Optimality or Sustainability

Presenter: Geoffrey Heal
Graduate School of Business
Columbia University

Discussant: Y.Hossein Farzin
Department of Agricultural and Resource Economics
University of California-Davis

American Economic Association Meetings
Washington, D.C., January 3, 2003

As should be expected, Geoff's paper is insightful and thought provoking. I very much enjoyed reading it. He picks up the sensitive and hotly debated issue of "sustainability" and throws considerable light on the issue by using some simple but intellectually powerful economic models.

Geoff concentrates on the important question: "Are optimal paths sustainable?" and explores the question by considering the optimal paths of the economy for various welfare criteria [*Discounted Utilitarian* (Koopmans), the *Green Golden Rule* (Beltratti, Chichilnisky, and Heal), The *Maximin* (Rawls), and *Overtaking* (Von-Weizacker)] and under different resource constraints (exhaustible and renewable). He comes out with the overall conclusion that "Optimal paths are sustainable provided that preferences and constraints reflect what we know about human society's dependence on environmental system". On this basis he argues that " sustainability is not a separate goal from optimality: rather optimality is a refined form of sustainability". Thus, he urges environmental economists to refine the concept of optimality generally used and ensure that it incorporates the ecological constraints imposed by the environmental system. Now, it is hard to disagree with Geoff's conclusion that a "real" or "full", or "informed" optimal policy should incorporate all the relevant ecological and biogeochemical constraints and that once this is done carefully such an optimal policy may turn out not to be so much in conflict with a goal of sustainability. However, *I believe that the conflict between, or possible coincidence of, the goals of optimality and sustainability depends more fundamentally on the specification of social preferences and definition of sustainability itself than on how well natural resource constraints are incorporated in calculating an optimal path.* I would like to illustrate my view by focusing on the case of the exhaustible resource economy considered in the first part of Geoff's paper, although I believe my argument holds true also for the case of an economy with renewable resources.

In fact, the Geoff's conclusion that "optimality is a refined form of sustainability" hinges on his specification of social preferences where social utility is assumed to depend not only on flow of resource consumption but also directly on the *size of the remaining stock*, i.e., $u(c_t, S_t)$. As he shows, when utility function is assumed to depend only on the

resource consumption flow, $u(c_t)$, there is *no sustainability,* neither in the sense of a constant consumption and hence utility flow nor in the sense of conserving some positive level of the stock forever. In fact, under his assumption that $\lim_{c\to 0} u'(c) < \infty$, the optimal policy is to consume exponentially declining rates of the resource and exhaust the entire resource stock over a *finite* period. In sharp contrast to this, when utility function is assumed to depend both on the resource flow and the remaining stock, the optimal policy, if it exists at all, calls for consumption to cease altogether after certain future date so that some positive level of the resource stock is conserved forever ($c_t = 0, S_t = S^* > 0, \quad \forall t \geq T$).

Further more, under the utility function Geoff uses, the existence of this optimal policy is by no means guaranteed. It requires a number of restrictive assumptions that question the generality of such a policy. These assumptions, as Geoff himself notes some of them, include:

(1) $\lim_{c\to 0} u'(c) < \infty$, implying that the resource must not be essential to life.

(2) The utility function is assumed to be additive separable, $\frac{\partial^2 u}{\partial c \partial S} = \frac{\partial^2 u}{\partial S \partial c} = 0$, implying that the marginal value of resource consumption is independent of how much resource is remaining for the future and similarly that the marginal valuation of the remaining stock is independent of how much of it, if any at all, is consumed currently. To better appreciate the degree of strictness of this assumption, one can think of it to imply that a starving individual endowed with a cake can obtain enjoyment merely from knowing that the cake exists but without ever consuming any of it, or inversely she is expected to derive the same enjoyment from consuming successive slices of the cake without ever worrying about the fact that the cake is son to finish. I believe it is reasonable to expect that in realistic situations the consumption flow and the remaining stock are complements in generating welfare ($\frac{\partial^2 u}{\partial c \partial S} = \frac{\partial^2 u}{\partial S \partial c} > 0$). But, for such cases the optimal policy may not have the steady-state characterized in the paper.

(3) For the optimal policy characterized in the paper to exist, the initial size of the resource stock, S_0 must be sufficiently large to satisfy the condition $\delta < u_2'(S_0)/u_1'(0) < u_2'(S^*)/u_1'(0)$. Otherwise, the optimal policy would entail zero consumption and conservation of the entire initial stock forever (a corner solution that coincides with GGR).

