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AGGREGATE RISK RESPONSE MODELS AND MARKET EQUILIBRIUM

Lindon Robison and Garth Carman

Aggregative economic models can be used to describe how the collective actions of individuals in a market economy allocate resources and determine prices. Most often these models ignore the influence of risk (or uncertainty) on the market behavior of individuals; hence, they cannot account for supply and demand responses to changes in risk. This omission, however, becomes critical when the aggregative model is supposed to measure demand or supply responses to a policy whose major impact is on the risk associated with demanding or supplying a good. Just (1975) argues that failure to account for a positive response to reduced price risk caused policymakers to underestimate supplies of agricultural products under various price stabilization schemes during the 1950s.

This paper develops a simple aggregative risk model which can account for the influence of risk on the collective actions of individuals who trade in an exchange economy. Then, equilibrium results from the model are used to infer welfare effects resulting from actions that influence the aggregate risk model. The welfare measure, which includes an explicit risk variable, is recommended in place of consumer and producer surplus measures which have been used in the past as welfare measures.

There are, of course, several complicating features of an aggregative risk model not present in aggregative certainty models. Resource allocation problems under uncertainty ultimately require information about decision makers' preference for income and their subjective probability assessments of outcomes. This information is difficult to obtain, and even more difficult to account for in an aggregative model because it varies by individual. Nevertheless, we suggest at least one method which can account for different income preferences, but requires all individuals to hold the same probability perceptions.

The paper's welfare measure, an extension of the aggregative model, faces the difficulty of all welfare measures: how to obtain interpersonally valid measures of utility or welfare. Unfortunately, such a measure does not exist; yet, policymakers make decisions which alter resources of one group in favor of another as though they knew that the net welfare effects were positive. Because such decisions must be made (with or without a valid welfare measure), we suggest a welfare measure and require that those who use it accept the value judgment that policies which increase (decrease) the certainty equivalents of market participants measured at equilibria be preferred (rejected).

But before examining welfare issues, we first demonstrate how to derive aggregate demand and supply curves from individual utility functions.

Literature Review

The analysis of markets in which risky assets are traded should answer at least three questions: can we obtain demand and supply functions for

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individual decision makers; can these demand and supply functions be aggregated; and can they be estimated. So far, at least, the last question has received the most attention. Behrman, Just, Traill, Ryan, and Lin have all introduced risk variables into positivistic models to more accurately measure supply responses under uncertainty. Behrman related the desired area planted in four major crops in small agricultural regions of Thailand to expected price, expected yield, and the standard deviations of prices and yields. Just introduced risk variables into his study of crop responses in California by assuming decision makers formed their expectations from geometrically weighted past observations or risk variables, taken to be the square of the difference between the explanatory variables and their expected values. Traill measured onion supply responses at the national level utilizing a polynomial lag of the absolute difference between actual and expected prices. Lin studied how the acreage of Kansas wheat responds to risk by utilizing the Almon lag structure of wheat prices along with a moving average standard deviation of past actual returns to acres. And, finally, Ryan demonstrated the importance of risk on the supply of pinto beans, using variances and covariances of pinto beans and sugar beet prices.

Having considered how risk models have been estimated in the past, this paper builds on these efforts by focusing on the questions: how to derive supply and demand functions from individuals' utility functions which can then be aggregated into a model amenable to econometric estimation. Moreover, it also considers how both forces of supply and demand interact in a risky asset market rather than focusing on a single risk response equation, as did earlier studies.

Problems With Utility Functions Under Uncertainty

The difficulty in obtaining supply and demand curves for a risky asset can be best illustrated by comparing how an input demand function is affected by the introduction of output price risk. Assume a decision maker uses input x acquired at a price p_x which he uses in a production process $f(x)$. Furthermore, assume that when output price is p_y , the decision maker earns profit π that he values according to his utility function $U(\pi)$. The decision maker then chooses x so as to maximize:

$$(1) \quad U(\pi) = U[p_y f(x) - p_x x]$$

Differentiating with respect to x yields:

$$(2) \quad U'[\cdot] (p_y f' - p_x) = 0$$

Since $U'[\cdot]$ is assumed positive, we can divide both sides by $U'[\cdot]$ to obtain the derived demand for x as:

$$(3) \quad p_x = p_y f'$$

which maximizes the decision maker's utility as long as $f'' < 0$. Notice, however, that parameters associated with the decision maker's utility of wealth do not enter his derived demand for x , and allows us to aggregate derived demand functions for x across individuals without consideration of individual

differences in the valuation of utility. But, let p_y be a random variable described by probability density function $g(x)$, and y utility function parameters associated with $U(\pi)$ remain in the derived demand for x as long as the object is to maximize expected utility. To illustrate, let the expression to be maximized equal:

$$(4) \quad E U(\pi) = \int_{p_y} U[p_y f(x) - p_x X] g(p_y) dp_y$$

where E is the expectation operator and the first order conditions equal:

$$(5) \quad E U'(\pi) = \int_{p_y} U'[\cdot] (p_y f' - p_x) g(p_y) dy = 0$$

Now cancellation of the marginal utility measure is ruled out because it must be weighted by the probability density function $g(p_y)$. Hence, the problem: how do you obtain individual derived demand curves from expected utility functions and how do you aggregate them once obtained?

