HICKSIAN VS MARSHALLIAN WELFARE MEASURES:

WHY DO WE DO WHAT WE DO?

Julian M. Alston and Douglas M. Larson

Although they are closely related fields, environmental economists often use Hicksian "exact" welfare measures whereas agricultural economists typically use Marshallian surplus measures. Our general objective is to explore the determinants of that choice. We begin with an illustration of the bias in Marshallian measures, based on the linear model. Then we review the differences in applied welfare economics as conducted by environmental and agricultural economists, including differences in the types of data, policy problems, and parameter estimates, that might account for their different approaches. We show that correcting a Marshallian measure for income effects may involve a loss of precision in order to reduce bias. We then turn to a more specific analysis, using the mean squared error criterion to evaluate the choice between exact Hicksian and Marshallian measures based on the semilog model.

Errors in Marshallian Welfare Measures

Figure 1 represents a commodity market with an initial equilbrium price and quantity of P_0 and Q_0 determined by the intersection of supply (S) and ordinary demand (D) and, when an output subsidy of t per unit is imposed, the price and quantity are P_1 and Q_1. The Marshallian welfare measures are the change in *consumer's surplus* (CS = area P_0abP_1), the change in *producer's surplus* (PS = area P_0acd), the change in *taxpayer's surplus* (TS = - area P_1bcd), and the net change in *national surplus* (NS = $CS+PS+TS$ = - area abc). Assuming linear supply and demand, defining the elasticities of supply and demand evaluated at the new equilibrium as ϵ and η, respectively ($\eta < 0$), and defining $\tau = t/P_1$,

$$CS = P_1Q_1\frac{\tau\epsilon}{\epsilon-\eta}\left[1+\frac{1}{2}\frac{\tau\epsilon\eta}{\epsilon-\eta}\right]; \ PS = -P_1Q_1\frac{\tau\eta}{\epsilon-\eta}\left[1+\frac{1}{2}\frac{\tau\epsilon\eta}{\epsilon-\eta}\right]; \ TS = -\tau P_1Q_1; \ NS = \frac{1}{2}P_1Q_1\frac{\tau^2\epsilon\eta}{\epsilon-\eta}. \quad (1)$$

The corresponding Hicksian measures differ only in the measure of consumer welfare change. The *compensating variation* measure of consumer welfare change is measured off the Hicksian demand $h(u_0)$ holding utility at u_0 (associated with P_0, Q_0): CV = area $P_0ab'P_1$. The *equivalent variation* measure is taken off $h(u_1)$: EV = area $P_0a'bP_1$. Taking a linear approximation to $h(u_1)$ between a' and b,

$$EV \doteq P_1Q_1\frac{\tau\epsilon}{\epsilon-\eta}\left[1+\frac{1}{2}\frac{\tau\epsilon\eta^H}{\epsilon-\eta}\right]. \quad (2)$$

Thus, the error in the Marshallian measure of consumer welfare change is:

$$EV - CS \doteq \tfrac{1}{2} P_1 Q_1 \left[\frac{\tau\epsilon}{\epsilon - \eta} \right]^2 (\eta^H - \eta) = \tfrac{1}{2} P_1 Q_1 k \eta_Y \left[\frac{\tau\epsilon}{\epsilon - \eta} \right]^2, \tag{3}$$

where η^H is the Hicksian (compensated) price elasticity, η_Y is the income elasticity of demand for the good, $k = P_1 Q_1 / Y$ is the fraction of total income (Y), spent on the commodity, and we have used the Slutsky equation in elasticity form for $k\eta_Y = \eta^H - \eta$. Willig showed that consumer's surplus will provide a good approximation for small k or small η_Y. Hausman pointed out that the same error would be relatively big as a fraction of the deadweight loss. The deadweight loss estimates are related as:

$$NS^{CV} \doteq NS^{CS} \left[1 + \left[\frac{\epsilon\eta}{\epsilon - \eta} \right] k\eta_Y \right]; \qquad NS^{EV} \doteq NS^{CS} \left[1 - \left[\frac{\epsilon\eta}{\epsilon - \eta} \right] k\eta_Y \right]. \tag{4}$$

A parallel research-induced shift from S to S-t has consumer and producer welfare impacts identical to the subsidy but, since there is no offsetting taxpayer cost, the total welfare change is very different:

$$NS = P_1 Q_1 \tau \left[1 + \tfrac{1}{2} \frac{\tau\epsilon\eta}{\epsilon - \eta} \right]. \tag{5}$$

The Hicksian measure is equal to this Marshallian measure plus or minus the same error as before.

