1996

Fuzzy Multiple Attribute Decision Making (MADM): A Tool for Agricultural and Resource Economics

Elizabeth G. Dunn, James M. Keller, and Leonie A. Marks
University of Missouri

Selected Paper Submission
1996 AAEA Meetings

ABSTRACT: Fuzzy sets and fuzzy logic can be incorporated into multiple attribute decision making (MADM) methods as a means to integrate multidisciplinary research on complex issues and to improve the handling of mixed, noncommensurate, and ambiguous data. The fuzzy MADM approach is applied to the problem of selecting sustainable farming systems.

Corresponding author: Elizabeth G. Dunn
Dept. of Agricultural Economics
200 Mumford Hall
University of Missouri
Columbia, MO 65211
Phone: (573) 882-8816 Fax: (573) 882-3958
e-mail: ssedunn@muccmail.missouri.edu

Paper presented at the AAEA annual meeting, July 28-31, 1996, San Antonio, Texas.

FUZZY MULTIPLE ATTRIBUTE DECISION MAKING (MADM): A TOOL FOR AGRICULTURAL AND RESOURCE ECONOMICS

Agricultural economists are increasingly called upon to collaborate with biological and physical scientists in developing an integrated, or systems, approach to solving real-world problems. With their traditional emphasis on the decision maker, agricultural economists can make an important contribution to these multidisciplinary efforts. However, their tools and approaches must fit the task of providing practical decision support in complex environments. The multifaceted nature of today's applied problems, such as in environmental economics, implies that single objective decision models may be inadequate. In addition, techniques that rely on increasingly precise data and complex models may be prohibitively expensive.

As scientists, policy makers, and the general public broaden the range of conditions that should be met for economic activities to be considered sustainable, there is a corresponding need for models that can incorporate a larger set of objectives. Constrained optimization techniques can provide only an asymmetric treatment of the decision maker's goals: one goal is optimized while the others are treated as constraints. Approaches based on the generation of trade-off surfaces between two or more goals (Haimes and Hall) have limited practical ability to handle a large number of goals or to provide the kind of prescriptive information that is needed in decision support. The most common approach that economists use to integrate economic and environmental goals is to place a monetary value on environmental outcomes so that these outcomes can be incorporated into standard, single-objective economic decision models (Lutz and Munasinghe; Cropper and Oates).

A related problem faced by applied scientists and engineers in general, and by agricultural economists in particular, is the difficulty of generating the increasingly "precise" and unambiguous information that is required for complex, equation-based models of reality. This problem stems from the fundamental assumption of bivalent (Boolean) logic built into our mathematics and our models. In this paper, an alternative logic, known as multivalent or "fuzzy" logic, is introduced and its use in multiple attribute decision making (MADM) is described. The thesis of the paper is that by incorporating MADM and fuzzy logic techniques into their tool kits, agricultural and resource economists can improve their ability to model complex issues, contribute to multidisciplinary teams, and provide decision support in areas such as sustainable agriculture.

In the section that follows, a prescriptive model of decision making and its relationship to multiattribute utility theory is presented. Multiple attribute decision making and fuzzy logic are introduced briefly in sections three and four, respectively. Section four goes on to show that key problems with the MADM approach can be addressed by incorporating fuzzy logic. In section five, the fuzzy MADM approach is applied to the development of a method for evaluating and selecting sustainable farming systems.

A PRESCRIPTIVE MODEL OF DECISION MAKING

Underlying the fuzzy MADM approach is a formal model of decision making that is normative or prescriptive in nature. In this model, the rational decision maker is assumed to select among the alternatives in order to maximize his or her satisfaction. The prescriptive (or choice) model of decision making can be defined as follows (Zimmermann, 6):

> Given the set of feasible actions, A, the set of relevant states, S, the set of resulting events, E, and a (rational) utility function, u--which orders the space of events with respect to their desirability--the optimal decision under certainty is the choice of that action which leads to the event with the highest utility; such a decision can be described properly by the quadruple {A,S,E,u}.

