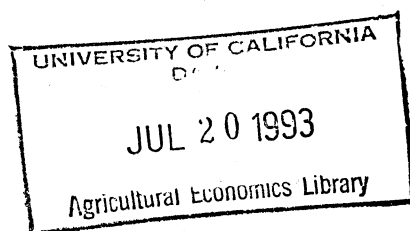# WEATHER DISTRIBUTIONS AND INPUT DECISIONS IN CROP PRODUCTION

by

Jerald J. Fletcher[1]

and

Qi Dai

Prepared for Presentation at the 1992 AAEA-NAREA Annual Meeting

August 9-12 in Baltimore, Maryland

---

[1]Jerald J. Fletcher is Associate Professor and Qi Dai is Research Associate, Division of Resource Management, West Virginia University, Morgantown, West Virginia.

A B S T R A C T

WEATHER DISTRIBUTIONS AND INPUT DECISIONS IN CROP PRODUCTION

Crop input decisions are often based on "normal" or average weather conditions. This paper compares the implications of using the mean of weather variables to using the entire distribution in determining optimal nitrogen fertilizer rates for corn in Indiana. In the case of a good (poor) weather-soil conditions for corn where the distribution skews to the left (right), using only the mean of weather variables in crop decision analysis would underestimate (overestimate) the expected crop yield. Either underestimating or overestimating expected yield implies input uses that may not be the most profitable. In this particular study, the economic losses implied by using only the mean rather than the whole distribution in nitrogen decision analysis for corn ranged from $5.48 to $22.77 per acre. Using the mean implies higher application rates and lower net returns.

WEATHER DISTRIBUTIONS AND INPUT DECISIONS IN CROP PRODUCTION

Determining optimal input levels in crop production has been a long term objective of agricultural scientists. Efficient input use in crop production is affected by many factors; some, such as weather, are stochastic. Decisions on inputs such as nitrogen fertilizer are often based on recommendations developed on the assumption of "normal" or average weather conditions (Heady and Dillon 1961; Olson 1984). In this case, either the values of the relevant weather variables in a "normal" year or the means of the weather variables over past years are used to represent weather conditions. However, given the complex nature of response functions, the variation in crop yields is affected by higher moments of the weather variables in addition to the mean. If only the mean is used in the development of input decisions, the effects of higher moments (e.g. variance and skewness) on expected yield are missing. The optimal input estimates obtained could be misleading.

This paper compares the implications of using the mean of weather variables to using the entire distribution in the analysis of optimal nitrogen rates for corn in Indiana using a stochastic response function approach. A soil moisture-stress index that reflects the interacting effects of weather and soil factors on crop growth is used to represent the stochastic weather effects. The probability distribution of the soil moisture-stress variable is estimated by maximum likelihood. Nitrogen use recommendations designed to maximize a farmer's profit from corn production in Indiana are developed using both an analysis based on the mean of the weather variables and one considering the entire distribution. Basing decisions on the mean results in nitrogen application rates 10.6 to 25.5 lb/ac higher and net returns from $5.48 to $22.77 per acre lower.

Weather Factors in Crop Production and Their Distributions

Various techniques have been developed to model the effect of weather related factors on crop yields. Baier (1973) provides a review of

earlier attempts to model the effects of weather on crop yield and analyses a number of unsuccessful studies. He concludes that a major problem in these studies is how to appropriately measure the interacting effects of weather and soil on crop yield development during the growing season. Agronomists and meteorologists have found that soil moisture is an appropriate measure of the interacting effects of weather and soil on crop yields (Dale and Shaw 1968). The use of concepts such as the ratio of actual to potential evapotranspiration and various related modifications were considered an important breakthrough in crop-weather analysis (Baier 1973). Baier and Robertson (1968) also reported that crop yield was more closely correlated with soil moisture than observations of rainfall and maximum and minimum temperatures. Andresen (1987) reported that corn yield in Indiana ranged from 112 bushels/acre on poorly-drained soils to nearly zero on the droughtier well-drained soils under the drought conditions of 1983. Soil moisture-stress was found to be the major determinant of corn yield. Such studies demonstrate that soil moisture is a better predictor of crop yields than specific climatological variables such as precipitation or temperature.

