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EX ANTE MODELING OF THE EFFECT OF IRREVERSIBILITY AND UNCERTAINTY ON CITRUS INVESTMENTS

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The study of investment has been an important concern within the disciplines of economics and finance, yet the process of investment is largely a mystery. Our traditional courses in financial management teach that the decision on whether or not to make an investment is based on a comparison between the present value of future inflows arising from the investment with the present value of outflows requirements for the investment. If the net present value of the investment is positive then the investment should be pursued. However, firms routinely fail to make investments that appear to be profitable considering the time value of money (Pagano). In addition, analysis of aggregate investment and asset value equations often find little effect of increases in market income on agricultural investment or asset values (Moss, Shonkwiler, and Reynolds).

These difficulties with the traditional net present value criteria have led to the search for other factors to explain investment. Clearly, the most fruitful area of research has been the presence of risk or uncertainty in the investment process. Typically, investments require significant and fairly certain outlays in exchange for risky returns that accrue over a number of years. Within this context several procedures have been suggested for integrating risk into the valuation of investment opportunities. The most straightforward approach is to use a CAPM or risk adjusted discount rate that discounts future returns for relative risk. Another approach is to use investor risk preferences with the distribution of future returns in either a mean-variance rule or a stochastic dominance approach. Finally, one could adopt a "pecking order" approach of adopting projects based on different criteria depending on the level of funding required, i.e. internal funds versus external funds (Myers).

From an operational vantage point, the "pecking order" and risk preference approaches require extensive knowledge about the individual. Further, the CAPM approach is typically severely hampered in agriculture by a lack of complete markets for agricultural equity. A more tractable approach for *ex ante* analysis proposed by Pagano can be derived from the investment under uncertainty and irreversibility literature. The empirical model presented in this paper builds on the theoretical model developed by McDonald and Siegel and proposed more recently by Pindyck and Dixit. This literature emphasizes that the investment process is seldomly a "now or never proposition." Instead, the investor typically has the ability to postpone the investment. The ability to postpone the investment changes the investment problem to one of deciding not only whether or not to execute an option, but also when to execute an option.

The purpose of this paper is to outline an empirical framework for analyzing the effect of uncertainty and irreversibility on an investment alternative *ex ante*. First, the paper presents the underlying model of investment under irreversibility and uncertainty. Next, the empirical

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procedure used to analyze an investment decision in Florida citrus is presented. Finally, we will comment on the implications for models of aggregate investment.

Derivation of the Value of Waiting

This section of the paper outlines the theoretical model of investment under uncertainty and irreversibility. It follows Pindyck's (1991) development fairly closely then points out the simplifying assumptions required to derive Dixit's formulation. The development of the investment rule under uncertainty and irreversibility parallels the derivation of option pricing models such as the Black-Scholes model. However, there are several differences between the Black-Scholes framework and the model of investment developed here. Most notably, the Black-Scholes model is for European options that cannot be exercised before the expiration date. Under the framework presented in this section, the option to invest is assumed to be infinitely lived. Thus, we are primarily interested in the conditions of early exercise, otherwise the farmer would never invest.

As stated in the introduction, one of the primary contributions of the investment under uncertainty and irreversibility literature was the explicit recognition that investment is very rarely a now or never proposition. Investment in citrus, for example, may be delayed until the end of the NAFTA vote to ascertain additional information regarding future fruit prices implied by different trading regimes. Given the alternative to delay an investment, the question of whether or not to invest becomes similar to the decision to exercise a call option. If the investor plants an acre of citrus, he exercises a call option of the investment. He then receives one year's dividends or return on the grove, but forfeits any gain in the option resulting from an increase in the value of the investment:

> "When a firm makes an irreversible investment, it exercises, or 'kills,' its option to invest. It gives up the possibility of waiting for new information to arrive that might affect the desirability or timing of the expenditure; it cannot disinvest should the market condition change adversely." (Pindyck, 1991)

The important question is then whether the single year's dividend is greater than the option to invest in the future.

Thus, the modified decision rule on investment becomes to invest when the net present value of asset is greater than the value of the option to invest in the future. The effect of irreversibility and uncertainty doubles the required returns on many investments (Pindyck, 1988). Leading Pindyck to conclude: "As an emerging literature has shown, the ability to delay an irreversible investment can profoundly affect the investment decision" (Pindyck, 1991). Pagano found that irreversibility and uncertainty increased the hurdle rate for investment in free-stall dairy technology by 2.28 times.

