A PERSPECTIVE ON DUALITY AND PRODUCTION ECONOMICS

by

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As I understand my assignment, I am to provide some retrospective on duality in the same spirit as a somewhat prospective session which occurred in 1982. I say somewhat prospective because dual methods were at least forty years old then (Samuelson, Hotelling). The first edition of Shephard's seminal book was in 1953. Varian's text was published in 1978 and lecture notes on duality were floating around long before that. However, in 1982 dual methods were relatively new to agricultural economics and they certainly were to me (I don't believe that I ever heard the word duality in the sense used here during my graduate training). There clearly has been much technological change in the way we model and measure economic responses. Duality is now a central theme in many introductory graduate courses in microeconomics. Yet, this seems to be changing as texts emphasize more general ways of thinking about behavior (e.g., game theory) and take duality concepts for granted.

I trust that an assignment such as this allows the opportunity to be informal. The basic question at hand then and now seems to be the proper role of duality in the tool kit of agricultural production economists. Faced with this topic, one could review the many impressive conceptual and successful empirical studies using duality. Yet there are many excellent applied treatments of the duality in texts and notes (Varian, Cornes, Chambers, Fare). One could develop some appropriate dual for a new problem. If I could do so, this might seem inappropriately narrow for this session. Hence, what seems left for me is to give a bit of historical and contemporary perspective. These almost universally fail cost-benefit tests. This paper is no exception.

For the most part, production economics methodology can be divided into mathematical programming and econometric approaches. Both are routinely applied to farm level and aggregate analyses. Both are extensively applied in primal and dual form. It appears to me that there has been a reduction in demand for programming methods and an increase in demand for econometric approaches. The areas, where programming methods have seen increased use is in nonparametric "tests" of structure and efficiency (e.g.,Chavas and Cox, Fried et al.).

Both econometric and programming approaches have their strengths and it seems that in an ideal world that the two approaches would be merged. However, as long as researchers wish to test hypotheses, the synthesis of the two approaches will be difficult because the statistical properties of estimators subject to inequality constraints is very difficult to specify. Later, I will suggest one way aggregate econometric analyses may incorporate one of the strengths of programming models. Of necessity, my discussion focuses on methods amenable to econometric analysis.

The plan of the paper is to lay out the basic tools and definitions of duality in the next section. Then the applicability and strengths of dual methods are discussed. I take the position that duality has been a tremendous advance but may have caused the Profession to be lax in careful and interesting modeling of primal problems. The paper concludes with a series of suggestions and questions which might motivate further clarification of concepts or advancements in measurement.

II. Background and Notation

It is useful to consider the following problem:

$$(1) \qquad \max_{x} \{F(x,w,\beta)\,|\,x \in T\}$$

where T defines a non-empty closed convex set of feasible choices and F denotes the objective to be maximized, x are decisions, and w are exogenous variables, and β are parameters. Conceptually, T might be described by the input distance function greater than or equal to one or by a transformation function (when it exists) as indicated by $g(x,\zeta) \geq 0$. Additionally, there may be other constraints on behavior $H(x,\nu) \geq 0$. It seems to have become fashionable to ignore such constraints in contrast to programming approaches but it is difficult to dismiss them entirely. Admittedly, imposing such constraints may often substitute for careful modeling of the complete decision problem. However, there are likely time, land, and machinery allocation constraints that need to be considered. The feasible set might be described by $G(x,w,\gamma) \geq 0$ with the efficient subset of T described by the equality. Observationally, it is difficult to differentiate the feasible set defined by technology from that defined by the intersection of technology and the ancillary constraints on choice. Thus, it may be difficult to untangle inherent non-jointness in technology but constrained choice, from jointness due to other factors. Technological change may be improperly interpreted. In most cases, differentiation of the two phenomenon will not be of interest. The first order conditions are:

$$(2)\quad F_x + \lambda\, G_x = 0;\ G=0,$$

where λ is the Lagrange multiplier. For the most part, estimation of (2) is what most people call the primal approach to production analysis. Some might call the estimation of G the primal approach. Comparative statics on (2) is what many consider the essence of the primal approach to economic theory. Until the dual approach dominated, comparative statics guided the interpretation of reduced form marginal effects of exogenous (passive) variables on choice (active) variables.

