbur. 1991. "Risk Analysis for Proprietors with Limited Liability: A Mean-Variance Safety-First Synthesis." *Western Journal of Agricultural Economics* 16: 156-162.

Elton, E. J., and M. J. Gruber. 1975. *Finance as a Dynamic Process*. Englewood Cliffs, New Jersey: Prentice-Hall Inc.

Fama, E. 1970. "Multi-period Consumption-Investment Decisions." *American Economic Review* 60: 163-174.

Featherstone, A. M., P. V. Preckel, and T. G. Baker. 1990. "Modeling Farm Financial Decisions in a Dynamic Stochastic Environment." *Agricultural Economic Review* 50: 80-99.

Gwinn, A. S., P. J. Barry, and P. N. Ellinger. 1992. "Farm Financial Structure Under Uncertainty." *Agricultural Finance Review* 52: 43-56.

Langemeir, L. N., and G. F. Patrick. 1990. "Farmer's Marginal Propensity to Consume: An Application to Illinois Grain Farms." *American Journal of Agricultural Economics* 72: 309-316.

Larson, D. K., M. S. Stauber, and O. R. Burt. 1974. "Economic Analysis of Farm Firm Growth in North Central Montana." Research Report No. 62. Montana Agricultural Experiment Station. Montana State University, Bozeman, Montana.

Merton, R. 1969. "Lifetime Portfolio Selection Under Uncertainty: The Continuous Time Case." *Review of Economics and Statistics* 50: 247-257.

Moss, C. B., A. M. Featherstone, and T. G. Baker. 1987. "Agricultural Assets in an Efficient Multiperiod Investment Portfolio." *Agricultural Finance Review* 47: 82-94.

Patrick, G. F., and B. F. Brake. 1981. "Measurement and Modelling of Farmer's Goals." *Southern Journal of Agricultural Economics* 12: 199-204.

Robison, L. J., and P. J. Barry. 1987. *The Competitive Firm's Response to Risk*. Macmillan Publishing Company, New York.

Robison, L. J., and L. Lev. 1986. "Distinguishing Between Indirect and Direct Outcome Variables to Predict Choices Under Risk or Why Woody Chip Went to the Air." *North Central Journal of Agricultural Economics* 8: 59-67.

Samuelson, P. 1969. "Lifetime Portfolio Selection by Dynamic Portfolio Selection." *Review of Economics and Statistics* 50: 239-246.

Young, R. P., and P. J. Barry. 1987. "Holding Financial Assets as a Risk Response: A Portfolio Analysis of Illinois." *North Central Journal of Agricultural Economics* 9: 77-84.