Table 1 shows the error in consumer's surplus for a 10 or 20% subsidy and for an equivalent research-induced supply shift ($P_1 Q_1 = 100$). For low values of either the income share ($k = 5\%$) or the income elasticity ($\eta_Y = 0.2$) the error is negligible (much less than 0.1%), compared with both the consumer welfare change and the net research benefits, and tolerable compared with the deadweight loss from a subsidy (less than 6.7%); it is large as a share of the deadweight cost of the subsidy but tolerable relative to consumer or national research benefits, even when $\eta_Y = 1$ and $k = 25\%$.

There may be small benefits from correcting the Marshallian measures for income effects. However, the costs are likely to be small, too, and there seems to be little justification for using a biased measure. In agricultural economics applications it must be felt that—as a reflection of the characteristics of the data, policy problem, the model being used, or the quality of measures of the parameters—either the costs of making the corrections are relatively high, or the benefits are relatively low, or both.

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Current Practice in Agricultural and Resource Economics

The three main types of applications of welfare analyses in environmental economics are: (a) analysis of large price changes to represent provision or removal of a valued private good related to an environmental amenity (e.g., recreational use of public lands), (b) changes in the quality of a private good which corresponds to changes in environmental quality (e.g., in relation to the demand for housing), and (c) changes in the quantity of a public good which is available for consumption. When Hicksian measures are used it is typically in the environmental economics literature. Marshallian surplus has been used almost exclusively in the three main types of agricultural economics applications: (a) analysis of commodity market distortions, especially those associated with domestic farm programs or border distortions for traded commodities, (b) the benefits and costs of research, and (c) political economy models of commodity programs or investments in agricultural R&D (in which measures of welfare impacts are data). The contrast between these two sets of literature involves differences in the types of questions being asked, the types of data available for the analysis, and the types of models being used.

Firstly, often agricultural economists analyze policies that introduce or modify market distortions (i.e., relatively small price changes) while environmental economists often study policies that may preclude, or make possible, certain types of uses of natural resources (i.e., large price changes, to represent provision or removal of a good). Secondly, agricultural economists often analyze aggregated time series data, whereas environmental economists most often use cross-section data. Goodness of fit is often very high in the former and very low in the latter. This has led to differences in problems emphasized (e.g., aggregation in agricultural economics; precision of estimates in environmental economics). Thirdly, the study of commodity markets has led to more interest in the implications of multi-market measures and the significance of partial versus general equilibrium. In contrast, environmental economists study non-marketed commodities for which prices are non-existent and for which data are not systematically collected on likely substitutes (dictating a single market approach).

A fourth area of similarity and difference is the study of non-price changes and measuring welfare changes in related markets. In commodity studies, such changes involve measuring areas between curves as they shift, and it is necessity of the relevant input or output to production that permits these areas to be taken as the full benefits of the parameter change (Just, Hueth, and Schmitz). The analogous condition in environmental economics is "weak complementarity" of the non-price argument with a set of private market goods. Input or output necessity is often reasonable in agricultural economics applications, but not appropriate in environmental economics problems where "nonuse value" is suspected to be large.

Differences such as these in the problems being addressed can account for the emphasis on different measurement issues in environmental and agricultural economics applications of welfare economics methods. Still, a question which we often encounter in casual discussions with applied researchers in both fields is: "Given my estimated function(s), which welfare measure should be calculated, the Hicksian or Marshallian measure?"

Mean Squared Error Comparison of Marshallian and Hicksian Welfare Measures

There seems to be little disagreement that in principle Hicksian measures should be used, as they are the defensible measures based on the Kaldor-Hicks compensation criterion. Nevertheless, the Hicksian measures are rarely used, and when they are it is typically in the environmental economics applications. The rationale for consumer's surplus often heard in informal discussions is twofold. First, Willig's results suggest that, for price changes, errors in approximating compensating variation by consumer's surplus are likely to be small in many empirical situations. Second, since the divergence of Hicksian from Marshallian demands is due to an income effect which is measured imprecisely, more "error" (i.e., statistical noise) is introduced into the Hicksian measure and this may outweigh the advantages of reduced bias. Thus it is natural to think of selection of a welfare measure in terms of its mean squared error.

This approach recognizes that it may be desirable in some contexts not to be a Hicksian purist, because of difficulties with measuring income, and the income slope, with adequate precision. It also

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has the added advantage of somewhat greater generality than previous approaches. Definitive comparisons can be made based only on three random variables: consumer's surplus, the change in quantity demanded implied by a given policy, and the income coefficient. Correct functional form is a maintained hypothesis. However, it is not essential to the analysis to assume specific parameterizations of other demand covariates, as has been necessary in previous simulation work.

