QUANTIFYING LONG RUN AGRICULTURAL RISKS AND EVALUATING FARMER RESPONSES TO RISK

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RISK AVERSION, INPUT USE, AND HETEROSKEDASTIC SUPPLY

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Econometric supply analysis recently has stressed the relation between a firm's technology and its response to prices. Assumptions about technology structure have been tested in the course of supply estimation or combined with sample data to improve estimation efficiency. For example, supplies and input demands often are derived from an indirect profit function using duality theorems. If sufficiently flexible production functional forms are employed, production convexity and other properties may be tested in the course of profit function estimation. The firm's price responses may, of course, be linked to its technology using primal methods as well.

Unfortunately, risk and nonneutral risk preferences greatly complicate the task of developing a correspondence between observed technology and economic choice. Production relations must be expanded to include inputs' effects on the variance and (possibly) higher moments of yield. Numerous higher moment effects may be involved, depending on utility form. More importantly, the firm's objective function cannot be fully specified *a priori* because utility shape is one of the unknowns to be determined. Adequate methods have not yet been found of formulating and restricting a dual objective function in a manner consistent with these unknown preferences (Pope 1982b).

Attention therefore has turned to primal -- but piecemeal -- models of positive economic behavior under risk. The present paper considers afresh the relationship between producer technology and risk preference estimation. It begins with a brief look at earlier efforts in this area, then specializes attention to technologies in which marginal yield moments are independent. We explore implications of this technology class for tests of consistency between production function and input demand parameters and for joint inference of preferences and technology. Applications are provided to Iowa corn production and our estimates compared with those in earlier studies.

Results show that joint estimation of utility and production function parameters is indeed feasible using primal methods. Insofar as it provides more efficient parameter estimates, such inference would be preferred to isolated inspection of a utility or production function. Parameters estimated jointly are in some cases significantly different from those derived from the production function alone.

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Earlier Literature

Estimation of stochastic production functions received a stimulus a decade ago when Just and Pope showed that standard methods of heteroskedasticity correction could be used to draw inferences about input-risk relations (Harvey). A key element of their approach was to assume a normal yield distribution and to permit an input's effect on yield risk to be independent of its effect on yield mean. The procedure involved initially estimating the mean portion of the production surface, fitting the risk portion through a regression of the log absolute residuals, then improving efficiency of the mean estimates through Aitken-type weighting. Buccola and McCarl showed that for small samples these estimates are biased and inefficient and that performance can be improved through repeated iteration.

Nelson and Preckel considered instead a beta yield distribution, which may be substantially skewed. They used maximum likelihood methods to estimate marginal impacts of selected farm inputs on the beta's parameters. Such impacts were used to derive an input's influence on successive yield moments. Although a beta's moments are not independent of one another, some separation was achieved in an input's effects on yield mean and variance by permitting the input to have independent effects on the beta distribution's parameters. A result is that fertilizer use, say, might increase yield mean but reduce yield variance.

A number of studies have estimated risk preference parameters by fitting input demands, constrained by an exogenously estimated production surface, to a sample of economic behavior (Pope 1982a). Wiens, Paris, and Brink and McCarl used mathematical programming frameworks to solve for optimal input allocations subject to restrictions involving absolute aversion. Once the stochastically efficient input combinations were known, Wiens and Paris substituted actual producer input usages into the restrictions to derive an estimate of the average risk aversion coefficient. Brink and McCarl chose the risk aversion coefficient minimizing the distance between actual and programmed allocations.

Antle imposed on a set of input demands prior parameter estimates of his moment-based stochastic production function, then used the constrained demands to fit parameters of the population distribution of risk attitudes. His approach preserves the independence of marginal yield moments implicit in Just and Pope's model; but the yield distribution may be flexibly nonnormal. Both Antle and the authors of mathematical programming models treat absolute risk aversion as essentially constant over wealth.

Loehman and Vandeveer lately have returned to a production specification violating the independence of marginal yield moments. They show that an interactive biological growth model requires choice variables fixed early in a production process to interact with random events occurring later. Marginal mean and marginal variance of yield become interdependent. The authors use prior estimates of the production function to simulate a parameter partially reflecting risk preference. Unfortunately this parameter reflects, in an indistinguishable way, both profit moment and risk preference factors.

The order of more known to less known in the mathematical programming, Antle, and Loehman and Vandeveer studies might well have been reversed. Prior estimates of risk preference factors (from, say, farmer interviews) could have been used together with first order conditions to estimate production function parameters. A difficulty of treating either set of parameters as prior and exogenous to the input demands is that the imposed parameters may be inconsistent with the ones input users actually had in mind. If the imposed values ill-reflect decision maker assumptions, parameters residually estimated also will ill-reflect their assumptions. A way around this dilemma is to estimate preference and technology parameters jointly when the input demands are fitted. Feasibility of doing so depends upon whether all parameters are identified and whether the criterion surface is well behaved.

Issues in Joint Estimation

Prospects for joint estimation are influenced by utility functional form, yield distribution form, and assumptions about the independence of marginal yield moments. For instance, considering marginal yield moments interdependent as in Loehman and Vandeveer implies the general production function form

$$Y = f^*(X)h(W,u) \tag{1}$$

where Y is output, X a vector of nonstochastic inputs, W a vector of stochastic inputs such as rainfall, and u a random error. At planting time, when many inputs X are committed, $h(W|u)$ may be considered part of the risk and the farmer faces simply $Y = f(X)\epsilon$, in which $f(X) = E[f^*(X)h(W,u)]$ and in which $\epsilon = f^*(X)f^{-1}(X)h(W,u)$ has unit mean. Just and Pope's criticism of this form is well known. In addition, to avoid large chances of negative yields, ϵ must have positive skew. An indeterminately large number of marginal yield moments thus enter the input demands and prospects for joint preference-technology estimation fade rapidly.

Enforcing independence between marginal mean and variance implies

$$Y = f_1^*(X) + f_2(W) + h(X)\epsilon. \tag{2}$$

Prior to W's occurrence, (2) has the Just-Pope form

$$Y = f(X) + h(X)\epsilon \tag{2'}$$

where $f(X) = f_1^*(X) + E[f_2(W)]$ and $\epsilon \sim N(0, 1)$. Despite its inconsistency with dynamic growth models and with skewed yield distributions, this form is popular and seems to hold the best potential for mutual inference of preferences and technology. We therefore concentrate on Just-Pope form (2′) and, to permit tractable results, assume negative exponential utility $U(\pi) = -\exp(-\lambda\pi)$ where π is net return and λ is absolute risk aversion. Comparative statics of (2′) in conjunction with nonincreasing risk aversion have been investigated by

Pope and Kramer. Babcock, Chalfant, and Collender used a moment generating function to derive more specific input demands in the case of (2′) and exponential utility.

Optimality Conditions

Optimal input levels for producers facing (2′) are found by

$$\underset{X}{\text{Max}}\ E[U(\pi)] = \underset{X}{\text{Max}}\ EU[Pf(X) + Ph(X) - r'x] \tag{3}$$

where P and r are (assumed known) output price and input price vector, respectively. For the sake of minimizing the number of estimable parameters, we employ multiplicative forms of f and h, which in the two-variable-input case reduce (2′) to

$$Y = AX_1^{a_1}X_2^{a_2} + BX_1^{b_1}X_2^{b_2}\,\epsilon \tag{4}$$

where A, a_1, a_2, B, b_1, b_2 are parameters and $\epsilon \sim N(0, 1)$. Substituting (4) into (3) and deriving first-order conditions gives

$$\begin{aligned} &E\{U'(\pi)[PAa_1X_1^{a_1-1}X_2^{a_2} + PBb_1X_1^{b_1-1}X_2^{b_2}\epsilon - r_1]\} = 0 \\ &E\{U'(\pi)[PAa_2X_1^{a_1}X_2^{a_2-1} + PBb_2X_1^{b_1}X_2^{b_2-1}\epsilon - r_2]\} = 0. \end{aligned} \tag{5}$$

Taking expectations and dividing through by $E[U'(\pi)]$,

$$\begin{aligned} &PAa_1X_1^{a_1-1}X_2^{a_2} + PBb_1X_1^{b_1-1}X_2^{b_2}t = r_1 \\ &PAa_2X_1^{a_1}X_2^{a_2-1} + PBb_2X_1^{b_1}X_2^{b_2-1}t = r_2 \end{aligned} \tag{6}$$

where $t = E[U'(\pi)\epsilon]/E[U'(\pi)]$.

