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Admissible Conjectures and Consistent Conjectures

Abstract

Firms' conjectures are consistent if their *ex post* behavior rationalizes their *ex ante* beliefs. Admissible conjectures are those that satisfy the necessary conditions for consistency. Competition is inadmissible unless aggregate output is stationary. Relaxing this restriction, admissibility eliminates Cournot behavior and constrains conduct to be collusive.

Holloway, Garth J.

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Key words: consistent conjectures, admissible conjectures, collusive behavior.

Admissible Conjectures

by

Garth J. Holloway*

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Admissible Conjectures

This paper derives hitherto unrecognized restrictions on a popularly-invoked framework in industrial organization—the conjectural-variations model of oligopoly. The conjectural-variations model has provided impetus for a large number of empirical investigations of noncompetitive behavior. Although it has been the subject of some well-merited criticism (e.g., Dixit),[1] it continues to provide an attractive framework for depicting departures from price-taking behavior. In agriculture the model has been applied repeatedly, primarily to assess departures from competitive pricing in the food industries. These applications have spawned a rather extensive literature that attests to the model's popularity. Examples of applications can be found in Gollop and Roberts, Sumner, Lopez, Roberts, Sullivan, Schroeter, Holloway, Schroeter and Azzam, Azzam and Pagoulatos, Azzam, Durham and Sexton, Wann and Sexton, and Chen and Lent.[2]

An important criticism of the conjectural-variations model stems from the observation that, in general, the *ex ante* conjectures of firms are not realized *ex post* (Fellner). Consequently, the model affords a degree of irrationality in firm behavior that one believes would not persist in long-run equilibrium. It follows that a topic of importance—one of shared significance to both conceptual and empirical practitioners—is the set of conditions under which the actual behaviors of the agents in question are consistent with those predicated by the model. In empirical investigations this question can be resolved through the application of various statistical criteria. In conceptual analyses the criteria are generally less clear.

One notion of rationality that has gained general acceptance is the concept of *consistent conjectures*. A firm's conjecture is deemed consistent when its *ex post* behavior—as determined by the comparative-static properties of the initial equilibrium—is consistent with its *ex ante* beliefs. This well-defined concept has led to an extensive literature (e.g., Laitner; Bresnahan, 1981; Perry; Boyer and Moreaux; Kamien and Schwartz; Daughety; Makowski) that examines the conditions under which particular conjectures are consistent. In general, the results depend on the values of various structural parameters, including the elasticity of demand for the product, the number of incumbent firms in the market, and the ease of entry and exit as depicted by the size of any fixed costs.

The authors of these papers consider conditions that are both necessary and sufficient for consistency. In this paper I propose a preliminary set of conditions that are necessary but not sufficient for consistency. I term conjectures that meet these criteria *admissible conjectures*. These conditions can be used to eliminate certain types of conjectures from the feasible domain of alternatives prior to considering the complete set of necessary and sufficient conditions. This is advantageous because the latter are frequently intractable and can be made more amenable to investigation by appealing to the admissibility criteria. As the analysis shows, we are able to rule out certain types of commonly invoked behaviors. In addition, satisfaction of the admissibility conditions illustrates the folly of invoking assumptions that are implicit in almost all of the empirical models that employ the conjectural-variation framework. I discuss these findings following the presentation of a standard model of industry equilibrium, some comparative statics, and the derivation of the main theorems of the paper.

Industry Equilibrium

Consider an industry producing aggregate output Q and facing price $p = D(Q,\sigma)$, where $D(\cdot)$ denotes a quantity-dependent schedule of prices and σ represents an exogenous variable that shifts demand. Let q_i $i \in \{1,2..n\}$ denote firms' output levels and define the corresponding cost functions by $c_i(q_i,\tau_i)$ $i \in \{1,2..n\}$, where τ_i $i \in \{1,2..n\}$ denote firm-specific, exogenous effects. Additionally, assume that firms form conjectures, $\theta_i \equiv (\partial Q(\cdot)/\partial q_i)(q_i/Q)$ $i \in \{1,2..n\}$, about how aggregate industry output responds to adjustments in the firms' own output levels. The parameters θ_i $i \in \{1,2..n\}$ denote the *conjectural elasticities* of each of the firms. They are defined over the unit interval and contain the particular reference points $\theta_i = 0$, corresponding to competitive behavior; $\theta_i = 1$, corresponding to monopolistc behavior; and $\theta_i = q_i/Q$, corresponding to Cournot behavior. Under this scenario we shall assume that an equilibrium exists for the $n+2$ endogenous variables—p, Q, and q_i $i \in \{1,2..n\}$—and that is defined by the $n+2$ equations:

$$p = D(Q,\sigma), \tag{1}$$

$$Q = \sum_{i=1}^{n} q_i, \tag{2}$$

$$p(1 + \theta_i/\eta) - \partial c_i(q_i,\tau_i)/\partial q_i \equiv g_i(q_i,\sigma,\tau_i) = 0, \qquad i = 1,2..n; \tag{3}$$

where η denotes the elasticity of demand for the product; $\partial c_i(q_i,\tau_i)/\partial q_i$ $i \in \{1,2..n\}$, denote firm's marginal costs, and $g_i(\cdot)$ $i \in \{1,2..n\}$ denote implicit supply schedules.