In the choice model of decision making (figure 1), the decision maker selects from among the alternative actions (A) that are under consideration. When an action is taken, it leads to a certain event (E). The event can be thought of as a vector of outcomes, with each element in the vector representing the realization of an economic, biophysical, social, or other type of result. The link between the actions and the events is influenced by the relevant states (S), which represent exogenous factors.

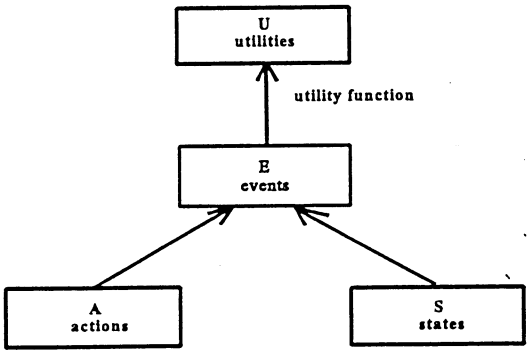

Figure 1 Choice Model of Decision Making.
(source: Zimmermann 1987, 6)

The sets of actions, states, and events are unordered. Through the introduction of a utility function, it is possible to order the events relative to the preferences or utilities (U) of the decision maker. This evaluation component can be an important feature of a decision support system. By providing a complete ranking of the events relative to the preferences of the decision maker, a decision support system offers a prescriptive solution.

Multiattribute utility theory (MAUT) provides a theoretical framework for modeling the preferences of decision makers within the context of alternatives that are characterized by multiple attributes (Dillon and Perry; Keeney and Raiffa). The MAUT framework can be used to model decision making either under certainty or uncertainty. In addition to providing guidance in the specification of multiattribute utility functions, MAUT provides procedures for eliciting the preferences of a decision maker (von Winterfeldt and Edwards, p. 272).

MULTIPLE ATTRIBUTE DECISION MAKING

Multiple attribute decision making (MADM) is the process of selecting a preferred alternative from among a set of given alternatives, where each alternative is described in terms of multiple attributes. The *alternatives* are the choices that the decision maker is considering. In MADM problems, the alternatives are finite and discrete.[1] The alternatives in the MADM framework correspond to the actions (A) in the choice model of decision making depicted in figure 1. For example, in the application presented later, the alternatives are the different farming systems that the farmer is considering for implementation.

The *attributes* in the MADM problem are the variables that the decision maker uses as the basis for evaluating alternatives. In other words, the attributes represent the decision *criteria* or the *goals* of the decision maker. The *measurements* or *data* for the MADM problem are the specific realizations of the attributes for each alternative. For example, when a consumer is selecting an automobile to purchase, price is likely to be one of the attributes included in the evaluation. The measurement on price for one automobile might be $12,000 while the

---

[1] This is the principle distinction between MADM and multiple objective decision making (MODM), since MODM problems are formulated so as to optimize over a continuous decision space. Both MADM and MODM fall under the more general category of multiple criteria decision making (MCDM).

$$\begin{array}{c|c|c|c|c|}
 & X_1 & X_2 & X_3 & X_4 \\ \hline
A_1 & x_{11} & x_{12} & x_{13} & x_{14} \\ \hline
A_2 & x_{21} & x_{22} & x_{23} & x_{24} \\ \hline
\end{array}$$

Figure 2 Decision Matrix for MADM Problem

measurement on price for an alternative automobile might be $14,400. Returning to the prescriptive model of decision making, an event (E) can be understood as the outcome from a given action or alternative; this outcome is represented by a vector of measurements on the attributes.

A *decision matrix* is a convenient way to summarize the elements of a MADM problem (figure 2). It is an nxm matrix whose elements consist of the data for the problem. There are n rows in the decision matrix, one for each alternative A_i (I=1,...,n), and m columns, one for each attribute X_j (j=1,...,m). Each element of the matrix, x_{ij}, represents the realization or measurement on the i^th^ alternative with respect to the j^th^ attribute.

