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Bellcore

Bell Communications Research

1992

(Prices) - Analysis

Economics Research Group

Morristown, New Jersey

AAEA 1992

Bellcore Economics Discussion Paper No. 79

SELF RATIONING WITH NONLINEAR PRICES

by

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May 1992

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Abstract:

This paper is concerned with pricing problems that arise when a service and the capacity to consume that service are jointly sold to consumers with random demand. It builds on the work of Oren, Smith & Wilson (1985) and Panzar & Sibley (1978). We show that profit maximizing firms that can use fully nonlinear prices will satisfy the usual right end point condition: the last unit sold to the largest consumer will be marginally priced at marginal cost. We also provide conditions under which marginal prices exceed marginal cost. When the firm is restricted to using a partly nonlinear price structure, selling capacity at a linear price and the service itself at a nonlinear price, we show that marginal prices will generally lie below marginal cost over some portion of their domain. The intuition underlying this result can be found in the loss leader problem first studied by Allen.

INTRODUCTION

This paper is concerned with questions raised by the simultaneous consideration of two important issues in public utility economics: nonlinear price structures and capacity constraints. There is now a large literature on each of these issues.1 A recent paper by Oren, Smith & Wilson (1985) brings these two strands of the literature on public utility pricing together. They obtain some very interesting results on the relationship between prices and costs. In particular, they show that, for a class of examples, capacity will be priced below cost by profit maximizing firms. In this paper we are concerned generally with the relationship between prices and costs and more specifically with the conditions under which prices will be below costs when capacity constraints are important and prices are nonlinear.

Our work is related to the work of Oren, Smith & Wilson and that of Panzar & Sibley (1978). Panzar & Sibley have provided a feasible, practical and efficient solution to the problem of public utility pricing under risk. The central construct introduced and analyzed by them is the notion of self- rationing. Briefly, a self-rationing scheme is one which requires consumers to subscribe to capacity. In the context of the market for electricity, a self-rationing scheme can be implemented through devices such as fuses or circuit barbreakers, which place an upper bound on how much electricity a consumer can use at any point in time. In a self-rationing scheme, consumers are required to purchase a fuse (as large as they desire, with larger fuses costing more) before the random factors affecting demand are observed. The utility then installs capacity equal to the total fuse sizes purchased by all consumers. No consumer can express a demand greater than his fuse size, so that demand fluctuations are no longer a reason for the utility to consider rationing schemes. Self-rationing arises naturally in telecommunications markets as well. Most business customers select the number of lines connecting their PBXs to the central office serving them. Standard tables are used by businesses to determine how many lines they need. These tables show the number of lines needed for any given combination of peak load and desired blocking probability. Residential customers make a similar, but coarser, choices in determining how many lines to order, and whether or not to subscribe to Call Waiting.

Panzar & Sibley showed that the welfare maximizing prices are equal to the appropriate marginal costs: the price of electricity is equal to the marginal operating cost and the price of a unit fuse is equal to the marginal capacity cost. If, as they assumed, each of these costs is constant, the utility breaks even. Moreover, as the utility does not ration consumers it does not need to design a revelation mechanism that reveals the personalized information that is required to implement the efficient rationing scheme: the one that allocates capacity to those with the highest willingness to pay. A final bonus: separability of the demand functions guarantees that the self- rationing scheme will generate the highest possible level of welfare, equal to that generated by the efficient rationing rule.

Oren, Smith and Wilson (1985) argue that many other services (such as electronic mail and computer services) share the essential characteristics of the public utility model described above. The costs of the seller and the benefits to the buyer do not depend only on the quantity produced and sold; other factors such as timing in the peak load context, speed of delivery in the electronic mail context, and capacity in the electricity context (size of fuse) or telecommunications context (size of switchboard or number of lines) are relevant. These related issues are treated by them under the combined heading of "Capacity Pricing". They introduce a novel framework that allows them to treat these issues as a nonlinear pricing problem, and derive the result that the profit maximizing utility will, for one class of examples, set the price of capacity below its cost.

