QUANTIFYING LONG RUN AGRICULTURAL RISKS AND EVALUATING FARMER RESPONSES TO RISK

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WHAT WE KNOW ABOUT DECISION MAKING UNDER UNCERTAINTY AND WHY WE DO NOT USE WHAT WE KNOW

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This paper reviews two major approaches used in the past for risk analysis—the expected utility approach and the use of safety rules—and endeavors to reconcile their applicability and use in light of the recent nonexpected utility risk literature. This leads to the identification of several "reduced form" hypotheses that hold under a variety of theoretical structures and to a discussion of some empirical evidence that we have in view of these hypothesis. We conclude that, in spite of the conceptual confusion, we have identified several "down to earth" relationships that allow prediction of outcome and choices under uncertain conditions. Economic studies of choices under uncertainty also established that the expected utility approach is an appropriate normative tool for risk management, especially in cases where variability associated with risk affects income but may not lead to disastrous situations such as bankruptcy. The major lesson of recent research of individual behavior under uncertainty is that it is not always consistent with the expected utility approach; in short, there is not a generic model for evaluating behavior under uncertainty.

Key words: Expected utility, multiattribute utility, safety rules.

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Understanding the effects that considerations of uncertainty and risk have on the choices of economic agents as well as the development of effective tools to address decision making under uncertainty have been major objectives of economic research over the last 60 years. While these issues have been given much effort and have been the foci of major research literature, we are still confused, unsure, and quite far away from satisfying solutions. As economists, we have made much progress—both in our understanding of human reactions to and measurement of risk and in the development of mechanisms and policies to address it. Much of this progress has hinged on the expected utility (EU) hypothesis, although recent developments have shown its limitations and flaws, forcing us to reassess our methodologies and to retool and take a somewhat different path. As we prepare for these changes, it is worthwhile to take stock and identify what we have learned and accomplished, i.e., what we know about behavior under uncertainty, how we can apply this knowledge effectively, and how it can be used to develop better methodologies and understanding of risky choices.

This paper is an attempt at this overview and assessment. It reviews two major approaches used in the past for risk analysis—the EU approach and the use of safety rules—and endeavors to reconcile their applicability and use in light of the recent non-EU risk literature. This leads to the identification of several "reduced form" hypotheses that hold under a variety of theoretical structures and to a discussion of some empirical evidence that we have in view of these hypothesis. We conclude that, in spite of the conceptual confusion, we have identified several "down to earth" relationships that allow prediction of outcome and choices under uncertain conditions.

Behavioristic Models of Choice Under Uncertainty

Before and parallel to the development of the von Neumann-Morgenstern EU models, economists and management scientists developed simple tools to incorporate considerations of uncertainty in economic decision choice models. The formal justification for this framework is not always clear; these tools are ad hoc in their nature. Still, they reflect formal presentations of behavioral rules that the authors deemed to be reasonable when uncertainty holds. They are also rather simple and practical in their computational and data requirements. Simon's notions of "bounded rationality" may explain the introduction and application of this set of "behavioristic" approaches.

Keynes introduced such a behavior rule in his analysis of investment under uncertainty. He suggested that the discount rate is corrected for risk aversion and uncertainty considerations in analyzing risky choices. The greater is the risk, the greater is the "risk premium." Although Keynes did not present a formal model to elaborate this practical notion, this idea has had substantial impact on the development of policy for capital formation. Feder and Just apply this approach to evaluate riskiness of investment in different countries by comparing the interest rates that they are charged for loans.

Another set of behavioristic approaches is represented by the safety models. Safety rules are based on the idea that decision makers are primarily concerned with avoiding unfavorable outcomes such as bankruptcy or starvation. The nature of this concern leads one to consider utility functions with a discrete element. There are three general forms of safety rules:

1. Roy: Safety-first rule—minimize $P(\pi \leq d^*)$.
2. Telser: Safety-fixed rule—maximize $E(\pi)$ subject to $P(\pi \leq d^*) = \alpha$.
3. Katoka: Maximize d^* subject to $P(z \leq d^*) \leq \alpha$.

In the safety-rule models above, π represents profit (or income), d^* is some critical level of profit, α is some critical probability level, and $P(x)$ denotes the probability of occurrence for event x. Katoka's model is a mini-max model; it chooses the distribution that has the highest value in the event of a crisis outcome. While safety rules have not been utilized in many studies of behavior under risk, they are an important model for evaluation of behavior under potential crisis outcomes.

Both Telser's and Katoka's models are quite similar to the practices of classical statistics. Telser uses the notion of a significance level for controlling the likelihood of a Type I error in much the same way that this likelihood is controlled by using the standard hypothesis-testing procedure of classical statistics. The prevalence of use of the hypothesis-testing procedure can serve as witness to the appeal of safety rules for decision making under uncertainty.

The significance levels used by decision makers who follow the Telser and Katoka models serve as measures of their aversion to risk. Note that, with both models, when returns are normally distributed, these decision criteria in essence are equivalent to maximizing a linear combination of mean and standard deviation of profits (or income). The standard deviation coefficient estimate is negative and increases in absolute value as the decision maker becomes more risk averse. While safety rules are ad hoc decision rules, they (or some variant of them) may be followed by large numbers of the population, justifying them as "positive" models.

The Expected Utility Approach

The EU approach has been the mainstay of the analysis under uncertainty in the last 40 years. Two major lines of research established the prominence of this approach. Von Neumann and Morgenstern were the major contributors to a large body of work (see Appendix A) that provides normative justification for the use of EU by rational decision makers. This literature views decision making under uncertainty as a choice between alternatives, each consisting of a vector of outcomes, $\boldsymbol{x}$ (representing income or wealth levels), with a corresponding probability vector, $\boldsymbol{p}$. The dimensions of $\boldsymbol{x}$ and $\boldsymbol{p}$ are n; the summation is over n throughout the paper. Decision makers are assumed to have a preference ordering defined over these alternatives for which the order, independence, and continuity axioms (given in Appendix A) hold. Under these assumptions, alternatives can be evaluated using the EU preference function, $\Sigma u(x_i)p_i$, which is linear in probabilities but not necessarily linear in outcomes. The utility function is assumed to be S-shaped (concave for low income levels and convex beyond a certain income).