Previous research has provided useful insight into the selection of an appropriate density function for the distribution of soil moisture factors. Ravelo and Decker (1979) reported that the beta density function provides a good statistical model of the frequency distribution of the soil moisture variable in Columbia, Missouri. Yao (1969) also found the beta density to reflect adequately the distribution of monthly and weekly soil moisture factors. An attractive feature of the beta density is that it imposes a minimum of a priori restrictions on the shape of the distribution. The data, not the functional form, determine the direction of skewness for a particular analysis. Dai (1991) compares a number of

distributions to the beta using the data on which this paper is based and finds the beta the appropriate choice.

The beta distribution for the random variable (S) can be expressed as:

(1)
$$P(S) = \frac{(S-a)^{\alpha-1}\,(b-S)^{\beta-1}}{(b-a)^{\alpha+\beta-1}}\quad \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)}\qquad a \leq S \leq b$$

where $\Gamma(\bullet)$ is the gamma function defined as:

$$\Gamma(c) = \int_0^{\infty} \exp(-u)\,u^{c-1}\,du.$$

Given the physical minimum and maximum of s (a and b), this distribution has two parameters, α and β, that can be estimated using the method of maximum likelihood. Because the process of maximizing the likelihood function involves evaluating variations of the gamma function which have no analytical expression, a numerical approximation routine from the IMSL library is used to calculate the gamma function, the log of the gamma function, and the derivative of the log of the gamma function (IMSL User's Guide 1987).

Weather Distributions and Profit Maximization

Traditional crop production research has often assumed that farmers attempt to choose input levels that maximize profit (Heady and Dillon 1961). Munson and Doll (1959) discussed several approaches to profit maximization. Weather variables can be implicitly assumed in the error term or explicitly included in the crop response function. Simple profit maximization is not stochastic and the combination of inputs that maximize profit are based on a single, mean value of weather variables. Expected profit maximization, however, maximizes profit by considering the entire distribution of the appropriate weather variables. The probability of facing "bad" or "good" weather conditions and the implications for input

use under each scenario are taken into account. The two alternative approaches to profit maximization can be expressed mathematically as follows:

1. Profit maximization with the mean of the weather variables:

(2)
$$\underset{x}{Max}\ \pi = pf(x,\ E(S)) - w'x$$

2. Expected profit maximization using the distribution of weather variables:

(3)
$$\underset{x}{Max}\ E(\pi) = E\ \{pf(x,S) - w'x\} = \underset{x}{Max} \int_a^b (pf(x,S) - w'x)\phi(S)dS$$

where x is a vector of input variables, S is the stochastic weather variable which can vary from a to b, π is profit or net return, p is output price, w is a vector of input prices, $E(\bullet)$ is the expectation operator, and $f(x,S)$ is the crop response function that depends explicitly on the stochastic weather parameter. All prices are assumed known with certainty at the time the input decision is made.

Depending on the distribution of weather variables such as precipitation and temperature and the soil type, the distribution of the soil moisture-stress variable can have quite different characteristics. Figure 1 depicts two alternative scenarios for the distribution of the soil moisture-stress variable and possible associated distributions of corn yield for a particular level of input use. In panel A, two different distributions of the soil moisture-index are presented. Lower levels of the soil moisture-stress index describe a "dry" weather condition, while higher levels describe a "wet" weather condition. Panel B depicts the probability distributions of corn yields that correspond to the two different distributions of soil moisture-stress in panel A. At a specific input use level, lower levels of the soil moisture-stress index indicate

the lack of moisture in the soil which is associated with lower corn yields. On the other hand, at higher levels of soil moisture-stress, crops experience little moisture stress and yields are higher. As reflected in Figure 1, different distributions of soil moisture-stress will lead to different distributions of corn yield, and consequently, to different optimal input use levels. In general, the more skewed the distribution of the soil moisture-stress index, the greater the difference between the yields obtained at the mean of the soil moisture-stress variable and the mean of the yields obtained over the distribution.