Consider the semilog demand, written for individual i as

$$x_i = e^{\alpha z_i + \beta p_i + \delta m_i}, \tag{6}$$

where p_i is own price, m_i is income, and z_i is a vector of nonprice shifters that includes the intercept, prices of substitutes, while α, β, and δ are corresponding parameters. For a change in p or z, the consumer's surplus measure is $CS_i = -\Delta x_i/\beta$, where $\Delta x_i \equiv x_{1i} - x_{0i}$ is the induced change in quantity. The corresponding formula for compensating variation is $CV_i = (1/\delta) \ln(1 + \delta CS_i)$, which can be used to compute both the point estimate and the precision of CV_i, given knowledge of the first two moments of δ and CS_i and their covariance.

Suppose the researcher has estimated CS_i and its variance, σ_{CS}^2. From the results above, one can calculate $Bias \equiv CS - CV$, and obtain the mean squared error:

$$MSE_M \equiv \sigma_{CS}^2 + Bias^2. \tag{7}$$

Letting $CV = f(CS,\delta)$, a first-order Taylor's series approximation to $var(CV)$ is given by:

$$\sigma_{CV}^2 \doteq (f_{CS})^2 \sigma_{CS}^2 + 2 f_{CS} f_\delta \sigma_{CS,\delta} + (f_\delta)^2 \sigma_\delta^2, \tag{8}$$

where the subscripts on f refer to partial derivatives. Then, defining $\sigma_{CS,\delta} \equiv \rho \sigma_{CS} \sigma_\delta$ (where ρ is the correlation of estimates of CS and δ, and σ_{CS} and σ_δ are their respective standard errors), yields

$$MSE_H \doteq \frac{\sigma_{CS}^2}{(1+\delta CS)^2} + \frac{2\rho\sigma_{CS}}{(1+\delta CS)} \frac{Bias^*}{t_\delta} + \frac{Bias^{*2}}{t_\delta^2}, \tag{9}$$

where $Bias^* \equiv CS/(1+\delta CS) - CV$, and $t_\delta \equiv \delta/\sigma_\delta$ is the Student's t-statistic for the hypothesis that the

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income coefficient is zero. Thus the precision of the Hicksian compensating variation—based on measures of δ and CS, their precision (σ_{CS} and t_δ), and their correlation ρ—can be compared with the MSE for the Marshallian measure in terms of the same variables.

We simulated comparisons for a range of settings for each parameter. The t-statistic on the income coefficient is intuitive and we chose $t_\delta = 0.5$, 1.0, and 2.0. A range of plausible values for income slope are obtained from $\delta = \eta_Y/m^0$ with income elasticities of $\eta_Y = 0.5$, 1.0, and 1.5, and reference income $m^0 = \$25,000$/year, resulting in $\delta = 2 \cdot 10^{-5}$, $4 \cdot 10^{-5}$, and $6 \cdot 10^{-5}$. Similarly, by selecting the fraction of income that consumer's surplus represents (we considered values of $\phi = .01$, $.1$, and $.5$), we chose corresponding values of CS of $\$250$, $\$2,500$, and $\$12,500$. The range of σ_{CS}^2 is generated from CS by considering values for the coefficient of variation for CS (V_{CS}). We chose values 0.1, 0.8, and 4.0 for V_{CS} (see Kling). Finally, we allow ρ to take the extreme values -1, 0, and $+1$.

Table 2 presents a subset of the comparisons of MSE_M and MSE_H for the ranges of ϕ, t_δ, V_{CS}, and ρ just noted, with $\eta_Y = 1$ (Alston and Larson provide more detailed results). Three cases, representing three levels of ϕ, are presented. The bias CV -CS is constant for each case. Reading across the table, the three values for V_{CS} (.1, .8, and 4.0) with corresponding value of σ_{CS}^2 are given, along with the root mean squared error ($RMSE$) for the Marshallian consumer's surplus. Then, for each of the values of t_δ, the $RMSE$ for the Hicksian measure is calculated, along with the percentage change in MSE that comes from using the Hicksian instead of the Marshallian measure. The three Hicksian measures for each Marshallian CS correspond to $\rho = -1$, 0, and $+1$. The patterns of simulation results are broadly consistent with intuition, yet contain some surprises. For a CS change relatively small in relation to income ($\phi \leq .01$, say), the MSE's are quite comparable in most cases. Because the bias is fairly small, the $RMSE$ for the Marshallian measure is virtually indistinguishable from the variance of CS, while the $RMSE$ is slightly smaller. The differences between the Hicksian and Marshallian measures are more pronounced as the welfare area increases (going from *Case I* to *Case III*).

