QUANTIFYING LONG RUN AGRICULTURAL RISKS AND EVALUATING FARMER RESPONSES TO RISK

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Acreage Decisions Under Risk: The Case of Corn and Soybeans

by

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Abstract:

An acreage supply response model is developed under expected utility maximization. The resulting framework is used to specify and estimate a system of risk-responsive acreage equations for corn and soybeans in the U.S. Particular attention is given to the truncation effects of government price supports on the distribution of corn and soybean prices. Also, a wealth variable is included in the acreage equations. The empirical results indicate that risk and wealth variables play an important role in corn-soybean acreage decisions. The analysis also shows that cross-commodity risk reduction is important in acreage allocation decisions.

Key words: Corn-soybean acreage, expected utility maximization, risk, truncation.

Introduction:

Much research has focused on acreage response functions in agriculture. Following the relative success of the Nerlovian approach (e.g. Askari and Cummings), recent developments have attempted to strengthen the link between empirical supply response and economic theory either in a static framework (e.g. Weaver; Shumway; Antle) or in a dynamic framework (e.g. Vassavada and Chambers; Howard and Shumway). At the same time, evidence is increasing that risk and/or risk behavior are important in agricultural production decisions (e.g. Behrman; Just; Lin et al.; Traill). However, the implications of decision theory under risk have played only a minor role in supply response analysis. A wide gap exists between the economic theory of risk behavior and aggregate supply analysis.

The objective of this article is to develop an acreage supply response model under expected utility maximization and to investigate its empirical implications for U.S. corn and soybean acreages. After the presentation of an expected utility model for acreage decisions, testable hypotheses of economic behavior under risk are developed. Multiple sources of revenue uncertainty are modeled and linkages between government price support programs and the subjective probability distributions of uncertain output prices for decision makers are investigated. This is done by modifying the

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bounded price variation models considered by Maddala (1983a, 1983b), Shonkwiler and Maddala, and others to include multivariate price distributions. The implications of the theory are then incorporated into the specification and estimation of a system of risk responsive acreage decision functions for corn and soybeans in the U.S. and the results are discussed.

The Model:

Consider a farm household producing n crops where A_i is the number of acres devoted to the ith crop and Y_i is the corresponding yield per acre, $i=1,\ldots,n$. Letting p_i be the market price of the ith crop, then agricultural revenue is given by

$$R = \sum_{i=1}^{n} p_i Y_i A_i .$$

Denoting the cost of production per acre of the ith crop by c_i, then the total cost of agricultural production is

$$C = \sum_{i=1}^{n} c_i A_i .$$

In the present case, revenue (R) is a risky variable since output prices $p = (p_1,\ldots,p_n)$ and crop yields $Y = (Y_1,\ldots,Y_n)$ are not observed by the household when production decisions are made. Alternatively, input prices and per acre costs (c_i) are known at the time crop acreages are allocated.

The household then faces the budget constraint

$$I + R - C = q\,G$$

or

$$I + \sum_{i=1}^{n} p_i Y_i A_i - \sum_{i=1}^{n} c_i A_i = q\,G \qquad (1)$$

where I denotes exogenous income (or wealth) and G is an index of household consumption of goods purchased with corresponding price index q, q G denoting household consumption expenditures. Equation (1) states that exogenous income (I) plus farm profit (R-C) is equal to consumption expenditures (q G).

Let the constraints on acreage decisions be represented by

$$f(A) = 0 \qquad (2)$$

where $A = (A_1, \ldots, A_n)$. Assume that the household preferences are represented by a von-Neumann Morgenstern utility function $U(G)$ satisfying $\partial U/\partial G > 0$. If the household maximizes expected utility under competition, then the decision model is

$$\max_{A,G} E\, U(G) \text{ s.t. (1) and (2)}$$

where E is the expectation operator over the random variables. After substituting the budget constraint into the utility function, the maximization problem is expressed as:

$$\max_{A} \{EU[\frac{I}{q} + \sum_{i=1}^{n} (\frac{p_i}{q} Y_i - \frac{c_i}{q})A_i]\} \text{ s.t. (2)},$$

or

$$\max_{A} \{EU(w + \sum_{i=1}^{n} \pi_i A_i)\} \text{ s.t. (2)} \qquad (3)$$

where $w = (I/q)$ is normalized initial wealth and $\pi_i = (p_i/q)\, Y_i - (c_i/q)$ denotes normalized profit per acre of the ith crop, $i=1,\ldots,n$, and all prices are deflated by the consumer price q.

This formulation illustrates that the acreage decision A is made under both price and production uncertainty. Both yields Y and output prices p are random variables with given subjective probability distributions.[1/] Consequently, the expectation E in (3) is over the uncertain variables p and Y and is based on the information available to the household at planting time.