Several alternative ways exist for aggregating across derived demand curves obtained from individual utility functions. One approach which Traill used, assumed there is only one large producer; hence, only one utility function and therefore no aggregation problems. Moreover, if one chooses the right functional form of the utility function, the derived demand curves may be tractable. But, how one estimates the utility function for one large nonexistent producer may prove to be a difficult task.

A second approach assumes all decision makers have the same functional form for their utility function. Of course, the tractability of this approach also depends on the form of the utility function chosen. But, tractability of the derived demand curves hardly seems an acceptable criterion, even aside from the indefensible assumption that all decision makers possess the same utility functions.

The third approach, the one this paper uses, avoids the difficulty associated with utility functions by focusing instead on the efficient set of choices. Consider the function below where λ and \bar{w} are constants and $W(x)$ and $\sigma^2(s)$ are portfolio expected wealth and variance determined by a vector of choice variables x .

$$(6) \quad W(x) = \bar{W} + \lambda \sigma^2$$

This function, or this function rearranged, has the following interesting properties:

- (a) When properly constrained, the solution is a member of the EV set at a point where the trade-off between mean and variance equals λ (Hadley).
- (b) If the distributions are normal, it is the expected utility for a constant absolute risk averter with risk aversion equal to 2λ (Freund).
- (c) If the variance is small, it provides the first order approximation of the risk premium for a decision maker whose average coefficient of absolute risk aversion is 2λ (Pratt).

- (d) It is the equation for a line tangent to the EV set where the slope equals λ (Robison and Barry).
- (e) It measures the certainty equivalent (\bar{w}) exactly for constant absolute risk averse decision makers facing normal distributions (Weins) or is a first order approximation if the distributions are not normal and the risk aversion is not constant (Pratt).

The properties of equation (6) allow us to use it in a convenient way to characterize the preferred solution for all decision makers, regardless of their utility functions. Assume a decision maker selects from an EV set his expected utility maximizing solution. The preferred choice, of course, occurs at the point of tangency between an isoexpected utility \bar{U} and the EV set AB in Figure 1. Assume this tangency occurs at a point on the EV set where the slope equals λ . Then, rather than maximizing expected utility we could maximize the expression in (6) and obtain the same solution for the inputs x that produce expected wealth and variance. Moreover, the only parameter in the model is λ , which is readily interpretable and valid for interpersonal comparisons, since it is both the slope of the preferred plan and also the coefficient of absolute risk aversion.

In summary, we can characterize the solution for any investor who chooses his preferred plan from an EV set by maximizing (6), providing we choose correctly the value of λ . Moreover, the expression we maximize in (6) has a convenient parameter that is equal to both the slope of the solution on an EV set and approximates the average risk aversion coefficient of the decision maker who makes that selection. Alternatively, it is the expected utility function for an investor with constant absolute risk aversion.

Justifying the EV Approach

The advantages and limitations of the EV criterion are probably well-known, but some summary comments seem called for. First, the EV set is guaranteed to include a decision maker's expected utility maximizing solution if his utility function is quadratic or if the decision maker is risk averse and the probability distributions are normal. On the other hand, apart from low-expected wealth, low variance solutions which are rarely preferred, the discrepancies between the EV set and the more general stochastic dominance criteria, are small. Hence, for practical people, the EV set is a useful, usually too large, efficient set. Finally, in empirical work, summary measures of the probability distributions are required--and the variance or a measure like variance is used with expected returns in all empirical studies reviewed in this paper. Since positivistic models rely on variance and expected returns, our theoretical model, derived in terms of variance and expected returns, seems complementary.