For single product competitive profit maximization, let p be output price, y output, x a vector of inputs and r the corresponding vector of input prices. F and G combine to yield first order conditions in share form:

$$(3)\quad S_x = \epsilon(x,\alpha),$$

where S_x is the revenue share of x or rx/py, and ϵ is the partial production elasticity $\partial y/\partial x_i$. Thus, large factor shares are associated with large partial production elasticities. In the primal problem x is thought of as an active or endogenous variable while r and p are passive or exogenous variables or parameters. The function ϵ is estimated parametrically or nonparametrically. Parametric specification aids in hypothesis formulation but restricts the maintained hypothesis on functional form. Occasionally G is appended to (3) for efficient estimation.

The dual problem often starts from the indirect objective function:

$$(4) \quad F^{*}(w,\gamma) = \max_{x,\lambda} \{L = F(x,w,\beta) + \lambda G(x,w,\alpha)\}$$

where $\gamma = (\alpha,\beta)$. A variety of papers examine the implications of (4). These include the possibility of recovering G or F from F^{*}, and of deriving useful expressions on the slope and curvature of F^{*}. Although recoverability seems central to theory, it doesn't seem as central to most applications. Likely, the most useful and definitive modern work on optimization theory for the applied is Luenberger (I have read the first chapter several times). It seems to me that a very useful reference on duality in production is McFadden who reinterpreted and extended the pioneering work of Shephard. The composition results which determine functional forms are extensive. A simple and useful, if not general, way of looking at the implications of this problem are contained in the vintage primal-dual formulation associated with Samuelson, Silberberg, and Hatta. In such case, the problems in (2) and (4) are subsumed by:

$$(5)\ \min_{x,w,\lambda} L^* = F^* - L$$

Hence, L^* directly measures economic inefficiency. First order conditions are:

$$(6)\ -F_x - \lambda G_x = 0,$$

$$(7)\ F^*_w - F_w - \lambda G_w = 0,$$

$$(8)\ -G = 0.$$

The first and third of these are the primal first order conditions in (5) and (7) is the well known envelope theorem. Because (6) - (8) represent a minimum with respect to these variables, second order sufficient conditions are that the quadratic form $tHt \geq 0$ for all t orthogonal to $(G_x : G_w)'$, where H is the Hessian of L^*. This implies that $vL^*_{ww}v = v(F^*_{ww} - F_{ww} - \lambda G_{ww})v \geq 0$ subject to $vG_w = 0$, $v \neq 0$. For many problems in economics, F_{ww} and G_{ww} are zero leaving very clear properties of F^*_{ww} (i.e., F^* is quasi-convex in w). There are some special cases that are immediately useful when $G \epsilon R$:

Result 1: If G(x,w) is convex in w and F=F(x), then F^* is quasi-convex in w (i.e. $vF^*_{ww}v \geq 0 : vG_w = 0, v \neq 0$).

Result 2: If F is convex in w and G=G(x), then F^* is convex in w (i.e. $vF^*_{ww}v \geq 0, v \neq 0$).

These are the heart of neoclassical production applications.[1]

Interpreting F in Result 1 as production and G as expenditure (i.e., w is a vector of prices) or cost yields the indirect production function and the well-known result that the indirect production function is quasi-convex in input prices (r). In Result 2, interpreting F as profit and G as "technology" yields the result that profit is convex in input and output prices. Restricting profit by constraining output to be fixed yields the cost function which is concave in input prices. Finally, restricting inputs to be fixed yields the revenue or "national product" function which is convex in prices. Fixed input can also be explicitly considered. These plus the envelope conditions are the heart of applications of duality. They involve slope and curvature interpretations and restrictions and are found in a myriad of applications in various fields of economics (see Diewert).