Comparative Statics

Allowing for displacements in each of the exogenous variables, we can express these, and the equilibrating adjustments in each of the endogenous variables as

$$\tilde{p} \;=\; \eta^{-1}\,\tilde{Q} \;+\; \nu\,\tilde{\sigma}, \tag{4}$$

$$\tilde{Q} \;=\; \sum_{i=1}^{n} (q_i/Q)\,\tilde{q}_i\,, \tag{5}$$

$$\xi_i\,\tilde{q}_i \;+\; \rho_i\,\tilde{\sigma} \;+\; \kappa_i\,\tilde{\tau}_i \;=\; 0, \qquad i = 1,2..n; \tag{6}$$

where tildes denote proportional changes (i.e., $\tilde{x} \equiv \Delta x/x$), $\nu \equiv (\partial D(\cdot)/\partial\sigma)(\sigma/D(\cdot))$ denotes the elasticity of the price schedule with respect to the demand shifter σ, and the parameters $\xi_i \equiv (\partial g_i(\cdot)/\partial q_i)q_i$, $\rho_i \equiv (\partial g_i(\cdot)/\partial\sigma)\sigma$, and $\kappa_i \equiv (\partial g_i(\cdot)/\partial\tau_i)\tau_t$, denote the respective effects on the first-order condition of changes in the arguments of the implicit supply schedules. In general, the parameters ξ_i and ρ_i are functions of the corresponding conjectural elasticity θ_i.

The specific structure of equations (4)-(6) allow us to solve recursively for the equilibrium movements in the price variable, upon solving initially for the movements in firm and aggregate output. The latter are the solutions to:

$$\Psi\,\tilde{q} \;=\; \Omega\,\tilde{\delta}\,, \tag{7}$$

where Ψ denotes a square matric of order $n+1$ containing the market-share and supply-response parameters $\{q_i/Q, \xi_i\}$ $i \in \{1,2..n\}$; Ω denotes a square array of the parameters ρ_i and κ_i $i \in \{1,2..n\}$; and $\tilde{q}$ and $\tilde{\delta}$ denote respectively the vectors $\tilde{q} \equiv (\tilde{Q}\,, \tilde{q}_1, \tilde{q}_2, .., \tilde{q}_n)^T$ and $\tilde{\delta} \equiv (\tilde{\sigma}\,, \tilde{\tau}_1, \tilde{\tau}_2, .., \tilde{\tau}_n)^T$.

As is customary, we shall assume local uniqueness of the original equilibrium and that a nontrivial solution exists. The latter we denote by:

$$\tilde{q} \;=\; \Psi^{-1}\,\Omega\,\tilde{\delta}\;. \tag{8}$$

Admissibility

We now consider values for the parameters $\theta_1, \theta_2 \ldots \theta_n$ that are *admissible*. Our objective is to rule out all modes of conduct that are inconsistent with the comparative-static properties of industry equilibrium. Our main results will utilize only the definition of admissibility, which we contrast with that of consistency:

Definition 1: Admissible Conjectures A set of conjectures $\{\theta_1, \theta_2 \ldots \theta_n\}$ is admissible if each firm's conjecture is consistent with the ratio of proportional changes in firm and industry output $\theta_i = \frac{\tilde{Q}}{\tilde{q}_i}$ $i = 1,2..n$ given that aggregate output adjusts according to $\tilde{Q} = \sum_{i=1}^{n} (q_i/Q)\, \tilde{q}_i$.

Definition 2: Consistent Conjectures A set of conjectures $\{\theta_1, \theta_2 \ldots \theta_n\}$ is consistent if it is admissible and, in addition, satisfies $\theta_i = \frac{\tilde{Q}(\theta_1, \theta_2 \ldots \theta_n)}{\tilde{q}_i(\theta_i)}$ $i = 1,2..n$ given $\tilde{Q}(\theta_1, \theta_2 \ldots \theta_n) = \sum_{i=1}^{n} (q_i/Q)\, \tilde{q}_i(\theta_i)$.