Characterizing the Market for a Risky Asset

Individuals may exchange safe assets (money) for a risky asset (e.g., money for bonds and stocks), for at least two reasons:

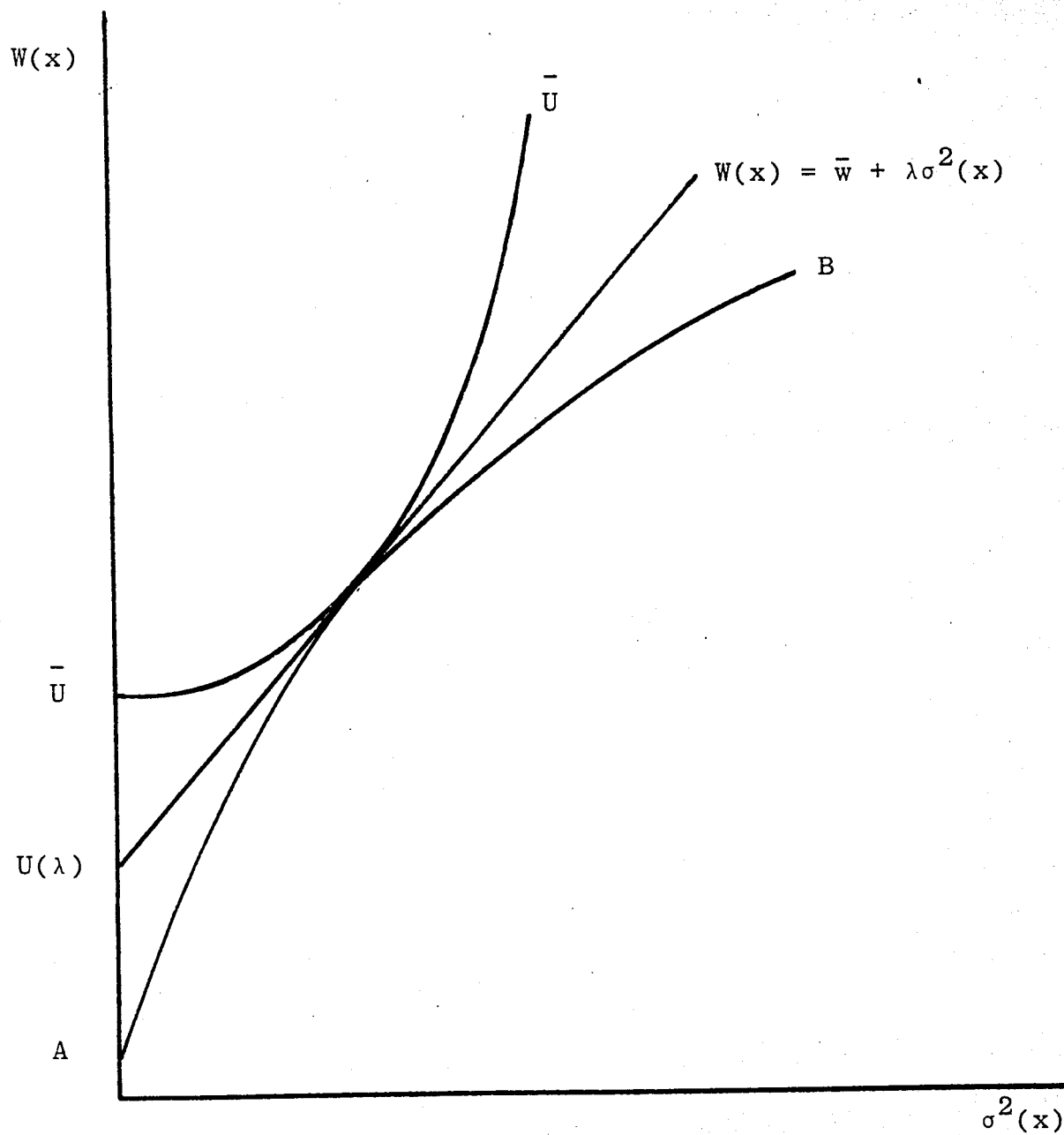


FIGURE 1. Equilibrium Between an EV Efficient Set and a Decision Maker's Isoexpected Utility Function.

- (a) They do not have the desired balance between safe and risky assets; hence, they may wish to balance their portfolios by trading safe assets for risky assets or vice versa.
- (b) They hold different expectations about the future performance of the risky asset and may have different investment opportunities for investing their safe assets.

Our model accounts only for the first motivation for trade--a desire to modify the risk-return character of one's portfolio. That is, decision makers A and B may have different average absolute risk aversion (e.g., $\lambda_A \neq \lambda_B$ in equation (6)), which may lead them to prefer different combinations of risky and safe assets even when they view safe investment opportunities and future performance of the risk asset the same.

We now derive demand and supply curves for a risky asset traded in an exchange economy by assuming the simplest kind of a market. Let two traders with average absolute risk aversion λ_A and λ_B begin the trading period with endowments of the safe asset x_1^0 and risky asset x_2^0 , respectively. Moreover, assume both have investment opportunities which can earn r_1 rate of return on the safe asset and expected rate of return r_2 for the risky asset but with a variance of return on the risky asset equal to σ^2 .