What are the economic implications of the optimization problem (3) for the acreage decision A? Letting A^* denote the optimal acreage choice in (3), such a choice depends on normalized initial income (or wealth) w, expected normalized profits per acre $\bar{\pi}_i = E\{(p_i/q)\, Y_i - (c_i/q)\}$, as well as second and (possibly) higher moments of the distributions of normalized profits per acre π_i, $i = 1,\ldots,n$, denoted here by $\underline{\sigma}$. In other words, the optimal acreage decision can be written as $A^*(w; \bar{\pi}; \sigma)$, where $\bar{\pi} = (\bar{\pi}_1, \ldots, \bar{\pi}_n)'$.

The acreage decision under risk $A^*(.)$ is homogenous of degree zero in (w, p, c, q). While this result does not depend on risk preferences U(.), this well known homogeneity property involves output price p, input cost c <u>and</u> initial wealth w and the consumer price q. The homogeneity condition implies that acreage decisions can be expressed as functions of the relative prices w/q, p/q and c/q (or their probability distributions). However, unless additional restrictions are imposed on risk preferences (see Pope), it does <u>not</u> imply that the acreage function $A^*(.)$ is homogenous of degree zero in output and input prices (p,c). In other words, the classical result of riskless production theory stating that production decisions depend only on input-output price ratios does not hold in general under uncertainty.

Properties of the Acreage Decision:

The empirical implications of expected utility maximization have been investigated by Sandmo, Ishii, Chavas and Pope, Pope, and others. In this section we focus on the theoretical restrictions implied by (3) which can be tested and/or imposed in the empirical specification and estimation of the acreage decision $A^*(.)$.

First, Sandmo and others have examined the relationship between wealth effects, $\partial A^*/\partial w$, and the nature of risk preferences. In particular, a zero wealth effect, $\partial A^*/\partial w = 0$, corresponds to constant absolute risk aversion. Alternatively, $\partial A^*/\partial w \neq 0$ corresponds to non-constant absolute risk aversion. Non-zero wealth effects are of interest here to the extent that decreasing absolute risk aversion is a maintained hypothesis in much of the economic literature (e.g., Arrow).

Second, the optimization hypothesis (3) implies symmetry restrictions on the slopes $\partial A^*/\partial \bar{\pi}$. These symmetry restrictions take the form

$$\frac{\partial A^c}{\partial \bar{\pi}} = \frac{\partial A^*}{\partial \bar{\pi}} - \frac{\partial A^*}{\partial w} \cdot A^{*\prime} \qquad (4)$$

where A^c is the wealth compensated acreage decision, holding utility constant. The matrix of compensated effects $\partial A^c/\partial \bar{\pi}$ in expression (4) is symmetric, positive semi-definite (Chavas). Expression (4) also indicates that the slope of the uncompensated function $\partial A^*/\partial \bar{\pi}$ can be decomposed as the sum of two terms: the compensated slope (or substitution effect) $\partial A^c/\partial \bar{\pi}$ which maintains a given level of utility plus the wealth effect $(\partial A^*/\partial w . A^{*\prime})$. These results are quite general since equation (4) holds for any risk preferences.

Under constant absolute risk aversion, the wealth effect vanishes implying that $\partial A^c/\partial \bar{\pi} = \partial A^*/\partial \bar{\pi}$. In this case, compensated and uncompensated choice functions have the same slope with respect to $\bar{\pi}$ and $\partial A^*/\partial \bar{\pi}$ is a symmetric, positive semi-definite matrix from (4). This illustrates the influence of risk preferences on acreage choice functions since non-zero wealth effects reflect non-constant absolute risk aversion. Also, note from (4) that non-negative wealth effects $(\partial A^*/\partial w \geq 0)$ are sufficient conditions to guarantee that an increase in expected returns per acre of the ith crop will result in an increase in the optimal acreage of that crop i.e., $\partial A_i^*/\partial \bar{\pi}_i \geq 0$.

Finally, Chavas and Pope (p. 229) and Pope have derived homogeneity restrictions in the context of the expected utility model (4). In particular, rewriting expression (2) as $f(A) = A_1 - g(A.) = 0$, where $A = (A_1, A.)$, Chavas and Pope have shown that the following restriction holds at the optimum under any risk preferences

$$\frac{\partial A^*}{\partial \bar{\pi}} \left(\frac{\partial f(A)}{\partial A}\right)' - \frac{\partial A^*}{\partial w} \cdot \frac{\partial f(A)}{\partial A} \cdot A = 0 \tag{5a}$$

Let the first-order conditions associated with (3) be $E(\partial U/\partial w.\pi) + \lambda.(\partial f/\partial A) = 0$, where λ is the Lagrange multiplier associated with the constraint (2) and $\partial f/\partial A$ is a (1 x n) of vector. Given $\lambda \neq 0$, substituting these conditions into (5a) yields

$$\frac{\partial A^*}{\partial \bar{\pi}} (\bar{\pi} + \delta) - \frac{\partial A^*}{\partial w} (\bar{\pi}' + \delta')A = 0 \tag{5b}$$

where $\delta = COV(\partial U/\partial w,\pi)/E(\partial U/\partial w)$ is a (nx1) vector. Under risk neutrality, $\partial A^*/\partial w = 0$ and $\delta = 0$, implying from (5b) that the acreage decision function A^* is homogeneous of degree zero in π_j,

$$\sum_{j=1}^{n} \frac{\partial A_i^*}{\partial \bar{\pi}_j} \bar{\pi}_j = 0.$$

This homogeneity restriction of classical production theory states that production decisions are not affected by proportional changes in all input and output prices. However, under risk aversion, $\delta \neq 0$ and (5b) implies that this homogeneity-like restriction takes a different form.[2/]