Parameter t is, like Loehman and Vandeveer's γ, a function of both risk and risk aversion. If profit is expressed as $\pi = \mu + \sigma\epsilon$ in which μ is profit's mean and σ is its standard deviation, then under exponential utility t's denominator is $E[U'(\pi)] = \lambda E[\exp(-\lambda\pi)] = \lambda E[\exp(-\lambda\mu - \lambda\sigma\epsilon)]$, that is, λ times the expectation of a lognormally distributed variate with parameters $-\lambda\mu$, $\lambda^2\sigma^2$. As Johnson and Kotz show, the latter expectation is just $E[\exp(-\lambda\mu - \lambda\sigma\epsilon)] = \exp(-\lambda\mu + \lambda^2\sigma^2/2)$. t's numerator similarly may be expressed as $E[U'(\pi)\epsilon] = \lambda E[\exp(-\lambda\mu - \lambda\sigma\epsilon)\epsilon] = [\lambda\exp(-\lambda\mu)]\,E[\exp(-\lambda\sigma\epsilon)\epsilon] = [\lambda\exp(-\lambda\mu)]\,[-\lambda\sigma\exp(\lambda^2\sigma^2/2)] = -\lambda^2\sigma\exp(-\lambda\mu + \lambda^2\sigma^2/2)$.[1] Dividing numerator and denominator gives $t = E[U'(\pi)\epsilon]/E[U'(\pi)] = -\lambda\sigma$. The fact that absolute risk aversion λ appears in (6) only as a multiple of

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[1]The fact that $E[\exp(-\lambda\sigma\epsilon)\epsilon] = -\lambda\sigma\exp(\lambda^2\sigma^2/2)$ can be proven by noting that the moment generating function of ϵ is $m(-\lambda\sigma) = E[\exp(-\lambda\sigma\epsilon)] = \exp(\lambda^2\sigma^2/2)$. Differentiating with respect to $-\lambda\sigma$ gives $m'(-\lambda\sigma) = E[\exp(-\lambda\sigma\epsilon)\epsilon] = -\lambda\sigma\exp(\lambda^2\sigma^2/2)$. An alternative, but lengthier, proof is to write out the density function for ϵ and complete the square in the exponent as Freund does in a related derivation.

profit risk σ implies the two terms never can be separately identified in the input demands without introducing information from production function (6) itself.

From (2′) and (4), profit standard deviation $\sigma = P[Var(Y)]^{\frac{1}{2}} = PBX_1^{b_1}X_2^{b_2}$. Substituting the latter into (6) and combining terms gives

$$PAa_1X_1^{a_1-1}X_2^{a_2} - P^2\lambda B^2b_1X_1^{2b_1-1}X_2^{2b_2} = r_1 \qquad (6'a)$$

$$PAa_2X_1^{a_1}X_2^{a_2-1} - P^2\lambda B^2b_2X_1^{2b_1}X_2^{2b_2-1} = r_2 \qquad (6'b)$$

which differ from the Babcock-Chalfant-Collender input demands only in suppressing acreage terms. Inasmuch as $\lambda\sigma^2/2 = P^2\lambda B^2X_1^{2b_1}X_2^{2b_2}$ is the producer's risk premium, marginal risk premia (MRP) are $\delta RP/\delta X_1 = P^2\lambda B^2b_1X_1^{2b_1-1}X_2^{2b_2}$, precisely the second RHS terms in (6′). Pope and Kramer and MacMinn and Holtmann show that the risk averter's optimal use of both inputs exceeds (falls short of) that in the riskless or risk neutral case if the inputs are complementary and if $\delta h/\delta X_1$, $\delta h/\delta X_2 < 0$ (> 0). For production form (4), these conditions are satisfied if b_1, $b_2 < 0$ (> 0), that is if yield is heteroskedastic such that yield variance decreases (increases) in the inputs. Clearly input demands need not undergo parallel shifts when λ changes; in fact, the demands may become positively sloped if risk aversion is very high (Pope and Kramer, pp. 495-6).[2]

Input Demand-Production System

All parameters in (6′) are in principle separately identifiable. Feasibility of estimating any parameter, including product λB^2, on the basis of just (6′) depends on the associated error structure. Unfortunately, no error structure in the first order conditions can be determined from (4). A complicating factor is that no closed-form expression for the optimal inputs exists. Solving (6′) as far as possible for the optimal variable factors gives

$$\begin{aligned} X_1 &= [r_1/(PAa_1X_2^{a_2} - P^2\lambda B^2b_1X_1^{2b_1-a_1}X_2^{2b_2})]^{1/(a_1-1)} \\ X_2 &= [r_2/(PAa_2X_1^{a_1} - P^2\lambda B^2b_2X_1^{2b_1}X_2^{2b_2-a_2})]^{1/(a_2-1)} \end{aligned} \qquad (7)$$

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[2]Pope and Kramer (pp. 492-3) argue that second-order conditions corresponding to (2′) are satisfied given risk aversion and concavity of $Y = f(X) + h(X)\epsilon$. However, their assumption that $E[U''(\delta\pi/\delta X_i)^2] < 0$ for risk averters appears incorrectly to suppose that $\delta\pi/\delta X_i$ is nonstochastic. Our own formulation of the Hessian for (6′) shows second order conditions could be violated if the b_i are large and prices are inappropriately scaled.

so that optimal factors only can be determined simultaneously. Heuristically, the reason for the simultaneity is that relative factor levels affect yield variance differently from the way they affect yield mean and each effect must be taken into account when determining the optimal allocation. All information about the optimal level of a given factor cannot be determined from the price levels alone.

Simultaneity in (7) makes clear that, even if optimization errors are specified additively, each error generally is correlated with both input choices. Rather than arbitrarily add errors to (7), we suppose that optimization mistakes occur in the form of random failure to satisfy first order conditions (6′). Assuming choices are optimal on average, errors v_1, v_2 are added to (6′a) and (6′b) such that $v_1 \sim N(0, s_1^2)$, $v_2 \sim N(0, s_2^2)$.

There are several reasons why production function (4) should be estimated as a system along with (6′). First, this would allow one to test whether technology parameters in (6′) differ from those in (4), that is, whether parameters which producers assume in the course of optimization differ from those in the input-output relation itself. Second, combining (4) with (6′) includes information about actual yield levels and so potentially improves estimation efficiency over the use of (6′) alone. Third, including (4) is necessary -- and sufficient -- for separately identifying λ and B^2 in (6′).

To make clear how (4) may be used for these purposes, observe that the heteroskedasticity in its error can be removed by dividing each term by $X_1^{b_1}X_2^{b_2}$. The result is

$$YX_1^{-b_1}X_2^{-b_2} - AX_1^{a_1-b_1}X_2^{a_2-b_2} = B\epsilon \tag{4'}$$

with homoskedastic error $B\epsilon \sim N(0,B^2)$. Equations (4′) and (6′) now may be fitted jointly once an error covariance structure for the errors $(v_1, v_2, B\epsilon)$ has been specified. Tests of the implied restrictions that the a_i, b_i are not different across equations then may be conducted. If the null hypothesis is not rejected, combining (4′) with (6′) improves efficiency. Relation (4′) also serves to distinguish risk aversion coefficient λ from risk parameter B^2. Substituting into (4′) consistent estimates of A, a_i, b_i that were derived from simultaneously fitting (4′), (6′) gives $B\epsilon^*$, a consistent estimate of the sample production errors. Log absolute value of $B\epsilon^*$ is $\ln|B\epsilon^*| = \ln B + \ln|\epsilon^*|$, the expectation of which is $\ln B + E \ln|\epsilon^*| = \ln B - 0.6352$. Adding 0.6352 and exponentiating gives a consistent estimate of B, which when squared and divided into the consistent estimate of λB^2 from (4′), (6′) gives a consistent estimate of absolute risk aversion λ.