Several studies have contributed to the emergence of the EU approach as a major tool of positive analysis, i.e., an approach to explain actual behavior of farmers. Friedman and Savage demonstrated its usefulness for explaining diversifying behavior and phenomena, e.g, when people both hold insurance and gamble; they also introduce basic notions such as certainty equivalents and risk premiums. Tobin relied on Friedman and Savage's work to develop a major literature on portfolio analysis, while Arrow established basic notions and concepts associated with the emergence of insurance contracts (e.g., moral hazard). Furthermore, Arrow (1958, 1974) and Pratt (1964) introduced basic risk attitude measurement concepts (measures of absolute and relative risk aversion). Rothschild and Stiglitz and others developed theoretical foundations to quantify and measure risk. Sandmo presented an analytical framework for analyzing producers' choices under uncertainty and used it to explain losses in production and supply associated with existence of risk (see Appendix A for a more extensive treatment of the EU approach).

Criticisms of the Expected Utility Approach

The Flaws of the Expected Utility Approach

In spite of the major achievements toward understanding choices under uncertainty that have been made with the EU approach, experiments have revealed significant limitations in this approach. Situations have been identified where choices are contrary to the predictions of the EU approach. Expansion of the range of applications of this approach has identified important elements that are not being considered under the EU model. The EU approach provides only partial explanation of choices under uncertainty; the stage is set for new, more general and realistic approaches to augment and replace the EU model.

Before some of the failings of the EU approach are presented, a rather general discussion of two of the major flaws is given below.

The first problem with the EU approach is that it presents rational choices, i.e., not taking into account the impacts of anxieties and worry associated with random outcomes or choices. The EU literature refers to individuals who have concave utility functions as risk averse which, on the surface, sounds as if the person experiences disutility from risk. Technically, however, the term only implies a decreasing marginal utility of income,[1] a property that is likely to be common to almost all economic agents.

An agent with a concave utility function loses utility under the EU model as income becomes random since, instead of receiving μ_x (the expected value of income) with certainty, he receives distributions of income around x; this utility loss is not because he is "afraid of risk" but because the variability of realized income results in average utility that is smaller that μ_x. The uncertainty per se does not matter—what affects EU is the instability and variability of income.

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[1]We mention income here, but the argument also holds for profits and wealth.

When uncertainty exists, the correct choice criteria of individuals who do not suffer from fears, anxieties, etc., should be to maximize EU. However, the use of the term "risk averse" to describe individuals with decreasing marginal utility of income is somewhat misleading since their well-being is not reduced because of anxiety and fear associated with the uncertainty. Their losses (in terms of EU) associated with introduction of risk are not because of lack of information but because of the instability of the outcomes.

Thus, the EU model seems to be a useful normative model since it determines choices under uncertainty ignoring the effects of "irrational" factors such as fear and anxiety. However, it may not be a good model for assessing actual behavioral patterns encompassing fears and anxiety that may affect actual choices under uncertainty. Development of more realistic models that incorporate these factors may require interdisciplinary efforts and inputs, especially those of psychologists.

A second flaw of the EU approach is its failure to include elements other than income in the utility function. In particular, there is a lack of consideration of discrete variables that have a strong impact on the quality of life, that may be affected by the randomness of income, and consequently have a strong impact on decision making. For example, a farmer's utility from a certain income level may be quite different in situations depending on whether he is solvent or not; fear of bankruptcy has a substantial impact on choices, but it is not incorporated in standard EU models.

The following is a detailed criticism of the axioms leading to the EU approach and a presentation of two new models that augment it and attempt to overcome its major flaws.

Systematic Paradoxes of Predictions of the Expected Utility Model

On a more technical level, the existence of systematic behavior violating the independence axiom, or paradoxes, has cast doubt on the viability of the EU model as a basis for prediction. There are two primary types of violations of the independence axiom that have been revealed under experimental conditions—the common consequence effect and the common ratio effect. The common consequence effect arises when lottery payoffs are adjusted in the following manner, where p, q, r, and t are probabilities over outcomes; $\alpha, s \in (0, 1)$; and X and Y are outcome (usually cash) levels:

Initial choice between lotteries:

A: $(\alpha)p + (1 - \alpha)p$ or B: $(\alpha)q + (1 - \alpha)r$.

Second choice between lotteries:

C: $(\alpha)p + (1 - \alpha)\mathrm{t}$ or D: $(\alpha)q + (1 - \alpha)t$.

The common ratio effect arises from changes in lottery choices of the form:

Initial choice between lotteries:

A: $(p)X + (1 - p)0$ or B: $(q)Y + (1 - q)0$.

Second choice between lotteries:

$$C: (sp)X + (1 - sp)0 \quad \text{or} \quad D: (sq)Y + (1 - sq)0.$$

The EU independence axiom states that in both cases agents should choose either (A, C) or (B, D); however, numerous experiments have shown a systematic tendency for violations of this expectation. Additional treatment of these paradoxes is given in the Appendix; summaries of these and other paradoxes are given in Machina (1983, 1987) and Fishburn (1988a, 1988b).

A rather disturbing set of empirical findings regarding the validity of EU theory are examples of preference cycles. These systematic intransitivities have been found over both monetary and nonmonetary outcomes and are of the form:

$$p >^* q >^* r >^* p, \quad \text{where } >^* \text{ denotes a preference ordering.}$$

One type of intransitivity, the preference reversal, occurs if the lottery denoted by a vector of probabilities over outcomes $\boldsymbol{p}$ is preferred to a lottery $\boldsymbol{q}$ over the same outcomes; but the certainty equivalent $c(\boldsymbol{p})$ is less than the certainty equivalent $c(\boldsymbol{q})$. This means that an agent would prefer to hold $\boldsymbol{p}$ over holding $\boldsymbol{q}$ but would sell (or purchase) the right to hold $\boldsymbol{p}$ for less than the right to hold $\boldsymbol{q}$. Numerous experimental studies have found that the preference reversal is a persistent phenomena; see Fishburn (1988a, 1988b) for a discussion of this paradox and more general types of intransitivities.

The existence of intransitive behavior suggests that a clever broker could profit by offering the preferred outcomes in turn for a small fee at each choice level for an infinite number of times. This line of argument is characterized by the person being a "money pump"; the decision maker, not realizing that he is being bilked, continues paying to exchange for the outcome he currently holds. Machina (1989b) evaluates this argument using game-theoretic concepts to show that even naive agents in this dynamic setting would recall their previous payments, effectively limiting the amount involved in trades on the intransitivities.