The underlying intuition for this result is not developed in their paper. It is not clear whether the differences between their result and that of Panzar & Sibley depend on the technical details of their model, or on the assumed objective of the firm, or on some other distinguishing feature of the two approaches. One objective of our paper is to clarify the conditions under which utilities will price below cost and to look at determinants of the direction of the cross subsidy.

In this paper, we use the assumptions of the Panzar-Sibley model on details of demand and costs, and differ from them in two important dimensions. First, we will look at the price structure that results from profit maximization, rather than welfare maximization as they did. Practical concern is the primary motivation for this assumption. Most public utilities in the U.S. are privately owned and profit maximization may be a better description of their behavior than welfare maximization. Also, the profit maximizing model can be more readily adapted to accommodate regulation. Second, we look at nonlinear price structures, rather than the linear price structures they did. Once again, we are motivated by the observation that most public utilities, which provide services that are hard to resell, use nonlinear prices. In these last two particulars, our model conforms to that of Oren, Smith & Wilson. We differ from them, however, in our description of preferences, costs and admissible price structures; on these matters our formulation is that of the standard public utility model.

Section 1 introduces the assumptions and models the behavior of the consumer. Section 2 looks at the problem of the firm and derives results on a fully nonlinear pricing structure. Our approach in these two sections differs in some details from earlier work in this area. Whereas the earlier literature used the dual approach pioneered by Mirrlees (1971), we develop a primal approach which is a variant of the more recent work by Goldman, Leland & Sibley (1984). The benefits and limitations of this framework are briefly discussed. Section 2 also establishes a standard right end point condition on the price structure and presents sufficient conditions for marginal prices to be greater than or equal to marginal costs. Section 3 looks at a mixed linear- nonlinear pricing problem and provides a range of cases for which marginal prices will lie below marginal cost. It appears that the basic mechanism at work is a modified form of Allen's (1971) loss leader strategy. Section 4 concludes with some remarks on possible generalizations.

1. THE CONSUMER'S PROBLEM

Following Panzar & Sibley, we assume that the market contains a continuum of consumers indexed by a variable θ that takes on values in the interval $[\underline{\theta},\overline{\theta}]$ according to a continuous density function $g(\theta)$. $G(\theta)$ is the associated distribution function. Demand is affected by a random variable, called temperature, indexed by a variable t which takes on values in the interval $[\underline{t},\overline{t}]$ according to the density function $f(t)$. $F(t)$ is the associated distribution function.

It is assumed that there are no income effects and that preferences can be fully represented by an inverse demand or willingness-to-pay function, $W(q,t,\theta)$. W represents the amount that consumer θ will pay for another unit of the good at temperature t, given that he is already consuming q units of electricity. It is assumed that $W_q<0$ and that $W_t, W_\theta>0$. The latter monotonicity assumptions are crucial to all nonlinear pricing problems.

The consumer faces a two stage decision problem. Before t is observed he must decide on a fuse size, A, which will then bound his ability to consume electricity during the period. After t is observed he must decide on how much electricity to consume, subject to the constraint imposed by his preselected fuse size. The most general price structure in this context would make the consumer's total payment an arbitrary function of his consumption bundle, $T(q,A)$. This problem is hard to solve. Considerable insight into the price cost relationship can be obtained by looking at simpler problems. We look at two special cases. In the first case, which we develop in this section, the marginal price schedule for electricity (fuses) is not a function of the fuse size (amount of electricity) bought by the consumer. We restrict our attention to additively separable total outlay functions, $T(q,A)$. In the second simplification, developed in Section 3, we look at the still more special case in which fuses are sold at a fixed (linear) price while electricity is sold according to a nonlinear price schedule.

For the first case, it is easiest to solve the problem recursively by considering the ex post problem first. Let $P(q)$ be the marginal price of electricity. The consumer's ex post demand for electricity is the solution of the following program:

$$\max_{q^*} \int_0^{q^*} [W(q,t,\theta)-P(q)]dq$$

In general, this problem may not be well-behaved and its solution will be hard to characterize using standard programming techniques. We reduce the problem to manageable size by restricting our attention to a class of price schedules for which the consumer's problem is well-behaved; that is, the global optimum is fully identified by the first order conditions for the problem.