It is useful to note that, inasmuch as inputs X_i in (4′) are endogenous, fitting (4′) along with (6′) equivalently estimates the risk averse firm's per-acre supply function

$$Y^* = f(X^*) + h(X^*)\epsilon \tag{8}$$

where X^* are the optimal input levels. Since, through (6′), X^* depends on prices P and r, supply must be heteroskedastic in the prices whenever yield is heteroskedastic in inputs.

Procedures

In this paper, we fit a multivariate extension of system (4′), (6′) to corn-soybean farm data in Iowa. Production function estimates are compared with (a) those derived with the method of Just and Pope and (b) those estimated by Nelson and Preckel with the use of the conditional beta distribution. We contrast risk averse with risk neutral supply and input demands and discuss implications for supply heteroskedasticity.

The data include farm-level information on per-acre corn yield; nitrogen, phosphorus, and potassium application; soil slope and clay content; and two dummy variables indicating respectively whether a legume preceded the corn crop by one or two years. Information from 1964 through 1969 was taken from farms in Fayette, Linn, and Muscatine County, Iowa.[3] Iowa input prices were drawn from annual issues of USDA's Agricultural Prices and Agricultural Resources: Situation and Outlook Report. Corn price was the average of high and low March closing prices of the Chicago corn futures contract maturing the following September. All prices were deflated with the CPI.

Nitrogen, phosphorus, and potassium were assumed to be the only purchased inputs, so equations (6′) were extended to accommodate three input demands. Soil slope and clay and the two dummy variables were included as exogenous factors, along with prices P, r, in all equations. These variables along with the expected price of soybeans were used as instrumental variables for first stage estimation. Expected soybean price was calculated to parallel the expected corn price variable. This extended version of (4′), (6′) was estimated with nonlinear 3SLS combining the indicated cross-equation restrictions on parameters and holding λB^2 fixed at trial values. (TSP Version 4.1B). A grid search then was conducted on alternative λB^2 values and the parameter set selected which minimized system sum square errors. Substituting optimal A, a_i, b_i estimates back into (4′) gave consistent estimates of $|B\epsilon|$, the adjusted mean log of which provided a consistent estimate of B and, through the NL3SLS estimate of λB^2, a consistent estimate of λ. The NL3SLS estimator provides consistent and sometimes asymptotically efficient estimates of the parameters. Further, it is generally more robust against nonnormality of errors than are maximum likelihood estimators (Jorgenson and Laffont, Gallant, Amemiya).

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[3]These data, collected by the Iowa Agricultural Experiment Station, are described in more detail in Nelson and Preckel. We wish to thank Carl Nelson for kindly furnishing the data to us.

Just-Pope-type estimates of (4) were derived by applying NLS to obtain stage 1 estimates of A and the a_i's. Logs of absolute values of residuals $u = BX_1^{b_1}X_2^{b_2}\ldots\epsilon$ then were regressed against $\ln|BX_1^{b_1}X_2^{b_2}\ldots\epsilon|$ to derive stage 1 estimates of B and the b_i's. A stage 2 estimate of A and a_i was obtained by applying NLS to the weighted regression (4′) and a stage 2 estimate of B and b_i was derived by repeating the log linear regression on residuals.[4] A Just-Pope-type routine provides not only a point of comparison for the NL3SLS system estimates, but a useful set of starting values for these estimates as well.

Results

Results of the systems estimates for Linn, Muscatine, and Fayette Counties are shown in the right column of tables 1-3, alongside the Nelson-Preckel and Just-Pope-type estimates in the left and center columns. Dummies signifying previous legume crops were not significant in any of the regressions and were dropped. A wide range of values for λB^2 were tried for each county (Appendix figure). In all cases, the optimal value of λB^2 fell in the positive range, although for Fayette County this might have been only a local minimum. NL3SLS estimates of the technology parameters, A, a_i and b_i were only modestly sensitive to parameterized λB^2 levels.

Parameter Estimates

The system estimates suggest that in each county one fertilizer type or another most significantly affects mean corn yield. In Linn County, phosphorus has the greatest influence on mean yield, followed by potassium. In Muscatine County, nitrogen is the most influential, while in Fayette County the dominant positive effect is from phosphorus, followed closely by nitrogen. Interestingly, in Fayette County potassium appears to have a fairly large negative effect on mean yield. A ten percent increase in potassium application results in a 24 percent decrease in expected yield in that county. Without exception, the effect on mean yield of soil slope is small. Clay content appears to play a more significant role in determining acreage yield, with an elasticity of 0.21 for Linn County and 0.30 for Muscatine. Clay does not significantly affect mean yield in Fayette County.

Mean effects estimated from the primal systems closely resemble those from Just-Pope-type estimation. A notable exception occurs in Fayette County, where all three estimators produce fairly divergent results. In general, the system and Just-Pope estimators suggest more

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[4]With small samples, this last log-linear regression likely improves bias in B estimates and efficiency in a_i estimates (Buccola and McCarl, pp. 735-7). Ln (B) also was adjusted by +0.6352 to account for residual asymptotic bias resulting from the fact that $E|\epsilon| = -0.6352$ in each log-linear regression.

Table 1. Elasticity of Corn Yield Mean and Standard Deviation with Respect to Selected Inputs, Linn County, Iowa, 1964-1969.[a/]

Input	Conditional Beta (Nelson and Preckel)[b/]	Just-Pope Method	Primal System
		- - Yield Mean - -	
A[c/]		48.85 (3.89)	40.36 (4.19)
Nitrogen (N)	0.46	0.05 (2.40)	0.02 (1.41)
Phosphorus (P)	-0.35	-0.01 (-1.61)	0.07 (2.05)
Potassium (K)	0.13	0.05 (0.91)	0.03 (1.90)
Slope	-0.06	0.04 (1.14)	0.07 (1.97)
Clay	0.99	0.18 (2.87)	0.21 (4.20)
		- - Yield Standard Deviation - -	
B[c/]		4.87	0.21
Nitrogen (N)	0.29	0.06 (0.59)	0.09 (1.64)
Phosphorus (P)	0.19	0.56 (1.71)	0.80 (6.67)
Potassium (K)	-0.22	-0.22 (-0.84)	0.35 (4.12)

a/ Numbers in parentheses are t-values. Sample size is 103.

b/ Values for the mean were derived from Nelson and Preckel's table 3. Values for the standard deviation are one-half of Nelson and Preckel's table 5 figures.

c/ Yields are measured in bushels per acre, fertilizers in lbs per acre.
Sample means were N = 83.1, P = 50.1, K = 48.8, Slope = 3.3, Clay = 22.1, Yield = 122.7.

Table 2. Elasticity of Corn Yield Mean and Standard Deviation with Respect to Selected Inputs, Muscatine County, Iowa, 1964-1969.[a/]

Input	Conditional Beta (Nelson and Preckel)[b/]	Just-Pope Method	Primal System
		- - Yield Mean - -	
A[c/]		46.90 (3.54)	26.32 (2.97)
Nitrogen (N)	0.38	0.10 (2.73)	0.12 (2.68)
Phosphorus (P)	0.10	-0.0003 (-0.003)	0.05 (0.56)
Potassium (K)	-0.03	0.01 (0.164)	0.01 (0.24)
Slope	-0.27	-0.09 (-3.41)	-0.06 (-1.40)
Clay	1.22	0.20 (2.63)	0.30 (3.18)
		- - Yield Standard Deviation - -	
B[c/]		17.40 (2.36)	1.35
Nitrogen (N)	-0.22	-0.26 (-2.01)	0.44 (5.33)
Phosphorus (P)	0.30	-0.16 (-0.36)	0.15 (1.70)
Potassium (K)	0.04	0.56 (1.83)	0.01 (0.48)

a/ Numbers in parentheses are t-values. Sample size is 55.

b/ Values for the mean were derived from Nelson and Preckel's table 3. Values for the standard deviation are one-half of Nelson and Preckel's table 5 figures.

c/ Yields are measured in bushels per acre, fertilizers in lbs per acre.
Sample means were N = 98.5, P = 43.5, K = 34.2, Slope = 3.5, Clay = 23.4, Yield = 126.2.