Merely stating that the cost function is increasing and concave in input prices does not convey whether or in what sense it is strictly concave. Because costs have been shown to be linearly homogeneous, the cost function is not strictly concave. This is most often stated: conditional demand functions are homogeneous of degree zero. Such nullity conditions appear in virtually all economic models and are considered by many to be the most important of economic predictions of theory. In any model of behavior these nullity conditions generally increase with increased parametrization (Pope,1982). Chavas and Pope present a general way to determine the dimension and form of these nullity restrictions essential to integrability. As models of risk and technical change become increasingly general, these restrictions should prove useful.

Analogous to (3) is the single product demand equations in dual share form (where profit is assumed nonzero):

(9) $S_r = \sigma(P, r, \alpha, \beta)$,

where S_r = is the profit share of x, or rx/F^* and σ is the elasticity of profit with respect to r. Thus, large profit shares are associated with large profit elasticities. Equation (5) or (6) in gradient or elasticity form are the most commonly applied so called dual representation. Additionally given convexity of T, it follows that F^* or profit is weakly convex in r and p with homogeneity giving the nullity conditions for the gradient of F^*. Revenue or cost functions can be obtained by restricting various decisions in (1) and creating conditional demand or supply functions. Again, often F^* is appended to (5) for "efficient" estimation.

Comparing (3) and (9) shows that the major difference between the two first order conditions are the right hand side variables. In the primal approach, these are generally inputs (and outputs) whereas under the dual method, the right hand sides of econometric equations are prices. In general, F^* is not of the same functional form as F; thus, there are often differences between the two methods in functional form unless F and F^* are self-dual. My view of the primal approach is that increasingly optimization errors are appended to first order conditions so that econometric equations are written in the implicit form $L_x = u$, where u is an econometric error (Antle). In contrast the dual methods lead to $x^* = m(\gamma, w) + u$ (or u is appended to shares). Thus, there are almost always differences between the two methods due solely to differences in econometric specification (McElroy).

A rather major research activity involves placing restrictions on L and deducing the restrictions on L^* and vice versa (e.g., McFadden, Lau). A second useful area of research involves examining various schemes of compensated behavior: wealth, output price or land rents.[2]

To conclude this section, it seems necessary to define the phrase "dual method". It is defined here as having the following components:

1. Specification of an indirect function having the properties of received theory,
2. Using the envelope theorem to derive estimable behavioral responses (more than one).

Generally, the method carries a further characteristic:

3. Using properties of indirect functions and behavior to infer something about the properties of the direct objective (F) or (G).

III. The Comparative Advantage of Dual Methods

The purpose of this section is to give a brief overview of the progress and direction of dual methods and assess them.

A Short History of Production Time

One of the earliest research objectives in the econometric approach to production economics was the characterization of output supply and factor demand responses or elasticities. This appears to be the central focus of the excellent work of Nerlove and other recent works (e.g., Shumway). Of particular early concern, were attempts to incorporate frictions in adjustment and price expectations. Models were developed which were econometrically viable and seemed to improve on previous methods [Nerlove (1958), Nerlove (1965), Muth). Attempts were made to recognize that government programs altered the price incentives and government entered in a serious way into the specification of production response models (Houck and Ryan). Irreversibilities and dynamics of supplies (fixity of assets) were also discussed at length (Johnson). Also, Nerlove using a self-dual form made popular specification of cost functions which are fully consistent with cost minimization (Nerlove). Lau and Yotopolous used the profit function to integrate supply and demand estimation and examine economic efficiency. Mundlak was a major contributor in multiproduct and panel specification.

Refinements followed in a variety of directions. These include attempts to: specify the dynamics more generally (Epstein), more fully specify or relax the structure of production under various structural restrictions (Shephard, Diewert; Christensen,Jorgenson, and Lau; Ball,Varian), and to incorporate risk response (e.g., Just, Coyle).