Specifically, we restrict our attention to price schedules that are continuous and *single crossing*. This allows us to use the first-order approach; see Rogerson (1985) and Brown & Sibley (1986), pp. 208-215. Under these conditions the set of global optima for the consumer's problem is the same as the set of solutions to his first barorder conditions. The Kuhn-Tucker condition for the consumer's ex post problem is:

$$W(q^*,t,\theta)-P(q^*)\leq 0, q^*\geq 0, (W-P)\cdot q^*=0. \tag{1}$$

Figure 1 illustrates interior and corner solutions to the problem. It is clear that the second order condition $W_q-P_q\leq 0$ will be satisfied automatically for all single crossing schedules. Therefore, equation 1 implicitly defines the ex post demand functional for electricity, $q^*(t,\theta,P)$.

For an arbitrary fuse of size A, the demand functional can be used to define the rationing temperature, $\hat{t}$, as follows: $q^*(\theta,\hat{t},P)-A=0$, or equivalently, $W(A,\hat{t},\theta)-P(A)=0$. The functional $\hat{t}(\theta,A,P)$ defines the lowest temperature at which θ will be rationed, given the price schedule, if he had a fuse of size A. The expected consumer surplus of θ can now be defined as:

$$ECS=\int_{\underline{t}}^{\hat{t}}\int_{0}^{q^*}[W(q,t,\theta)-P(q)]dqdF+\int_{\hat{t}}^{\bar{t}}\int_{0}^{A}[W(q,t,\theta)-P(q)]dqdF-\int_{0}^{A}R(a)da. \quad (2)$$

where $R(.)$ is the (nonlinear) marginal price schedule for fuses. We assume that R is single crossing with respect to the ex ante demand curve for fuses, which is given by the first two expressions in equation 2. Once again, this assumption allows us to fully characterize the demand for fuses by the Kuhn-Tucker condition for the ex ante problem:

$$\max_{A\geq 0} ECS(\theta,R,P,A).$$

The first order condition for interior solutions reduces to:

$$\int_{\hat{t}}^{\bar{t}}[W(A^*,t,\theta)-P(A^*)]dF-R(A^*)=0. \quad (3)$$

The first term in (3) is the demand price (or marginal value) of fuses: it represents the additional (expected) value of electricity in those states of nature where the consumer's ex ante capacity choice prevents him from purchasing electricity. Consumer θ buys capacity to the point where its expected benefit is equal to its marginal price, and, with a single crossing price schedule, his choice is uniquely determined by this first order condition. The demand functional for fuses, $A^*(\theta,P,R)$, is implicitly defined by equation (3). As with any option demand it is derived from the underlying demand for the good, $W(q,t,\theta)$.

A useful property of the functional $\hat{t}$ can now be established. Given a positive marginal price of fuses, consumers will choose to ration themselves. We state this formally as:

PROPOSITION 1 : If $R(A)$ is positive then $\hat{t}(\theta,A,P)<\bar{t}$.
Proof: Suppose not. Then $\hat{t}=\bar{t}$. By (3), $\partial ECS/\partial A=-R(A^*)<0$. The Kuhn-Tucker condition then requires $A^*=0$, which contradicts $\hat{t}=\bar{t}$.

1.1 PROPERTIES OF DEMAND FUNCTIONALS.

Later when we look at the firm's problem we will need consumer responses to changes in the price schedules. We now develop these responses. Given a price schedule $P(q)$, construct a new price schedule $P(q)+\varepsilon\cdot\eta(q)$ where ε, a scalar, and $\eta(q)$ are restricted so that $P(q)+\varepsilon\cdot\eta(q)$ is also a single crossing schedule. For this new schedule the ex post demand for electricity is:

$$W(q^*,t,\theta)-P(q^*)-\varepsilon\cdot\eta(q^*)=0. \quad (4)$$

Differentiating implicitly we obtain

$$\partial q^*/\partial\varepsilon=\eta(q^*)/[W_q-P_q-\varepsilon\cdot\eta'(q)].$$