Table 3. Elasticity of Corn Yield Mean and Standard Deviation with Respect to Selected Inputs, Fayette County, Iowa, 1964-1969.[a]

Input	Conditional Beta (Nelson and Preckel)[b]	Just-Pope Method	Primal System
	- - Yield Mean - -		
A[c]		43.32 (3.45)	39.78 (1.89)
Nitrogen (N)	0.93	0.10 (3.70)	0.23 (3.65)
Phosphorus (P)	0.12	-0.07 (-1.14)	0.27 (1.92)
Potassium (K)	0.05	0.07 (1.66)	-0.24 (-1.19)
Slope	0.02	-0.03 (-1.21)	0.07 (1.56)
Clay	0.12	0.20 (2.40)	0.01 (0.14)
	- - Yield Standard Deviation - -		
B[c]		61.97	2.03
Nitrogen (N)	0.04	-0.16 (-1.40)	0.41 (3.38)
Phosphorus (P)	0.19	-0.86 (-2.56)	0.23 (2.36)
Potassium (K)	-0.10	0.72 (2.64)	0.06 (0.36)

a/ Numbers in parentheses are t-values. Sample size is 106.
b/ Values for the mean were derived from Nelson and Preckel's table 3. Values for the standard deviation are one-half of Nelson and Preckel's table 5 figures.
c/ Yields are measured in bushels per acre, fertilizers in lbs per acre.
Sample means were N = 76.7, P = 52.1, K = 57.5, Slope = 4.2, Clay = 21.4, Yield = 113.3.

sharply diminishing expected returns to scale than do the parameter estimates from Nelson and Preckel's conditional beta.

The greatest differences among estimators is in their variance effects. In some cases the system estimator produces results contrary to both Nelson-Preckel and Just-Pope. For Linn County, Nelson-Preckel and Just-Pope-type estimates indicate that potassium has a negative effect on yield risk while the system estimator indicates it has a substantial positive effect. For Muscatine County, the system estimate implies, contrary to the Nelson-Preckel and Just-Pope models, that nitrogen application increases yield risk. On the other hand, our model agrees with Nelson and Preckel's in showing a modest positive yield risk influence for phosphate and potassium in Muscatine County. In Linn County, our model most closely agrees with Just and Pope's in showing a positive yield risk influence for nitrogen and phosphate. Like the estimated mean yield effects, estimated variance effects for Fayette County vary widely across the three estimators.

Two tests were performed to determine significance of differences between the Just-Pope estimates and those obtained using NL3SLS. The null hypothesis that the mean-yield elasticities obtained from the two estimators are the same was rejected at the 0.05 level for all counties. Calculated test statistics were 1199.91, 28.51, and 31.81 for Linn, Muscatine, and Fayette County, respectively, compared to a tabled Chi-square value of 7.81 for a 5% test level with three degrees of freedom. The null hypothesis that variance-of-yield elasticities were the same between the two estimators also was rejected at the 0.05 level for all counties. Calculated statistics were 1365.24, 286.55, and 9547.33 for the three counties compared to a tabled Chi-square value of 12.59 in a 5% test level with six degrees of freedom. It should be noted, however, that the absolute magnitudes of the differences in mean-yield effects are small, while the absolute magnitudes of the differences in the variance-of-yield effects are large.

Substituting the elasticities in tables 1-3, column (3), back into (4′) and adding 0.6352 to the mean log of $|B\epsilon|$ gave B estimates of .20884, 1.35260, and 2.03086 for Linn, Muscatine, and Fayette County, respectively. Combining these consistent estimates of B with our estimates of λB^2 from NL3SLS allowed us to calculate coefficients of absolute risk aversion, λ, for each county. Our estimates of λ are 0.016 for Linn, 0.538 for Muscatine, and 0.140 for Fayette County (1967 dollars per acre basis). This is consistent with the common assumption that producers tend to be risk averse.

Per-Acre Supply and Input Demands

System estimates of Linn County's production function and risk aversion parameters were substituted into (6′) and per-acre fertilizer demand quantities calculated at alternative price levels. Fertilizer quantities demanded then were substituted into (8) along with parameter estimates to determine, for each price combination, means and standard deviations of per-acre corn quantity supplied. Since (6′) cannot be expressed in explicit form, MathCAD 2.0 was used to find iterative solution values for the optimal inputs given a set of starting values. Responses to nitrogen price changes are shown in table 4 and responses to expected corn price changes in table 5. Each table shows the optimal responses assuming (a) risk neutrality, and (b) the estimated absolute risk aversion of 0.016.

The risk-neutral nitrogen demand function (figure 1) is negatively sloped and strongly elastic (-2.02 near sample price means). Under risk aversion the demand shifts left and steepens, although elasticity at sample price means remains about the same. Despite appearances of both curves, elasticity is lower at low nitrogen prices than at high prices. Both demand curves pass to the left of the sample mean nitrogen use rate of 83 lbs per acre.

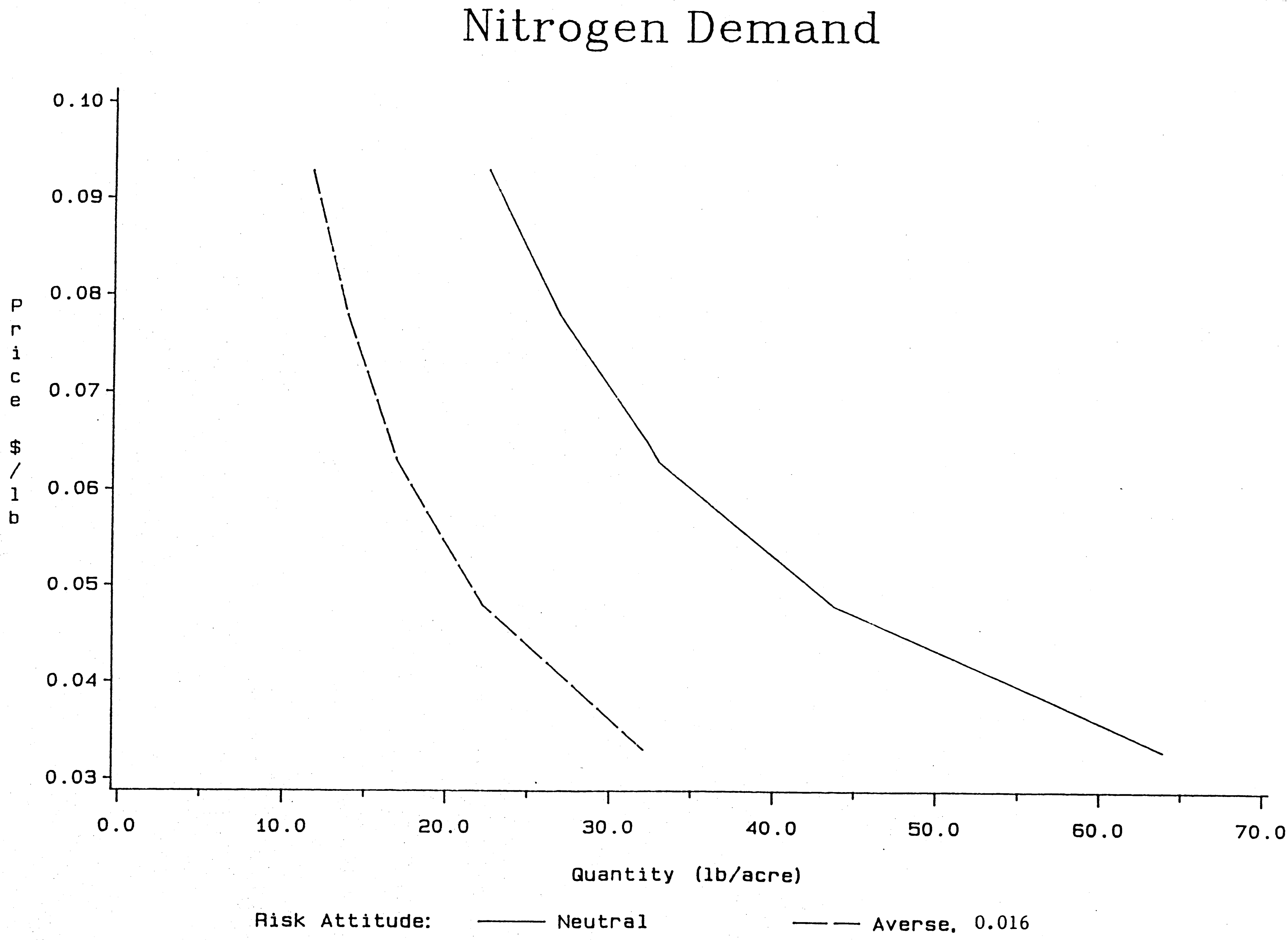
Phosphate and potash demand (figures 2 and 3) follow the same general pattern as that of the nitrogen demand. The risk averters' demand is steeper and to the left of the risk neutral demand. With these two fertilizers, however, risk neutral demands are well above sample mean use rates (50.1 lbs per acre for phosphorus and 48.8 lbs per acre for potash) whereas the risk averse demands approximate these use levels quite well. Inelasticity of risk averters' phosphorus and potash demands follows from the rather high coefficients on P and K in the risk portion of Linn County's production function (table 1). Increased fertilizer use in response to fertilizer price declines is mitigated by the positive effect of fertilizer use on yield risk.