Some empirical implications of specific forms of risk preferences have been presented by Pope. In particular, under constant relative risk aversion, a positive scaling of wealth does not alter optimal decisions (Sandmo).[3/] This implies that decisions functions are almost homogeneous of degree one in initial wealth, degree one in mean returns π, degree two in moments of order two, and degree s in moments of order s of π. Similarly, under constant partial relative risk aversion a positive scaling of profit does not alter optimal choices. This implies that decision functions are almost homogeneous of degree one in mean returns π, degree two in moments of order two of π, and degree s in moments of order s of π. (See Pope for details).

Finally, it is well known that $\partial A^*/\partial \sigma = 0$ and $\partial A^*/\partial w = 0$ under risk neutrality. Alternatively, $\partial A^*/\partial \sigma \neq 0$ and/or $\partial A^*/\partial w \neq 0$ implies a departure from risk neutrality. In particular, under risk aversion, risk will influence the allocation of resources in agriculture.

An Application under Government Price Support Programs:

The acreage decision model (3) involves uncertainty about prices p and yields Y. In this section the influence of government programs on the subjective probability distribution of output prices p is considered by focusing on a price support program which places a floor under the market price. The resulting truncation of the subjective probability distribution of prices will affect expected prices as well as second and higher moments of the price distribution. Thus, a price support program will influence both price expectations and the riskiness of revenue.

Since the effects of multivariate truncation are best understood in the context of a normal distribution (see Johnson and Kotz; Maddala, 1983a), we limit our discussion to the normal case.[4] Let $X = (X_1, X_2, \ldots)$ be a vector of normally distributed random variables with mean $\bar{X} = (\bar{X}_1, \bar{X}_2, \ldots) = E(X)$ and variance $V(X) = E(X-\bar{X})'(X-\bar{X}) = \{\sigma_{ij}\}$, where E is the expectation operator. Now, assume that each random variable X_i is truncated from below at a level H_i. Define the truncated random variables

$$x_i = \begin{cases} H_i \text{ if } X_i < H_i \\ X_i \text{ if } X_i \geq H_i \end{cases}, \quad i = 1, 2, \ldots$$

Consider the standardized random variable $e_i = (x_i - \bar{X}_i)/\sigma_{ii}^{1/2}$ and define $h_i = (H_i - \bar{X}_i)/\sigma_{ii}^{1/2}$. The mean and variance of e_i are derived in the Appendix. The expected value of e_i is

$$\bar{e}_i = E(e_i) = \phi(h_i) + h_i\,\Phi(h_i) \tag{6a}$$

where $\phi(.)$ and $\Phi(.)$ are the standard normal density function and distribution function, respectively. The second moments of e_i are given by (see Appendix)

$$M_{ii} = E(e_i^2) = 1 - \Phi(h_i) + h_i\,\phi(h_i) + h_i^2\,\Phi(h_i) \tag{6b}$$

and

$$\begin{aligned} M_{ij} = E(e_i\,e_j) &= F(h_i,h_j)\,\rho_{ij} + [(1-\rho_{ij}^2)/2\pi]^{1/2}\,\phi(Z_{ij}) \\ &+ h_i\,\phi(h_j)\,\Phi(k_{ij}) + h_j\,\phi(h_i)\,\Phi(k_{ji}) \\ &+ h_i\,h_j\,\Phi(h_i,h_j), \quad i \neq j \end{aligned} \tag{6c}$$

where $F(h_i,h_j) = \text{Prob}(X_i \geq H_i, X_j \geq H_j)$, $\rho_{ij} = \sigma_{ij}/(\sigma_{ii}\,\sigma_{jj})^{1/2}$, $Z_{ij} = \{(h_i^2 - 2\,\rho_{ij}\,h_i\,h_j + h_j^2)/(1-\rho_{ij}^2)\}^{1/2}$, $k_{ij} = (h_i - \rho_{ij}\,h_j)/(1-\rho_{ij}^2)^{1/2}$, and $\Phi(h_i, h_j) = \text{Prob}(X_i < H_i, X_j < H_j)$. It follows that the mean, variance, and covariance of $x = (x_1, x_2, \ldots)$ are

$$\bar{x}_i = E(x_i) = \bar{X}_i + \sigma_{ii}^{1/2}\,\bar{e}_i \tag{7a}$$

$$V(x_i) = E(x_i-\bar{x}_i)^2 = \sigma_{ii}\,(M_{ii} - \bar{e}_i^2) \tag{7b}$$

and

$$COV\,(x_i, x_j) = E(x_i-\bar{x}_i)\,(x_j-\bar{x}_j) = (\sigma_{ii}\,\sigma_{jj})^{1/2}\,(M_{ij} - \bar{e}_i\,\bar{e}_j) \tag{7c}$$

Expressions (7) provide an analytical evaluation of the truncation effect of a price support program on the mean, variance, and covariance of commodity prices. These results will be used to investigate the influence of government programs on corn and soybean acreage decisions.