Some Questions

It seems important to mention some of the central production economic questions being asked.

q1. What are the magnitude of price elasticities of supply and demand? Interest here is motivated generally by policy questions (trade, commodity and environmental)?

q2. Which firms and/or technics of production are technically and price efficient and why?

q3. What are the productivity gains in agriculture? Often the policy issues here relate to public research and development expenditures. Ancillary issues are the cause of the productivity gains. For example, do prices "cause" innovation?

q4. How do policies affect the structure of agriculture? Policies affect risk, and economies of scope and scale, and market structure.

q5. What are the characteristics of demands for policies? For example, crop insurance, CRP, and other programs require active enrollments for program participation.

q6. How do policies affect producer and factor rents or resource prices?

q7. How do production practices lead to environmental degradation?

In addition, there were procedural questions associated with aggregation of crops, functional forms, fixity of factors, and tests of various parts of received theory (Chambers, Shumway).

Duality and Comparative Advantage

During the sixties and earlier, most attempts to study the questions listed above used single equation reduced form approaches. Indeed my short account of developments suggests that most innovations addressed new problems in either primal or single-equation reduced forms. These were refined and extended in the dual framework. Under the adopted definition, single equation reduced forms do not fall under the dual approach. This approach has to a large extent been supplanted by the dual complete systems approach (many call such equations ad hoc). It seems that one now sees in the literature so-called ad hoc reduced form (single equation) demands or supplies or else dual complete systems. Primal systems are rare in most areas of agricultural economics but seem less rare in other fields of economics (e.g., labor,finance). If one takes a survivorship approach to analyzing efficiency, one might forecast the virtual demise of primal single-equation or systems approaches in agricultural economics.

It is interesting to note that virtually all of the basic theory on duality, works from the primal to the dual. This tells us that the most natural form for problem specification is initially in decision space. Then, assumptions on primal structure generate the dual structure. The latter usually takes some thought and proof but often does not shed light on the basic economic issues.[3] In spite of the primal being the most natural starting place, it is not the most convenient place to end up econometrically. Clear advantages to dual methods include:

d1. dependent variables are generally decision variables of interest (or simple transformations). That is, reduced forms are more immediate and easily to specify as linear in parameters even in second order flexible characterizations.

d2. independent variables are prices. In inelastic agriculture, prices tend to vary much more than their respective quantities thus perhaps reducing standard errors of estimates. In some cases, it might be easier to defend prices being orthogonal to econometric errors.

d3. classic primal problems which are nondifferentiable yield differentiable dual forms. Thus calculus is saved.

d4. rent or welfare analysis is particularly convenient with dual methods.

d5. In some cases, input quantities or output supplies need not be measured. Log-linear dual profit function models require only input prices and expenditures or revenues (see (5)).

d6. Some would argue that duality rids itself of the portion of technology which is economically uninteresting. More avid dualists might argue that it rids the profession from ever explicitly representing the feasible set T.

Reviewing the problems of interest and general advantages of dual systems, it seems clear that each of the problems presented above can be appropriately studied in a dual framework. Because the message of duality is that under fairly weak conditions, the primal and dual contain the same economic information, the issue as to comparative advantage does not deal with feasibility of an approach but to finite comparative costs. Regarding the first question (q1) supply and demand elasticities can be much more conveniently modeled within a dual system framework. Presumably, standard errors of slopes and elasticities are more easily obtained in the dual approach (though they are often not presented). However, the advantages of the system approach in elasticity estimation seems less well documented. If one wishes to trade increased parametrization of risk, dynamics, or market integration in a single equation framework with increased parametrization due to a dual system, I know of no study which substantiates the dual system is to be preferred. Thus, it isn't apparent that the Profession had a sound basis to reject the advances early in early single equation representations as it embraced dual systems.[4] This is an argument, not fundamentally about duality but about theoretically extensive systems versus empirically extensive single-equation representations.

A typical application of the dual econometric approach using non-experimental data, uses a feasible restricted GLS approach to estimation. Dual approaches typically have prices as right hand side variables, whereas primal approaches have quantities on the right hand side. If prices are more likely to be orthogonal to econometric errors, this might give a gentlemen's nod to the dual approach. For most cases in agriculture, it seems more likely that prices are "more exogenous" than quantities. One exception to this conclusion may be inputs and outputs which are contracted.