For both risk neutral and risk averse farmers, nitrogen use responds positively to increases in the expected corn price (figure 4). The risk averters' demand again is lower than that of the risk neutral farmer. Demand also is less elastic: risk neutral elasticity is roughly constant near 1.00, while the risk averter's elasticity is about 0.48 near the sample mean. The steeper slope of the risk averter's response reflects the fact that profit risk rises with corn price increases. Inasmuch as nitrogen use moderately increases risk as well, the farmer optimally limits additional nitrogen use more than he would if he were risk neutral.

As one would expect from the relatively large marginal risk effects of phosphorus and potassium, demands for these fertilizers are even less positively responsive to corn price than is the demand for nitrogen (figures 5 and 6). Phosphate demand is virtually inelastic and potash demand responds slightly <u>negatively</u> to corn price increases. Possible backbending of the input demand-output price relationship is one reason why comparative statics of the risk averse firm are so difficult to generalize (Pope and Kramer).

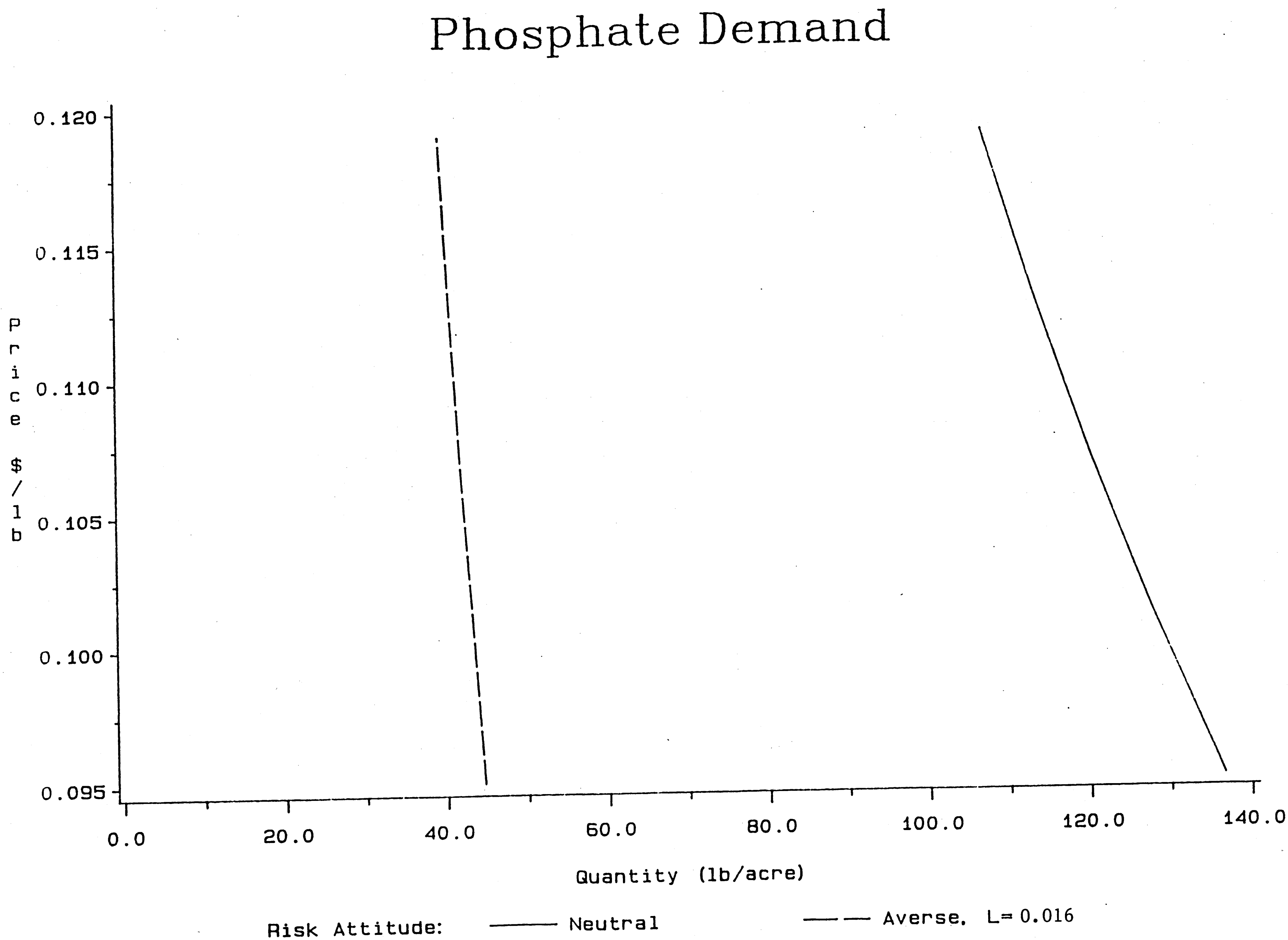
Tables 4 and 5 make clear that, for both risk averse and risk neutral farmers, expected yield is quite insensitive to fertilizer or corn price. Part of this insensitivity results from some net substitutability in fertilizers. That is, phosphorus and potassium demand rises slightly with increased nitrogen prices. But most of the insensitivity comes from the low coefficients on N, P, and K in the expected-yield portion of Linn County's production function, in turn probably reflecting high fertilizer use rates in that county.

Figure 1



Nitrogen Demand
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
Price $/lb
0.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
Quantity (lb/acre)
Risk Attitude: Neutral Averse, 0.016

Figure 2



Phosphate Demand
0.120
0.115
0.110
0.105
0.100
0.095
Price $/lb
0.0
20.0
40.0
60.0
80.0
100.0
120.0
140.0
Quantity (lb/acre)
Risk Attitude:
Neutral
Averse, L= 0.016