As in the case of stock options, the value of the option to invest is a derived asset whose value is dependent on the value of original investment in much the same way that the value of a stock option is derived from the value of the stock. Thus, the first step in defining the option to invest is to specify how the value of the original investment moves over time. In the irreversibility literature, the value of the investment alternative is assumed to be a geometric stochastic process just as assumed in the derivation of stock options:

$$dV = \alpha V dt + \sigma V dz \tag{1}$$

where V is the value of the investment into perpetuity α is the nonstochastic change in the asset value over time, dt, σ is the instantaneous standard deviation of the stochastic element event, dz. Given the stochastic process depicting the evolution of asset values over time, we assume that there exists a perfectly correlated asset that obeys a similar stochastic process:

$$\begin{gathered} dx = \mu x dt + \sigma x dz \\ \mu = r + \phi \rho_{vm} \sigma \end{gathered} \tag{2}$$

where x is the value of the correlated asset, μ is the return (dividends plus capital gains). The returns on the correlated asset can then be expressed as a combination of the risk free rate of return, r, and a market price for risk.

Comparing equations (1) and (2) leads to a comparison of α and μ. The relationship between these two values gives rise to the execution of the option. To discuss the implications of these two values we define $\delta = \mu - \alpha$ to be the dividend associated with owning the asset. Specifically, α is the capital gain or change in the value of the asset while μ is the total return associated with an asset with the same risk profile. If δ is less than or equal to zero, the option will never be exercised. It is more profitable to hold the option than the stock. Thus, in order for investment to occur $\delta = \mu - \alpha > 0$.

Following the traditional option pricing formulation the next step to developing the option to invest is the construction of a riskless portfolio containing one unit of the option to invest and some level of short sale of the original asset. Specifically, we can construct a portfolio

$$P = F(V) - F_V(V)V \tag{3}$$

where P is the value of the riskless portfolio and F(V) is the value of the option. $F_V(V)$ is the derivative of the option price with respect to the value of the original asset. In this case, it is also the amount of option held short. Dropping the Vs from the notation, we differentiate (3) to obtain the return on the portfolio. To this differentiation, we append two assumptions. First, the rate of return on the short sale over time must be $-\delta V$. This simply states that the short sale must pay at least the expected dividend rate on holding the asset. Second, the return on the risk-free portfolio must equal the risk-free return on capital $r(F - F_V V)$. The result of these two assumptions are

$$dF - F_V dV - \delta V F_V dt = r(F - F_V V)dt. \tag{4}$$

Embedded in condition (4) is the zero profit condition for riskless arbitrage. In other words, any solution for F satisfying equation (4) will yield zero profit. The next step is to combine this condition with a restriction implying zero risk.

Combining equation (4) with the original geometric process and Ito's lemma we derive the combined zero-profit and zero-risk condition as:

$$\left(\frac{1}{2}\right)\sigma^2 V^2 F_{VV} + (r-\delta)VF_V - rF = 0. \tag{5}$$

In addition to the differential equation, we have three boundary conditions on the solution:

$$\begin{aligned} &F(0)=0 \\ &F(V^*)=V^*-I \\ &F_V(V^*)=1. \end{aligned} \tag{6}$$

The first condition states that the option on a valueless asset is also valueless. The second condition identifies the exercise price by stating that at some asset value, the value of the option is equal to the value of the asset less the cost of investment. The third condition is the "smooth pasting" condition that rules out the possibility of arbitrage at the point of exercise.

The solution to equation (5) given the conditions in equation (6) is then

$$\begin{aligned} &F(V)=aV^{\beta} \\ &a=\frac{(V^*-I)}{V^{*\beta}} \\ &V^*=\frac{\beta}{(\beta-1)}I \\ &\beta=\frac{1}{2}-\frac{(r-\delta)}{\sigma^2}+\left\{\left[\frac{(r-\delta)}{\sigma^2}-\frac{1}{2}\right]^2+2\frac{r}{\sigma^2}\right\}^{\frac{1}{2}}. \end{aligned} \tag{7}$$

One key number in this solution is $\beta/(\beta-1)$ which is the number of times that the present value of investment must exceed its cost to be viable under uncertainty and irreversibility. First, since $\beta>1$ the range of this multiplier is 1 to infinity. Intuitively, as β goes to ∞ then $\beta/(\beta-1)$ goes to 1 and the optimal investment rule approaches net present value. Given a finite dividend ratio and risk-free cost of capital, the β can only approach infinity by σ^2 going to zero or the investment becoming riskless. A simplified version of equation (7) is derived by Dixit by letting $r - \delta = 0$. This assumption yields

$$\beta = \frac{1}{2}\left\{1 + \left(1 + \frac{8r}{\sigma^2}\right)^{\frac{1}{2}}\right\}. \tag{8}$$