Data and Estimation:

Assuming that aggregate behavior can be represented by a representative farm household making decisions according to model (3), we propose to specify and estimate the acreage function $A^*(w,\pi,\sigma)$ from aggregate data. This is done by analyzing annual time series data for U.S. corn (i=1) and soybean (i=2) acreage decisions from 1954-1985. The acreage variables A_1 and A_2 measure acreage planted to each crop (in millions of acres) and were obtained from various USDA publications. The market prices (p_1 and p_2) are average prices received by farmers and were also obtained from USDA publications. The costs of production per acre (c_1 and c_2) were obtained from USDA's Cost of Production surveys for the 1975-1985 period. For years prior to 1975, the cost of production data reported by Gallagher were used. Following Houck et al. and Gallagher, effective diversion payment and support price (p_1^s and p_2^s) variables were used to quantify the price support and acreage set-aside provisions of government programs.5/ The consumer price q was measured by the consumer price index as reported by the Bureau of Labor Statistics. Yields per acre were obtained from USDA publications.

To analyze supply behavior under risk, assumptions about the (untruncated) expectations of prices and yields are needed. We use adaptive expectations for the untruncated normalized prices. That is,

$$E_{t-1}\left(\frac{p_{it}}{q_t}\right) = \alpha_i + \frac{p_{i,t-1}}{q_{t-1}} \tag{8a}$$

where

$$\alpha_i = E\left(\frac{p_{it}}{q_t} - \frac{p_{i,t-1}}{q_{t-1}}\right)$$

as measured by the corresponding sample mean. Similarly, the variance measure used for untruncated normalized prices is

$$\mathrm{Var}\left(\frac{p_{it}}{q_t}\right) = \sum_{j=1}^{3} \omega_j \left[\frac{p_{i,t-j}}{q_{i,t-j}} - E_{t-j-1}\left(\frac{p_{i,t-j}}{q_{i,t-j}}\right)\right]^2 \tag{8b}$$

where the weights ω_j are .5, .33, and .17.[6/] The assumption stated in (8a) that expected prices are a function of the average price of the previous year has been successfully employed in previous research (e.g. Houck et al.; Chavas, Pope, and Kao).

Expression (8b) states that the variance of price is a weighted sum of the squared deviations of past prices from their expected values, with declining weights. These measurements of price risk are also consistent with those used previously in the literature (e.g. Lin; Traill; Brorsen, Chavas, and Grant).

Expressions (8) give the untruncated mean and variance of the price distributions. These results, along with the expressions in (7), determine the mean and variance of the truncated multivariate price distributions associated with price supports p_1^s and p_2^s.

To measure yield expectations, actual yields were regressed on a trend variable. The resulting predictions were taken as expected yields, and the estimated residuals were used to generate the variance of yield and the covariance between price and yield. For simplicity, both the variance of yield and the correlation between price and yield were assumed constant over time.[7/]

The farm value of proprietor equity was used as a proxy for initial wealth. Farm equity of corn-soybean producers was obtained by multiplying the U.S. farm value of proprietor equity by the percentage of U.S. farm acreage planted to corn and soybeans.[8/]

The acreage equations $A^*(w, \bar{\pi}, \sigma)$ were specified using these data. Consider the first order Taylor series expansion

$$A_{it} = a_i + (\partial A_i/\partial w)\, w_{t-1} + \sum_{j=1}^{2} (\partial A_i/\partial \bar{\pi}_j)\, \bar{\pi}_{jt} + \sum_{k\geq j}^{2} \sum_{j=1}^{2} (\partial A_i/\partial \bar{\sigma}_{jk})\, \bar{\sigma}_{jkt}$$

$$+ \theta_i t + \eta_i D83_t + u_{it}, \quad i = 1,2, \tag{9}$$

where A_{it} is the number of acres planted to the ith crop at time t,

$$\bar{\pi}_{jt} = E_{t-1}(p_{jt}/q_t)\, Y_{jt} - (c_{jt}/q_t) \mid p_t \geq p_t^s),$$

is the truncated mean return per acre of the jth crop,

$$\bar{\sigma}_{jjt} = \mathrm{Var}(p_{jt} \mid p_t \geq p_t^s)$$

and

$$\bar{\sigma}_{jkt} = \mathrm{COV}(p_{jt}, p_{kt} \mid p_t \geq p_t^s)$$

are the truncated variances and covariances of output prices, and u_{it} is an error term. A trend variable is included to capture the systematic effects of any omitted variables on acreage decisions over time. Lastly, a dummy variable $D83_t$ is included to discount the effects of the payment-in-kind program offered in 1983.