For the market to be active, decision maker A must trade some of his safe assets for part of decision maker B's risky assets. The market mechanism, of course, determines the amount of trade and rate of exchange between the two assets. If asset x_1^0 is valued at price p_1 and asset x_2^0 is valued at price p_2 , then the value of the safe asset traded ($p_1 x_1$) must equal the value of the risky asset traded ($p_2 x_2$), or alternatively:

$$(7) \quad x_1 = \frac{p_2}{p_1} x_2$$

To simplify, let p_2/p_1 equal p , the exchange ratio between the risky and safe assets. Of course, where p_1 is the price of a dollar--always equal to 1, p_2 equals p .

Now, characterize the expected utility maximizing solution for both decision makers A and B by rearranging (6), assuming some trade occurs. If a trade occurs, A exchanges x_1 of his safe asset in return for x_2 of B's risky asset which valued in units of x_1 equals px_2 . So, A now subtracts x_1 from his safe holdings and invests px_2 in risky assets. B meanwhile acquires x_1 of the safe asset, and gives up x_2 amount of his risky asset. But for the certainty equivalents for A and B (\bar{U}_A and \bar{U}_B), we convert the units of x_2 to x_1 units by multiplying them by p . So, decision makers trade so as to maximize their respective certainty equivalents equal to:

$$(8) \quad \bar{U}_A = r_1(x_1^0 - x_1) + r_2px_2 - \lambda_A p^2 x_2^2 \sigma^2$$

$$(9) \quad \bar{U}_B = r_1x_1 + r_2p(x_2^0 - x_2) - \lambda_B p^2 (x_2^0 - x_2)^2 \sigma^2$$

where $r_1(x_1^0 - x_1) + r_2px_2$ and $r_1x_1 + r_2p(x_2^0 - x_2)$ are portfolio expected

wealth for decision makers A and B, respectively, while $x_2^2 p^2 \sigma^2$ and $(x_2 - x_2^0)^2 p^2 \sigma^2$ are the variances associated with their portfolios.

The variable x_1 can be eliminated from (8) and (9) by introducing the restriction that x_1 and x_2 must be exchanged at rate p . So, we next substitute px_2 for x_1 and then differentiate both (8) and (9) to obtain the derived demand and supply for x_2 . The amount of x_2 demanded by A equals:

$$(10) \quad x_2 = \frac{r_2 - r_1}{2\lambda_A \sigma^2 p}$$

and the amount of x_2 supplied by B equals:

$$(11) \quad x_2 = (r_1 - r_2)/2\lambda_B p \sigma^2 + x_2^0$$

In equilibrium, equations (10) and (11) must be equal; setting them equal also permits us to solve the reduced form equation for p equal to:

$$(12) \quad p = (\lambda_A + \lambda_B)(r_2 - r_1)/2\lambda_A \lambda_B x_2^0 \sigma^2$$

The equations are sketched in Figure 2 and conform to our expectations regarding supply and demand curves, but with some richer information regarding the slope and location of the curves. For example, price p and quantity traded x_2 are certain--just as in our certainty models, but the expectations regarding the performance of x_2 , the opportunity cost of x_1 and the average risk aversion of decision makers A and B all influence the equilibrium price and quantity. The reduced form expression for p also allows us to examine how changes in the parameters influencing market equilibrium will affect market prices:

$$(13) \quad \frac{dp}{dr_1} = \frac{-(\lambda_A + \lambda_B)}{2\lambda_A \lambda_B x_2^0 \sigma^2} < 0$$

$$(14) \quad \frac{dp}{dr_2} = \frac{\lambda_A + \lambda_B}{2\lambda_A \lambda_B x_2^0 \sigma^2} > 0$$

$$(15) \quad \frac{dp}{dx_2^0} = \frac{-p}{x_2^0} < 0$$

$$(16) \quad \frac{dp}{d\sigma^2} = \frac{-p}{\sigma^2} < 0$$

$$(17) \quad \frac{dp}{d\lambda_A} = \frac{-\lambda_B p}{(\lambda_A + \lambda_B)\lambda_A} < 0$$

$$(18) \quad \frac{dp}{d\lambda_B} = \frac{-\lambda_A p}{\lambda_B(\lambda_A + \lambda_B)} < 0$$