Generalizations of Expected Utility Theory

It seems that a major reason for the "paradoxical" behavior contradicting the prediction of the EU approach is the failure of approach to incorporate the loss of welfare (due to anxieties and worry) that individuals experience when they face uncertainty. It seems that these factors may reduce the welfare obtained under uncertainty from the level indicated by the EU model.

Generalizations of the EU framework have been developed in order to account for the systematic paradoxes revealed through experimental studies. Models addressing the common consequence and common ratio effects seek to generalize EU theory by introducing some form of a decision weight as a function of probabilities, giving a measure of utility marginal in the decision weights:

(1) $$V(\boldsymbol{p}, \boldsymbol{x}) = u(\boldsymbol{x}) f[\boldsymbol{p}(\boldsymbol{x})].$$

In equation (1), $f[\boldsymbol{p}(\boldsymbol{x})]$ is a decision weight on the vector $\boldsymbol{p}$ given the values of the outcome vector $\boldsymbol{x}$. Therefore, these models are still a marginal utility measure but on a transformed probability axis, $f[p(x)]$.

The general result of these models is shown in figure 1. People's desire to avoid losses causes them to place high probability weights on low outcomes but low weights to high outcomes, with one probability level (p^*) having no change from stated to perceived levels. The stated (prior) probabilities are transformed by the weighting method, giving the perceived (posterior) probabilities as a function flatter than the 45° line. This weighting method is somewhat akin to the Bayesian approach of using a nonquadratic loss function with different estimation losses for outcomes above and below a certain outcome. Like the Bayesian loss function approach, this nonlinear weighting of probabilities has not been a common model used for estimating behavioral attitudes toward risk.

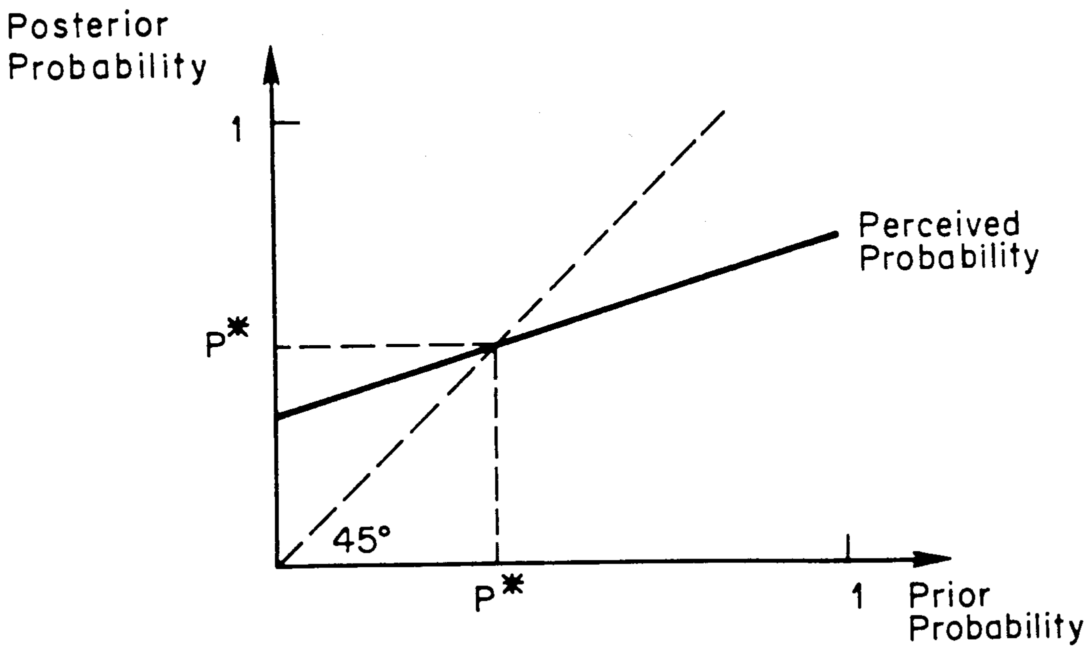

Figure 1. Probability Weighting Transformation

Note that one of these generalization approaches, the "prospect theory, " has been introduced by two psychologists (Kahnemann and Tversky) who also have conducted many of the experiments that revealed behavioral paradoxes. In the prospect theory case, the sum of the decision weights is $[\Sigma_i f(p(x_i))] < 1$, which indicates the "psychological" cost associated with the loss of certainty. The fixed cost of optimization under risk is reflected by this downward bias in the weighting of outcomes. One can view this idea in light of a decomposition of the loss of welfare associated with the introduction of uncertainty, $[u(x) - v(\boldsymbol{x}, \boldsymbol{p})]$, into two elements:

1. $u(x) - Eu(x)$, the welfare loss due to the introduction of variability of income determined by the curvature properties of the utility function.

2. $Eu(x) - v(\boldsymbol{x}, \boldsymbol{p})$, the welfare loss due to "psychological effect" (anxiety) associated with uncertainty.

A list of generalized EU models using transitive preference orderings and their developers is given in table 1.

Table 1

A. "Prospect Theory" $\Sigma v(x_i)\ \pi(p_i)$	Edwards Fellner Kahnemann and Tversky
B. "Subjective Weighted Utility" $[\Sigma v(x_i)\ \pi(p_i)]/[\Sigma \pi(p_i)]$	Karmarker
C. "Weighted Utility" $[\Sigma v(x_i)\ p_i]/[\Sigma \tau(x_i) p_i]$	Chew and MacCrimmon Chew Fishburn (1981, 1983)
D. "Anticipated Utility" $\Sigma v(x_i)\ [f(j{=}1\ \Sigma_{j=1}^{i} p_j) - [f(j{=}1\ \Sigma_{j=1}^{i-1} p_j)$	Quiggin

The approach used in A and B above requires decision makers to form probability weights based on the objective probabilities alone; in these cases, equiprobable outcomes would be given equal weights. The marginal approach in C and D uses decision weights that depend on both the probabilities and the outcomes, x_i.

Yaari's (1987) dual model is an interesting twist on the marginal approach; in his model, the utility function is marginal over the outcomes x instead of marginal over the probabilities as in the EU model:

$$V(x, p) = f[p(x)]\ u(x) = f[p(x)]x. \tag{2}$$

This model allows for behavior shown by the EU paradoxes but has its own paradoxes that are, in turn, answered by the EU theory. The salient point of Yaari's dual model is it reveals that the common paradoxes under the EU model arise from the marginal (with respect to probability) nature of the EU model.