Although the two soil moisture-stress distributions in Figure 1 are obviously different, both are assumed to have the same mean value. If only the mean value is used in the derivations of expected yield and the associated optimal input use, the effects of variance and skewness are missing and the estimated corn yields would be biased. In the case of good weather-soil conditions where the distribution skews to the left (A-1), using only the mean of the distribution would tend to underestimate the expected corn yield ($Y < Y1$) because the positive distribution effects of a good weather-soil condition on the expected corn yield are missing. In the case of a poor weather-soil condition where the distribution skews to the right (A-2), using only the mean of weather variables would tend to overestimate expected corn yield ($Y > Y2$) because the negative distribution effects of a poor weather-soil condition on the expected corn yield are missing. Either underestimating or overestimating the expected yield could imply input use levels that may not be the most profitable.

One approach to quantifying the possible economic loss from an incorrect analysis is as follows. Assume that x_m is the optimal input use from a non-stochastic profit maximizing analysis based on the mean of the moisture-stress variable S (represented in equation 2) and x_d is the optimal input use obtained using the entire distribution of S in

calculating expected profit (see equation 3). Let $E[\pi(\bullet)]$ represent expected profit evaluated at specific input levels. $E[\pi(x_m,S)]$ is the value of the profit equation when using x_m, and $E[\pi(x_d,S)]$ is the value when using x_d. By construction, $E[\pi(x_d,S)] \geq E[\pi(x_m,S)]$ since x_d represents the expected profit maximizing input level. The expected economic loss of using the mean value rather than the entire distribution of the weather variable is the difference between the two, $E[\pi(x_d,S)] - E[\pi(x_m,S)]$.

Data and Procedures

In this study, a soil moisture-stress index is used to represent the interacting effects of weather and soil factors on corn yield. The daily soil moisture-stress index is defined as the ratio of actual to potential evapotranspiration (ET/PET). If soil moisture is not limiting, ET is equal to PET, the ratio (ET/PET) is equal to 1, and the corn experiences no moisture stress. If ET is less than PET, the ET/PET ratio is less than 1, and the crop suffers moisture stress. Stuff and Dale (1978) developed a SIMulation of the soil water BALance (SIMBAL) model, which uses specific soil information in addition to daily precipitation and pan evaporation to calculate the daily soil moisture-stress. SIMBAL was used in this study to generate 15 different soil moisture-stress indices based on 12 different soil types and daily weather data at 4 different locations in Indiana from 1960 to 1986. These daily indices of soil moisture-stress are then summed over the 37-day critical period of the corn growing season to provide a single soil moisture-stress index defining the joint effect of the weather and soil related factors in corn production for the respective year and soils. The data on soil types and daily weather used in this study were obtained from Andresen (1987). The maximum likelihood method is used to estimate the parameters of soil moisture-stress distributions. The log of the likelihood function contains the term "log(b-S)", and the condition $S < b$ has to be met to insure $(b-S) > 0$

during the maximization process. Since 5 of 15 soil moisture-stress indices have some observations that are exactly 37 (i.e. S = b), the condition S < b implies the loss of 10 percent of the observations for these 5 soil moisture-stress indices and may result in misleading conclusions. One alternative is to assume a mixed beta distribution defined as (Johnson and Kotz 1970):

$$
(4) \qquad \phi(S_i) = \begin{cases} \dfrac{(S_i - a)^{\alpha-1} (b - S)^{\beta-1}}{(b - a)^{\alpha+\beta-1}} \; \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} \; \dfrac{N-n}{N} & \text{if } a \leq S < 37 \\[3ex] n/N & \text{if } S = 37 \end{cases}
$$

where S_i is the soil moisture-stress index for soil i, a is the physical minimum limit of S ($S_{min} = 0$), b is the physical maximum limit of S ($S_{max} = 37$), n is the number of observations of S that are exactly 37, and N is the total number of observations. If $S_i = 37$, then $\phi(S) = (n/N)$, a constant, and no parameter estimation is needed. The parameters must be estimated when $S_i < 37$. The maximum likelihood (ML) estimates of the beta parameters and their standard errors for each of 15 soil moisture-stress indices are presented in table 1.