While all well-defined MADM problems can be represented in terms of a decision matrix, there are numerous distinct solution methods that can be used to evaluate the alternatives. There are several key issues that generally arise in selecting an appropriate method for the types of problems faced by agricultural economists in applied, multidisciplinary settings. These issues include 1) the type of ranking provided; 2) mixed data; 3) noncommensurate data; and 4) the presence of vague or imprecise information.

The MADM solution approaches vary with respect to how much information they provide on the final ranking of alternatives. Some methods provide a complete (full) ranking of alternatives, while other methods provide only a partial ranking of alternatives. Methods that provide a complete ranking of alternatives require cardinal information on the decision maker's preferences toward the decision criteria.[2] A complete ranking of alternatives is a desirable feature in a prescriptive decision support system.

The problem of mixed data refers to the existence of both quantitative and qualitative information in the decision matrix. Non-fuzzy solution methods that provide a complete ranking of alternatives require the conversion of the qualitative data to quantitative scales. Because there are problems with justifying the procedures used for generating and converting qualitative data, qualitative information is often ignored or deleted from the formulation of the problem. However, such information may be critical to the analysis of complex systems.

A related problem is noncommensurate data, which occurs when the available information is measured in different types of units (e.g. dollars, acres, tons, years). The transformation of the data through some type of normalization procedure is essential for many MADM approaches, particularly those that permit trade-offs between attributes, otherwise known as compensatory approaches. The three most commonly used normalization methods--vector normalization, linear scale transformation, and range transformation--differ in the extent to which they preserve the intra-attribute and interattribute preference structure (Howard). Even though the choice of normalization procedure can considerably affect the ranking of alternatives, there are no clear theoretical or empirical justifications for selecting between them.

---

[2] Yoon and Hwang (pp. 5-6) categorize MADM solution methods according to the preference information provided by the decision maker: no preference information; information on thresholds or standards; ordinal preference information; or cardinal preference information.

Finally, it is not always possible to express the decision criteria or the data in terms which are precise and unambiguous. Nevertheless, it may be important to include these criteria in the decision matrix when they are relevant to the decision maker's assessment of the alternatives. Three specific types of problems can be identified: vagueness, imprecision, and ambiguity (cf. Dunn, Keller and Marks; Klir and Folger; Zimmermann). The following section introduces the concept of a fuzzy set, which can be used to model vague or ambiguous criteria. The problem of vagueness, as well as the problems of mixed and noncommensurate data, and the need for a complete ranking of the decision alternatives, have all led to the emergence of fuzzy approaches to the formulation and solution of MADM problems.

FUZZY SETS AND FUZZY MADM

Fuzzy logic refers to an inference system that is based on the concept of multivalence. In contrast to bivalent logic, in which a statement must be either true or false, multivalent logic allows for a statement to have degrees of truth. For example, the simple statement, "it is cloudy," can have varying degrees of truth, depending on the appearance of the sky. To say that a statement has a degree of truth is not the same, however, as saying that it has a certain probability of being true. To refer to an existing mixture of clouds and blue sky, the appropriate statement would be "it is partially cloudy" rather than "it is probably cloudy". While probability theory plays an important role in modeling the uncertainty associated with future or unknown events, fuzzy logic can deal with the ambiguities that exist in known or realized events.

Fuzzy Sets and Fuzzy Logic

Fuzzy sets are the basic building blocks in fuzzy logic. A *fuzzy set*, A, can be described as a set of ordered pairs (Zimmermann, p. 11):

$$A = \{ (x, \mu_A(x)) \mid x \in X \}$$

where x is an object in the collection of objects X, and $\mu_A(x)$ indicates the *degree of membership* of x in A. The mapping μ_A is called the *membership function* for the fuzzy set A. For every object in X, the membership function for A returns a value from zero to one ($\forall x \in X, 0 \leq \mu_A(x) \leq 1$).