Letting $\beta_{ij} = \partial A_i^c/\partial\bar{\pi}_j$ be the compensated slopes with respect to $\bar{\pi}$ and using (4), it follows that equation (9) can be expressed alternatively as

$$A_{it} = a_i + \alpha_i \left(w_{t-1} + \sum_j A_j \bar{\pi}_{jt}\right) + \sum_j \beta_{ij} \bar{\pi}_{jt} + \sum_{k\geq j} \sum_j \gamma_{ijk} \bar{\sigma}_{jkt}$$

$$+ \theta_i t + \eta_i D83_t + u_{it},\ i=1,2, \tag{10}$$

where $\alpha_i = \partial A_i/\partial w$ and $\gamma_{ijk} = \partial A_i/\partial\bar{\sigma}_{jk}$. In the absence of a priori information about functional form, equation (10) provides a local approximation to the decision function $A^*(.)$. Also, the symmetry of (4) implies that $\beta_{ij} = \beta_{ji}$, $i \neq j$. Thus equation (10) is convenient for testing and/or imposing the symmetry restrictions (4).

Equation (10) can be used directly for an empirical analysis of acreage decisions for soybeans. The corn acreage equation (i=1) is specified according to (10) except that corn diversion payments (DP) are also included as an intercept shifter. The model parameters are estimated by seemingly unrelated regression.

Results and Implications:

The econometric model (10) is used to test various hypotheses about economic behavior under risk. The first hypothesis examined is the symmetry restriction (4) implied by expected utility maximization. This test is general since the symmetry restriction holds for any risk preferences. The null hypothesis associated with (4) is H_o: $\beta_{12} = \beta_{21}$ and the F-value for the test was $F(1,45) = 0.002$. Thus, the symmetry restriction cannot be rejected at any usual levels of significance. The implication is that acreage decisions, as represented by equation (10), are consistent with the symmetry restriction implied by expected utility maximization.

With the symmetry restrictions imposed, the parameter estimates of equations (10) are reported in table 1, along with several key measures of model fit and performance. The estimated model explains historical variations in corn and soybean acreages well as indicated by the high R-squares. Serial correlation is also apparently not a problem as reflected by the single-equation Durbin-Watson statistics. Many of the parameter estimates are large relative to their standard errors and have signs consistent with the theory.

The compensated own-revenue elasticities are .068 and .279 for corn and soybean acreage, respectively (table 1). Alternatively, the compensated own-price elasticities for corn and soybeans are .158 and .441, respectively.[9/] These mean-response elasticities appear reasonable and compare favorably with those reported elsewhere (e.g., Gallagher; Lee and Helmberger; Tegene, Hoffman, and Miranowski). The risk elasticities are generally small, although soybean acreage appears more risk responsive than corn acreage. This result is not surprising because government intervention has been less important in the soybean market than in the corn market. Finally, the elasticities with respect to initial wealth are .087 and .270, for corn and soybean acreage, respectively.

Having found evidence in favor of the expected utility model (3), the next step is to test for the nature of risk preferences taking the symmetry restriction $\beta_{12} = \beta_{21}$ in (10) as maintained. The hypothesis of risk neutrality is tested as H_o: $\gamma_{ijk} = 0$ and $\alpha_i = 0$ for all i, j, k. The F-value for this test was F(8,46) = 7.367, which implies that the null hypothesis can be rejected at the 5 percent level.

The hypothesis of constant absolute risk aversion can be tested as H_o: $\alpha_1 = \alpha_2 = 0$. The F-value for this test was F(2,46) = 17.589, indicating a rejection of the null hypothesis at the 5 percent level. This result provides evidence that the risk preferences of corn and soybean growers are not characterized by constant absolute risk aversion over the period of analysis.

The empirical results presented in table 1 also show positive wealth effects $\partial A^*/\partial w > 0$. In the single product case Sandmo has shown that a positive wealth effect in supply response implies decreasing absolute risk aversion (DARA). To the extent that Samdmo's result holds in the multiproduct case, our analysis indicates that farmers are decreasingly absolute risk averse.[10/] While it is well accepted that agricultural producers may exhibit DARA, this appears to be the first empirical illustration of positive (and significant) wealth effects in an aggregate agricultural supply model.

From equation (4), having wealth elasticities $\partial \ln A^*/\partial \ln w$ that are different from zero or one implies that the uncompensated slope matrix $\partial A^*/\partial \bar{\pi}$ is not symmetric.[11/] Yet, the symmetry of uncompensated price slopes has been imposed as a maintained hypothesis in many previous studies of aggregate supply response (e.g. Shumway; Antle). Our analysis indicates that the implications of riskless production theory may not apply to supply response analysis under risk.

Finding evidence against the hypothesis of constant absolute risk aversion also raises questions about the appropriateness of mean-variance risk analysis. Indeed, the mean-variance approach is typically motivated under constant absolute risk aversion and normality which imply zero wealth effects. Our results suggest a need to incorporate a wealth variable in programming models of risk.