One necessary step in crop decision analysis is to estimate crop response functions that describe crop yield response to economic inputs (e.g. nitrogen fertilizer) as well as the stochastic soil moisture-stress variable. Long-term experimental plot data for corn from 1960-1986 are used in this study (Barber and Mengel 1988). These experiments were documented with respect to variables such as the nitrogen application rates, corn yield, previous year's crop and previous nitrogen rates. The process relating corn yield to nitrogen use is complicated. In addition to the direct effect of the fertilization rate, nitrogen available to the plant in the soil also depends on several factors such as the carryover from the previous year's nitrogen application and the residual effect from

previous year's nitrogen-fixing crop if any. In this study, the nitrogen available to corn is defined as:

(5) $$x_t^a = (x_t + \tau * x_{t-1}) * [(1-C)/(1-\tau^2)] - (x_t + \delta) * C$$

where x_t^a is nitrogen available in year t, x_t is nitrogen applied in year t, x_{t-1} is nitrogen applied in year t-1, τ is the fraction of the nitrogen applied in t-1 carried over to year t, δ is a parameter reflecting the nitrogen-equivalence provided by a previous crop of soybeans, and C is binary variable for the previous year's crop: C=0 if corn; C=1 if soybeans. The expression of available nitrogen (5) assumes that the carry-over effects prior to the previous year are the same percentage of the previous year's available nitrogen (i.e., τ is constant over years). In this analysis, coefficients describing the nitrogen carryover from the previous year's applications (τ=0.27) and the residual nitrogen from any previous year's N-fixing crop (δ=39.5 lb/ac) are based on Merz's estimates from the same data set. These estimates were found to be consistent with agronomic expectation (Merz 1988).

While numerous algebraic forms can be used to specify crop yield response functions, some functional forms are more flexible than others. Selection of the appropriate algebraic form may be guided by previous empirical studies as well as the scientific theories involved. In this study, the quadratic function, the cubic function, and the Mitscherlich are used to estimate the corn-nitrogen response functions. Given the stochastic corn response models and the distribution of the stochastic factor (4) in crop production developed previously, the profit maximization model can be applied to the nitrogen rate decision for corn production using both the mean and the full distribution of the stochastic weather-related variable.

The prices of corn and nitrogen fertilizer are assumed to be known at the time the fertilizer decision is made and are therefore exogenous in this decision model. The means of corn and nitrogen fertilizer prices from 1980 to 1986 are used in this study.

Substituting the mixed beta distribution (4) into the expected profit maximization model (3), the objective function is then rewritten as:

$$(6) \qquad \underset{x}{Max} \left\{ \frac{(N-n)}{N} \left[\int_{0}^{36.99} (pf(x;S_i)-w'x)\phi(S_i)dS \right] + (pf(x;S_{max})-w'x)\frac{n}{N} \right\}$$

where p is the price of corn, w is the price of nitrogen fertilizer, x is the nitrogen application rate (pound/acre), S_i is the soil moisture-stress index for the ith weather and soil condition, $f(x;S)$ is the stochastic crop response model and $\phi(S_i)$ is the distribution of the S_i index for the ith weather and soil condition over the continuous portion of the distribution. The last term gives the contribution at the end point with positive probability n/N. In the optimizing process, the integrals in (6) are approximated by the FORTRAN subroutine DQDAG from IMSL (IMSL User's Guide 1987). The nonlinear optimization was carried out using the Modular In-core Nonlinear Optimization System (MINOS) (Murtagh and Saunders 1983).

Results and Implications

Three corn-nitrogen-moisture response functions were estimated. The estimated equations (with t values in parentheses) are as follows:

A. The Quadratic Function:

$$Y = 1918.99 + 0.73*N - 0.002*N^2 - 110.64*S + 1.60*S^2 + 0.009*S*N$$

$$\quad\quad (3.23) \quad (1.31) \quad (-6.65) \quad\quad (-3.08) \quad\quad (2.96) \quad\quad (0.61)$$