Figure 3

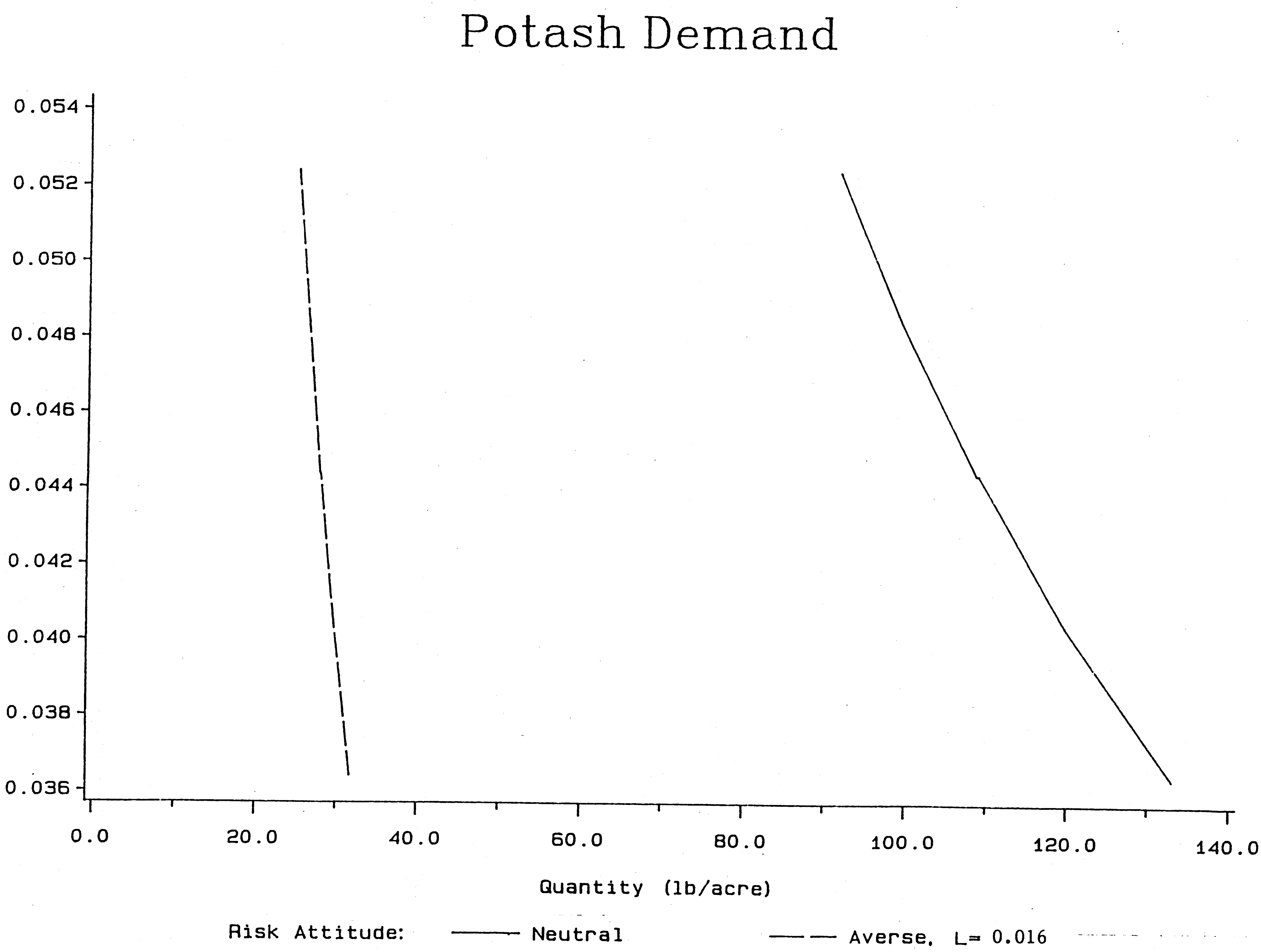

Potash Demand
0.054
0.052
0.050
0.048
0.046
0.044
0.042
0.040
0.038
0.036
Price $/lb
0.0
20.0
40.0
60.0
80.0
100.0
120.0
140.0
Quantity (lb/acre)
Risk Attitude: Neutral
Averse, L= 0.016

