QUANTIFYING LONG RUN AGRICULTURAL RISKS AND EVALUATING FARMER RESPONSES TO RISK

Proceedings of a Seminar sponsored by
Southern Regional project S-~~180~~232
"Quantifying Long Run Agricultural Risks and Evaluating
Farmer Responses to Risk"
Sanibel Island, Florida
April 9 - 12, 1989

Agricultural Economics Department
Texas A&M University
College Station, Texas

July 1989

EQUILIBRIUM LAND PRICES UNDER RISK

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I. Introduction

The behavior of a competitive firm facing a random output price has been modeled extensively. The impact of changing the mean or the riskiness of the output price distribution has been determined. In addition, the effect of changing a nonrandom parameter, such as a tax level or fixed cost, has been shown to differ depending on whether output price is random or not. These comparative static results are the basis for much agricultural policy analysis.

As Appelbaum and Katz note, a difficulty with this comparative static analysis of the competitive firm is that equilbrium in the competitive industry has not been considered. That is, how the individual firm reacts to changes in certain parameters is calculated without determining whether or not those parameter changes are consistent with equilibrium for the competitive industry.

Appelbaum and Katz remedy this for a constant cost industry by allowing the mean output price to depend on the aggregate output level. Parameter changes which affect the welfare of the individual firm lead to entry or exit from the industry and thus alter the mean output price. This implies that changes, such as an increase in the riskiness of the output price distribution or a change in the firm's fixed costs, result in a shift in the mean output price. Appelbaum and Katz show that including these effects significantly alters the comparative static results obtained in models without them.

Appelbaum and Katz's extension of the earlier partial equilibrium work applies to the constant cost industry case. That is, entry or exit are assumed to affect output price, but not costs to the firm. Only output price adjusts so as to attain industry equilbrium. This constant cost assumption is typical in the analysis of a competitive industry.

Even though constant cost competitive industries are often analyzed, the increasing cost case is an important one as well. For increasing cost industries, the individual firm's cost of production rises with industry output level. This is a second mechanism for adjusting to industry equilibrium.

In the extreme case, all adjustment to equilibrium occurs through these cost increases. Typically, industries where firms are required to

*This is Michigan Agricultural Economic Staff Paper No. 89-52. Funding was provided jointly by the Michigan Agricultural Experiment Station and Economic Research Service, U.S.D.A. Jack Meyer and Lindon J. Robison are Professors of Economics and Agricultural Economics, respectively, Michigan State University.

have a license to operate, and the number of licenses is fixed, are cited as examples where the extreme case applies. Under these conditions, industry equilbrium is attained through a price adjustment in these licenses. The pricing of taxicab medallions, television broadcast licenses, or seats on a stock exchange are often modeled in this fashion.

For the agricultural sector, this second mechanism for adjusting to industry equilibrium is a particularly important one. This is because land is an input which is not competitively available. In fact, under the simplest assumption land is available in fixed supply much like broadcast licenses or stock exchange seats. In addition, empirical evidence indicates that land prices do adjust to offset other parameter changes which affect the profitability of agricultural production (Robison, Lins, and VenKataraman). It is important to model this significant factor in maintaining equilibrium in the agricultural industry when parameters in the model are random variables.

This research develops a model of a competitive firm facing a random output price in an increasing cost competitive industry. The model is formulated so that the price of a single input, land, rises or falls, so that equilibrium in the industry is maintained. As in the constant cost case, the comparative static effect of various parameter changes are significantly altered by this industry equilibrium consideration.

A variety of comparative static questions can be addressed using the model presented here. The following are considered. First, how do land prices reflect the risky nature of the farm activity? Second, how do the effects of various parameter shifts or policy actions differ from those implied in the traditional models?

The paper is organized as follows. First, the literature is reviewed briefly, and then a very simple model of a competitive firm which uses land as an input is presented. The next section presents conditions for equilbrium in this competitive industry when output price is random and land is in fixed supply. Since profits are random, the simple zero profit condition from the certainty case is not appropriate. Finally, in the last section of the paper the simple model is expanded so that relevant comparative static calculations can be carried out.

II. Literature Review

A typical model of a competitive firm assumes the firm chooses output level x to maximize profit, where profit $\pi = p \cdot x - c(x) - B$. In this model p represents the output price, and $c(x)$ and B represent the variable and fixed cost, respectively. When there is no randomness, the industry in which this firm operates is assumed to be in equilibrium when the firm earns zero profit.