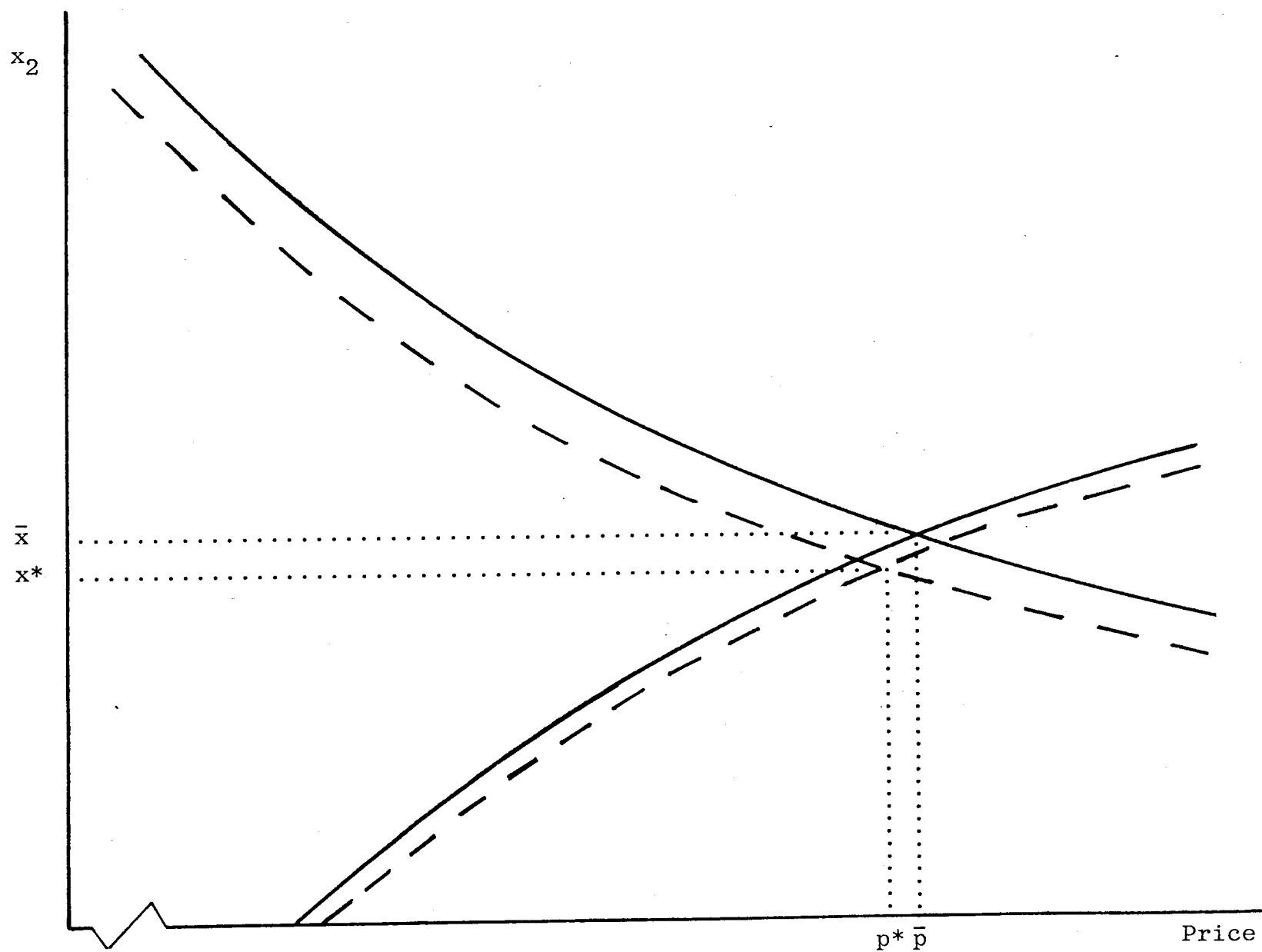


FIGURE 2. The Demand and Supply Functions for Risky Asset x_2 Exchanged for the Safe Asset x_1 at Rate p .

The results are intuitively acceptable: increasing the opportunity cost of x_1 by increasing r_1 lowers the rate at which x_1 is traded for x_2 ; increasing the average risk aversion of either decision maker A or B, increasing x_2^0 and σ^2 all reduce the value of the risky asset x_2 , relative to x_1 . And, only increasing r_2 produces the opposite effect of increasing the desirability of x_2 relative to x_1 .

Multiplicative Errors and Aggregation

The model formulated in the earlier section assumed risk was additive. That is, \tilde{r}_2 , a random variable, was assumed equal to $r_2 + \theta$ where θ was a random variable with expected return and variance equal to 0 and σ^2 , respectively. With this model, the expected returns and variance of \tilde{r}_2 are r_2 and σ^2 , respectively. We could alternatively assume that risk entered in a multiplicative manner where $\tilde{r}_2 = r_2 \gamma$ where the expected value and variance of γ is 1 and σ^2 , respectively. Under this assumption the expected return and variance of \tilde{r}_2 equals r_2 and $r_2^2 \sigma^2$, respectively. The different expression for the variance \tilde{r}_2 required changing equations (8) and (9) which were maximized earlier to obtain demand and supply response functions for x_2 . The change could be made by adding in place of σ^2 , $r_2^2 \sigma^2$ in equations (8) and (9). The new demand and supply equations obtained after allowing for multiplicative risk equal:

$$(19) \quad x_2 = (r_2 - r_1) / 2\lambda_A \sigma^2 r_2^2 p \quad (\text{Demand})$$

$$(20) \quad x_2 = (r_1 - r_2) 2\lambda_B p \sigma^2 r_2^2 + x_2^0 \quad (\text{Supply})$$