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The answer to the question of which *MSE* is smaller depends on both ρ and how precisely *CS* and the income slope δ are measured. Not surprisingly, the Marshallian measure has smaller *MSE* (the % Difference is positive) nearly always when *CS* is measured very precisely ($V_{CS} = 0.1$) and δ is measured very imprecisely ($t_\delta = 0.5$). This is also the case where the impact of ρ is greatest on the difference in *MSE*'s. However, the Hicksian measure consistently has smaller *MSE*, regardless of ρ, when either the precision of *CS* decreases or the precision of δ increases relative to this extreme case. As an example, when V_{CS} is 0.8, the Hicksian *MSE* is smaller, even when the Student's t on the income coefficient is insignificant (e.g., $t_\delta = 0.5$ or 1.0), for virtually all values of ρ. When t_δ is 1.0 or better and V_{CS} is 0.8 or higher, the Hicksian *MSE* is smaller regardless of ρ. The magnitude of the difference increases with income elasticity and the magnitude of *CS* relative to income.

The implication is that one of the standard rationales for using Marshallian welfare measures may be weaker than generally expected: calculating the Hicksian measure does not necessarily introduce a lot of noise into the welfare estimate. On the contrary, unless *CS* is very tightly measured, or the income coefficient is very poorly measured, one gains in terms of both reduced bias and reduced variance by using the Hicksian measure, at least for the case of willingness to pay measures using the common semilog demand form. Why does this occur? Since *CV* is a concave transformation of *CS* when the income slope is positive, for willingness to pay measures (i.e., for positive *CS* and *CV*) the first term in (9) is less than σ_{CS}^2 since the denominator (the Jacobian of the *CV-CS* transformation) is less than one. Thus, for fixed δ, using *CV* instead of *CS* reduces variance. The income slope in fact is not fixed, and the second and third terms on the right side of (9) account for the additions to variance due to random δ, which depends on the correlation between δ and *CS*. On balance, however, the first term dominates across a fairly wide range of the parameter values we simulated.

These findings for willingness to pay measures are essentially reversed when willingness to accept measures (*EV* for a price decline or *CV* for a price increase, for example) are considered. The reason

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again is straightforward: from (9), the denominator of the first term is less than unity, so that even for fixed δ, the transformation from CS to CV is variance-increasing. Further interesting contradictions arise in the comparisons of mean squared errors of Marshallian and Hicksian measures of deadweight loss.

Some Concluding Remarks

In applied welfare analysis, in both agricultural and environmental economics, a common question at a practical level remains one of whether it is a good idea to use a Marshallian (biased) or a Hicksian (unbiased) measure of welfare change. This can be viewed as a tradeoff of bias and precision: when the (small) extra effort is taken to calculate the Hicksian measure from the Marshallian one, bias is reduced but variance may increase.

Using the semilog model, when consumer's surplus is a small fraction of the total budget (e.g., 1% or less), there is little difference in mean squared errors. However, the Hicksian measure of consumer welfare has a smaller mean squared error even when the income coefficient is not measured with high precision. This improvement in mean squared error increases as the precision of the income coefficient increases, as the precision of consumer's surplus decreases, and as the size of the welfare area increases. The question of which measure to use for deadweight loss calculations in practice is less clear.

These findings are suggestive rather than definitive, because much remains to be done to research the bias-variance tradeoff more fully. In particular we have explored the tradeoff only for a single commonly-used demand specification under the maintained hypothesis that it is the correct functional form, and we have ignored the small-sample bias in the estimate of CV obtained from the model. While the curvature of the CS-CV transformation is likely to be an important determinant of which measure (Hicksian or Marshallian) has the greater mean-squared error in other cases, we cannot say that the mean-squared error criterion always will favor one welfare measure over the other. The mean-squared error approach is quite useful, though, in identifying the combinations of relative magnitudes and precisions of consumers surplus and the income slope that will tend to favor one measure or the other in practice.