Eventually we will be interested in the demand response evaluated at $\varepsilon=0$ because we will be considering variations around the optimal price schedule. For reference we note that

$$\partial q^*/\partial\varepsilon\,|_{\varepsilon=0}=\eta(q^*)/[W_q-P_q]. \tag{5}$$

The variational exercise underlying equation 5 highlights the true value of the single crossing assumption combined with the assumption of zero income effects. We can see that a variation in the schedule limited to an interval dq evokes a demand response only in those consumers whose ex post purchases lie in that dq interval. Consumers who buy more or less than this amount are not affected by the variation $\eta(q)$ because $\eta(q^*)$ will be zero for them.

Later notation will be considerably simplified if we define $\partial q^*/\partial P=1/[W_q-P_q]$, so that $\partial q^*/\partial\varepsilon=\eta(q^*)\cdot\partial q^*/\partial P$. The notation is suggestive; if we treat q^* and P as variables in equation 4 and differentiate implicitly our result would correspond to the definition given above. Needless to say, this latter procedure is meaningless in the context of the problem solved by the consumer because changing the price schedule at a single point has no effect on the integral representing consumer surplus, and hence no effect on demand. However, the notational convenience is useful. Additionally, the expression denoted by $\partial q^*/\partial P$ captures that part of the demand response that is common to all variations in the price schedule, while $\eta(q)$ captures that portion of the demand response that is due to the particular variation under consideration.

Standard application of the Implicit Function Theorem to equation 4 yields: $\partial q^*/\partial t=-W_t/[W_q-P_q]$; $\partial q^*/\partial\theta=-W_\theta/[W_q-P_q]$. The second order condition for the consumer's problem guarantees that q^* will be monotone increasing in t and θ.

The properties of the demand for fuses can be derived in a similar way. Once again, begin with a perturbed price schedule $P(q)+\varepsilon\cdot\eta(q)$. The first order condition for choice of fuse size is:

$$\int_{t(\theta,A^*,P+\varepsilon\cdot\eta)}^{\bar{t}} [W(A^*,t,\theta)-P(A^*)-\varepsilon\cdot\eta(A^*)]dF-R(A^*)=0.$$

Implicit differentiation yields:

$$\partial A^*/\partial\varepsilon=\frac{\int_{t}^{\bar{t}}\eta(A^*)dF+[W(A^*,t,\theta)-P(A^*)-\varepsilon\cdot\eta(A^*)]\cdot f(t)\cdot\partial t/\partial\varepsilon}{\int_{t}^{\bar{t}}[W_q-P_q-\varepsilon\cdot\eta_q]dF-R_A-[W(A^*,t,\theta)-P(A^*)-\varepsilon\cdot\eta(A^*)]\cdot f(t)\cdot\partial t/\partial\varepsilon}$$

$$= \frac{\eta(A^*)[1-F(t)]}{\int_t^{\bar{t}}[W_q - P_q - \varepsilon\cdot\eta_q]dF - R_A}$$

because $W(A^*, t, \theta) - P(A^*) - \varepsilon\cdot\eta(A^*) = 0$ by definition of t. Evaluate at $\varepsilon = 0$ to obtain:

$$\partial A^*/\partial\varepsilon\,|_{\varepsilon=0} = \frac{\eta(A^*)[1-F(t)]}{\int_t^{\bar{t}}[W_q - P_q]dF - R_A} = \eta(A^*)\cdot\partial A^*/\partial P. \tag{6}$$

where $\partial A^*/\partial P$ is defined and interpreted exactly as $\partial q^*/\partial P$ was. As before it is clear that if $P(q)$ is perturbed in a small interval, dq, then only those consumers whose purchased fuse sizes fall in that dq interval will change their demand for fuses. This is abundantly clear from (6): $\eta(A^*)=0$ outside the dq interval under consideration.