Machina's (1982) model allows for EU maximization without the independence axiom by postulating a high degree of smoothness and uses a local two-factor utility function given by a marginal over p:

$$(3) \qquad U(x; F) = E_F[R(x)] + 1/2[E_F(S(x))]^2,$$

where E_F is the expectation over the cumulative distribution function, $F()$. The smoothness of preferences in Machina's model allows for the use of calculus to find linear approximations to the local nonlinear utility functions that are close to the results given by the linear EU approach.

The models in table 1 and the additional generalizations suggested by Machina, while not an exhaustive treatment of the recent important contributions in generalized utility theory, present important advances in the development toward more general models of behavior under risk. All of these models allow for the behavior indicated by the common consequence and common ratio paradoxes but not for intransitivities such as the preference reversal paradox.

Chew and Epstein have developed a unifying approach to the generalized EU models in table 1 and for additional models. Their implicit rank-linear utility model develops axioms to represent the major thrusts of the generalized EU models and points to some additional untested models within the new framework.

Multicomponent Extensions of the Expected Utility Model

Regret/Rejoice Theory

The EU theory uses the term "risk aversion" to describe the shape of the utility function: concavity for risk aversion or convexity for risk-preferring behavior. However, the curvature of the utility function over income or goods alone does not reflect all of the "costs" of risk. People may have constant marginal utility of income [$u'(x) = c$], described by the EU theory as risk neutrality, but still prefer a certain to a risky outcome.

Planning for, and even worrying about, the actual outcome under an uncertain situation may entail costs that are nontrivial. This notion of costs of risk goes beyond the EU preference reversal paradox and other intransitive behavior. Normative models of behavior under uncertainty should allow for some notion of the cost (or disutility) experienced from being involved in the uncertain situation itself.

One idea capturing some of the notions of disutility from partaking in a gamble is a model developed by Loomes and Sudgen (1982, 1987) and Bell called the regret/rejoice model. In this model, the agent chooses an action and then experiences regret (rejoice) if the outcome of his chosen action is dominated by (dominates) an alternative action's outcome. This model can account for the intransitivities experienced in the preference reversal paradox in addition to the behavior under the common consequence and common ratio effects.

The regret/rejoice preference ordering gives a relationship between action x_g and x_h such that $x_g \geq * x_h$ implies that $\Psi(x_g, x_h) \geq 0$ for a real valued function $\Psi(,)$. Thus, the preference ordering is not only on the realized outcomes but, rather, it is on both the probabilities of the realized and the alternative outcomes.

Status Model

Recently, there has been a renewed interest in decision making under uncertainty where factors in addition to income enter into the utility function. One approach to modeling such behavior has been to consider multiattribute utility functions; for example, preferences over outcome distributions may depend on consumption as well as income. Pratt (1988) and Finkelshtain and Chalfant (1989, 1990) have examined multiattribute utility models and have obtained behavioral results using an approach parallel to that of Sandmo in the univariate case. These models use a smooth von Neumann-Morgenstern utility function in their analysis.

For some choices under uncertainty, a discrete "crisis" outcome, such as bankruptcy or starvation, may affect the utility from income. The possibility of such a crisis occurring presents real threats to the life and livelihood of the decision-making agents. These crisis outcomes become an issue for entire communities, particularly in the not unexpected instances where agents' realized outcomes are not independent of other agents' outcomes. Concern with these potential outcomes may well overwhelm decisions regarding the marginal conditions that are so prevalent in models of economic behavior.

Of particular interest is to develop a model that combines the treatment of discrete problems by safety rules and the axiomized treatment of decisions under uncertainty by the EU and generalized EU models. The status model incorporates the discrete behavioral approach in a normative model. Consider an indicator variable D, having the value 1 if an adverse event occurs and 0 if the event does not occur, representing the discrete nature of many crises situations. Suppose that the probability of the adverse situation occurring depends on income y, i.e., $P(D = 1) = f(y)$. Also, let the cardinal utility function (which is obtained through a well-defined preference ordering in the sense of von Neumann and Morgenstern), $v_i(y) = v(y/D = i)$, depend on the value of D ($i = 1, 0$) in addition to the level of income. For example, $v_1(y)$ may represent utility function of a farmer who retains ownership of his land while $v_0(y)$ represents the utility function of a farmer who gives up landownership and must rent the land. The function $f(y)$ denotes the likelihood that the farmer will need to sell his/her land (or declare bankruptcy) when his/her income equals y.

Then, the agents' maximization problem becomes:

(4a) $$EU(D, x) = \int_y v_1(x)[f(x)]\, g(x)dx + \int_y v_0(x)[1 - f(x)]\, g(x)\, dx.$$

(4b) $$= \int_y v_1(x)\, \phi_1(x)dx + \int_y v_0(x)\, \phi_0(x)dx,$$

where $g(x)$ is the density function for x, $\phi_1(\ ) = [f(x)]g(x)$, and $\phi_0(\ ) = [1 - f(x)]g(x)$. The independence axiom could hold for a constant level of D but, if both levels of D could occur, the independence axiom will in general not hold. If the integrals are Rieman-Stieljes (we assume that $\phi_1(\ )$ and $\phi_2(\ )$ are nondecreasing in x), then (4b) becomes:

$$EU(D, x) = \int_y v_1(x)\, d\phi_1(x) + \int_y v_0(x) d\, \phi_0(x). \tag{5}$$

In (5), the integration must be done for the $D = 1, 0$ cases separately, rather than over all states as in Ravid, because of the discrete nature of the function, $U(D, x)$.

The status model has the following as special cases:

a. Safety-rule models: $v_1(x)$ is extremely negative and $f(x) = 1$ if $x \leq t$ for some level of income t.

b. Target models: $D = 1 \Rightarrow x \leq t$; $v_1(x) = k(t - x)^{\alpha}$; $v_0(x) = (x - t)^{\beta}$.

c. EU model: $v_1(x) = v_0(x)$, $g(x)$ are objective probabilities.

d. Non-EU models: Weights $w[g(x)] = \frac{v_1(x)}{u(x)} \cdot \phi_1(x) + \frac{v_2(x)}{u(x)} \phi_2(x) = \sum_i \frac{v_i(x)}{u(x)} \phi_i(x)$, reflecting the state-dependent utility levels, $v_i(x)$, relative to the von Neumann-Morgenstern utility, $u(x)$, and the income/state density function, $\phi_i(x)$.