Remark: The functions $\tilde{Q}(\theta_1, \theta_2 \ldots \theta_n)$ and $\tilde{q}_i(\theta_i)$ $i = 1,2..n$ are the elements of the solution vector $\tilde{\mathbf{q}}$. The conjectures $\{\theta_1, \theta_2 \ldots \theta_n\}$ are embedded in the parameters $\{\xi_1, \xi_2 \ldots \xi_n\}$ and $\{\rho_1, \rho_2 \ldots \rho_n\}$ which appear in $\Psi^{-1}\Omega$ on the right-hand side of equations (8). The specific forms of the functions $\tilde{Q}(\theta_1, \theta_2 \ldots \theta_n)$ and $\tilde{q}_i(\theta_i)$ $i = 1,2..n$ result from the recursive structure of this system. A set of conjectures that satisfy Definition 2 must also satisfy Definition 1, but not the converse. Hence, admissibility is a necessary condition for consistency. The following result will prove useful in subsequent analysis.

> ***Theorem 1: The Inadmissibility of Competitive Conjectures*** *Competitive conjectures by any single firm are inadmissible unless aggregate output is stationary. If aggregate output adjusts then so too must the output levels of each of the firms. These adjustments must occur in the same direction.*

Proof: Competitive conjectures require $\{\theta_1, \theta_2 \ldots \theta_n\} = \{0, 0 \ldots 0\}$. For these conjectures to be admissible, either $\tilde{Q} = 0$ and $\tilde{q}_i \neq 0$ $i = 1,2..n$, or $\tilde{Q} \neq 0$ and $\tilde{q}_i \to \pm\infty$ $i = 1,2..n$. Suppose $\tilde{Q} = 0$ and $\tilde{q}_i \neq 0$ $i = 1,2..n$. This can occur if and only if there exist offsetting adjustments in some firms' quantities. This is permissible with $\tilde{Q} = 0$.

When $\tilde{Q} \neq 0$ the non-negative domains of the conjectural elasticities require firm and industry output to move in the same direction. *Q.E.D.*

Remark: Note that we have previously ruled out the trivial solution $\tilde{Q} = 0$ and $\tilde{q}_i = 0 \;\; i = 1,2..n$. In the unlikely case where this solution does exist the expression $\tilde{Q} / \tilde{q}_i$ tends to one as both $\tilde{Q}$ and $\tilde{q}_i$ tend to zero. In the absence of firm-specific quotas stationarity of aggregate output is unlikely. Although there may exist restrictions on technology that could ensure existence of this case, the more interesting scenario is one in which firm and aggregate outputs adjust. Several important observations follow from examining this case.

Theorem 2: The Admissibility of Collusive Conjectures *When firm and industry output adjust, the set of admissible conjectures $\{\theta_1, \theta_2 ... \theta_n\}$ must satisfy the condition:*

$$\prod_{i=1}^{n} -\theta_i \; + \; \sum_{i=1}^{n} (q_i/Q) \prod_{j \neq i}^{n} -\theta_j \; = \; 0.$$

Proof: With non-zero movements in firm and industry output we can manipulate the first n admissibility conditions to yield $\tilde{Q} = \theta_i \, \tilde{q}_i \;\; i=1,2...n$. Combining these with the aggregation condition yields an equation system of the form $\Sigma \, \tilde{\mathbf{q}} = \mathbf{0}$, where $\mathbf{0}$ denotes the n+1 null vector and Σ is defined by:

$$\Sigma \equiv \begin{pmatrix} 1 & -q_1/Q & -q_2/Q & .. & -q_n/Q \\ 1 & -\theta_1 & 0 & .. & 0 \\ 1 & 0 & -\theta_2 & .. & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 0 & 0 & .. & -\theta_n \end{pmatrix}.$$

In order for this system to yield nontrivial solutions in the vector $\tilde{\mathbf{q}}$ the determinant of this matrix must equal zero. This determinant is: $|\Sigma| \; \equiv \; \prod_{i=1}^{n} -\theta_i \; + \; \sum_{i=1}^{n} (q_i/Q) \prod_{j \neq i}^{n} -\theta_j$. *Q.E.D.*

Lemma 1: Monopoly Monopolistic behavior by all firms is admissible. If any n-1 of the firms colludes then so too must the n^{th} firm. The first observation is derived by setting $\theta_i = 1$ for all $i = 1,2...n$; the second follows from setting $\theta_i = 1$ for any n-1 subset of the n firms and deriving the constraint on the behavior of the n^{th} firm.

Lemma 2: Cournot Cournot behavior is inadmissible unless the equilibrium is such that it supports only a single firm. This follows from setting each of the conjectural elasticities equal to its corresponding market share; that is $\theta_i = q_i/Q$ $i = 1,2...n$, yielding $|\Sigma| = (1 - n) \prod_{i=1}^{n} (-q_i/Q)$.