Empirical *Ex Ante* Analysis of Investment Opportunities

Given the results in equation (7) and the simplified results for β in equation (8) we see that the difference between the net present value criteria and the criteria given irreversibility and uncertainty is determined by the instantaneous variance in the geometric motion of the asset value equation. Methodologically, two procedures for calculating this variation would appear fruitful. First, the variance could be derived based on historical asset values. This approach has several shortcomings. Notably, the volatility of agricultural asset values may change over time. In fact, the hypothesis that the risk to agricultural assets may evolve over time has been used to fit debt models (Moss, Shonkwiler, and Ford). In addition, some of the most relevant uses of this approach may be to predict changes in investment patterns that may result from changes in the economic climate. For example, we may be interested in analyzing the effect of NAFTA policy implementation decisions on new citrus plantings or possible changes in the tax code on equipment demand. In either case a viable alternative is to quantify the instantaneous variance using simulation.

In developing a measure of instantaneous variation, we begin with the traditional present value of investment:

$$V_t = \sum_{i=t}^{N+t} \frac{CF_i}{(1+r)^i} \tag{9}$$

where V_t is the present value of the investment in period t, N is the life of the investment, CF_t is the cash flow in period t and r is the appropriate risk adjusted discount rate. The annualized present value of the investment can then be used to calculate the present value of the infinite stream

$$APV_t = \frac{V_t}{\dfrac{1 - \dfrac{1}{(1+r)^{-N}}}{r}} \tag{10}$$

$$V_t^* = \frac{APV_t}{r}$$

where APV_t is the annualized present value at period t and V_t^* is the present value of the infinite stream with the same annualized present value.

Next we assume that the cash flows in equation (9) are drawn from a random distribution that depicts the sources of volatility in the investment. Table 1 gives the @RISK spreadsheet used to simulate one acre of orange grove. The statistics for the geometric stochastic process can be approximated by:

$$\begin{aligned}\frac{dV}{V} &= d\ln(V) \\ &\doteq \ln(V_{t+1}) - \ln(V_t)\end{aligned} \tag{11}$$

where ln(V) is the natural logarithm of V.

To compute V_t and V_{t+1} we form an overlapping draw on, in this case, weather and prices. V_t is computed based on years 1-26 while V_{t+1} is computed based on years 2-27. The additional information available is whether or not a freeze event occurred in year 1 and the on tree orange price for year 1. Intuitively, the distribution is stationary so that the expected drift will be zero, but the instantaneous variance will be nonzero because of freeze incidence or price variation.

Application to Citrus

Citrus is an irreversible investment because of the large "up front" cost in land preparation and the life cycle of the asset. In addition, the cost of disinvesting in citrus groves and conversion of citrus acreage to alternative uses may be substantial. In our analysis of the citrus investment we start with a citrus budget for southwest Florida from Ford, Muraro, and Fairchild. The annual yields and operating cost for the grove are given in table 2. In addition, we assume two major sources of risk: Freeze risk and price volatility.

The freeze risk is modeled with a binomial distribution of five draws with a probability of .25 per draw. If the result of the draw was a 1 through 3, no freeze occurred (the spreadsheet result for freeze was "None"). If the draw was a 4, a "Light" freeze occurred. A light freeze reduced the harvest by 25% and stunted tree growth in that year. Finally, if the draw was a 5 a "Hard" freeze occurred. The hard freeze cost the tree 50% of the yield and one year growth. If the hard freeze occurs and the citrus tree is one year old, then the freeze drives the tree age to zero. When the tree age becomes zero because of a hard freeze a replanting cost of $400 per acre is accessed.

On tree orange prices were assumed to be normal with a mean of $6.00/box and a standard deviation of $1.57/box. These figures were calculated based on deflated historical on-tree orange prices for 1977-1991.

The simulated results indicate that the present value of orange production was $852.99/acre with a standard deviation of $179.88/acre. Clearly, this investment is not profitable given the initial investment of $3,950/acre for trees, land clearing, irrigation and permitting. The average log change as defined in equations (10) and (11) based on 7500 draws was .0084693 with a standard deviation of .0099294. Assuming that the mean of the log change is zero, the computed β for this scenario is 25.17 implying a $\beta/(\beta-1)$ of 1.0414. Hence, the risk adjustment raises the hurdle rate to $4,113.40. Alternatively, the value of the option to invest given the current scenario is $163.40/acre.

Implications for Aggregate Investment Models

While irreversibility did not affect the current orange investment, the forgoing example shows how the option to invest in the future may change the decision to invest. This procedure differs from other discussions of individual investment decisions because these results are independent of risk preferences. Thus, in aggregate we could expect investment behavior in agriculture to be sensitive to irreversibility. In particular, the option to delay investment will always decrease the investment schedule under uncertainty. Thus, we should not be surprised to find empirical models of aggregate investment incomplete. Specifically, the model above indicates that certain changes in investment behavior may be related to changes in risk even in the absence of risk aversion.