Figure 4

Change in Nitrogen Use in Response to Corn Price

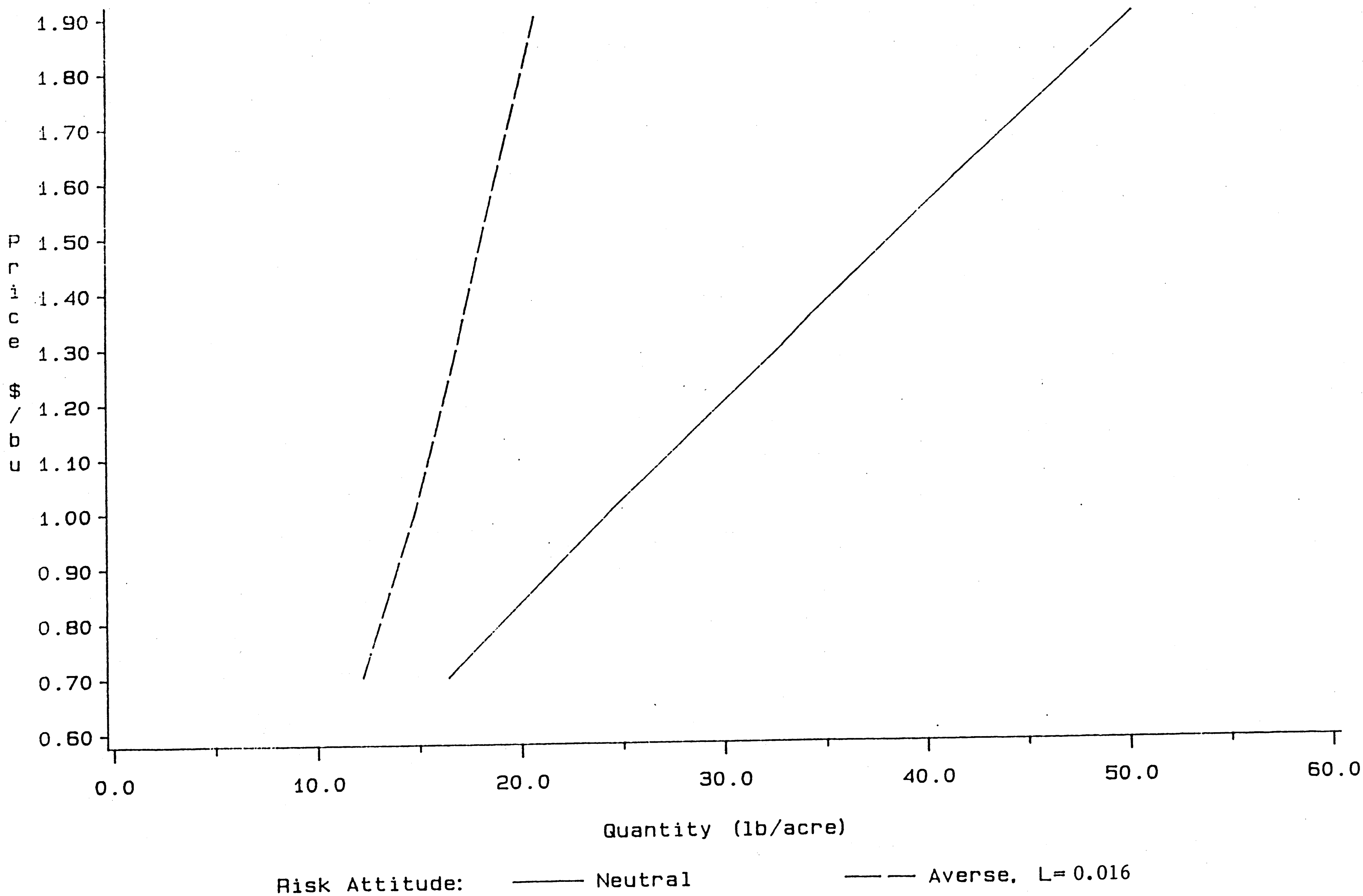


Figure 5

Change in Phosphate Use in Response to Corn Price

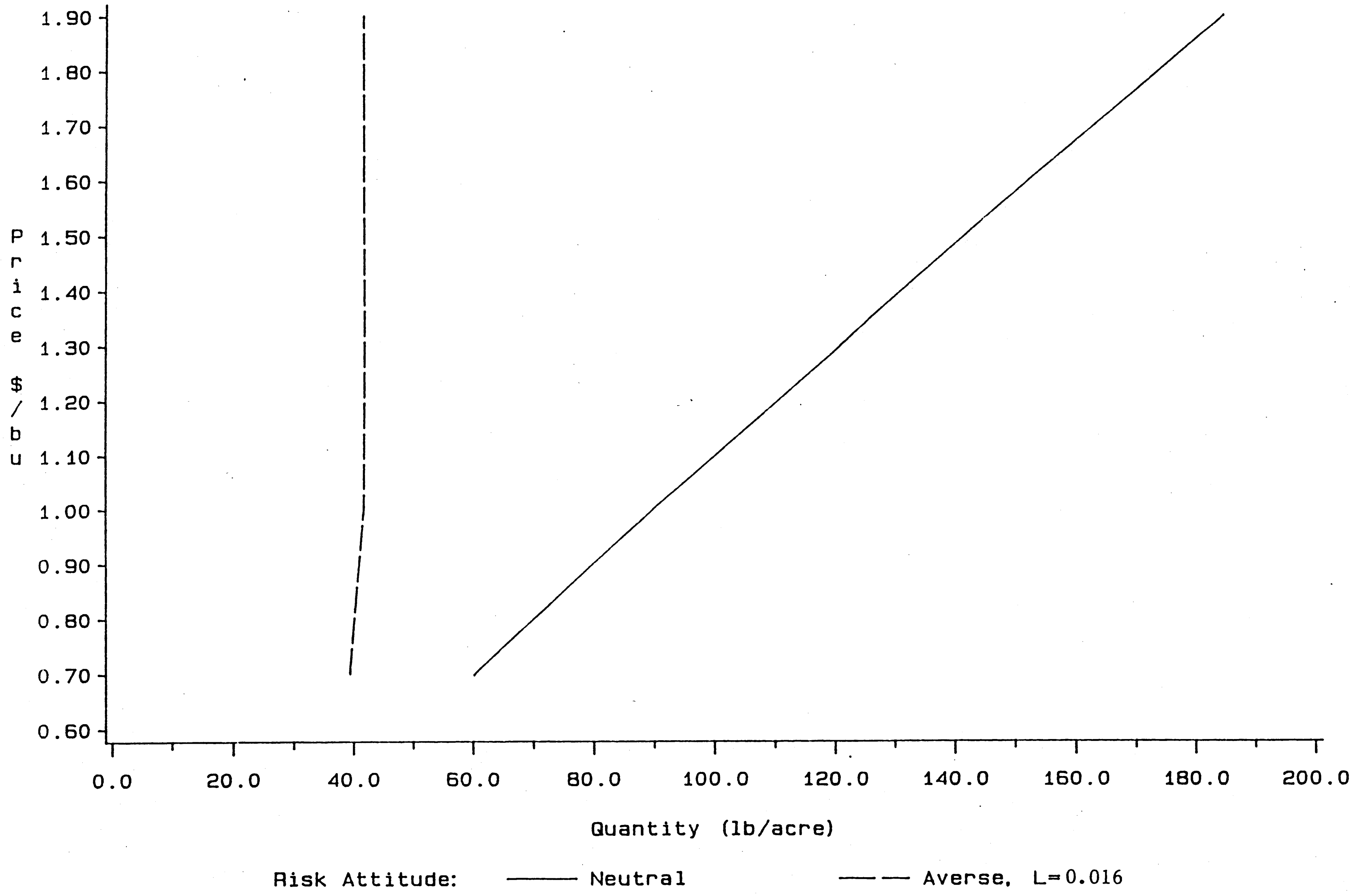


Figure 6

Change in Potash Use in Response to Corn Price

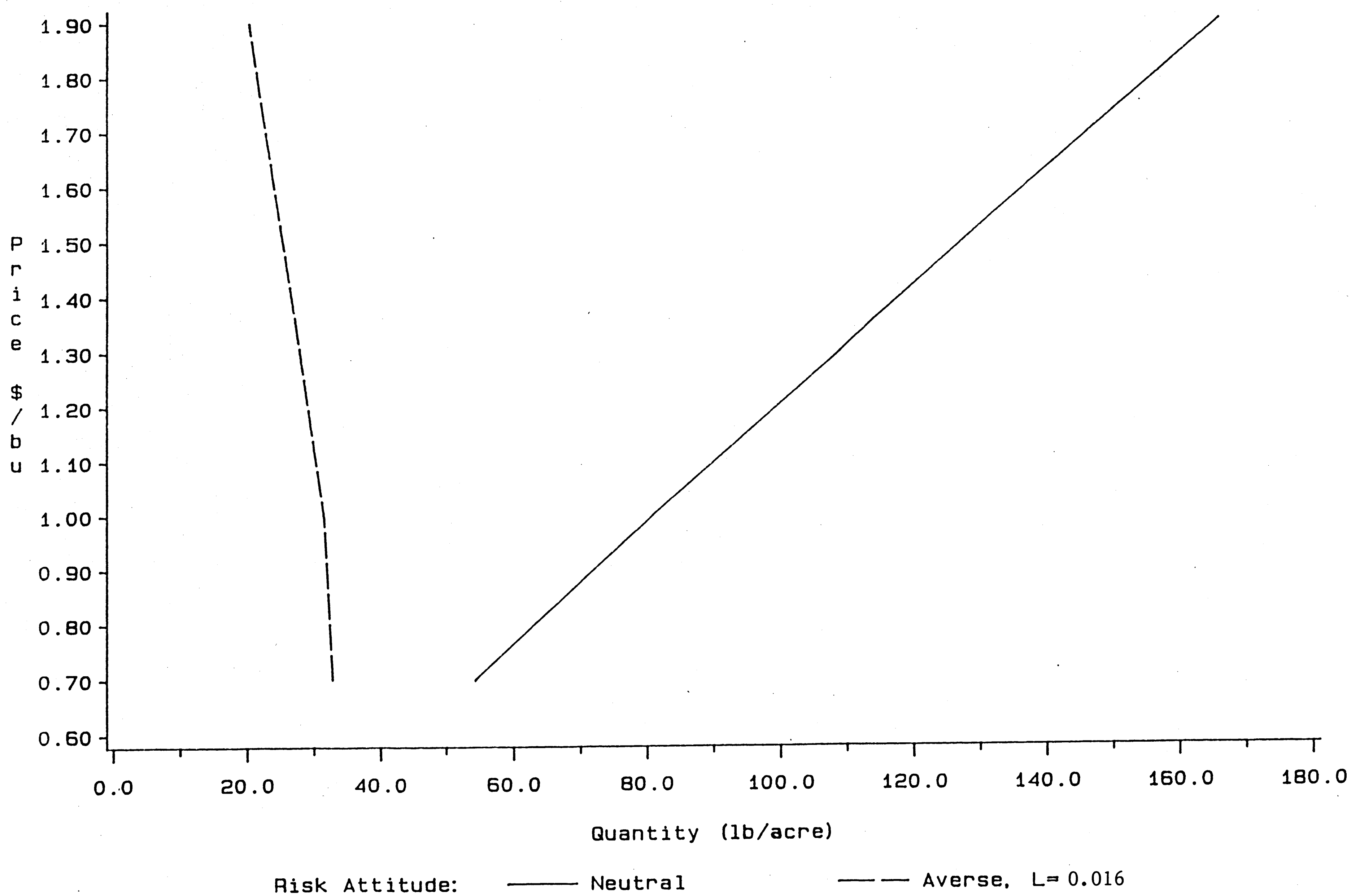


Table 4. Nitrogen Demand and Expected Per-Acre Corn Yield at Alternative Nitrogen Prices.[a/]

Nitrogen Price	Nitrogen Demanded[b/]		Expected Corn Supply[c/]	
(1967 cents/lb)	$\lambda = 0$	$\lambda = 0.016$	$\lambda = 0$	$\lambda = 0.016$
3.3	63.0	32.1	139.9	123.5
4.8	43.7	22.2	139.2	123.2
6.5	32.2	16.5	138.6	123.0
7.0	26.7	13.8	138.3	122.9
9.3	22.4	11.6	138.0	122.8

a/ Corn price, phosphorus price, and potassium price are held fixed at $1.30 per bushel, 10.6 cents per lb, and 4.4 cents per lb, respectively (1967 dollar basis).

b/ Nitrogen figures are in lbs per acre.

c/ Corn supply figures are in bushels per acre.