Studies of efficiency (q2) seem to be produced at least cost with dual methods. The very notion of economic efficiency encompasses within it a norm. That norm is virtually always some indirect objective function (F^*). Either F/F^* or it's reciprocal are common indices of efficiency. Generally F is directly measured and F^* must be estimated parametrically or nonparametrically (often as an envelope rather than an average, e.g. Fried, Lovell, and Schmidt).

I can think of no separate reason why productivity or innovation (q3) should be more conveniently measured using duality. It is by now firmly rooted in both approaches. It would seem that the measurement of Solow residuals based upon Cobb-Douglas production functions remain extremely common in other fields of economics.

For q4 ,q5 and q7, I see no clear separate advantage to dual methods. Ideas of policy demand, environmental effects and regulation, and economies of scale and scope are often most clearly developed within a cost function framework (Baumol, Panzer, and Willig). Yet, many issues involving risk, non-competitive and non-convex economies seem to be less natural in a dual framework (Romer).

Regarding q6, welfare measurement using reduced form demands and supplies preceded adoption of duality. However, it seems that dual methods have a clear comparative advantage. Quasi-rents are simply and directed measured as the change in restricted profit. No integrals need to be evaluated. Standard errors can be obtained by conventional or delta methods.

With all of these compelling reasons to adopt dual applied methods, is there any reason left to commend primal methods? First, some problems are still adequately modeled in a parsimonious primal framework where more is initially known about the structure of the primal problem than it's dual or "inverse". Production work on nutrients is typical of this type (Lanzer, E. and Q. Paris). To be sure, the structure of profit or cost can be worked out in nearly all of these cases. Some problems are still very complicated in primal form and the complete structure of the profit or cost function is not apparent. That is, it is easy to lose information going from either method to the other. If one imposes structure on the primal, then it is important to specify the dual with necessary and sufficient conditions to generate the primal structure. A good example of this is the current and past practice in virtually all empirical dual models to reduce the dimensionality of T to the production possibility curve. Without measuring and imposing the appropriate primal information, one can only recover the production possibility curve.

Second (clearly related to the first), the researcher may know more about the appropriate stochastic specification of the primal. Thus, at the very least one may need to work primal stochastic specifications through to the dual if appropriate dual stochastic specifications are used (McElroy).

Third, implicit nonlinear econometric methods (generalized methods of moments) are increasingly available and convenient (at least in theory). This obviates the necessity of having decision variables as independent variables in a regression context. Indeed, primal systems are ubiquitous in other economic settings (Burgess). Thoughtful primal systems are established and first order conditions or Euler conditions are estimated.

Progress of biological and agronomic models may imply, sometimes in the present but likely in the future, that we will have much better knowledge of parameters of the production elasticity than the elasticity of demand (supply) with respect to an input (output) price. It is always surprising to me how few quantitative stylized facts about behavior we have developed in agricultural economics. Perhaps science does a comparably poor job of representing technology.

Finally, in this age of relatively unconstrained computing, it is an easy matter to simulate or solve for demand, supply, and welfare responses even for the primal form in (2) and (3).

The discussion above suggests that dual vs. primal is not the most interesting of methodological questions. Both are appropriate tools and under particular yet mostly reasonable ssumptions yield similar theoretical information. However, in empirical work there may be vast differences in parameter estimates due mainly to econometric issues. The widespread adoption of dual methods suggests that it produces ways of organizing and answering questions such as those posed above at a lower cost than primal approaches when the problems are rather standard. Apparently, use of these dual methods primarily under static profit maximization with flexible functional forms replaced much of the reduced form demand/supply estimation with linear and nonlinear generalizations of the static model but without integrability restrictions. That is, the search for dynamics, integration of production with other relevant markets, risk, policy and other innovations to specification was largely abandoned when dual methods were introduced. Only relatively recently have dual flexible form models added some of these "previously thought important" elements (e.g., Chambers and Vasavada, Fernandez-Cornejo, et al.)