Adj. R = 0.873.   F value = 1065.0

B. The Cubic Function:

$$Y = -27339 + 10.38*N - 0.00141*N^2 - 0.5 \times 10^{-7}*N^3 + 2806.2*S$$

$$(-3.39) \quad (2.01) \quad (-0.37) \quad (-0.03) \quad (2.238)$$

$$- 69.57*S^2 + 0.6677*S^3 - 0.5838*S*N - 0.000025*S*N^2 + 0.009*N*S^2$$

$$(-3.16) \quad (3.09) \quad (-1.87) \quad (-0.16) \quad (1.93)$$

Adj. R = 0.895.   F value = 875.6

3. The Mitscherlich Function:

$$Y = 155.49 * (1 - \exp(-0.01*N)) * (1 - \exp(-0.07*S))$$

$$(8.35) \quad (-12.25) \quad (-2.27)$$

Adj. R = 0.850.   F value = 1795.1

Optimal nitrogen application rates for 15 different weather and soil conditions in Indiana determined by maximizing returns using both the mean of the soil moisture-stress index are presented in table 2 and the results using the full distribution are presented in table 3. With the quadratic function, the first order condition is linear in S so that using the mean or using the full distribution implies the same optimal nitrogen rates. Since the net returns equation is not linear in s, however, the values of expected net returns differ.[1] With the exception of the quadratic form using only the mean of the stochastic variable usually leads to higher input use levels but lower net returns than that from using the entire distribution. For instance, with the Chalmers soil in central Indiana and a cubic response model, using only the mean implies an optimal nitrogen rate of 197 pounds per acre and net returns of $ 318 per acre, while using the entire distribution implies an optimal nitrogen rate of 185 pounds per acre and net returns of $342 per acre. The economic losses implied by using the mean value rather than the whole distribution in nitrogen decision analysis for corn using the cubic function are reported in table 4. It should be noted that the empirical results are based on reasonably good weather-soil conditions where the distributions of the soil moisture-

stress variable are all skewed to the left; this is typical of Indiana growing conditions. If the analysis was applied to "bad" weather-soil conditions where the distributions of the soil moisture-stress variable were skewed to the right, the opposite results would be expected.

Summary

Optimal input decisions in crop production are often developed under the assumption of "normal" or average weather condition using only the mean of the appropriate random variable. This study compares the implications of using the mean of weather related variables to using the entire distribution in crop decision analysis. Nitrogen use recommendations that maximize a farmer's profit for corn production in Indiana were determined using both the mean and the entire distribution of such a weather related variable, a soil moisture-stress index. Except for the quadratic form, using only the mean results in higher optimal nitrogen rates and lower net returns than implied by the use of the entire distribution. In the case of a good weather-soil conditions for corn where the distribution skews to the left, using only the mean of the soil moisture-stress index in crop decision analysis would underestimate the expected crop yield. In the case of poor weather-soil conditions where the distribution is skewed to the right, using only the mean of the stochastic variable would overestimate expected yield. Either underestimating or overestimating expected yield implies input uses that may not be the most profitable. In this study, the economic losses implied by using only the mean rather than the whole distribution in nitrogen decision analysis for corn ranged from $5.48 to $22.77 per acre.

A substantial amount of research is being conducted on how to reduce nitrogen fertilizer use in crop production while maintaining farm profitability. Understanding the consequences of using average or mean values rather than the full distribution of uncertain parameters in

decision analysis will help farmers avoid unnecessary losses and achieve the objective of reducing nitrogen fertilizer use while maintaining profitability.

REFERENCES

Andresen, J. *Corn Yield Prediction with a Daily Energy-crop Growth Variable for Counties in Indiana.* Unpublished Ph.D dissertation, Purdue University, 1987.

Baier, W. "Crop-weather Analysis Model: Review and Model Development." *J. of Applied Meteorology* 12(1973):937-947.

Baier, W. and R. W. Robertson. "The Performance of Soil Moisture Estimates as Compared with the Direct Use of Climatological Data for Estimating Crop Yields." *Agricultural Meteorology* 5(1968):17-31.

Barber, S. and D. Mengel. "Long-term Nitrogen Experiment Data from 1960 to 1986." Unpublished Experiment Plot Data, Department of Agronomy, Purdue University, W. Lafayette, Indiana, 1987.

Dai, Q. *Nitrogen Management Decisions in crop Production Under Different Weather and Soil Conditions*, unpublished Ph.D. dissertation, Purdue University, 1991.

Dale, R.F. and W. L. Shaw. "Effect on Corn Yields of Moisture Stress and Stand at Two Different Fertility Level." *Agronomy Journal*, 57(1965): 475-479.

Heady, E. O. and J. L. Dillon. *Agricultural Production Function*, Iowa State University Press, Ames, Iowa, 1961.