Due to its discrete nature over D, the reduced form required for estimation will differ according to the situation at hand. Familiarity with the nature of the decision maker's situation will be needed for the utilization of the status model.

The models presented are applicable in different situations; generalized EU theory may work well for portfolio analysis where there is no crises outcome. Safety rules are advantageous when potential crises situations are involved. The status model incorporates both the discrete and continuous nature of choice under uncertainty, but the status model would require considerable knowledge of the nature of the decision problem. Of interest for future research is the effect of the aggregation of behavior under this model; we would like to determine if behavior under a potential discrete crisis poses an answerable empirical question.

Empirical Methods to Infer Decision Making Under Uncertainty and Their Findings

There have been a growing number of empirical studies of choices under uncertainty. These studies can be divided according to their methodologies into the following groups:

(A) Experimental Studies

The notion of alternative gambles in decision making under uncertainty lends itself well to inferring behavior based on experimental gambles such as those carried out by Kahnemann and Tversky. Respondents are asked to choose between lotteries (usually simple) that differ in payoffs and probabilities or both. Games only occur once, so no learning is involved. Many of the paradoxes, such as the Allais paradox, have been clearly and consistently shown by these experiments. These experiments are relatively inexpensive to carry out and are quite flexible. The major problem with this experimental approach is that it is not representative of the decision problems actually faced by agents.

Binswanger (1980, 1981) carried out an extended experiment, allowing learning, where agents chose between various simple lotteries in multiple plays. Payoff magnitudes were increased as the game progressed in order to give incentives for learning and to study income effects. Such extended experiments are more expensive to carry out, but they offer a greater degree of representation of the agents' decision-making and learning process than the one-shot experiments. However, these simple lottery choices that are divorced from actual decisions are also not very representative of the agents' approach to decisions under risk. Binswanger found that farmers' behavior is consistent with the assumption of constant relative partial risk aversion and that risk preferences tend to vary substantially among the population.

(B) Programming Studies

Programming methods have been used by many economists (Lin, Dean, and Moore) to estimate risk aversion in a normative model of activity analysis (usually continuous). Such approaches generate an efficient set in risk and return space under the production structure used for the model. The problem with this method of estimation is that the coefficients are quite sensitive to variable measurement, while general testing procedures for estimates from programming models have not been developed. Another problem with the programming approach is that all deviations from the model's optimal outcome are attributed to risk behavior.

(C) Econometric Studies

Econometric studies can be divided into two groups—structural and nonstructural. Most of the econometric studies thus far have been structural. Estimation of a general structural model assuming neoclassical production technology and EU approach is very challenging. This approach requires joint estimation of the structure of a firm's technology and input decision rules; the structural approach is needed to test hypotheses about input use or for policy analysis in addition to risk measurement. This econometric method requires input use and price data for individual farms. Some problems with the structural econometric method are that all of the observed deviations from the mean are attributed to risk response and that data for farm's individual input use and price are seldom available; see Antle (1989) and Pope for further treatment of the structural approach.

Two interesting structural models were presented by Antle. Antle (1987a) estimated the risk behavior of California tomato growers, explicitly modeling their sequential choice of pesticide strategy. Antle (1987b) estimated the risk attitudes of farmers in India. Both studies estimated a moment-based version of the EU approach, and the results were heterogenous risk attitudes in the populations, with a high tendency to be Arrow-Pratt absolute and relative risk averse and to have downside risk aversion.

Calvin and Bar-Shira assume fixed proportion technologies and applied the Just-Zilberman portfolio framework to farmers' choice in Iowa and Israel. Calvin found that risk considerations have had very small impacts on crop choices by Iowa farmers. These choices were more affected by labor constraints and crop rotation considerations. Risk considerations were instrumental, however, in explaining farmers' decisions regarding participation in the commodity programs. Bar-Shira found that risk considerations played a significant role in explaining land allocation among crops in Israel. Both studies verified Arrow's hypothesis that absolute risk aversion tends to decline with wealth, while relative risk aversion tends to increase with wealth.

Moscardi and de Janvry used results from Katoka's safety-first rule and the assumption of a generalized production function to develop a model for the risk-aversion parameter from the certainty equivalent identity. They use a structural econometric approach to estimate the risk-aversion parameter.

Antle (1989) has developed a nonstructural economic approach for estimation of the degree of risk aversion. The nonstructural approach narrows the scope of analysis to a focus on risk attitude measurement only, allowing for fewer assumptions and the use of aggregate market data. The nonstructural approach assumes optimal portfolio management by the agents; therefore, farmers' risk attitudes induce patterns over time for the changes in the moments of their net returns distribution. Antle shows that it is possible to draw inference concerning risk attitudes from these patterns in the producer population by a decomposition into overall effects and individual effects. The individual effects are assumed to follow a distribution, allowing testing and confidence interval estimation. Antle carries out 2SLS of the first moment differences (between time periods) on the higher moment differences using pooled data, obtaining parameter estimates used to infer risk behavior.

All of the econometric studies tend to verify that risk matters in decision making. We have not had sufficient data of high enough quality to use econometric techniques to select between alternative models of choices under uncertainty. Up to now, the experimental studies have played the most crucial role in helping to sort out between alternative models. Much data collection and modeling effort is required to make econometric analysis a more useful tool for inferring between alternative modeling frameworks.

Conclusion

Economic research over the last 60 years has identified and established the important role that risk management considerations have in the economy. Behavioral patterns such as insurance holding and portfolio diversification, together with high premiums and interest rate levels, have been related to risk considerations and concerns. Moreover, statistical estimation methods have been developed that allow quantification of the impacts of changes in measures of risk (e.g., variance) on choice variables. Research using these statistical methods confirmed that behavior of more affluent individuals is likely to be less affected by risk avoidance considerations, at least in the absolute sense.

Economic studies of choices under uncertainty also established that the EU approach is an appropriate normative tool for risk management, especially in cases where variability associated with risk affects income but may not lead to disastrous situations such as bankruptcy. Application of the EU approach for normative purposes requires care as production inflexibilities and the discrete nature of the utility function over certain outcomes may have considerable influence on the decision-making process; in short, there is not a generic model for evaluating behavior under uncertainty.