The response of demand for capacity to changes in $R(A)$, the marginal price schedule for capacity, can be similarly derived. Begin with the price schedule $R(A)+\mu\cdot\psi(A)$ and the associated first order condition:

$$\int_{t(\theta, A^*, P)}^{\bar{t}} [W(A^*, t, \theta) - P(A^*)]dF - R(A^*) - \mu\cdot\psi(A^*) = 0.$$

Implicit differentiation yields

$$\partial A^*/\partial\mu = \frac{\psi(A^*)}{\int_t^{\bar{t}}[W_q - P_q]dF - R'(A) - \mu\cdot\psi'(A)}$$

which can be rewritten as

$$\partial A^*/\partial\mu\,|_{\mu=0} = \psi(A^*)\cdot[\int_t^{\bar{t}}[W_q - P_q]dF - R'] = \psi(A^*)\cdot\partial A^*/\partial R \tag{7}$$

where $\partial A^*/\partial R$ is defined and interpreted as $\partial q^*/\partial P$ and $\partial A^*/\partial P$ were.

Finally, another implicit differentiation yields:

$$\partial A^*/\partial\theta = -\frac{\int_t^{\bar{t}} W_\theta dF}{\int_t^{\bar{t}}(W_q - P_q)dF} > 0. \tag{8}$$

Larger θ-types buy larger fuses.

2. THE PROBLEM OF THE FIRM.

We retain the assumptions on costs made by Panzar & Sibley: the firm faces a constant average cost, c, of producing electricity and a constant average cost, k, of installing capacity. The firm is constrained to instal productive capacity equal to the total rated fuse sizes of all customers. Reliability is assumed to be 100%. Finally, we assume that the firm maximizes expected profit.

Our first task is to construct the expression for the expected profit of the firm. The key insight, due to Goldman, Leland & Sibley (1984), is to realize that each infinitesimal dq segment along the price schedule can be treated as a different good whose demand is independent of the price charged for any other dq segment. This feature of demand functionals showed up in our comparative static study of demand. The natural procedure is to obtain the profit from each dq segment along the price schedule and integrate over the domain of the price schedule.

Consider first the profit obtained from the sale of electricity. Pick a small segment $(q', q'+dq)$ along the price schedule, $P(q)$. The market demand for this segment will consist of all consumers whose ex post demand for electricity exceeds $q=q'+dq$ and whose preselected fuse sizes allow them to purchase at least q units. We proceed in two steps: first we obtain the set of potential customers for the dq segment, that is, the set of customers whose fuses are big enough to allow the purchase of q units of electricity per period. Next, we obtain the actual purchasers of the dq segment at each temperature by looking at ex post demand.

At each q define $\tilde{\theta}$ as the customer whose fuse size permits him to consume no more than q units of electricity per period:

$$A^*(\tilde{\theta}, P, R) - q = 0. \tag{9}$$

Equation (9) defines $\tilde{\theta}(q, P, R)$. Implicit differentiation and earlier results yield:

$$\partial\tilde{\theta}/\partial\varepsilon\,|_{\varepsilon=0} = -\frac{\partial A^*/\partial\varepsilon}{\partial A^*/\partial\theta} = \frac{\eta(q)\cdot[1-F(f)]}{\int_f^{\bar{t}} W_\theta dF} = \eta(q)\cdot\partial\tilde{\theta}/\partial P \tag{10}$$

and

$$\partial\tilde{\theta}/\partial\mu\,|_{\mu=0} = -\frac{\partial A^*/\partial\mu}{\partial A^*/\partial\theta} = -\frac{\psi(A)}{\int_f^{\bar{t}} W_\theta dF} = \partial\tilde{\theta}/\partial R. \tag{11}$$

It is important to remember that f in these derivatives is evaluated at $(\tilde{\theta}, A^*, P)$. Thus we are evaluating the expression at the rationing temperature of the consumer whose fuse size permits him to consume q units.