Of course, in equilibrium (19) and (20) are equal, which permits again the derivation of the new reduced form equation for the exchange rate p , equal to:

$$(21) \quad p = (\lambda_A + \lambda_B)(r_2 - r_1) / 2\lambda_A \lambda_B \sigma^2 r_2^2 x_2^0$$

Of course, we could again differentiate p with respect to the parameters in the reduced form equation. The signs from the resulting equation are, however, the same as when risk was additive, with the exception of the derivative of p with respect to r_2 . We can no longer disassociate changes in r_2 and σ^2 --increasing r_2 also increases portfolio variance. So, the impact on the exchange price p of increasing r_2 , given in equation (22) below, is indifferent.

$$(22) \quad \frac{dp}{dr_2} = \frac{r_2 - 2(\lambda_A + \lambda_B)(r_2 - r_1)}{2\lambda_A \lambda_B \sigma^2 r_2^2 x_2^0}$$

Aggregating the Models

The earlier models are now generalized to show how results could be obtained in a two-asset exchange market with m suppliers of x_2 and n demanders, all of whom hold the same expectations about the future performance

of the risk variable and have the same opportunities for investing the safe asset. We now have one more restriction, however, that being the total amount demanded by the n demanders must equal the total value received in exchange from the m suppliers:

$$(23) \quad \sum_{i=1}^n x_{1i} = p \sum_{j=1}^m x_{2j}$$

This restriction, however, is redundant if all trades take place at the same price p , so aggregating across all traders must produce the result that value supplied equals value demanded. Thus, identifying the amount of x_2 demanded by the i -th decision maker as x_{2i} with a corresponding absolute risk aversion λ_i and the amount supplied by the j -th supplier as x_{2j} , we sum equations (10) and (11) to obtain the aggregate demand and supply for risky asset x_2 as:

$$(24) \quad \sum_i x_{2i} = [(r_2 - r_1)/(2\sigma^2 p)] \sum_i \lambda_i \quad (\text{aggregate demand})$$

$$(25) \quad p = (r_2 - r_1) \sum_j \lambda_j / (\sum_j x_{2j}^0 - \sum_j x_{2j}) 2\sigma^2 \quad (\text{aggregate supply})$$

The equations can be transformed into linear expressions for convenience in estimation by taking the log transformation. The expressions after transformation become:

$$(26) \quad \log \sum_i x_{2i} = \log \sum_i \lambda_i + \log(r_2 - r_1) - \log 2\sigma^2 - \log p$$

$$(27) \quad \log p = \log \sum_j \lambda_j + \log(r_2 - r_1) - \log 2\sigma^2 - \log(\sum_j x_{2j}^0 - \sum_j x_{2j})$$

It is interesting to compare the above results with the aggregative models used by Just and Behrman, which also included expected returns and terms representing variances. Their models were linear and are represented in a simplified form as:

$$(28) \quad \sum_i x_{2i} = \alpha_0 + \alpha_1 r + \alpha_2 \sigma^2$$

where α_0 , α_1 , and α_2 are constants. Except for the constant term α_0 in place of $\sum_i \lambda_i$ and the absence of the logarithmic transformation, the expressions are similar. Hence, the theoretical arguments in this paper lend support to their work, with the exception of their use of a constant term.

Recall that λ 's represented the average absolute risk aversion of market participants; alternatively, it was also the slope of linear tangent lines drawn to EV frontier at points of equilibrium in any one period. But as variances, expected returns and exchange rates between risky and safe assets change, we cannot assume that individual λ 's remain constant--unless we assume all decision makers possess constant absolute risk aversion.

Allowing the aggregate demand to vary with changes in $\Sigma l/\lambda_i$ is equivalent to including income or wealth effects in the model. Robison and Barry have shown that if a linear tangent line is used to approximate the investor's isoexpected utility curve, as we have in this paper, then the income or wealth effect can be accounted for as the change in demand associated with a changed value of $\Sigma l/\lambda_i$. Furthermore, they showed that for decreasingly risk averse investors an empirically valid assumption (see Cohn, et al.), then for any shift to the right (left) in the EV frontier, the equilibrium slope λ on the EV frontier will increase (decrease). Therefore, to exclude λ or its instrument from a positive risk model is equivalent to ignoring the income effect on the demand for a risky asset.