However, this possibility raises certain difficulties in the development of the model under uncertainty and irreversibility. Specifically, an implicit assumption in our analysis is that volatility or the standard deviation of the stochastic process remains constant over time. Given that the variance of agricultural assets in general or citrus assets in particular were constant, then $\beta/(\beta-1)$ will be constant over time and the difference between the investment decision under irreversibility and uncertainty cannot be distinguished from the traditional net present value rule. In order for the $\beta/(\beta-1)$ to be identified there must be a change in risk. However, to be completely consistent with the theoretical formulation this change must be unanticipated.

Conclusions

This study demonstrates one approach to *ex ante* analysis of investment under uncertainty and irreversibility. We derive the effect of the option to postpone investment following Pindyck and relate that solution to Dixit's formulation. Next, we described how the change in investment rules can be derived using simulation analysis. Finally, we briefly discuss the potential effects of irreversibility on aggregate models of investment in agriculture.

To demonstrate the application of simulation to valuing the option to invest, we use a hypothetical investment in one acre of citrus. The results are somewhat anticlimactic since the investment is not profitable in the riskless scenario. However, the procedure does indicate that the option to postpone investment is worth $163.40/acre to the firm.

Table 1. Stochastic Spreadsheet for Investment Analysis

	Plant in Year 1					Plant in Year 2				Cash Flow	
Year	Freeze Draw	Freeze Intensity	Tree Age	Replant Cost	Yield per Acre	Tree Age	Replant Cost	Yield per Acre	Price per Box	V_t	V_{t+1}
0			0						4.26		
1	2	None	1	0	0.00	0			6.81	-15.00	
2	2	None	2	0	0.00	1	0	0.00	4.56	-1537.00	-15.00
3	3	None	3	0	0.00	2	0	0.00	5.99	-526.00	-1537.00
4	0	None	4	0	62.00	3	0	0.00	7.44	-87.78	-526.00
5	2	None	5	0	89.00	4	0	62.00	3.05	-250.30	-359.72
6	2	None	6	0	143.00	5	0	89.00	5.44	197.32	-38.21
7	2	None	7	0	219.00	6	0	143.00	6.83	910.54	397.19
8	3	None	8	0	305.00	7	0	219.00	5.37	1016.46	590.47
9	2	None	9	0	374.00	8	0	305.00	5.66	1462.94	1105.20
10	2	None	10	0	412.00	9	0	374.00	5.16	1467.62	1275.48
11	2	None	11	0	419.00	10	0	412.00	5.21	1496.91	1488.42
12	2	None	12	0	424.00	11	0	419.00	8.62	2965.94	2926.82
13	3	None	13	0	429.00	12	0	424.00	6.70	2181.21	2151.69
14	4	Light	13	0	321.75	12	0	318.00	7.31	1656.81	1633.40
15	1	None	14	0	432.00	13	0	429.00	3.75	895.97	914.71
16	4	Light	14	0	324.00	13	0	321.75	4.83	839.28	858.42
17	2	None	15	0	436.00	14	0	432.00	4.92	1413.54	1398.88
18	2	None	16	0	436.00	15	0	436.00	7.22	2412.26	2418.26
19	1	None	17	0	419.00	16	0	436.00	6.67	2054.05	2173.49
20	4	Light	17	0	314.25	16	0	327.00	5.05	844.12	914.47
21	2	None	18	0	402.00	17	0	419.00	6.31	1788.48	1902.79
22	2	None	19	0	386.00	18	0	402.00	7.14	1998.13	2120.33
23	2	None	20	0	371.00	19	0	386.00	7.37	1968.05	2086.56
24	3	None	21	0	356.00	20	0	371.00	5.56	1204.04	1296.38
25	3	None	22	0	342.00	21	0	356.00	6.66	1494.59	1597.86
26	1	None	23	0	328.00	22	0	342.00	7.75	1746.19	1865.65
27	4	Light	23	0	246.00	22	0	256.50	4.12	219.02	273.30
						23	0	328.00	7.71		1733.37

Table 2. Annual Yield and Operating Cost per Acre

Tree Age	Yield per Acre	Operating Cost per Acre
0	0	0
1	0	15
2	0	1537
3	0	526
4	62	549
5	89	522
6	143	580
7	219	586
8	305	622
9	374	655
10	412	659
11	419	687
12	424	691
13	429	695
14	432	725
15	436	730
16	436	736
17	419	742
18	402	749
19	386	757
20	371	765
21	356	774
22	342	784
23	328	795
24	315	808
25	302	821

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