IMSL, Inc. *IMSL User's Manual (version 2.0)*, IMSL, Inc., Houston, Texas, 1987.

Johnson, N. L. and S. Kotz. *Continuous Univariate Distribution*, volume 2, John Wiley, New York, 1970.

Merz, L. *A Method of Combining Several Nitrogen Experiments Over Time*, Unpublished M.S. thesis, Purdue University, W. Lafayette, Indiana, 1987.

Munson, R. D. and J. P. Doll. "The Economics of Fertilizers Use in Crop Production." *Advances in Agronomy*, 11(1959):133-169.

Murtagh, B. A. and M. A. Saunders. *MINOS 5.0 User's Guide*, Stanford University Systems Optimization Laboratory Technical Report SOL 83-20, Stanford University, California, 1983.

Olson, B. R. "Efficient Use of Nitrogen in Crop System", in *Nitrogen in Crop Production*, edited by R. R. Hauck, et al., American Society of Agronomy, Madison, Wisconsin, pp.273-294, 1984

Ravelo, A. C. and W. C. Decker. "The Probability Distribution of a Soil Moisture Index." *Agricultural Meteorology* 20(1979):301-312.

Stuff, A. C. and R. F. Dale. "A Soil Moisture Budget Model Accounting for Shallow Water Table Influences." *Soil Sci. Soc. Amer. J.* 42(1978):637-643.

Yao, A. M. "The R-index for Plant Water Requirement." *Agricultural Meteorology* 6(1969): 259-273.

Table 1.   Estimated Beta Parameters for 15 Soil Moisture-Stress
            Indices

S Index	Soil Types	α	β	$\frac{n}{N}$	Mean	Standard Deviation
(1)	In Central Indiana:					
S1	Eel	7.49 (0.89)	1.06* (0.17)	0	29.77	2.85
S2	Miami	9.87* (1.27)	0.98* (0.19)	0	31.30	2.24
S3	Fox	7.05 (2.14)	2.31 (0.44)	0	26.22	3.08
S4	Morley	13.78* (1.37)	0.83* (0.16)	0	32.92	2.06
S5	Russell	7.85* (0.92)	0.91* (0.58)	0	30.43	2.40
S6	Chalmers	34.50* (1.74)	0.42* (0.15)	0.250	35.55	1.51
S7	Brookston	33.86* (1.38)	0.64* (0.21)	0.210	35.34	1.54
S8	Crosby	30.20* (1.65)	0.68* (0.21)	0.170	35.11	1.67
(2)	In Southern Indiana:					
S9	Alford	15.08* (1.85)	0.81* (0.58)	0	33.26	2.02
S10	Eel	7.85* (1.07)	0.95* (0.19)	0	30.32	2.97
S11	Vincennes	7.42* (1.36)	0.90* (0.42)	0	30.19	2.76
(3)	In Northwest Indiana:					
S12	Morley	12.48* (1.55)	0.87* (0.31)	0	32.49	2.09
S13	Rensselaer	29.45* (2.04)	0.69* (0.27)	0.125	35.05	1.75
(4)	In Northeast Indiana:					
S14	Morley	11.05* (1.71)	0.76* (0.35)	0	32.29	2.15
S15	Blount	31.10* (1.79)	0.54* (0.28)	0.21	35.31	1.61

Note: * indicates statistical significance at the 1% level, asymptotic
standard errors are in parentheses.

Table 2. Optimal Nitrogen Rates by Soil Moisture-Stress Index for 15 Weather and Soil Combinations in Indiana for Three Functional Forms Based on the Mean Value of the Soil Moisture-Stress Index Distribution.