The major lesson of recent research of individual behavior under uncertainty is that it is not always consistent with the EU approach. Economists are accustomed to using normative behavior as a starting point for positive analysis. This approach is likely to result in misleading results when applied to risk choices. Behavior under uncertainty is affected by anxieties and worries that are not incorporated in the traditional EU model. Quantification of these elements is not easy and might require assistance and methodologies from other disciplines. The success of experimental studies in deciphering behavioral patterns under uncertainty suggests that econometrics is not necessarily the preferred empirical tool for a study of individual's choices under uncertainty. We may need to use more experimental methods such as questionnaires and interviews in order to understand how people react to risk. Econometrics seems to be preferred for studying the impact of risk parameters at the aggregate level, e.g., the impacts of changes in variance of price on aggregate output supply. Thus, different aspects of the study of choice under uncertainty—normative, positive, and aggregate—may need to proceed almost independently in the near future, each using its own appropriate techniques.

One area we neglected to discuss is that of learning and individual estimation of risks. This is an area we know little about, and it requires much conceptual and, more importantly, empirical research. The consequences of the type of expectation formation, i.e., rational or adaptive, could be critical for the effects of changes in risk levels and attitudes.

APPENDIX A: Expected Utility Theory

Standard EU theory has progressed along two paths: (1) the normative axiomatic approach for behavior under uncertainty based on game theory and (2) the positive summary measure approach using summary measures based on some general properties of utility functions. This section will review the status of the EU approach along these two paths.

Normative Approach

The initial normative approach to behavior under uncertainty was set forth by early probability theorists using simple outcome maximization. Let x_i be the outcome and $p(x_i)$ denote the probability of outcome x_i, $i = (1, \ldots n)$; then consider:

(A1) $$E(x, p) = \Sigma x p(x).$$

Gabriel Cramer and Daniel Bernoulli recognized that utility is not necessarily linear in wealth. Bernoulli considered $v(w)$ a cardinal utility measure on wealth in the maximization of the following summation.

(A2) $$\Sigma v(w_i)p(w_i) \text{ or, alternatively, } \Sigma v(w_0 + x_i)p(x_i).$$

In the model in equation (3), w is the final wealth, w_0 is the initial wealth, and x is the value of the new outcome. Bernoulli's approach required the existence of a well-defined underlying cardinal measure of utility over w; an assumption that is now considered unattractive for the evaluation of decisions under uncertainty.

An ordinal EU model defined on a binary relation (preference ordering) rather than directly assuming the existence of a cardinal utility function was developed by von Neumann and Morgenstern. If and only if axioms regarding the binary relation $>^*$ on a nonempty set P of probability measures $(p, q, \ldots)$ hold, there is a linear functional $u()$ on P such that, for all $p, q \in P$, $p >^* q \Leftrightarrow u(p) > u(q)$; here $u()$ is unique up to a positive linear transformation. In other words, if and only if certain axioms for a preference ordering hold, von Neumann and Morgenstern show that there is a real valued function $u()$ that indicates a preference measure.

There are three axioms in the von Neumann-Morgenstern utility (EU) theory for a preference ordering $>^*$ on P for all $p, q, r \in P$ and for all λ, $0 \leq \lambda \leq 1$:

O (Order): $>^*$ on P is a weak ordering. This implies that $\geq^*$, $>^*$, and $\sim$ are transitive and also that $[p \sim q, q >^* r] \Rightarrow p >^* r$ and that $[p >^* q, q \sim r] \Rightarrow p >^* r$.

I (Independence): $p >^* q \Rightarrow \lambda p + (1 - \lambda)r >^* \lambda q + (1 - \lambda)r$.

C (Continuity): $[p >^* q, q >^* r] \Rightarrow \{\alpha p + (1 - \alpha)r >^* q$ and $q >^* \beta p + (1 - \beta)r$ for some $\alpha, \beta \in (0, 1)\}$.

According to Fishburn (1988b), Axiom O is the basis for the economic conception of rationality; violations of 1 are usually considered as aberrations. Axiom I is the linearity assumption and is closely associated with similar properties referred to as substitution principles, cancellation conditions, additivity axioms, and sure thing principles. Axiom C is the Archimedean axiom, a condition that is in keeping with the properties of the real number system; it states that nothing is infinitely preferred to another. There is also a convergence notion of prospects and probabilities in C; convergence in distribution is used.

The main representation and uniqueness theorem for the von Neumann-Morgenstern utility (EU) is:

Theorem 1: Axioms O, I, and C hold over an appropriately defined nonempty convex probability set P iif there exits a linear functional $u(\ )$ on P such that, for all $p, q \in P, p >^* q$ iff $u(p) > u(q)$. Moreover, such a $u(\ )$ is unique up to a positive affine transformation.

Therefore, when the Axioms (O, I, and C) hold for a set P, we can express preferences in terms of a cardinal function $u()$, which is unique up to a positive affine transformation.

The anticipated utility form as a method of evaluation of choice under risk follows from the EU axioms. For an n-vector of outcomes $\boldsymbol{x}$ with a corresponding vector $\boldsymbol{p}$, we have an EU measure from the Independence Axiom given by:

(A3) $$V(\boldsymbol{x}, \boldsymbol{p}) = V(x_1, p_1; x_2, p_2; \dots ; x_n, p_n) = \Sigma^n p_i u(x_i).$$

We see that the EU preference function for evaluating alternatives, $\Sigma u(x_i)p(x_i)$, is linear in probabilities, while nonlinear in outcome x. The linearity of the preference ordering $V(\boldsymbol{x}, \boldsymbol{p})$ implies that it is additively separable for outcomes x, or separable across mutually exclusive events x. The general attributes of separability can be broken into two properties (Machina, 1989b)—replacement separability and mixture separability.

Replacement separability requires that, if an individual prefers a lottery $\{y_1, p_1; x_2, p_2; \dots ; x_n, p_n\}$ where the pair (x_1, p_1) was replaced by the pair (y_1, p_1) to the initial lottery $\{x_1, p_1; x_2, p_2; \dots ; x_n, p_n\}$, then the individual would prefer such a replacement in any other lottery of the form $\{x_1, p_1; x_{*2}, p_{*2}; \dots; x^*_m, p^*_m\}$. Mixture separability requires that the lottery $\{y_1, p_1; x_2, p_2; \dots; x_n, p_n\}$ be preferred to the lottery $\{x_1, p_1; x_2, p_2; \dots; x_n, p_n\}$ if and only if y_1 is preferred to x_1, i.e., including additional identical events with identical probabilities in the choice sets does not affect the underlying preferences.