In the constant cost industry case, $c(x)$ and B are exogenous, and thus only output price can adjust to maintain zero profit. The standard assumption is that p depends on the industry output level, and entry or exit occur until industry output is at the appropriate level. That is, entry or exit occur until aggregate output Q satisfies $p(Q) = (c(x) + B)/x$, and the firm's profit is zero.

In industries where input prices are not exogenous, an individual firm's costs can also rise or fall to attain a zero level of profit. This is the increasing or decreasing cost industry case. In the extreme, all adjustment to a zero profit equilibrium occurs through this mechanism. To present a model of such an industry, the above profit function must be written as a function of input levels rather than output level.

Consider the simple case of one input, L, and one output. Let profit be given by $\pi = p \cdot L - \phi \cdot M \cdot L$. This reformulation of the profit function assumes constant returns to scale, that fixed cost is zero, and that input and output are measured in units so that one unit of input produces one unit of output. This is done for simplicity purposes, and is generalized in the last section. Also, since inputs, such as licenses to operate, seats on a stock exchange, or land, have a long or infinite life, a per period cost is given. This is written as $\phi \cdot M$, where ϕ is the interest rate and M is the price of the durable input.

When there is no randomness, industry equilibrium is characterized by the zero profit condition. To focus on how industry equilibrium affects the input price, output price is assumed to be exogenous. Zero profit implies the standard capitalization formula
$M = p/\phi$ as the equilibrium price for the durable input. In this very simple model, the three parameters, p, ϕ, and M are linked by this equation.

Sandmo and others extend these models of the competitive firm to situations where the output price is a random variable. Typically though, the industry is not assumed to be in equilibrium in the comparative static analysis which they conduct. Recently, Appelbaum and Katz, ask whether or not assuming the industry is in equilibrium would significantly alter the results from that analysis. They do this for the constant cost industry case.

Appelbaum and Katz assume that profit is $\pi = p \cdot x - c(x) - B$, where output price is a random variable. Specifically, $p = f(Q) + \epsilon$ where ϵ is random with a zero mean, and f(Q) is a nonrandom term giving the mean output price as a decreasing function of industry output level Q. Costs are assumed to be exogenous. Equilibrium in the industry is attained when no firm desires to enter or exit the industry.

Since profit is a random variable, profit cannot be set equal to zero to determine industry equilibrium. Instead, Appelbaum and Katz assume that firms can attain a reservation level of expected utility in other activities, and choose to enter or leave this industry if the expected utility from profit exceeds or falls short of this reservation level.

They are able to show that requiring the industry to be in equilibrium creates a link between mean output price and all other parameters in the model. Thus, shifts in those parameters have additional effects, and these can significantly alter the comparative static results found by Sandmo and others. Their main finding is that comparative static results no longer depend on assumptions concerning absolute and relative risk aversion.

III. A Random Output Price and A Fixed Supply of One Input

The model considered here deals with the increasing cost industry case when output price is random. It assumes a competitive firm faces a random output price in an industry where an input is in fixed supply. This case appears to be of interest in agriculture since land has characteristics which approximate an asset available in fixed supply. Furthermore, in agriculture, output price is often unknown and hence modeled as random.

The simple model introduced earlier, where profit is given by $\pi = p \cdot L - \phi \cdot M \cdot L$, is used, except now p is a random variable. The firm is assumed to choose L so as to maximize expected utility of profit. Since constant returns to scale is assumed, strict risk aversion is required if the solution is to be a finite level of L.

Before formally portraying the firm's decision problem, it is advantageous to recognize that the output price p is the only source of randomness in this model, and that profit is a positive linear transformation of p. This implies that all potential profit distributions are location and scale (LS) transformations of the distribution of output price. This property allows the firm's decision concerning the optimal L, and a large portion of the comparative static analysis, to be conducted in a mean-standard deviation (MS) framework without violating the expected utility maximization assumption or imposing other special assumptions.[1]

The MS formulation of the decision model with $\pi = p \cdot L - \phi \cdot M \cdot L$ assumes the competitive firm chooses L to maximize $V(\sigma,\mu)$ where mean $\mu = \mu_p \cdot L - \phi \cdot M \cdot L$ and standard deviation $\sigma = \sigma_p \cdot L$. The notation μ_p and σ_p represent the mean and standard deviation of output price. Solving for L and substituting, this pair of constraints reduces to the linear restriction given as equation (1). The firm's choice of L is a selection of some point on this straight line in (σ,μ) space. This constraint, and hence the firm's choice of L, depends on all of the parameters in the decision model.

$$\mu = (\mu_p - \phi \cdot M) \cdot \sigma / \sigma_p. \tag{1}$$

1. Sinn and Meyer describe the implications of this LS property, and Meyer and Robison use it to analyze the hedging behavior of the competitive firm.