S Index	Soil Type	Quadratic $N = x_m$ lb/ac	Cubic $N = x_m$ lb/ac	Mitscherlich $N = x_m$ lb/ac
(1)	In Central Indiana:			
S1	Eel	160.25	185.40	182.02
S2	Miami	169.82	187.14	184.16
S3	Fox	146.73	181.37	181.22
S4	Morley	171.08	189.89	184.79
S5	Russell	168.53	186.35	183.60
S6	Chalmers	188.12	196.83	186.71
S7	Brookston	187.65	196.81	186.61
S8	Crosby	185.96	196.64	186.15
(2)	In Southern Indiana:			
S9	Alford	173.17	190.63	184.93
S10	Eel	158.06	182.38	181.60
S11	Vincennes	164.48	186.20	182.34
(3)	In Northwest Indiana:			
S12	Morley	170.82	189.10	184.68
S13	Rensselaer	185.13	196.01	185.87
(4)	In Northeast Indiana:			
S14	Morley	170.49	187.86	184.47
S15	Blount	186.42	196.28	186.36
Mean		172.44	189.91	184.37
STD		11.90	5.21	1.79

Table 3.   Optimal Nitrogen Rates by Soil Moisture-Stress Index for 15 Weather and Soil Combinations in Indiana for Three Functional Forms Based on the Full Distribution of the Soil Moisture-Stress Index (S)

| S Index	Soil Type	Quadratic $N = x_d$ lb/ac	Cubic $N = x_d$ lb/ac	Mitscherlich $N = x_d$ lb/ac
(1)	In Central Indiana:			
S1	Eel	160.25	165.04	174.83
S2	Miami	169.82	170.27	177.16
S3	Fox	146.73	155.92	174.24
S4	Morley	171.08	171.80	178.38
S5	Russell	168.53	169.19	176.64
S6	Chalmers	188.12	185.43	183.83
S7	Brookston	187.65	184.95	182.65
S8	Crosby	185.96	184.09	180.92
(2)	In Southern Indiana:			
S9	Alford	173.17	172.27	178.67
S10	Eel	158.06	158.82	175.15
S11	Vincennes	164.48	165.35	175.92
(3)	In Northwest Indiana:			
S12	Morley	170.82	171.45	178.16
S13	Rensselaer	185.13	183.76	180.48
(4)	In Northeast Indiana:			
S14	Morley	170.49	171.26	177.92
S15	Blount	186.42	184.62	181.79
Mean		172.44	172.94	178.44
STD		11.90	9.36	2.84

Note: x_m is the optimal nitrogen rate from maximizing profit with the mean value of the stress index, S (see equation 2) and x_d is the optimal nitrogen rate from maximizing expected profit with the entire distribution of S (see equation 3).

Table 4.   Economic Costs of Using the Mean of Soil Moisture-Stress
            in Optimal Nitrogen Use Calculations by Soil Type and
            Region - Cubic Model.

S Index	Soil Type	$E[\pi(x_d,S)]$ ($/acre)	$E[\pi(x_m,S)]$ ($/acre)	Cost of Using x_m ($/acre)
(1)	In Central Indiana:			
S1	Eel	193.17	173.67	19.50
S2	Miami	218.25	206.20	12.06
S3	Fox	163.63	140.86	22.77
S4	Morley	230.86	220.67	10.19
S5	Russell	203.76	189.93	13.83
S6	Chalmers	342.01	336.53	5.48
S7	Brookston	330.67	324.97	5.70
S8	Crosby	305.29	298.59	6.70
(2)	In Southern Indiana:			
S9	Alford	231.42	221.62	9.80
S10	Eel	189.25	168.07	21.18
S11	Vincennes	197.71	179.42	18.29
(3)	In Northwest Indiana:			
S12	Morley	224.93	214.44	10.49
S13	Rensselaer	322.79	315.40	7.39
(4)	In Northeast Indiana:			
S14	Morley	223.16	212.06	11.10
S15	Blount	310.90	304.67	6.23

Note: x_d is the optimal nitrogen rate from maximizing expected profit
with the entire distribution of s (see equation 3), x_m is the optimal
nitrogen rate from maximizing profit with the mean value of the stress
index, s (see equation 2), and $E[\pi(K)]$ is expected profit. In
particular, $E[\pi(x_d,S)]$ is expected profit when using x_d, and $E[\pi(x_m,S)]$
is expected profit when using x_m. By construction, $E[\pi(x_d,S)] \geq$
$E[\pi(x_m,S)]$ since x_d is the expected profit maximizing input level.
The economic cost of using the mean value rather than the entire
distribution of the weather variable is the difference between the
two, $E[\pi(x_d,S)] - E[\pi(x_m,S)]$.