Consider a lottery with outcomes $x_1 < x_2 < x_3$, with the respective probabilities p_1, p_2, p_3; $(\Sigma p_i = 1)$. Lotteries over these three outcomes can be represented by the unit triangle in figure A1 as in Machina (1982) or, alternatively, by an equilateral triangle as in Fishburn (1988b). In figure A1, lotteries with

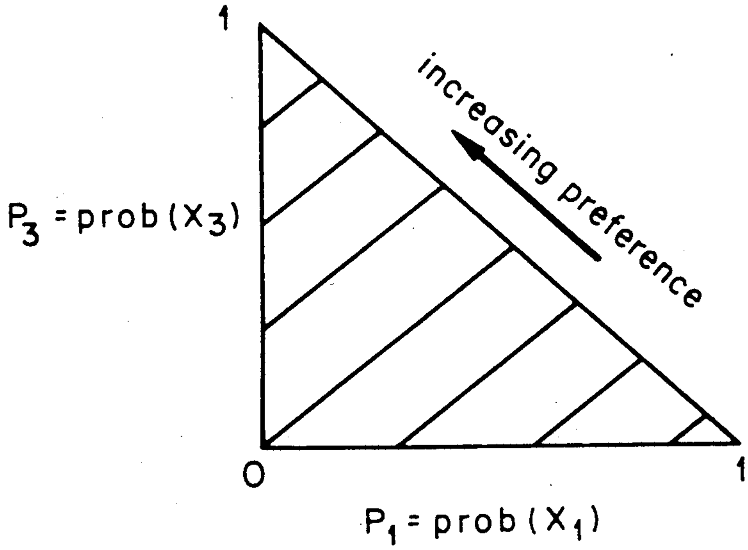

Figure A1. Expected Utility Indifference Curves Represented in a Unit Probability Triangle

Source: Mark J. Machina. "Choice Under Uncertainty: Problems Solved and Unsolved." *Economic Perspectives*, Vol. 1, No. 1 (Summer, 1987), pp. 121-154.

well-defined probability triples (p_1, p_2, p_3) can be described by a point assigned to them in the unit triangle since $p_2 = 1 - p_3 - p_1$. Movements in the triangle toward p_3 (away from p_1 and/or p_2) will increase utility since $x_3 > x_2 > x_1$; in general, northwest movements in the diagram are movements of increasing preference. The indifference curves under EU theory are linear in the probability space; their slope indicates the degree of preference among the outcomes.

The indifference curves in the EU triangle are parallel, with their slope given by $[u(x_2) - u(x_1)]/[u(x_3) - u(x_2)]$. Therefore, knowledge of the slope of one of the indifference curves in the triangle is sufficient for knowledge of the decision maker's preferences over any such three outcome lottery well defined by a probability triple (p_1, p_2, p_3).

Machina (1983, 1987, 1989a) uses the triangle diagram to indicate the degree of risk aversion. In figure A2, the solid lines are EU indifference curves as before, while the dashed lines are iso-expected value lines. Since the expected value does not change as one moves along the iso-expected value lines, but the variance of outcomes increases for movements along these curves to the northeast (tail probabilities p_1 and p_3 increase), these movements represent mean-preserving spreads. The relatively steep EU indifference curves in figure A2a represent a risk-averse decision maker; the relatively flat indifference curves in figure A2b indicate the preferences of a risk-preferring person.

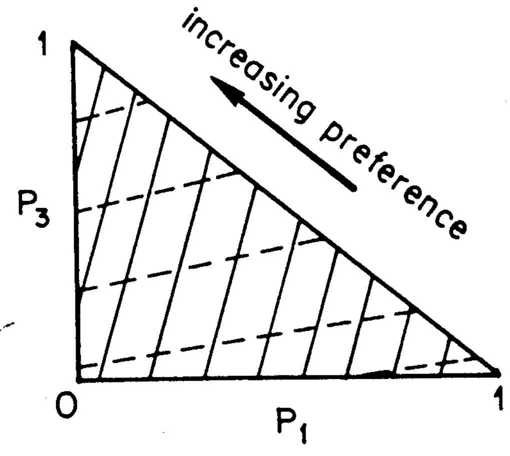

Figure A2a. Relatively Steep Indifference Curves of a Risk Averter

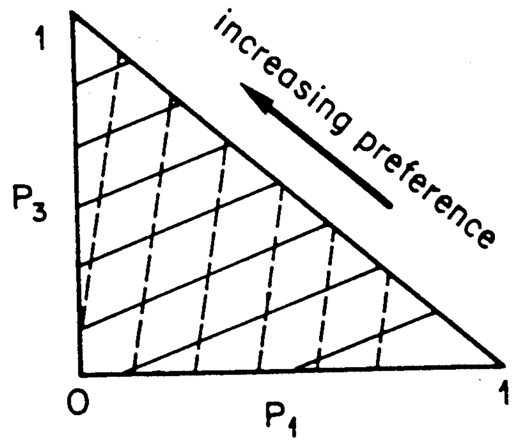

Figure A2b. Relative Flat Indifference Curves of a Risk Preferrer

Source: Mark J. Machina. "Choice Under Uncertainty: Problems Solved and Unsolved." *Economic Perspectives*, Vol. 1, No. 1 (Summer, 1987), pp. 121-154.

The normative model set forth by the EU theory is a simple, elegant model with convenient extensions for evaluating agents' behavior under uncertainty. The standard EU model has been used as a basis for studies of decision-making behavior by agricultural, finance, and other applied economists since its introduction by von Neumann and Morgenstern.

Positive Approach

The EU theory is a vehicle for describing a decision maker's behavior over risky outcomes. Many authors have used some of the conditions of the normative EU theory to develop workable summary measures for agents' behavior under uncertainty. Savage (1954) was a forerunner of many of these generalized forms when he considered the uncertainty case where estimates of probabilities are required. Pratt (1964), Arrow (1965, 1974), and other authors gave conditions on the behavior of the utility curve under particular forms of risk behavior.

In Savage's model, agents form subjective probability assessments over a set of states of the world S. These agents assign acts through a function $f() \in F$ from s to x. There is a preference relation $>^*$ on x from a preference relation $>^{**}$ on F through constant acts: $x >^* y$ if $f() >^{**} g()$ when $f(s) = \{x\}$ and $g(s) = \{y\}$. Also, $f >^* g <=> {}_0\int^\infty [f(s)]d\pi(s) > {}_0\int^\infty u[g(s)]d\pi(s)$, where $\pi(s)$ gives a subjective probability for state s to occur.