Next define the temperature f at which $\tilde{\theta}$ is exactly rationed by his chosen fuse size:

$$W(q,\tilde{t},\tilde{\theta}(q,P,R))-P(q)=0. \tag{12}$$

Equation (12) implicitly defines $\tilde{t}(q,P,R)$. We note that :

$$\tilde{t}(q,P,R)=\tilde{t}(\tilde{\theta}(q,P,R),A^*(\tilde{\theta},P,R),P). \tag{13}$$

Differentiation of (12) (rewritten for the price schedule $P(q)+n\cdot\eta(q)$) yields

$$\partial\tilde{t}/\partial\varepsilon\,|_{\varepsilon=0}=\eta(q)\cdot[1-W_\theta\cdot\partial\tilde{\theta}/\partial P]/W_t\equiv\eta(q)\cdot\partial\tilde{t}/\partial P$$

and

$$\partial\tilde{t}/\partial\mu\,|_{\mu=0}=-\frac{\psi(A)\cdot W_\theta}{W_t\cdot\int_{\tilde{t}}^{\bar{t}}W_\theta dF}\equiv\psi(A)\cdot\partial\tilde{t}/\partial R.$$

When $t>\tilde{t}$, individual $\tilde{\theta}$ will purchase exactly q units as he will be constrained by his fuse , which, by definition is of size q. All consumers with $\theta>\tilde{\theta}$ will also purchase the dq segment under consideration as A^* and q^* are increasing functions of θ. All consumers with $\theta<\tilde{\theta}$ will not purchase the *qth* unit because their fuses are too small: they will be effectively rationed out of the market. The contribution of the dq segment to the expected profit of the firm in these states is therefore:

$$[P(q)-c]\cdot\int_{\tilde{t}(q,P,R)}^{\bar{t}}dF\cdot\int_{\tilde{\theta}(q,P,R)}^{\bar{\theta}}dG\;dq \tag{14}$$

When $t<\tilde{t}$, fuses play no role in determining purchases of electricity. $\tilde{\theta}$ was just rationed at $\tilde{t}$, at all lower temperatures his ex post demand will be smaller than his fuse because $\partial q^*/\partial t$ is positive. The set of purchasers will be determined only by ex post demand at these low temperatures. Let θ^* be the marginal purchaser, defined by $W(q,t,\theta^*)-P(q)=0$. This equation defines the functional $\theta^*(q,t,P)$. Its properties include:

$$\partial\theta^*/\partial\varepsilon\,|_{\varepsilon=0}=\eta(q)/W_\theta\equiv\eta(q)\partial\theta^*/\partial P.$$

For $t<\tilde{t}$, the contribution of the dq segment to expected profit is:

$$[P(q)-c]\cdot\int_{\underline{t}}^{\tilde{t}(q,P,R)}\int_{\theta^*(q,t,P)}^{\bar{\theta}}dGdF\;dq \tag{15}$$

Strictly speaking, we should limit consideration to those temperatures at which demand is positive. This can be done by defining t^* as the lowest temperature at which the largest consumer, $\bar{\theta}$, will purchase at least q units, $W(q,t^*,\theta bar)-P(q)=0$; and restricting the range of integration in (15):

$$[P(q)-c]\cdot \int_{t^*(q,P)}^{t(q,P,R)} \int_{\theta^*(q,t,P)}^{\bar{\theta}} dGdF\ dq \tag{16}$$

Combining (14) and (16) we obtain the following expression for the expected profit from the sale of electricity:

$$E\pi^1 = \int_0^{\bar{A}} \{[P(q)-c]\cdot[\int_t^{\bar{t}} dF \int_{\tilde{\theta}}^{\bar{\theta}} dG + \int_{t^*}^{t} \int_{\theta^*}^{\bar{\theta}} dGdF]\}\ dq$$

where $\bar{A}=A^*(\bar{\theta},P,R)$ is the largest fuse bought by any customer and $\overline{[0,A}]$ is the relevant domain of the price schedule. We have deliberately left out the arguments of the limits of integration in order to maintain clarity. Table 1 contains a list of all the relevant functionals showing their arguments and the equations defining them.