Hence, two other equations are needed to complete the model: approximating equations for $\Sigma l/\lambda_i$ and for $\Sigma l/\lambda_j$. We cannot know the exact equations for their estimation since they ultimately depend on individual decision makers' utility functions. We can only observe the effects of individual utility functions on the resultant equilibrium slopes. So, we choose convenient forms equal to:

$$(29) \quad \Sigma l/\lambda_i = \beta_0 (r_2 - r_1) \beta_1 (\sigma^2) \beta_2 p \beta_3 e^{\varepsilon_t}$$

where ε_t is assumed to be normally distributed. Then taking logarithmic transformations we obtain the expression:

$$(30) \quad \log \Sigma l/\lambda_i = \beta_0 + \beta_1 \log(r_2 - r_1) + \beta_2 \log \sigma^2 + \beta_3 \log p + \varepsilon_t$$

A similar expression could be obtained for estimating $\Sigma l/\lambda_j$, which after transformation would equal:

$$(31) \quad \log \Sigma l/\lambda_j = \gamma_0 + \gamma_1 \log(r_2 - r_1) + \gamma_2 \log \sigma^2 + \gamma_3 \log p + \gamma_4 \log Ex_{2j}^0 + U_t$$

Finally, aggregative risk models could also be developed under the assumption of multiplicative risk. The important difference would be the inclusion of the term $\log r_2^2$ in both the aggregate demand and supply equation. Moreover, this also implies that the simplified version of risk models, with multiplicative risks, should also include r_2^2 .

A New Approach to Welfare

As economists, we are in somewhat of a dilemma with regard to welfare analysis. On the one hand, we lack valid interpersonal measures of welfare to correctly evaluate the distributional impacts of most policies. Yet, on the other hand, if we refuse to discuss them our usefulness as policy analysts is severely reduced. Moreover, since the distributional effects will be decided anyway, economists should provide input since we are as qualified as others to make inferences. So, we do, but in a roundabout way. We substitute for welfare measures we can't obtain, measures we can obtain--such as consumer and producer surpluses. At first, we claimed the surplus measures were welfare measures, but recent extensions of the surplus notions do not claim as much. Instead they claim to measure whether gainers can bribe losers or whether losers can bribe gainers to forego the benefits or costs associated with the policy change (compensating and equivalent variations).

Willig has shown these measures can be approximated by changes in consumer surplus.

Unfortunately, we see little alternatives to measuring substitutes for welfare, if indeed, we are forced to make inferences about the welfare impact of alternative policies. In fact, little harm will likely result, as long as we recognize, as modern welfare economists do, that they are not in fact measuring welfare (Just, 1978).

Nevertheless, we suggest that alternatives to consumer and producer surpluses (compensating and equivalent variations) exist and should be considered. The alternative we propose is to measure the certainty equivalents of market participants' portfolios at equilibrium, then allow for perturbations in the market, while at the same time constraining prices and quantities to remain in equilibrium.

Assuming the risky asset market is in equilibrium, there is only one price combination experienced by market participants. It seems more correct to infer welfare (utility?) at that point versus subsequent equilibria than to compare areas and changes in areas resulting from price/quantity combinations not actually experienced as is the case with welfare measures now used.¹

To illustrate, recall the derived demand and supply functions, (10) and (11) that gave rise to equilibrium price \bar{p} and \bar{x}_2 , depicted in Figure 2. Should variance increase, the demand for the risky asset will be reduced (and shifted to the left), while the supply of x_2 will increase (and be shifted to the right) as depicted by the broken lines in Figure 2.

The welfare of the market participants, meanwhile, has changed. Where before they exchanged at price p and quantity x_2 , they now exchange at price p^* and x_2^* . It seems reasonable to us to measure equilibrium welfare (utility) not at all price/quantity combinations not experienced before and after the shift, but to compare their measure of welfare (really its substitute) at the two equilibrium combinations.

To do so recall that the functions from which derived demand and supply curves were obtained, among their other properties, measured certainty equivalents of the decision makers. Adding the two certainty equivalent measures together, of course, measures the sum of their respective certainty equivalents:

$$(32) \quad \bar{U}_A + \bar{U}_B = r_1 x_1^\circ + r_2 p x_2^\circ - \lambda_A p^2 x_2^2 \sigma^2 - \lambda_B p^2 (x_2^\circ - x_2)^2 \sigma^2$$

Next, the sum of certainty equivalents is constrained to be in equilibrium by substituting for p and x_2 in equation (32) their respective reduced form expressions obtained by imposing equilibrium conditions. The reduced form expression for p was given in equation (12), while the reduced form for x_2 can be found by solving equations (10) and (11) for p and then setting them equal. Solving the result for x_2 gives us the reduced form for x_2 :

$$(33) \quad x_2 = \frac{x_2^\circ \lambda_B}{\lambda_A + \lambda_B}$$

¹It may be the case, though, that this paper's substitute welfare measure produces results consistent with the surplus measures; if so, then perhaps our results add credence to them. But, comparing the two measures is a subject for later discussion.