IV. Some Ramblings

In this final section, I wish to raise a few issues which: (a) either seem useful but underexploited (b) which seem muddled (at least to me), or (c) I just want to complain about.

Data

It is always appropriate if not trite to complain about the economic data available. One hears this even from financial economists who arguably have the best data in the world. Many agricultural economists of my generation emerged into the Profession basking in the glory bestowed by Leontief: Agricultural economists were serious applied economists with outstanding data and commitment to data; "Official agricultural statistics are more complete, reliable, and systematic than those pertaining to any other major sector of our economy". I have come to reject every aspect of that attribution. To my knowledge, there is no public use panel of data on U.S. production. Yes, some experiment stations (two?) have supported proprietary data bases but there is nothing generally and publically available as in virtually every other field of economics. There are good reasons why this is so but we as a Profession haven't seemed willing to pay much of a price for change. Thus, most publically available data are very cleverly constructed, highly aggregated, and few agricultural economists are well trained on data or data analysis as say labor economists are. Methodological and more substantive economic issues cannot be capably addressed.

Multicrop Production

My early research was on the nature of farm diversification in a portion of California. I had reasonably good data with which to study the various possible explanations for the production patterns which were in the data. Factor analysis on returns and patterns suggested that risk and intra-seasonal demands on factors were key determinants of diversification. A common pattern was cotton or alfalfa and nut production, a result of the second factor, timing of factor demands. This form of input non-jointness would be especially prevalent in mathematical programming approaches to farm or regional production models. I proposed a simple but perhaps useful way to think of such factors in the absence of measureable allocations. Perhaps they should be modeled as public inputs. Managerial labor or machinery at the optimum is allocated in such a way that the value of the marginal products across enterprises equals the opportunity cost of the lumpy labor (rental rate on machinery). This would imply that profit conditional on the stock of these inputs would exhibit non-jointness.

Another facet of multicrop production methodology which seems muddled is how to represent joint technology. Mittelhammer, Matulich and Bushaw argue that the single product implicit representation mapping to R (rank $\leq$ 1) is substantially deficit. This seems correct to me (and obviously to them). They go on to argue for a representation of technology having rank greater than one. Yet, it seems that virtually all so-called modern treatments of production characterize technology by the input or output distance function technical efficiency measures which are real-valued functions (Faire). It seems that some in the profession have difficulty accepting the notion that a scalar valued function of multiple outputs and inputs is generally as useful as explicit representations of rank greater than one (Just and Pope). Shouldn't a knowledge of inputs (assuming they can be determined) directly determine a vector and not a locus of outputs?

Conditional and Mixed Functions (C-span)

Early in our training, we are taught the advantages of ceteris paribus mental and empirical experiments: "hold some things fixed and study adjustments. These effects will often be unambiguous. Then, fold these ceteris paribus adjustments into a more general equilibrium setting (where as nearly as I can figure theory says little (Sonnenhcein)." Yet this reasonable perspective seems to have been abandoned in some respects in agricultural production studies.

There seems to have been a movement away from cost to profit function estimation in recent years in agricultural economics. I would characterize the dual cost system as:

$$x = C_r(y,r,.),$$

$$C = C(y,r,.),$$

and perhaps

$$p=C_y(y,r,.)$$

where x represents factor demands, y is output(s), r represent input prices, p is output price(s), and C is cost. When questions can be answered with only cost or conditional demands, they should be on the basis of robustness. Note that many of the questions in q1-q7 can be answered with cost alone. It strikes me that production economists studying manufacturing use the cost function much more than agricultural economists (Berndt and Christensen). The reasons are unclear to me.[5] One difficulty is that the first two equations cannot reveal directly the supply function. Because supply functions are often at the heart of policy questions, this seems like a substantial drawback to the use of cost. Yet, one can solve for the supply curve under monotonicity of marginal cost. Also, one might have more intuition or information about C than profit, π. There was a time in agricultural economics when the shape of the cost function was greatly debated and perhaps well understood. Hence, the mixed dual system adding price equals marginal cost might for some problems be a useful way of studying supply.