(4) For the characterized optimal policy to exist, the utility discount rate should be sufficiently small to satisfy the condition $\delta < u_2'(S^*)/u_1'(0)$ (recall that the condition should hold as an inequality when $c_t = 0$); otherwise, it would be optimal to exhaust the entire stock of the resource *possibly in a finite time.* (This seems much in accord with the Mike Spence's (1974) paper on optimality of Blue Whales extinction.)

(5) Even when all the required conditions for the existence of the optimal policy involving conservation of some of the resource stock hold, there may be many optimal paths all satisfying the conditions $\delta < u_2'(S_0)/u_1'(0) < u_2'(S^*)/u_1'(0)$, in which case, what level of the resource stock is optimal to conserve is *indeterminate*.

Thus, the optimal policy is more likely to involve conserving some part of the resource stock forever the less essential is the resource to life, the larger is its initial stock, and the smaller is the society's utility discount rate. But, in reality it is precisely *the opposite conditions* that raise our concern for sustainability.

Now, leaving aside the issues of the existence and indeterminacy of the optimal policy, I have a problem with Geoff's *identifying the stationary-state of the optimal policy with the goal of sustainability*. After all, as he himself notes in the introduction of the paper, the question of sustainability arises mainly out of concerns for intergenerational equity. In contrast, the characterized optimal policy is intergenerationally neither equitable in terms of resource consumption flow, nor in terms of the size of the inherited stock and, consequently, nor in welfare levels both when we compare with each other the earlier generations that will be living during the period [0,T] -these generations will be consuming the resource at exponentially declining rates and will leave diminishing stocks for their heirs $\{c_t > 0, \frac{\dot{c}}{c} < 0, \dot{S} < 0 \quad \forall t \varepsilon [0,T)\}$- and when comparing any of these generations with post-conservation generations who will experience equity in welfare only by refraining from resource consumption altogether but deriving enjoyment from conserving the resource stock!

Furthermore, the Geoff's striking result that for all three welfare criteria, the Rawl's Maximin, the Green Golden Rule, and the Overtaking rule, the optimal policies coincide and involve zero consumption and conservation of the initial stock forever is also an artifact of the specification of the utility function he uses. In fact, as I noted above, this will also be the utilitarian optimal policy (a corner solution) when the initial stock size is sufficiently small.

Now, I would like to address the important point of the role of the *definitions of sustainability* in thinking about optimality versus sustainability. For consistency of the discussion, I continue to consider an exhaustible resource economy of the Hotelling type where social welfare is derived only from the consumption of the resource, and is indicated by a utility function $u(c(t))$ with standard properties. In a paper (published in FEEM Working papers Series, No.47.2002, Review of Development Economics forthcoming, 2003) I have considered two alternative definitions of sustainability dominating in the literature: (1) Maintaining a maximum *constant consumption level* forever, and (2) Keeping the *value of wealth constant*. The questions I have asked are: (i) Can the utilitarian optimal policy satisfy one of these sustainability criteria while not the

other, and (ii) How significantly the optimal paths arising from the two sustainability criteria differ from one another?

The utilitarian optimal policy is defined as usual

$$\max_{c(t)} \int_0^\infty e^{-\delta t}\, u(c(t))\, dt \qquad (1)$$

$$s.t. \quad \dot{S}(t) = -c(t) \geq 0, \quad S(t) \geq 0, \quad S_0 \ (given)$$

with the associated current-value Hamiltonian

$$H(c(t), \lambda(t)) = u(c(t)) - \lambda(t)c(t)$$

The optimal consumption path is characterized by the familiar condition

$$\frac{\dot{c}(t)}{c(t)} = -\frac{\delta}{\eta(c)} \qquad (2)$$

For the iso-elastic utility function, $u(c) = \dfrac{c^{1-\eta}}{1-\eta}$, $\quad 0 < \eta < \infty$, the optimal policy is

$$c(t) = c(0)\, e^{-\frac{\delta}{\eta} t} \qquad (3)$$

Now elsewhere in a very general framework where a dynamically optimizing economy may possess all different types of capital stocks (exhaustible, renewable, physical reproducible capital, human capital, etc.) and operates under various types of resource constraints, I have shown that a sufficient condition for the optimal policy be sustainable in the sense of maximum constant consumption path is that the current-value Hamiltonian be *constant* over time.