The next section discusses industry equilibrium when the input L is available in fixed supply. Industry equilibrium conditions result in a link between the price of L and other parameters in the model. Thus, when doing comparative statics, such changes as increasing the mean or variance of output price have a different impact than in the partial equilibrium case.

IV. Equilibrium in the Land Market and the Increasing Cost Industry

The competitive firm treats the prices of its inputs and the random price for output as exogenous variables. Models of the competitive industry however, view one or more of these as endogenous, with the level adjusted so as to make entry or exit from the industry a matter of indifference. In the constant cost industry case, only output price adjusts with input prices remaining exogenous. The increasing cost industry case treated here makes the opposite assumption. Output price is assumed to be exogenous, and one input price adjusts.

As in the case without randomness, the price of the input is assumed to adjust to attain the appropriate profit level. Since profit is now random, this appropriate level is somewhat more difficult to define. One approach is to treat the input as an asset generating a random return, and hence its equilibrium price can be determined using the Capital Asset Pricing Model(CAPM).[2] That is, the price of the input can be assumed to adjust until the mean rate of return it earns is equal to the risk free rate of return plus a factor which depends on the riskiness and the diversification possibilities for the asset.

Formally applying CAPM to the pricing of land requires that the mean return per dollar satisfy

$$\mu_p/M = \phi + \eta \cdot \sigma_p/M \tag{2}$$

where ϕ is the risk free rate of return. The η term[3] represents the asset's diversification possibilities and the return to risk in the market. Since one unit of output is produced per unit of land, μ_p and σ_p are the mean and standard deviation of return per unit of land as well as the mean and standard deviation of output price.

The equilibrium value for M which this pricing equation implies is given by

$$M = (\mu_p - \eta \cdot \sigma_p)/\phi = \mu_p/\phi - \eta \cdot \sigma_p/\phi \tag{3}$$

2. Barry has treated the pricing of agricultural land in this fashion.

3. The η term is not the traditional CAPM beta (β), but is related to it in the following way: $\eta = (\mu_m - \phi) \cdot \beta/\sigma_p$. Thus, η represents both the assets diversification possibilities and the return to risk in the market.

As in the certainty case, the price of the input depends on the other parameters in the model, but now this dependence is more complex since the output price is random. The industry equilibrium condition again results in a link between the price of the input and the other parameters in the model.

To see how the comparative static analysis in the model of the firm is altered by this link, it is sufficient to recall that changing μ_p, σ_p, ϕ, or M while holding the others fixed alters the firm's opportunities and hence its choices. If industry equilibrium is assumed however, M is linked to the other parameters by equation (3). When this value for M is substituted into (1), the constraint the firm faces becomes $\mu = \eta \cdot \sigma$. Thus, in industry equilibrium, the price of the input adjusts so that the firm's opportunities do not depend on the mean or variance of output price or any parameter other than diversification possibilities. In the more general model introduced later, the effects of industry equilibrium are not quite so simple, but are just as significant and dramatic.

Before going to this more general model, a second approach to the issue of equilibrium in this industry is worthy of discussion. In this approach a specific functional form for the firm's preferences is assumed so that an explicit demand function for land can be calculated. The equilibrium price for land is then obtained by aggregating these demand functions across firms and setting aggregate demand equal to the fixed supply.

In order to easily calculate the demand for land for a firm, the linear mean-variance utility function is assumed. That is, the firm is assumed to maximize $\mu - (\lambda/2)\sigma^2$. For the competitive firm $\mu = \mu_p \cdot L - \phi \cdot M \cdot L$ and $\sigma = \sigma_p \cdot L$. The parameter λ represents the risk aversion measure of the firm and is assumed to be positive. The firm's decision variable is L. Choosing L to maximize this function leads to

$$(\mu_p - \phi \cdot M) - \lambda \cdot \sigma_p^2 \cdot L = 0 \tag{4}$$

as the first order condition, with the second order condition satisfied since λ is positive.