Figure 1. Effects of the Distribution of Soil Moisture on Crop Yield.
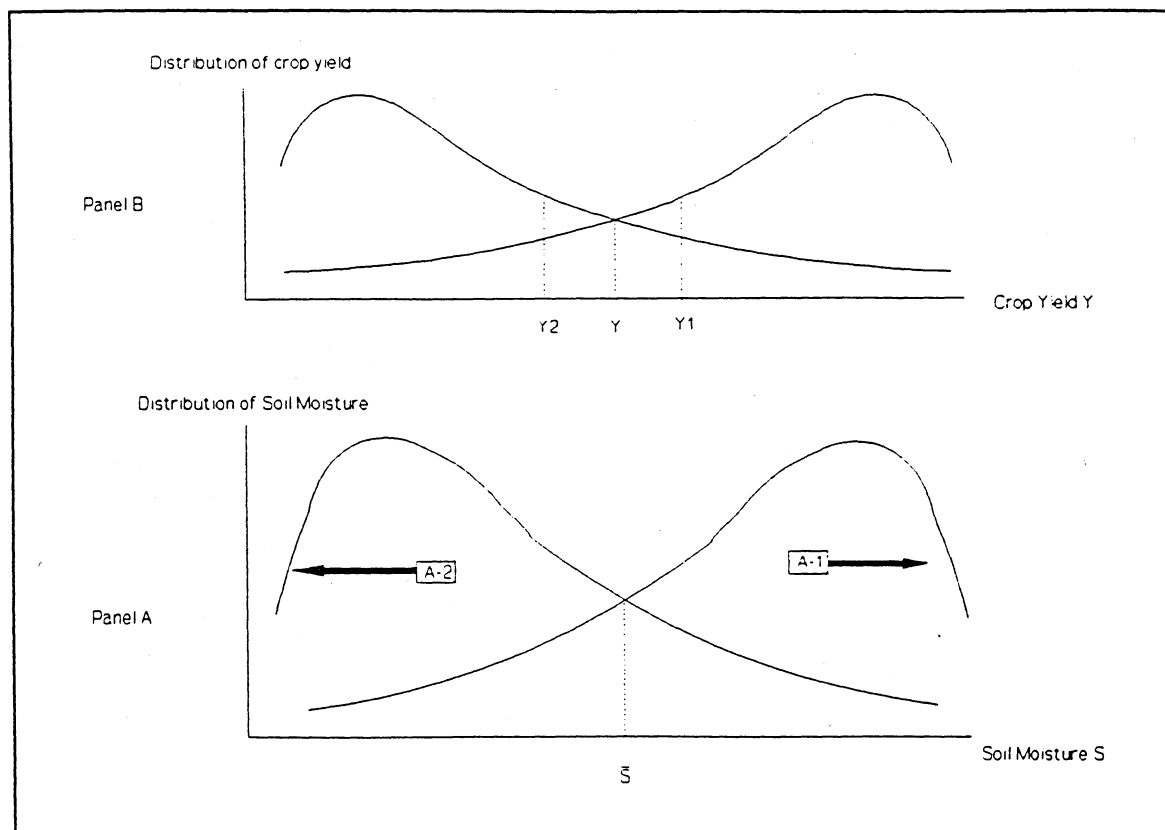
Note:

1. A-1 represents the distribution of soil moisture-stress for a poorly-drained silt loam soil with high water-holding capacity which reflects good weather-soil conditions for corn. A-2 represents the soil moisture-stress for a well-drained sandy soil with low water-holding capacity, a poor weather-soil combination for corn.

2. Panel B describes two distributions of corn yield that correspond to the two distributions of the soil moisture-stress in panel A with mean Y1 and Y2, respectively. Y is expected yield conditional on the mean of S. Using only the mean of the weather variable underestimates expected corn yield in the case of A-1 (Y1 > Y) and overestimates expected yield in the case of A-2 (Y2 < Y).

ENDNOTES

1. Since the expectation is a linear operator, if $f(x,s)$ is linear in s, $E[f(x,s)] = f(x,E[s])$. If f is a non-linear function of s, however, $E[f(x,s)] \rbrack f(x,E[s])$. With the quadratic function, the first order condition for profit maximization is linear in s, but the profit equation is not linear in s. In this case, using the mean or using the full distribution implies the same optimal nitrogen rates but different expected profits.