The existence of wealth effects may also have policy implications. If corn-soybean farmers exhibit DARA, then higher private wealth tends to offset their need for income and price protection. Hence, the existence of positive wealth effects could provide a possible justification for income transfers to corn-soybean farmers with low initial wealth.

In order to obtain additional insights into the nature of risk preferences, the tests proposed by Pope were performed at the mean values of the sample data. Testing the hypothesis of constant relative risk aversion (CRRA) consists of testing whether a rescaling of terminal wealth has a zero effect on acreage decisions. The F-value for the CRRA hypothesis was $F(2,46) = 28.429$. Also, testing the hypothesis of constant partial relative risk aversion (CPRRA) considers whether a rescaling of profit has a zero effect on acreage decisions. The F-value for the CPRRA hypothesis was $F(2,46) = 9.126$. Using normal significance levels, these results indicate that neither CRRA nor CPRRA characterize the risk preferences of corn-soybean producers. In short, many of the simple utility function representations are not supported by the data.[12/]

Finally, the supply models were simulated at alternative support price levels. Because of the truncation effects, changing the support price levels will influence the means, variances and covariances of producer prices (see (7)). Selected static simulation results for the effects of support prices on expected prices, price risk, and acres planted are reported in table 2. As expected, increasing the support price of a crop tends to expand its acreage, although the relationship is non-linear. For example, when the support price is much below the expected market price, the truncation effect is negligible and the price support program has only a limited impact on acreage decisions. Alternatively, as support price levels, the truncation effects become larger, and the resulting impact on acreage decisions is more pronounced.

The cross-commodity price effects reported in table 2 are of interest since increasing the support price for a commodity tends to increase its expected price and thus decrease the acreage of the substitute commodity. However, the risk reducing effect of a price support program also influences acreage substitution. The net effect of the soybean support price on corn acreage is negative (table 2). However, the net effect of the support price for corn on soybean acreage is positive for low price support levels (e.g., effective support prices less than $1/bu), and negative otherwise (see table 2). Thus, within some price range the risk reducing effect of corn support prices on soybean acreage is positive and dominates the mean price effect. This result emphasizes the importance of cross-commodity risk effects and the risk-reducing role of price supports.

Conclusions:

This study presented a framework for analyzing multiple acreage decisions under uncertainty. A household decision model that includes both output price and yield uncertainty was developed and the resulting behavioral relationships were tested with a system of U.S. corn-soybean acreage equations. The truncation effects of government price supports on the distribution of corn and soybean prices were carefully considered. Expressions relating the truncated means, variances, and covariances for joint normally distributed random variables were developed. Moreover, a wealth variable was included in the estimated acreage equations to facilitate tests of hypotheses about risk attitudes.

The empirical results indicate that both risk and wealth effects are important in corn-soybean acreage allocation decisions. The symmetry restriction implied by expected utility maximization could not be rejected; however, the results also suggest that many commonly used hypothesis about risk attitudes, including CARA, CRRA, and CPRRA, are not supported by the data. These results cast doubt on the use of CARA utility functions. They also suggest that targeting policy benefits toward low income producers may be warranted.

The importance of considering risk in a multicrop framework was illustrated by simulating the acreage models at various price support levels for corn and soybeans. The model simulations illustrate that cross-commodity risk reduction is potentially important since there is some range over which increasing the support price for corn will actually result in more acres planted to soybeans. Such results could not be obtained with a single crop focus.

While the results of this study shed new light on the role of risk in farmers' decisions, further study is required. For example, future research could consider the effects of voluntary participation on acreage response under risk. The analysis could also be expanded to include other sources of risk and a richer set of risk response than acreage adjustments alone. For example, issues pertaining to financial risk and financial management have become increasingly important in recent years. Finally, the present model could be couched in a rational expectations framework in order to evaluate price support policies in a market equilibrium context.

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TABLE 1 - Estimated Corn and Soybean Acreage Equations with Symmetry Imposed, 1954-85.[a]

	Corn Equation	Soybean Equation
intercept	80.981* (1.543)	1.978* (1.579)
corn diversion payments ($/acre)	-66.347** (3.865) [-0.075]	--
$\bar{\pi}_1$ ($/acre)	0.116** (0.046) [0.068]	-0.166** (0.043) [-0.164]
$\bar{\pi}_2$ ($/acre)	-0.166** (0.043) [-0.107]	0.255** (0.064) [0.279]
$\bar{\sigma}_{11}$	33.583 (39.285) [0.020]	-75.834* (43.886) [-0.077]
$\bar{\sigma}_{22}$	15.157** (6.677) [0.049]	-16.021** (7.472) [-0.087]
$\bar{\sigma}_{12}$	-74.420 (45.049) [-0.070]	92.282* (49.796) [0.147]
$w + \sum_j A_j \bar{\pi}_j$	0.066** (0.023) [0.087]	0.122** (0.025) [0.270]
t	-0.242** (0.109)	1.534** (0.106)
D83	-14.545** (2.045)	-1.880 (2.049)
R^2	0.941	0.989
Durbin-Watson	1.695	1.875

a/ Standard errors are in parentheses below the parameter estimates. Elasticities evaluated at the sample means are presented in brackets. A double (single) asterisk indicates significantly different from zero at the 5% (10%) significance level.