Solving for the optimal size of the firm, one obtains

$$L = (\mu_p - \phi \cdot M)/(\lambda \cdot \sigma_p^2) \tag{5}$$

Thus, the individual agricultural producers demand for land depends on the mean and variance of the price of output, the risk free rate of return, the price of land and the producer's risk aversion level. Only this latter term is firm specific. Indexing the risk aversion level to show that it depends on the firm, the demand functions for land can be easily added across firms. Doing so one obtains

$$L_D = ((\mu_p - \phi \cdot M)/\sigma_p^2)(1/\lambda_1 + \ldots + 1/\lambda_n) \tag{6}$$

as the aggregate demand function for land.

To find the equilibrium price of land this demand function is set equal to the given supply L_S. Solving for M one obtains

$$M = (\mu_p/\phi) - (L_S \cdot \sigma_p^2)/(\phi \cdot (1/\lambda_1 + \ldots + 1/\lambda_n)) \qquad (7)$$

Notice that the form of this pricing equation is similar to that presented earlier using the CAPM argument. That is, the price of land equals a capitalization of the mean return plus a term which discounts for the riskiness of the farm production activity. Again this pricing equation links the price of land to the other parameters in the model and thus alters the comparative static effect of shifting those parameters.

To see the impact of this industry equilibrium condition on the comparative statics in the firm model, substitute the value for M given in (7) into the first order condition (4). Doing so yields $L_S/(1/\lambda_1 + \ldots + 1/\lambda_n) - \lambda \cdot L = 0$ as the first order condition for the competitive firm. As in the previous CAPM based argument, the firm's decision no longer depends on such parameters as the mean or variance of output price. In this case, the firm's choice of size depends only on the available supply of land and the firm's risk aversion level relative to the average level in the industry.

Before analyzing these comparative static results in more detail, the model is augmented with several other parameters to make the comparative static analysis more relevant.

V. A More General Model

This model of the competitive firm facing a random output price is adequate to illustrate the effects of industry equilibrium on the price of an input whose supply is fixed. It is not, however, sufficiently well developed to allow interesting and relevant comparative static analysis to be carried out for the agricultural sector. Thus, two features are added to the model.

First, the initial wealth of the firm is assumed to be nonzero, and more importantly, a portion of this wealth is held in the form of the durable input land. Let $C_o + M \cdot L_o$ be the firm's initial wealth, where C_o represents the nonland portion, and $M \cdot L_o$ is the value of the land the firm holds. Introducing this term allows the changes in the price of land induced by industry equilibrium to impact the firm's wealth as well as alter the cost of using that input. This is particularly relevant since 73 percent of the agricultural sector's wealth is held in the form of land.

The second feature added to the model of the competitive firm allows more than one input to be used in the production of output. This addition implies that even if the quantity of land is assumed to be fixed, the industry output level can expand or contract by altering the levels of the other inputs. Again, this seems to more adequately reflect the situation in the agricultural sector.

Formally, output level is assumed to be given by $L \cdot f(s)$, where s represents another input. Its use is measured per acre of land. This production function represents a constant returns to scale process, but now output per acre depends on s. The function f(s) is assumed to be increasing and concave.

Since this is a one period model, the firm's objective can be written as maximizing expected utility from either income Y, or terminal wealth W. Terminal wealth is used here, and it can be written as

$$W = (1+\phi)(C_o + M \cdot L_o) + (p \cdot f(s) - \phi \cdot M - p_s \cdot s)L \tag{8}$$

where p_s is the price of input s. The firm takes all prices in the model as parameters outside its control. This wealth function is linear in the random variable and hence MS analysis is appropriate in this case also.

Casting this decision in the MS framework, the competitive firm chooses L to maximize $V(\sigma,\mu)$ where $\mu = (1+\phi)(C_o + M \cdot L_o) + (\mu_p \cdot f(s) - \phi \cdot M - p_s \cdot s)L$ and $\sigma = \sigma_p \cdot f(s) \cdot L$. Again, one can solve for L in the second restriction and substitute into the first to reduce the pair of restrictions to a single equation. This yields

$$\mu = (1+\phi)(C_o + M \cdot L_o) + (\mu_p \cdot f(s) - \phi \cdot M - p_s \cdot s)\sigma/(\sigma_p \cdot f(s)) \tag{9}$$

a linear restriction in (σ,μ) space. If the expected utility preferences display risk aversion, $V(\sigma,\mu)$ is known to be quasiconcave. This implies that the second order conditions for the maximization are satisfied under this condition.