Table 5. Nitrogen, Phosphorus, and Potassium Demanded and Expected Per-Acre Corn Supply at Alternative Corn Prices.[a/]

Corn Price (1967 $/bushel)	Nitrogen Demanded[b/]		Phosphorus Demanded[b/]		Potassium Demanded[b/]		Expected Corn Supply[c/]	
	$\lambda = 0$	$\lambda = 0.016$	$\lambda = 0$	$\lambda = 0.016$	$\lambda = 0$	$\lambda = 0.016$	$\lambda = 0$	$\lambda = 0.016$
0.70	16.4	12.2	60.1	39.3	54.2	32.7	128.5	122.6
1.00	24.4	14.8	89.7	41.7	80.8	31.4	134.4	123.2
1.30	34.2	17.2	125.6	41.8	113.2	27.2	139.3	123.8
1.60	41.4	18.8	152.0	41.6	137.0	23.9	142.2	122.6
1.90	50.3	20.8	184.4	41.5	166.2	20.2	145.2	122.2

a/ Nitrogen, phosphorus, and potassium prices are held fixed at 6.4 cents, 10.6 cents and 4.4 cents per lb, respectively (1967 dollar basis).

b/ Fertilizer demanded is in lbs per acre.

c/ Corn supply figures are in bushels per acre.

Figure 7 gives expected per-acre corn supply at various levels of expected corn price. The risk neutral grower's somewhat elastic fertilizer demands result in a slight increase in per-acre expected supply as corn price rises. In contrast, the risk averse farmers' somewhat inelastic fertilizer demands combine with low mean-yield coefficients to produce relatively inelastic corn price-corn supply relations. The risk averter's per-acre supply, in fact, backbends slightly, reflecting greater risk exposure at higher corn prices and positive marginal risk premia for all fertilizers. Just and Zilberman also have shown that negative supply elasticities reasonably can occur under constant absolute risk aversion, even when partial risk aversion is as low as 0.60. Partial risk aversion for Linn County is 2.34 at sample means.

The dotted lines in figure 7 indicate the extent to which yield heteroskedasticity in inputs leads to supply heteroskedasticity in price. (Inner dotted lines signify one-standard-deviation confidence intervals for the risk averse farmer and outer dotted lines signify one-standard-deviation intervals for a risk-neutral farmer.) Risk averters' steep input demands mean that corn price changes will little affect yield risk. For these farmers, yield variance is slightly lower at high corn prices than at low corn prices because input use is slightly lower at these prices as well. Risk neutrality, on the other hand, encourages greater input demand response and hence a per-acre supply variability that responds more strongly to output prices.

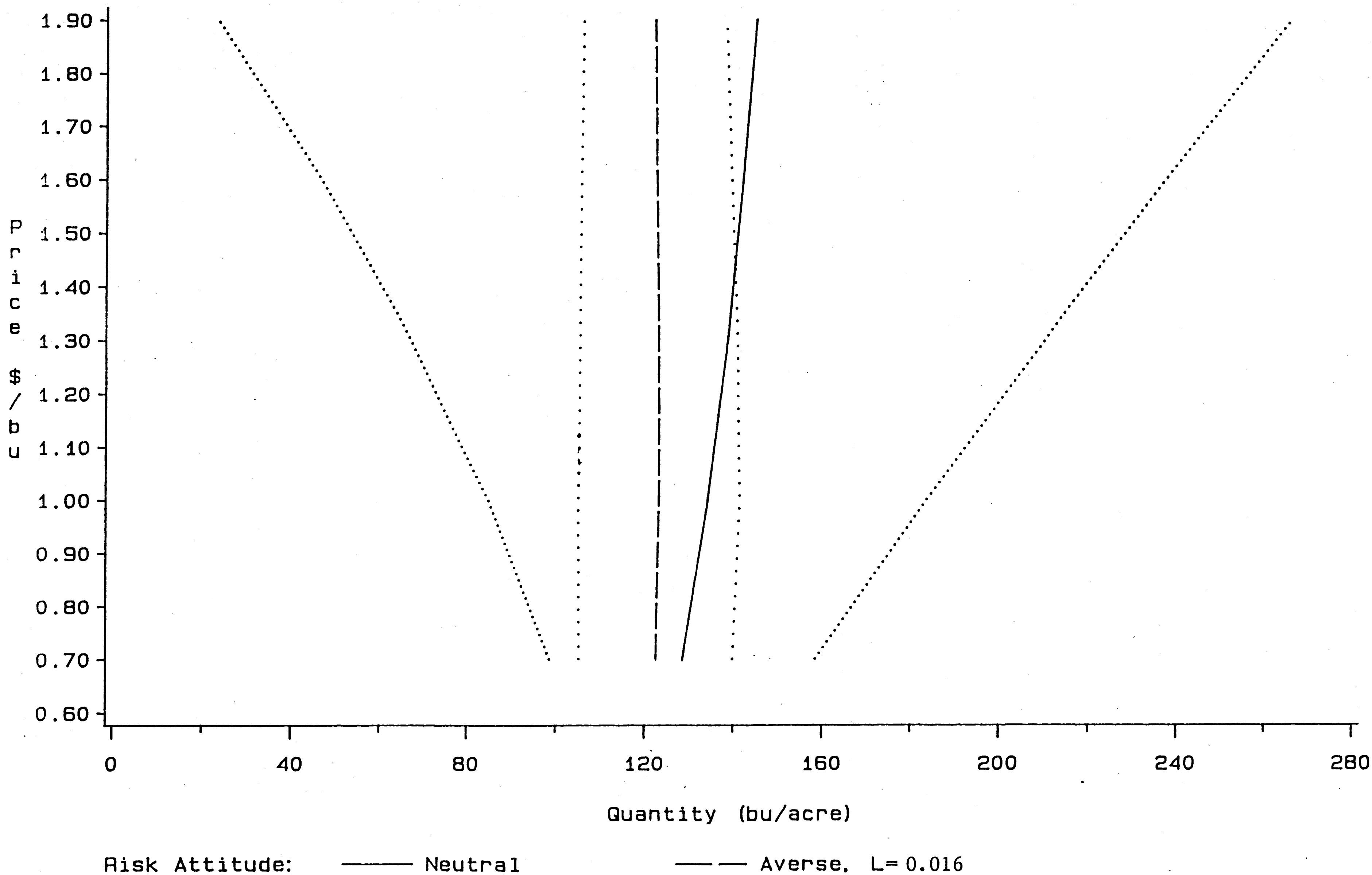
Conclusions

Adding risk and risk aversion to a farmer's input choice problem greatly complicates economic analysis. Applied economists' response to the increased complexity has been to separate risk measurement from risk preference assessment. Such separation raises practical difficulties about sample comparability and fails to take maximum advantage of available information. We have shown that a primal system of input demands and the production function may indeed be specified through which risk and risk preference can jointly be estimated. Assumptions employed for this purpose -- that production be additive in mean and variance and that risk aversion be constant in income -- are not more restrictive than used in many other applied studies.

An advantage of the system approach is that it enables one to assess farmers' revealed opinions about production function relationships, including inputs' marginal impacts on yield risk. Further, it can be used to derive utility or risk preference estimates that are mutually consistent with these revealed opinions. The approach is not without its cost. Extending such a system to a case of nonnormally distributed errors or nonconstant risk aversion would not be straightforward. Incorporating nonnormal distributions into a system context probably would require using a particular nonnormal family along the lines of Nelson and Preckel.

Figure 7

Change in Expected Yield in Response to Corn Price



Elasticities of yield mean estimated with the system approach closely approximated those derived with a Just-Pope-type estimator. However, elasticities with respect to yield variance often differed substantially from the Just-Pope-type estimates, falling in several cases closer to the Nelson-Preckel figures. Our estimate of Linn County farmers' mean risk aversion is moderate, although corresponding estimates for Fayette and Muscatine Counties appeared unrealistically high. Our production function specification may be improvable. We plan to test the model with additional data in Iowa and Oregon, to investigate the use of linear-quadratic rather than multiplicative functional forms, and to include weather data both as regressors and as instruments.

An important lesson from this research is that commonplace elasticities of yield mean and variance easily lead to relatively inelastic per-acre supplies and input demands even when risk aversion is moderate. This does not imply that total supply and input demand would be inelastic as well, since acreage may remain significantly elastic. In any event, the often-mentioned farm practice of maintaining fixed input proportions over time may well be a rational response to positive marginal risks rather than a demonstration of rule-of-thumb behavior.

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Appendix Figure