A second area where conditional functions seem to be of value is in the area of risk. Without doubt, there are or will be dual approaches to risk which are worth considering. In general, they will not have the elegance and generality that duality under certainty possesses because of the simple conjugate pairs nature of certainty problems. This allows dual methods great freedom because they are not contaminated directly with primal information. This is often possible for conditional problems under risk as well. For example, under output price uncertainty, ex ante cost minimization is well-known to be consistent with expected utility. Thus, the cost function can be used to infer or test much without resorting to the specification of supply behavior which will depend on the distribution of price and risk averse specifications. In many cases, simple cost functions also exist under production uncertainty. One such class consists of multiplicative errors of the form $y=f(x)\epsilon$. Any ex ante cost function minimizing cost subject to a moment of y (assuming convexity of the input requirement set) will be consistent with expected utility. Hence , cost functions can be usefully used and exploited to test for returns to scale, substitution possibilities, productivity, measure deviation from cost minimization behavior, and more. Once the cost function is successfully "discovered", then the second stage research activity can be performed to find the relationship between variations in the moment of y and price (e.g., expected supply as a function of output price and input price).

An alternative approach is to condition the problem more generally. It is reasonable in most cases to represent expected utility as depending on the moments of profit (Antle). Hence, expected utility of wealth might be represented by $E[U(W)] = L(w, E(\pi), v(\pi), ..., m(\pi))$ where the arguments are successive central moments of π ending with m and w is initial wealth. Assuming that moments up to v are relevant for convenience, the relevant first stage or conditional problem is:

$$\mathrm{Max}_x \; \{E(\pi) \,|\, v \geq v(\pi(x)) = h(x)\}$$

presuming that h(x) is quasi-concave. The second stage maximizes L over v.

The first stage conditional problem is the standard portfolio problem. It can be shown that $F^* = E(\pi^*)$ has the standard properties of the profit function under certainty. This is illustrated most simply with the heteroskedastic production function $y = f(x) + h(x)\epsilon$, $E(\epsilon) = 0$. Assuming only production uncertainty, the first stage problem can be written:

$$\mathrm{Max} \; \{E(\pi) \,|\, v \geq h(x)\} = F^*(p,r,v)$$

where v is proportional to the standard deviation of output given x. In this case, it follows from Result 2 that F^* is convex and linearly homogeneous in prices (p,r). Further, the envelope theorem implies that the gradient of F^* yields factor demands and expected supplies. Expected profit can be profitably (excuse the pun) used to study several phenomenon (Pope and Just). Yet there seems to have been little attention paid to this conditional problem. Partially this is so because neither v nor $E(\pi)$ are directly observable. However, one can't admit uncertainty into the model without expecting that more complicated inferences must be made.

Once the expected profit function is well understood, then the researcher can expand to research the optimal choice of v which will depend on the nature of the distribution of ϵ and risk preferences.

Aggregation

Most of the aggregation literature is either negative to the practice of economics period, or else, it attempts to rationalize the use of existing aggregate indices in aggregate work. Heterogeneity seems particularly important and common in most settings which we study. There are a host of ways to positively deal with heterogeneity (Theil,Ringwald,Stoker,Gorman). I do not think that the comparative strengths of the methods have been appropriately studied. In any case, it seems that we should know as much as possible about how aggregate data, e.g., prices and quantities are constructed. When this is done perhaps we will use either dual or primal techniques to better advantage. To illustrate, an issue which I find of concern is the practice of defining an aggregate U.S. price or price times yield for some commodity and then using some measure of variance of this average as relevant for aggregate supply. This can substantially bias econometric results. They can even cause fundamentally fallacious interpretations and inferences. There are some simple constructive ways to be logically consistent and more empirically relevant short of running many micro regressions (Pope,1990).

In summary, the distribution of things matter. They matter to policy makers, they matter for normative reasons as well as for the positive measurement.