In this decision model s only affects the slope of (9). Thus, all risk averse firms choose s to maximize this slope. Firms could choose different levels of L, however, and in doing so are selecting a particular point on the linear opportunity set.

The level of s which maximizes the slope of this linear constraint (9), satisfies $(\phi \cdot M + p_s \cdot s)/f(s) = p_s/f'(s)$. The left side and right side of this equality are the firm's average cost and marginal cost, respectively. Thus, all firms choose to operate where the average cost of production is at its minimum. Notice however, that changes in various parameters can affect this level of output per acre. Specifically, changes which induce increases in the price of land will cause the output per acre to rise.

If the price of s and the price of output are assumed to be exogenous to the industry, then only the price of land adjusts to maintain industry equilibrium. The resulting equilibrium price for land is given by an equation quite similar to (3) or (7) found earlier in the simple model.

The CAPM pricing argument requires the return to land to satisfy

$$(\mu_p \cdot f(s) - p_s \cdot s)/M = \phi + \eta \cdot \sigma_p \cdot f(s)/M \tag{10}$$

The return to land now is value of output per acre exlusive of other input costs, and the risk is the standard deviation of output price scaled by the output per acre.

Solving equation (10) for M, leads to a land pricing equation.

$$M = (\mu_p \cdot f(s) - p_s \cdot s - \eta \cdot \sigma_p \cdot f(s))/\phi \qquad (11)$$

Similarly if one assumes the explicit linear mean-variance preference function and requires that the demand for land be equal to a fixed supply, one finds that the price of land satisfies

$$M = ((\mu_p \cdot f(s) - p_s \cdot s)/\phi) - (L_S \cdot (\sigma_p \cdot f(s))^2)/(\phi \cdot (1/\lambda_1 + \ldots + 1/\lambda_n)) \qquad (12)$$

To determine the effect of industry equilibrium on the comparative statics in the firm model, one of these land pricing equations (11) or (12) must be used. Substituting the value for M from (11) into the firm's linear constraint (9) yields an opportunity set for the firm

$$\mu = (1+\phi)(C_o + (\mu_p \cdot f(s) - p_s \cdot s - \eta \cdot \sigma_p \cdot f(s)) \cdot L_o/\phi) + \eta \cdot \sigma \qquad (13)$$

In this linear constraint parameter changes, including changes in the mean or variability of output price, only shift the intercept. When the competitive industry is in equilbrium, the effects of changing parameters the firm faces are capitalized into the price of the input and this affects wealth. As mentioned before, changes in M can alter output per acre through the choice of s.

Wealth effects can alter the choice of L. These effects are represented by a parallel shift in the opportunity set in (σ,μ) space. The effect of this depends on the firms risk aversion characteristics. If the firm is constant absolute risk averse, no change is the optimal L occurs, while under decreasing(increasing) absolute risk aversion the firm increases(reduces) the number of acres it employs. This finding, that assumptions concerning risk aversion matter, is in direct contrast to that of Appelbaum and Katz.

If the solution for M arising from the linear mean-variance utility function is inserted into the firm's first order condition, one obtains $(1+\phi)(C_o + M \cdot L)L_S/(1/\lambda_1 + \ldots + 1/\lambda_n) - \lambda \cdot f(s)^2 \cdot L = 0$. Again, the effect of shifting parameters to the firm occur only through changes in M and are wealth effects to the firm.

VI. Conclusions

The paper extends the theory of the firm under risk by including industry equilibrium considerations. Specifically, the comparative statics are derived for a firm in an industry with a fixed supply of an input. The two approaches used to determine equilibrium prices for the input in fixed supply are the CAPM, and the linear mean-variance model. Both of these models are frequently used as deductive tools in agricultural economics.

The most important result is that many firm level responses to parameter changes which involve substitution effects are altered when industry equilibrium conditions are imposed. Specifically, these parameter changes produce only wealth effects when the industry equilibrium condition is imposed. This conclusion may have a useful application in agricultural policy analysis. Many agricultural programs are supposed to affect farm firms' behavior through substitution effects, but may only produce unanticipated wealth effects.

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