Fuzzy set theory is based on the idea that an object can have grades or degrees of (partial) membership in the fuzzy set.[3] If $\mu_A(x) = 0.7$, we say that x has degree of membership of 0.7 in set A. The higher the degree of membership of x in A, the more closely x corresponds to the idea expressed by the fuzzy set A. The connection between fuzzy sets and classical (i.e. crisp, bivalent, Boolean) sets can be understood in terms of the extreme values in the range of the fuzzy membership function. The case of $\mu_A(x) = 1$ corresponds to the statement "x is an element in A" ($x \in A$) in classical set theory. Similarly, $\mu_A(x) = 0$ corresponds to the statement "x is not an element in A" ($x \notin A$) in classical sets. Thus, classical sets require the statement "x is an element of A" to be either (totally) true or (totally) false, while fuzzy sets allow for the statement to be partially true and partially false (at the same time).

A linguistic variable is a variable with a range which consists of linguistic terms. Linguistic variables can be described by the triple (X, T, b), where X is the name of the variable, T is the term set for X, and b is the base variable for X. Linguistic variables can be thought of as being constructed of fuzzy sets since each of the terms in the term set (T) is a fuzzy set. An

---

[3] More complete introductions to fuzzy sets can be found in Klir and Yuan and in Yager et al. For an exposition on fuzzy rule inference, see Kosko.

example of a linguistic variable is provided in the application below.

Fuzzy logic is an inference mechanism built on fuzzy sets. In a fuzzy logic rule-based system, knowledge is represented by IF-THEN rules, just as in traditional expert systems. However, the symbols in the rules are values of linguistic variables modeled by fuzzy sets. The rules in the rule base are fired by a fuzzy inference engine to arrive at appropriate classification confidence values. In the standard implementation of fuzzy logic, the minimum degree to which the input values satisfy the antecedent clauses is used to produce a modification of the consequent clause fuzzy set (by truncation) for each rule. The modified consequents for all rules are added, and the actual numeric value of the output is given by the centroid of the output fuzzy set.

Fuzzy MADM

Fuzzy sets, linguistic variables, and inference mechanisms based on fuzzy logic can provide useful tools for formulating and solving problems in the MADM framework (cf. Chen and Hwang). Opportunities for combining fuzzy set theory and MADM to address applied problems in agricultural and resource economics appear very promising, although little work has been done to date.[4] Fuzzy techniques can be usefully incorporated into MADM problems at both the problem formulation stage and in the solution, or aggregation stage. During problem formulation, an approach for incorporating vague and imprecise decision variables into MADM models is to use linguistic variables, with their associated fuzzy sets on the terms of the variable. Through the selection of appropriate membership functions for the terms of the linguistic

---

[4] For a notable exception, see Munda; Munda, Nijkamp, and Rietveld. The use of a non-fuzzy MADM approach in agricultural economics can be found in Foltz et al.

variable, critical but "fuzzy" decision criteria can be modeled in a way that is both understandable to the decision maker and useful for inclusion in a quantitative solution approach.

An important contribution of fuzzy MADM is its ability to solve problems involving attributes that are measured in different units (noncommensurate data) without the necessity of transforming or rescaling the data. Fuzzy logic is an inference system based on a set of rules that manipulate the fuzzy sets associated with the terms of linguistic variables. These fuzzy sets are already commensurate in the sense that they all take on values from zero to one, reflecting degrees of membership. Thus, fuzzy logic can provide a compensatory approach without normalizing the variables. Finally, these techniques for incorporating fuzzy sets and fuzzy logic into MADM problems widen the set of available solution approaches that can lead to a full ranking of the alternatives.

FUZZY MADM APPLIED TO SUSTAINABLE AGRICULTURE

A fuzzy MADM approach, which included linguistic variables, fuzzy sets, and a fuzzy rule base, was applied to the problem of selecting between two alternative farming systems that might be found on a typical farm in the northern part of Missouri. The farm covers 800 acres and combines the cultivation of corn and soybeans with the raising of beef cattle in cow-calf pairs. In the first alternative, the cattle (30 head) are allowed to graze continuously on 120 acres of pasture, leaving 640 acres for crop production. In the second alternative, 100 acres are converted from crops to pasture and the cattle (100 head) are rotated to utilize the pasture more effectively.