Lemma 3: Collusion All firms' behaviors must be at least as collusive as the Cournot conjectures. This follows from observing that no single firm has a zero-valued conjectural elasticity. In this case we can multiply and divide by $-\theta_j$ in the expression $\prod_{j \neq i}^{n} -\theta_j$. This allows us to factor the term $\prod_{j=1}^{n} -\theta_j$, in the expression for the determinant of Σ. This yields $|\Sigma| = (1 - \sum_{i=1}^{n} \frac{q_i/Q}{\theta_i}) \times \prod_{j \neq i}^{n} -\theta_j$. The second term in this expression is nonzero by assumption. In the first term the expression $\sum_{i=1}^{n} \frac{q_i/Q}{\theta_i}$ can equal one if and only if each of the conjectural elasticities exceed the values of their market-share parameters.

Lemma 4: Homogeneity If beliefs are homogeneous then they are monopolistic. Setting $\theta_i = \theta$ $i=1,2...n$, factoring out the common term $(-\theta)^{n-1}$, and using the fact that $\sum_{i=1}^{n} (q_i/Q) = 1$, yields $|\Sigma| = (-\theta)^{n-1} (1 - \theta)$.

Lemma 5: Convergence The limit of a symmetric Cournot equilibrium converges to an admissible conjecture. Setting $\theta_i = q_i/Q = 1/n$ in the above expression yields $|\Sigma| = (-1/n)^{n-1} (1 - 1/n)$. This approaches zero as n increases without bound.

Discussion

Consistency between observed behavior and *ex ante* beliefs places restrictions on the admissible modes of conduct of firms in oligopoly. The objective of the above analysis has been to derive these restrictions explicitly. We can summarize the main findings through the set of restrictions: θ_i $i \in (q_i/Q, 1]$ $i = 1,2...n$. That is, the admissible domains of the conjectural elasticities are an open interval between the Cournot and the monopolistic conjectures. Whether these intervals do in fact contain conjectures that are consistent is another matter that should be considered.

Although a comprehensive examination of consistency lies outside the scope of the present work one should observe that the set of consistent conjectures are defined explicitly by a combination of the admissibility restrictions—implicit in the matrix Σ—and a set of additional restrictions implied by the solution vector $\tilde{\mathbf{q}}$. Substituting from equations (8) in the admissibility condition $\Sigma \tilde{\mathbf{q}} = \mathbf{0}$, we see that the set of consistent conjectures must provide solutions to the equations $\Sigma \Psi^{-1} \Omega \tilde{\delta} \equiv \Theta \tilde{\delta} = \mathbf{0}$. Since the conjectural elasticities enter this system in nonlinear fashion, deriving their precise intervals may be an intractable problem. However, we can place further restrictions on their domains by recalling that the matrix Σ is singular. This implies that the newly constructed parameter matrix Θ is also singular. It follows therefore, that we can proceed in a similar fashion to that used above, obtaining results from the zero-valued determinant $|\Theta|$. Thus, without retrieving the consistent conjectures explicitly it may be possible to further restrict the domains over which they may exist. More importantly however, it may be possible to derive insightful results about the types of technologies and demand structures that permit consistent conjectures to exist. Current work is proceeding with these intentions.

While the above exercise is theoretical in nature, a principal interest lies in the important consequences of these findings for empirical applications of conjectural variations in the food system. There are two key implications. The first stems from the homogeneity lemma in which the conjectures of each of the firms are constrained to be equal. This particular case is the one invoked in virtually all of the empirical studies that are cited. The finding that this case admits only the monopolistic conjecture also implies that, if a consistent conjecture exists, it must be the monopolistic one. This indicts rather severely most of the empirical works in which monopoly or cartel conjectures are ignored and, at least to the author's knowledge, this hypothesis is never tested. Most attentions are focused on testing the hypothesis that conjectures are competitive, but this case is relegated to an artifact by Theorem 1.

The second implication of the results pertains to the objectives and scope of future empirical work in industrial organization in the food system. In this context the derivation of consistent estimates of firm conduct remains one of the major challenges facing applied economists. Although the econometric application of this idea is still in its genesis, we should not consider it novel. The profession has, for some years now, imposed consistency conditions on both consumer demand and producer supply equations in an effort to reconcile the findings of empirical studies with those suggested by their theory. One cannot foresee why the technical details of these procedures should be less burdensome than their imposition under oligopoly. While many issues must be resolved before the concept can be implemented empirically, the results presented in this paper should at least assist in guiding such a study.

Footnotes

[1]As Dixit (p. 107) notes, these criticisms pertain to the model's static environment, within which the inherently dynamic concepts of conjectures and reactions are nebulous.

[2]For other empirical examples see the works cited in Bresnahan (1989); for other qualitative examples see Seade, Katz and Rosen, Quirmbach, and Dixit.

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