TABLE 2 - Simulation of the Effects of Support Prices for Corn and Soybean*

Support Price of Corn ($/bu)	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8
Expected corn price	1.30	1.30	1.30	1.30	1.31	1.35	1.45	1.61	1.80
Variance of corn price	.055	.055	.055	.054	.046	.029	.011	.002	.0002
Covariance corn/soybean prices	.085	.085	.085	.084	.078	.059	.030	.009	.002
Corn acres	74.119	74.119	74.120	74.147	74.426	75.611	78.021	80.734	82.959
Soybean acres	44.655	44.656	44.659	44.673	44.566	43.624	41.088	37.741	34.716

Support Price of Soybeans ($/bu)	.4	.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6
Expected soybean price	2.90	2.90	2.90	2.90	2.91	2.95	3.07	3.30	3.62
Variance of soybean price	.283	.283	.283	.279	.261	.207	.118	.043	.010
Covariance corn/soybean prices	.085	.085	.085	.084	.082	.072	.051	.026	.009
Corn acres	74.704	74.704	74.701	74.672	74.547	74.278	74.010	73.770	73.127
Soybean acres	44.313	44.314	44.317	44.345	44.460	44.691	44.925	45.309	46.438

* Untruncated expected prices are $1.30 for corn and $2.90 for soybeans while support prices are $1.0 for corn and $2.20 for soybeans. All other variables are set equal to their sample means.

Appendix

The mean of e_i:

Let $\phi(.)$ be the standard normal density function. We have

$$E(e_i) = h_i \int_{-\infty}^{h_i} \phi(y)\, dy + \int_{h_i}^{\infty} y\, \phi(y)\, dy$$

$$= h_i\, \Phi(h_i) + \phi(h_i)$$

where $\Phi(.)$ is the standard normal distribution function.

The second moments of e_i:

a/ M_{ii}:

$$E(e_i^2) = h_i^2 \int_{-\infty}^{h_i} \phi(y)\, dy + \int_{h_i}^{\infty} y^2\, \phi(y)\, dy \tag{A1}$$

But the second term in (A1) can be shown to be equal to: $1-\Phi(h_i) + h_i\, \phi(h_i)$ (e.g., see Maddala, 1983a, p. 365). It follows that $E(e_i^2) = h_i^2\, \Phi(h_i) + 1-\Phi(h_i) + h_i\, \phi(h_i)$.

b/ M_{ij}, $i \neq j$:

Let $\phi(.,.)$ be the bivariate standard normal density function. Then,

$$E(e_i e_j) = h_i h_j \int_{-\infty}^{h_j} \int_{-\infty}^{h_i} \phi(y,z)\, dy\, dz + h_i \int_{h_j}^{\infty} \int_{-\infty}^{h_i} y\, \phi(y,z)\, dy\, dz$$

$$+ h_j \int_{-\infty}^{h_j} \int_{h_i}^{\infty} z\, \phi(y,z)\, dy\, dz + \int_{h_j}^{\infty} \int_{h_i}^{\infty} z\, y\, \phi(y,z)\, dy\, dz, \tag{A2}$$

Note that the second term in (A2) can be written as

$$h_i \int_{h_j}^{\infty} \int_{-\infty}^{h_i} y\ \phi(y,z)\ dy\ dz = h_i \int_{h_j}^{\infty} \int_{-\infty}^{\infty} y\ \phi(y,z)\ dy\ dz$$

$$- h_i \int_{h_j}^{\infty} \int_{h_i}^{\infty} y\ \phi(y,z)\ dy\ dz. \qquad \text{(A3)}$$

The first term on the right hand side of (A3) is equal to $h_i\ \phi(h_j)$. From Rosenbaum, and using the notation defined in the text, the second term on the right hand side of (A3) can be written as

$$-h_i\{\phi(h_j)[1-\Phi(k_{ij})] + \rho_{ij}\phi(h_i)[1-\Phi(k_{ji})]\}$$

which implies that

$$h_i \int_{h_j}^{\infty} \int_{-\infty}^{h_i} y\ \Phi(y,z)\ dy\ dz = h_i\phi(h_j) - h_i\{\phi(h_j)[1-\Phi(k_{ij})] + \rho_{ij}\phi(h_i)[1-\Phi(k_{ji})]\}.$$

By symmetry, the third term on the right hand side of (A2) is given by

$$h_j \int_{\infty}^{h_j} \int_{h_i}^{\infty} z\ \phi(y,z)\ dy\ dz = h_j\phi(h_i) - h_j\{\phi(h_i)[1-\Phi(k_{ji})] + \rho_{ij}\ \phi(h_j)[1-\Phi(k_{ij})]\}.$$