A decision maker is described as risk averse around x if $u''(x) < 0$, risk neutral around x if $u''(x) = 0$, and risk preferring around x if $u''(x) > 0$, where $u''(x)$ is the second derivative of $u()$ with respect to x. The certainty equivalent $c(p)$ of a lottery with probability p is such that the agent is indifferent under the EU framework between the lottery p or receiving the certainty equivalent outcome $c(p)$, given by the identity:

$$u[c(p)] = \Sigma u(x)p(x). \tag{A4}$$

A risk premium $\pi(u, p)$ is given as the difference between the certainty equivalent of a gamble and its expected value:

$$u[c(p)] = u[E(x, p)] - \pi(u, p). \tag{A5}$$

Notions of stochastic dominance (second degree) and mean-preserving spread address preferences of risk-averse agents on probability distributions. Generalization of these ideas is attributed to Rothschild and Stiglitz, and others have led to properties of risk on the distribution functions. A cumulative distribution function $G(y)$ is second degree stochastically dominant over another cumulative distribution $F(y)$ if:

$$\int_0^y F(y)dy \geq \int_0^y G(y)dy, \quad \text{for all y} \in X. \tag{A6}$$

$F(y)$ is a mean-preserving spread of $G(y)$ if:

$$\int_0^y F(y)dy \geq \int_0^y G(y)dy, \quad \text{for all } y \in X, \tag{A7a}$$

and

$$\int_0^\infty [F(y) - G(y)]dy = 0. \tag{A7b}$$

The condition on the integrals of the functions indicates that $F()$ has more probability weight on the lower outcome levels.

Risk-aversion measures which are invariant to positive transformations of x have been developed. The measure of absolute risk aversion (R_A) is given by:

$$-u''(x)/u'(x). \tag{A8}$$

Their relative risk-aversion measure (R_R), which can also be thought of as an elasticity of risk-aversion measure, is given by:

$$-[u''(x)/u'(x)]x. \tag{A9}$$

There is also a partial risk-aversion measure (R_P):

$$-[u''(x_0)/u'(x_0)]x^*. \tag{A10}$$

In equation (10), x_0 is the initial income level and x^* is the outcome of the uncertain prospect.

These Arrow-Pratt measures are widely used to address extensions and consequences of behavior under risk; comparative static results for exogenous variable changes, such as by Sandmo (1971), often depend on the sign and magnitude of R_A and R_R.

APPENDIX B: Paradoxes

The Allais paradox is an example of the more general common consequence effect and was one of the first examples of systematic behavior violating the EU theory. We will illustrate this paradox by the results of a study reported in Kahnemann and Tversky that used modest payoffs. The number of respondents was 72; the percentage choosing each lottery is given in brackets.

Problem 1:

Lottery A: 2,500 with probability .33
[18] 2,400 with probability .66
0 with probability .01

Lottery B: 2,400 with certainty
[82]

Problem 2:

Lottery C: 2,500 with probability .33
[83] 0 with probability .67

Lottery D: 2,400 with probability .34
[17] 0 with probability .66

Kahnemann and Tversky reported that 61% of the respondents chose both B and C. This pattern of choice violates the EU axioms since, under the EU theory, we have (where $u(0) = 0$):

> Lottery B preferred to Lottery A => $u(2{,}400) > .33*u(2{,}500) + .66*u(2{,}400)$ or, equivalently, $.34*u(2{,}400) > .33*u(2{,}500)$ which is contradicted in the second lottery choice by 61% of the respondents.

The Allais paradox can be represented diagrammatically as in Machina (1987). In figure B1a, the EU model gives parallel indifference curves; the gambles in the choices above form the parallelogram ABCD. Lottery A is preferred to lottery B implies that lottery C is preferred to lottery D under EU using the linearity property.

However, we observe A preferred to B and D preferred to C, a violation of the EU theory. A form of indifference curves consistent with observed behavior are ones that "fan out" as shown in figure B1b. This fanning out implies that knowing the behavior (slope) of one indifference curve is not sufficient for describing the indifference curves in the entire triangle—each curve can differ in slope while linearity of the indifference curves is not required. Utility curves which fan out can also allow for the more general common consequence and common ratio effects.

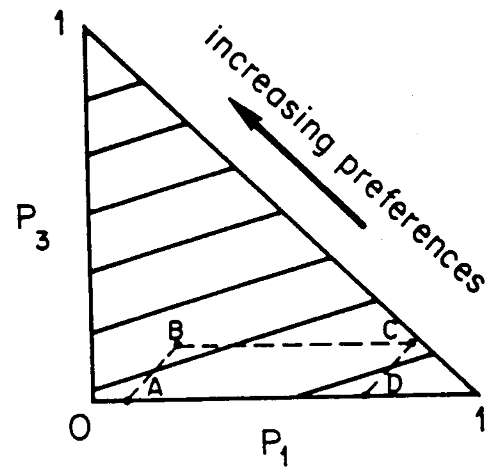

Figure B1a. The Allais Under Expected Utility Maximization

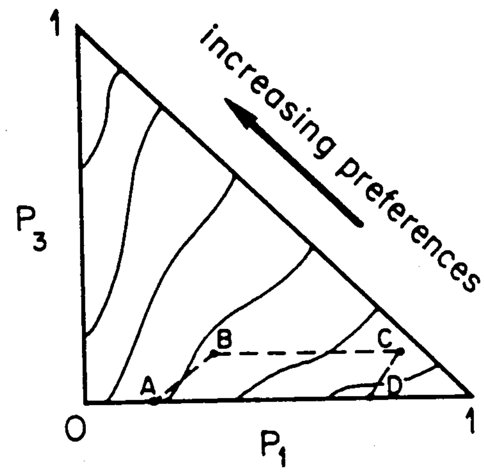

Figure B1b. The Allais Paradox and Indifference Curves which Fan Out

Source: Mark J. Machina. "Choice Under Uncertainty: Problems Solved and Unsolved." *Economic Perspectives*, Vol. 1, No. 1 (Summer, 1987), pp. 121-154.

1. One could also model the probability of the adverse event on variables in addition to y, i.e., variables such as consumption expenditures, negotiations with lenders, lobbying efforts, etc.

2. The use of triangles to represent choices among three outcome lotteries has been used by many authors; its first use is attributed to Marschak.

3. Other stochastic dominance notions and their applications are: First-degree agents have utility that increases in wealth and third-degree agents have declining risk aversion.

4. The payoffs refer to Israeli currency.

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