A set of nine decision variables were selected for use in evaluating the sustainability of the two alternatives. In the economic dimension, the decision variables were 1) return to unpaid labor, management, and equity; 2) cash-flow feasibility of the overall system; and 3) return on assets. The three decision variables in the environmental dimension were 1) soil erosion; 2) nitrogen losses; and 3) attractiveness of the landscape. The variables in the social dimension were 1) occupational safety; 2) self-esteem of the farm operator; and 3) lifestyle.

Two of the nine decision variables--return to unpaid labor, management, and equity and return on assets--were measured in monetary terms (although return on assets was then expressed as a percentage return). Soil erosion and nitrogen losses were expressed in physical measurements. The remaining five variables were qualitative in nature and were mapped to a 0-20 Likert-type scale, thus providing quantitative base variables for the linguistic variables. Data on all nine decision variables for each of the two farming system alternatives were provided by a University of Missouri research and extension specialist in forage-livestock systems.

An example of a linguistic variable is provided in figure 3. The name of the variable is "landscape attractiveness", the term set consists of five terms (VU:very unattractive, U:unattractive, N:neutral, A:attractive, and VA:very attractive), and the base variable is an interval scale from zero to twenty. Note that each of the terms in the term set for the linguistic variable is a fuzzy set described by a membership function. Consider a landscape that measures 11 on the base variable. The degree of membership of 11 in each of the fuzzy term sets is as follows: $\mu_{VU}(11)=0$, $\mu_{U}(11)=0$, $\mu_{N}(11)=0.75$, $\mu_{A}(11)=0.5$, $\mu_{VA}(11)=0$.

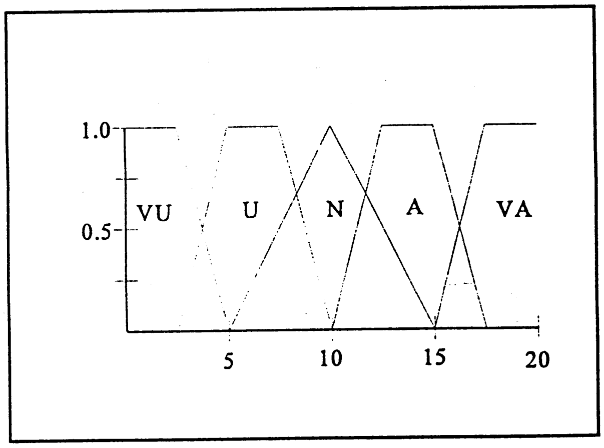

Figure 3. Attractiveness of the landscape as a linguistic variable on an interval scale

	Economic Dimension	**Environmental Dimension**	**Social Dimension**	**Overall Sustainability**
A_1	**.562**	**.672**	**.618**	**.591**
A_2	**.564**	**.803**	**.907**	**.765**

Table 1. Results of fuzzy MADM evaluation of alternative farming systems.

The sustainability of the two farming systems was evaluated using fuzzy logic on a commercially available software (CubiCalc by HyperLogic). After linguistic variables were created for each of the nine decision variables, a fuzzy rule base was constructed to link the measurements on the input variables to measurements on the output variable (sustainability). A hierarchical structure was used so that the systems were first assessed relative to economic, environmental, and social dimensions. These first-stage results were then aggregated, again using a fuzzy rule base, to derive a measurement for overall sustainability. The model results (table 1) indicate that the second farming system, the one using rotational grazing, was more sustainable overall and had a better outcome in each of the three measures of sustainability.

CONCLUSION

Agricultural and resource economists can play a key role in applied, multidisciplinary research if they can develop innovative methods for integrating information from different sciences. Fuzzy MADM is one such method, encompassing a broad range of techniques and solution algorithms. For any given application, the most appropriate fuzzy MADM approach to use depends both on the context of the problem and on the need to select an internally consistent set of aggregation operators, membership functions, and weighting mechanisms.