Likewise, following Rosenbaum the fourth term on the right hand side of (A2) can be shown to be

$$h_j \int_{h_j}^{\infty} \int_{h_i}^{\infty} zy\phi(z,y)dzdy = F(h_i,h_j)\rho_{ij} + \rho_{ij}h_i\phi(h_i)[1-\Phi(k_{ji})]$$

$$+ \rho_{ij}h_j\phi(h_j)[1-\Phi(k_{ij})]$$

$$+ [(1-\rho_{ij}^2)/2\pi]^{1/2}\ \phi(Z_{ij}).$$

After making the appropriate substitutions and collecting terms, (A2) can be shown to be

$$E(e_i e_j) = F(h_i, h_j)\,\rho_{ij} + [(1-\rho_{ij}^2)/2\pi]^{1/2}\,\phi(Z_{ij}) + h_i\,\phi(h_j)\,\Phi(k_{ij}) + h_j\,\phi(h_i)\,\Phi(k_{ji}) + h_i h_j\,\Phi(h_i, h_j)$$

where $F(h_i, h_j) = \Phi(h_i, h_j) + 1 - \Phi(h_i) - \Phi(h_j)$.

ENDNOTES

1/ The formulation in (3) is consistent with a yield function of the form

$$Y_i = \text{Min}(a_i(x_i),\ b_i(x_i)),\ i=1,\ldots,n,$$

where x_i is a variable input (e.g. fertilizer), and $\text{Min}(a_i(x_i),\ b_i(x_i))$ is assumed to be a concave function of x_i. This is a kinked yield function if $\frac{\partial a_i}{\partial x_i} > \frac{\partial b_i}{\partial x_i} \geq 0$ at the point where $a_i(x_i) = b_i(x_i)$. Moreover, optimum input use $\bar{x}_i$ is not responsive to changing relative prices at the kink (at least within some range of prices). This formulation has been found to provide a reasonable representation of yield functions (e.g. Anderson and Nelson; Ackello-Ogutu, Paris and Williams) In this context, letting r be the price of the input x_i, the variable cost of production per acre is $c_i = r\ \bar{x}_i$ in equation (3) (within some range of prices).

2/ Under constant absolute risk aversion, $\partial A^*/\partial w = 0$ and (5b) takes the form $\sum_{j=1}^{n} \frac{\partial A_i^*}{\partial \pi_j} (\bar{\pi}_j+\delta_j) = 0$. This illustrates the influence of risk preferences on the restrictions discussed by Chavas and Pope.

3/ The coefficient of relative risk aversion is defined as

$$\tau = (w + \sum_{i=1}^{n} \pi_i A_i)\kappa$$

where $\kappa = -(\partial^2 U/\partial w^2)/(\partial U/\partial w)$ is the coefficient of absolute risk aversion. Similarly, the coefficient of constant partial relative risk aversion is

$$\psi = (\sum_{i=1}^{n} \pi_i A_i)\kappa.$$

4/ The normal distribution has also been used widely for modeling truncation effects in a single commodity context. See, e.g., Shonkwiler and Maddala or Holt and Johnson.

5/ Following Gallagher, the effective support prices for corn during the non-program years (i.e., no set-aside requirements) 1974-77 and 1980-81 were determined as weighted averages of the loan rate and the target price. The weights in turn are derived from the proportion of total corn acreage planted eligible for target price protection.

6/ Several weighting schemes were used to gauge the sensitivity of the results obtained to the specification of the ω_j coefficients. The alternative weights used were (.8, .15, .05), (.7, .2, .1), (.5, .3, .2), and (.33, .33, .33). In all instances, the results compared favorably with those obtained using (.5, .33, .17) in terms of signs, significance of coefficients, and model fit. In addition, the conclusions regarding symmetry, wealth effects, and the nature of risk preferences were unaffected by the weighting scheme.

7/ The (untruncated) correlation between price and yield was estimated to be .320 for corn and .279 for soybeans. The (untruncated) correlation ρ between p_1 and p_2 was also assumed to be constant for all years. The estimated value was $\rho = .753$.

8/ Initial wealth was also constructed by multiplying U.S. farm equity by the proportion of net income from corn and soybean production to total net cash farm income. The results obtained using this alternative wealth measure were comparable to those obtained using acreage-weighted wealth. This result is encouraging since the wealth variable is only a proxy for the true equity positions of corn and soybean producers.

9/ The uncompensated own-revenue elasticities for corn and soybean acreage are .071 and .285, respectively. Similarly, the uncompensated own-price elasticities are .166 and .450, respectively, for corn and soybean acreage.

10/ This is also consistent with much of the economic literature (e.g. Arrow; Binswanger).

11/ The null hypothesis that the wealth elasticities equal one was also tested. It was rejected at usual significance levels.

12/ The acreage elasticities with respect to a proportional increase (rescaling) of terminal wealth were -.025 for corn and .368 for soybeans when evaluated at the sample means. Similarly, the acreage elasticities with respect to a proportional increase (rescaling) of profit were -.117 for corn and .082 for soybeans. While the test results indicate that corn and soybean producers do not exhibit CRRA or CPRRA, it is not clear whether relative risk aversion or partial relative risk aversion is increasing or decreasing.