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Table 1: **Approximate Percentage Errors Relative to Equivalent Variation in Marshallian Measures of Welfare Impacts of a Subsidy and a Research-induced Supply Shift**

Subsidy Rate (τ)	Income Elasticity (η_Y)	Income Share (k)	Error EV-CS	Error as a Percentage of Marshallian		
				Deadweight Loss	Consumer Surplus	Research Benefits
0.100	0.200	0.050	0.002	-1.333	0.034	0.023
		0.150	0.007	-4.000	0.102	0.068
		0.250	0.011	-6.667	0.169	0.113
	0.600	0.050	0.007	-4.000	0.102	0.068
		0.150	0.020	-12.000	0.305	0.203
		0.250	0.033	-20.000	0.508	0.339
	1.000	0.050	0.011	-6.667	0.169	0.113
		0.150	0.033	-20.000	0.508	0.339
		0.250	0.056	-33.333	0.847	0.565
0.200	0.200	0.050	0.009	-1.333	0.069	0.043
		0.150	0.027	-4.000	0.207	0.129
		0.250	0.044	-6.667	0.345	0.215
	0.600	0.050	0.027	-4.000	0.207	0.129
		0.150	0.080	-12.000	0.621	0.387
		0.250	0.133	-20.000	1.034	0.645
	1.000	0.050	0.044	-6.667	0.345	0.215
		0.150	0.133	-20.000	1.034	0.645
		0.250	0.222	-33.333	1.724	1.075

The research benefits from a τ percent shift of supply are calculated as being equivalent to the consumer's surplus plus the producer's surplus from a τ percent subsidy.

Table 2.   RMSEs of Hicksian (CV) and Marshallian (CS) Welfare Measures for an Exogenous Price Change, Semilog Demand With Varying Precision in Measuring CS and δ. (m=$25,000/year, and $\eta_Y = 1.0$).

			Precision in Measuring the Income Slope					
			$t_\delta=0.5$		$t_\delta=1.0$		$t_\delta=2.0$	
Precision of Measuring CS								
		RMSE	RMSE	% diff.	RMSE	% diff.	RMSE	% diff.
V_{CS}	σ^2_{CS}	Marsh	Hicks	in MSE	Hicks	in MSE	Hicks	in MSE
Case I:	*bias=1.2417;*	*CS=$250/year*						
.10	25.	25.	22.3	-20.7	23.5	-11.7	24.1	-7.0
.10	25.	25.	24.9	-1.2	24.8	-2.0	24.8	-2.2
.10	25.	25.	27.2	18.2	26.0	7.8	25.4	2.7
.80	200.	200.	195.6	-4.4	196.8	-3.2	197.4	-2.6
.80	200.	200.	198.0	-2.0	198.0	-2.0	198.0	-2.0
.80	200.	200.	200.5	.5	199.3	-.7	198.6	-1.4
4.0	1000.	1000.	987.6	-2.5	988.9	-2.2	989.5	-2.1
4.0	1000.	1000.	990.1	-2.0	990.1	-2.0	990.1	-2.0
4.0	1000.	1000.	992.6	-1.5	991.3	-1.7	990.7	-1.8
Case II:	*bias=117.25*	*CS=$2,500/year*						
.10	250.	276.	7.2	-99.9	117.3	-81.9	172.3	-61.0
.10	250.	276.	316.4	31.2	252.5	-16.3	233.8	-28.2
.10	250.	276.	447.3	162.0	337.3	49.2	282.3	4.5
.80	2000.	2003.	1598.	-36.3	1708.	-27.3	1763.	-22.5
.80	2000.	2003.	1831.	-16.4	1822.	-17.3	1819.	-17.5
.80	2000.	2003.	2038.	3.5	1928.	-7.4	1873.	-12.5
4.0	10000.	10001.	8871.	-21.3	8981.	-19.3	9036.	-18.3
4.0	10000.	10001.	9094.	-17.3	9092.	-17.3	9091.	-17.3
4.0	10000.	10001.	9311.	-13.3	9201.	-15.3	9146.	-16.3

Case III: bias=2363.4 CS=$12,500/year

.10	1250.	2674.	2773.	7.6	970.	-86.8	68.	-99.9
.10	1250.	2674.	3702.	91.6	1987.	-44.7	1228.	-78.9
.10	1250.	2674.	4440.	175.0	2637.	-2.7	1735.	-57.8
.80	10000.	10275.	3060.	-91.1	4863.	-77.5	5765.	-68.5
.80	10000.	10275.	7580.	-45.5	6906.	-54.8	6727.	-57.1
.80	10000.	10275.	10273.	-.0	8470.	-32.0	7568.	-45.7
4.0	50000.	50056.	29727.	-64.7	31530.	-60.3	32432.	-58.0
4.0	50000.	50056.	33528.	-55.1	33382.	-55.5	33346.	-55.6
4.0	50000.	50056.	36940.	-45.5	35137.	-50.7	34235.	-53.2

Figure 1: Price, Quantity and Welfare Impacts of an Output Subsidy (or R&D) in a Non-traded Commodity Market