Relative Responses

There are some problems which can be studied without imposing a great deal of structure yet are amenable to standard statistical tests. Some of these problems effectively use ratios of quantities of factor demands or output supplies. Indeed, much trade theory is presented in terms of quantity ratios. To illustrate, suppose that rent only adjusts in response to a price change. Then the changes in rent in response to an input price change is the ratio of the input to land in a Ricardian model. Tests for homotheticity under risk often can be couched as the ratios of inputs being independent of the distribution of output price parameters. Often only one or two ratios are of interest, particularly to reject the hypothesis. Tests of indirect separability can be stated as ratios of inputs or outputs being independent of particular prices (Lau). It appears to me that tests need not be saddled with the additional structure of integrability (to some problem) to be interesting. One ratio, robustly estimated, may be sufficient to reject the hypothesis.

Flexible Functional Forms

Much has been discussed about the wisdom of using functional forms with do not unduly constrain results. In earlier work, ϵ in (3) was considered a constant (e.g., Cobb-Douglas) and now it might be a translog making ϵ depend on x. An issue which concerns issues of aggregation and stabilization is the curvature of demands or supplies. Thus, there is a need for explorations which involve third order properties of profit or cost functions.

A second issue is the nature of the approximation. Typical earlier supply response models used first order models of behavior:Cobb-Douglas or linear models. Regarding crops, often acreage response equations were estimated, and yield responses were estimated. From these two components, one could attempt to infer the supply of each enterprise. This process never implied that supplies were first order approximations. In contrast, many so-called second order approximations to efficient production (the production possibility frontier) use no more parameters. To put a different (slightly) issue another way, a second order approximation to enterprise specific profit functions assuming non-jointness is far less parsimonious than a second order approximation to whole farm (economy) profit. Whether this is a substantive issue is unclear. It seems that the essential point is that we must be careful to specify what we mean by the technology which profit recovers.

IV. Conclusions

Duality has been a tremendous advance in economics. As McFadden has pointed out, the theory frees us from difficult constructive arguments in specifying indirect functions. There are a few production problems where dual methods are clearly superior e.g., welfare comparisons and supply/demand estimation. There are another set of problems where dual methods may not be as natural or useful (some non-linear models or non-conjugate pairs problems). In general, the power of dual methods are diminished where specific primal information is required to specify the dual (moving away from Results 1 or 2). It seems that duality has been very useful in analyzing rather standard problems. The profession is advanced by novel problems, and effective ways of modeling these problems or less novel problems. This proceeds from the primal to the dual. Some problems currently seem not to be empirically amenable to dual system methods (e.g., limited dependent variable systems beyond a few variables). For others, the current value of the marginal product of dual methods may be low. A case can be made that dual methods have led to a very convenient way of organizing production data but for some time this seemed to take research a very different direction than it was previously taking. This may well have been for the better but there doesn't seem to be convincing evidence that dual methods led to more satisfying empirical answers to many of the questions.

Finally, I suggested some methodological issues that still seem a bit confused or under-exploited.

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ENDNOTES

1. These are not the strongest apparent sufficient conditions. F need not be free of w. Note that F linear in w is sufficient for Result 1 to hold.

2. I don't know of a good reference here. Pope and Chavas tried to unify the major notions of compensation for the theory of the firm under risk.

3. Symmetry conditions seem substantially easier using dual theory. For example, the fact that

$$\partial y/\partial r_i = -\partial x_i/\partial P$$

for all inputs (i=1,...,n) in the theory of the firm requires some dexterity with matrix algebra and Hessians in the primal approach (comparative statics). However, differentiability of the profit function implies this symmetry condition directly via the envelope theorem.

4. I don't mean to imply that there hasn't been applications involving risk, government programs, dynamics using dual methods. However, these did not occur very early in the analysis. Partially this was so because these methods weren't entirely worked out using dual methods.

5. One argument for using the profit function rather than the cost function is that output is likely correlated with regression errors. However, this does not mean that supply must be fully characterized for consistency.