REFERENCES

Chen, S-J.. and C-L. Hwang. "Fuzzy Multiple Attribute Decision Making." *Lecture Notes in Economics and Mathematical Systems 375*. New York: Springer-Verlag, 1992.

Cropper, M.L., and W.E. Oates. "Environmental Economics: A Survey." *Journal of Economic Literature*. 30 (June 1992): 675-740.

Dillon, J.L., and C. Perry. "Multiattribute Utility Theory, Multiple Objectives and Uncertainty in Ex Ante Project Evaluation." *Review of Marketing and Agricultural Economics*. 45 (March and June 1977): 3-27.

Dunn, E.G., J.M. Keller, and L.A. Marks. "Integrated Decision Making for Sustainability: A Fuzzy MADM Model for Agriculture." Paper presented at the First International Conference on Multiple Objective Decision Support Systems for Land, Water, and Environmental Management: Concepts, Approaches, and Applications, Malama 'Aina, Hawaii, 1995.

Foltz, J.C., J.G. Lee, M.A. Martin, and P.V. Preckel. "Multiattribute Assessment of Alternative Cropping Systems." *American Journal of Agricultural Economics*. 77 (May 1995): 408-420.

Haimes, Y.Y., and W.A. Hall. *Multiple Objective Analysis in the Maumee River Basin: A Case Study on Level-B Planning*. Report No. SED-WRG-77-1, Case Western Reserve University at Cleveland, 1977.

Howard, A.F. "A Critical Look at Multiple Criteria Decision Making Techniques with Reference to Forestry Applications." *Canadian Journal of Forest Research*. 21 (November 1991): 1649-1659.

Hwang, C-L., and K. Yoon. "Multiple Attribute Decision Making Methods and Applications: A State-of-the-Art Survey." *Lecture Notes in Economics and Mathematical Systems 186*. New York: Springer-Verlag, 1981.

Keeney, R.L., and H. Raiffa. *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. New York: John Wiley and Sons, 1979.

Klir, G.J., and T.A. Folger. *Fuzzy Sets, Uncertainty, and Information*. Engelwood Cliffs, New Jersey: Prentice Hall, 1988.

Klir, G.J., and B. Yuan. *Fuzzy Sets and Fuzzy Logic*. New Jersey: Prentice Hall, 1995.

Kosko, B. *Neural Networks and Fuzzy Systems*. New Jersey: Prentice Hall, 1992.

Lutz, E., and M. Munasinghe. "Integration of Environmental Concerns into Economic Analyses of Projects and Policies in an Operational Context." *Ecological Economics* 10 (May 1994): 37-46.

Munda, G. *Fuzzy Information in Multicriteria Environmental Evaluation Models*. Free University of Amsterdam PhD dissertation, 1993.

Munda, G., P. Nijkamp, and P. Rietveld. "Qualitative Multicriteria Evaluation for Environmental Management." *Ecological Economics*. 10 (July 1994): 97-112.

Smith, P.G.R., and J.B. Theberge. "Evaluating Natural Areas Using Multiple Criteria: Theory and Practice." *Environmental Management*. 11(December 1987): 447-460.

von Winterfeldt, D., and W. Edwards. *Decision Analysis and Behavioral Research*. New York: Cambridge University Press, 1986.

Yager, R.R., S.V. Ovchinnikov, R.M. Tong, and H.T. Nguyen, eds. *Fuzzy Sets and Applications: Selected Papers by L.A. Zadeh*. New York: John Wiley and Sons, 1987.

Yoon, K.P., and C.-L. Hwang. *Multiple Attribute Decision Making: An Introduction*. Sage University Paper series on Quantitative Applications in the Social Sciences, 07-104. Thousand Oaks, CA: Sage, 1995.

Zimmerman, H.J. *Fuzzy Sets, Decision Making, and Expert Systems*. Boston: Kluwer Academic Publishers, 1987.