It can be shown that for the Hotelling economy: $\dfrac{dH}{dt} = -\delta\, \lambda(t)\, c(t) < 0, \quad \forall t \geq 0$

So, obviously, the economy is **not** sustainable by the first definition (constant consumption path).

On the other hand, defining the value of the wealth of the economy as

$$V(t) = \int_t^\infty e^{-\delta(\tau - t)} p(\tau) c(\tau)\, d\tau = p(t) S(t)$$

where *p(t)* is the shadow price of the resource, it can be shown that

$$\dot{V}(t) = (\eta - 1)\, p(t) c(t) \tag{4}$$

So, in general along the optimal path

$$\dot{V}(t) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0, \quad \forall t \geq 0 \qquad as \qquad \eta \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 1 \tag{5}$$

For $\eta = 1$ the utility function $u(c) = \dfrac{c^{1-\eta}}{1-\eta}, \eta > 0$, takes the logarithmic form of $u(c) = \ln c$.

Thus,

Proposition: *An exhaustible resource economy is sustainable in the sense of maintaining the asset value of the resource stock intact (i.e., $\dot{V}(t) = 0, \forall t \geq 0$), provided that it has a* ***logarithmic utility function****, $u(c) = \ln c$, or, equivalently, it consumes the resource at rates that decline over time at the* ***discount rate*** *($c(t) = \delta S_0 e^{-\delta t}$).*

How significantly the optimal paths arising from the two sustainability criteria differ from one another?

Constant Consumption Path Requires: $\dfrac{dH(t)}{dt} = 0 \quad \Leftrightarrow \quad \dfrac{\dot{V}(t)}{V(t)} = \delta, \quad \forall (t) \geq 0$

But, along the optimal policy: $\dfrac{\dot{V}(t)}{V(t)} = \delta (1 - \dfrac{1}{\eta})$

Example,

For $\eta = 1.125$ and $\delta = 0.05$, $\dfrac{\dot{V}(t)}{V(t)} = 0.005$ and $\dfrac{\dot{c}}{c} = -0.0444$ along the optimal path.

For $\eta = 0.75$ and $\delta = 0.05$, $\dfrac{\dot{V}(t)}{V(t)} = 0.0166$ and $\dfrac{\dot{c}}{c} = -0.0666$

For $\eta = 2.5$ and $\delta = 0.01$, $\frac{\dot{V}(t)}{V(t)} = 0.006$ and $\frac{\dot{c}}{c} = -0.004$ the optimal policy is *close to paths* satisfying *either* of the sustainability criteria.

Thus, for *some* preferences, and *some* definitions of sustainability, the goals of sustainability and optimality may coincide, but this cannot be generalized for all preferences and alternative definitions of sustainability. How significantly an optimal policy deviates from a sustainable optimal path depends, among other things, on the specification of preferences and the magnitudes of various parameters involved. Notice that this is not an argument against optimality as an objective. Rather, it seeks to identify possible sources of deviation of the goal of sustainability from that of optimality and possible ways that could lead to a narrowing of the gap between the two. What I have tried to argue is that one important source of the deviation is the *choice of the specification of social preferences, and the other is the choice of the criteria of sustainability itself.*

As Geoff quite soundly argues, one source of the deviation could be our poor understanding of the ecological and biogeochemical constraints that the environmental system imposes on us. So that once these constraints are better understood and properly treated in our calculation of an optimum, the gap between the objectives of optimality and sustainability may significantly narrow. Here, I would like to add the need for a better understanding and incorporation into our models the set of *institutional constraints,* whether of social, political, or economic type, and whether these constraints operate at local, regional, national, international, or global level. These institutional constraints include *cultural values*, *consumption habits*, *production technologies*, *legal institutions* and *institutions of property rights*. A better understanding and treatment of these set of constraints can importantly aid us in more informed specifications of preferences, values of various ethical, technological, and resource parameters, and above all, in a better understanding of the implications of various sustainability criteria.

Finally, I would like to emphasize two immediate policy actions that come out of Geoff's paper. These are: (1) We, (economists as well as natural scientists) should *improve our scientific knowledge of, and seriously account for, the natural and environmental constraints* in planning for the future. (2) We should aim at polices to *remove or mitigate as much as possible the market imperfections and distortions of various types* so that our optimal policies are calculated on the basis of correct economic prices for natural and environmental resources.