Substituting (33) into (32) and simplifying yields:

$$(34) \quad \bar{U}_A + \bar{U}_B = r_1 x_1^0 + r_2 p x_2^0 - p^2 \sigma^2 x_2^0 \left[\frac{\lambda_A \lambda_B}{\lambda_A \lambda_B} \right]$$

Next substituting the reduced form for p from (12) into (34) and simplifying produces the following sum of certainty equivalents:

$$(35) \quad \bar{U}_A + \bar{U}_B = r_1 x_1 + \frac{(\lambda_A + \lambda_B)(r_2^2 - r_1^2)}{4\lambda_A \lambda_B \sigma^2}$$

This expression allows us to analyze the impact of changes in the respective parameters and their associated impact of the summed certainty equivalents measured at market equilibrium.

$$(36) \quad \frac{d(\bar{U}_A + \bar{U}_B)}{dx_1} = r_1 > 0$$

$$(37) \quad \frac{d(\bar{U}_A + \bar{U}_B)}{dr_2} = \frac{2r_2(\lambda_A + \lambda_B)}{4\lambda_A \lambda_B \sigma^2} > 0$$

$$(38) \quad \frac{d(\bar{U}_A + \bar{U}_B)}{d\sigma^2} = -\frac{(\lambda_A + \lambda_B)(r_2^2 - r_1^2)}{4\lambda_A \lambda_B (\sigma^2)^2} < 0$$

$$(39) \quad \frac{d(\bar{U}_A + \bar{U}_B)}{d\lambda_A} = -\frac{r_2^2 - r_1^2}{4\lambda_A^2 \sigma^2} < 0$$

$$(40) \quad \frac{d(\bar{U}_A + \bar{U}_B)}{d\lambda_B} = -\frac{r_2^2 - r_1^2}{4\lambda_B^2 \sigma^2} < 0$$

$$(41) \quad \frac{d(\bar{U}_A + \bar{U}_B)}{dr_1} = x_1^0 - \frac{r_1(\lambda_A + \lambda_B)}{2\lambda_A \lambda_B \sigma^2}$$

Again, the results are intuitively acceptable: increasing the initial endowment of x_1^0 , or the expected return on the risky asset r_2 , will result in an increased certainty equivalent. On the otherhand, increasing the variance associated with the risky asset, or increasing the average risk aversion coefficient of decision maker A or B, reduces the certainty equivalent. The impact of changes in the safe rate of return (r_1) upon the certainty equivalent is not clear, although, in almost all cases we could expect x_1^0 to be greater than $\frac{r_1(\lambda_A + \lambda_B)}{2\lambda_A \lambda_B \sigma^2}$ and the resulting derivative to be positive.

$$\frac{r_1(\lambda_A + \lambda_B)}{2\lambda_A \lambda_B \sigma^2}$$

This welfare analysis so far has assumed that risk is additive. Relaxing that assumption and assuming that risk enters in a multiplicative way, as was done earlier, yields a new expression for the certainty equivalent:

$$(42) \quad \bar{U}_A + \bar{U}_B = r_1 x_1^o + \frac{(\lambda_A + \lambda_B)(r_2^2 - r_1^2)}{4\lambda_A \lambda_B \sigma^2 r_2^2}$$

This expression is identical to that obtained in equation (35) for additive risk with the exception of r_2^2 in the denominator. Consequently, except for changing r_2 , the certainty equivalents associated with changes in the various parameters have the same signs. However, in most cases, the magnitudes of those changes are influenced.

Summary and Conclusions

This paper has examined the equilibrium conditions in a risky asset market. The derivation centered around an approximating equation developed in alternative settings by Freund, Pratt, and Robison and Barry. The expression allowed us to obtain demand and supply response functions for a risky asset in an exchange economy. Moreover, we showed that the results could be easily aggregated for decision makers holding the same expectations about alternatives available for investing the safe asset and the future performance of the risky asset. Finally, we suggested a method for obtaining empirical estimates of the parameters affecting equilibrium in the market.

The parameters determining equilibrium in the risky asset market are for the most part unobservable. Expectations regarding future performance of the risky asset can only be inferred. Hopefully, an expectations model that predicts actual changes with more accuracy than an alternative model has a greater chance of being used. Yet, we can never be sure; and to complicate matters still more, these inferred values of variances and expected returns determine average risk aversion parameters, which also cannot be observed. Equilibrium price and quantity, fortunately, can be observed which provides the model with useful empirical content.

The last section introduced an alternative measure which can be used to infer welfare effects of alternative policies. The approach measured certainty equivalents at equilibrium and implicitly assumed that policies that increase certainty equivalents of market participants should be adopted; those which reduce certainty equivalents should be avoided. Finally, the results showed how average risk aversion of the market participants influence directly net certainty equivalents of any stabilization policies.

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