Total profit from the sale of fuses can be derived in exactly the same manner. It is non-stochastic because all the risk is borne by the consumer:

$$\pi^2 = \int_0^{\bar{A}} [R(A)-k]\cdot \int_{\tilde{\theta}(A,P,R)}^{\bar{\theta}} dG\ dA. \tag{18}$$

The expected profit of the firm from all sources is:

$$E\pi = E\pi^1 + \pi^2. \tag{19}$$

The problem facing the firm is to maximize (19) by appropriate choice of $P(q)$ and $R(A)$. The rules for optimal pricing are most easily obtained by using the standard variational approach. Let $P(q)$ and $R(A)$ be the profit maximizing choices and let $P(q)+\varepsilon\cdot\eta(q)$ and $R(A)+\mu\cdot\psi(A)$ be (feasible, single crossing) variations around them. Let the expected profit associated with the variation be $E\pi(P+\varepsilon\cdot\eta,\ R+\mu\cdot\psi)$. The assumed optimality of P and R will require that, for interior solutions:

$$\partial E\pi/\partial\varepsilon\,|_{\varepsilon=\mu=0} = \partial E\pi/\partial\mu\,|_{\varepsilon=\mu=0} = 0.$$

The expression for $E\pi(P+\varepsilon\cdot\eta, R+\mu\cdot\psi)$, with irrelevant arguments of the limits of integration suppressed, is:

$$E\pi(\varepsilon,\mu) = \int_0^{\bar{A}} \{[P(q)+\varepsilon\cdot\eta(q)-c]\cdot[\int_{t(\varepsilon,\mu)}^{\bar{t}} dF\cdot \int_{\tilde{\theta}(\varepsilon,\mu)}^{\bar{\theta}} dG + \int_{t^*(\varepsilon)}^{t(\varepsilon,\mu)} \int_{\theta^*(\varepsilon)}^{\bar{\theta}} dGdF +$$

$$[R(q)+\mu\cdot\psi(q)-k]\cdot[\int_{\tilde{\theta}(\varepsilon,\mu)}^{\bar{\theta}} dG]\}dq.$$

In writing this expression we have used the fact that q and A are dummy variables of integration that have the same range of integration and combined $E\pi^1$ and π^2. The necessary conditions for the maximization of $E\pi$ are derived in the Appendix. They are:

$$\int_{t}^{\bar{t}} dF \int_{\bar{\theta}}^{\bar{\theta}} dG + \int_{t^*}^{t}\int_{\theta^*}^{\bar{\theta}} dG dF - [P(q)-c]\cdot[g(\bar{\theta})\int_{t}^{\bar{t}} dF \cdot \partial\bar{\theta}/\partial P + \int_{t^*}^{t} g(\theta^*)\partial\theta^*/\partial P \; dF] -$$

$$[R(q)-k]\cdot[g(\bar{\theta})\partial\bar{\theta}\partial P] = 0. \tag{20}$$

$$[P(q)-c]\cdot[-\int_{t}^{\bar{t}} dF g(\bar{\theta})\partial\bar{\theta}/\partial R] + \int_{\bar{\theta}}^{\bar{\theta}} dG - [R(q)-k][g(\bar{\theta})\partial\bar{\theta}/\partial R] = 0. \tag{21}$$

3. PRICE COST RELATIONSHIPS

The relationships between prices and costs can be examined by analysing the first order conditions (20) and (21). These can be treated as two equations in the two unknowns $(P(q)-c)$ and $(R(q)-k)$. We now state and discuss a number of propositions on the price-cost relationship. The proofs can be found in Appendix 2.

PROPOSITION 2: $P(q) \geq c$.

In Appendix 2 we show that (20) & (21) can be solved simultaneously to yield:

$$P(q)-c = \frac{\int_{t^*}^{t}\int_{\theta^*}^{\bar{\theta}} dG dF}{\int_{t^*}^{t} g(\theta^*)\partial\theta^*/\partial P dF} = \frac{\int_{t^*}^{t}\int_{\theta^*}^{\bar{\theta}} dG dF}{\int_{t^*}^{t} g(\theta^*) W_{\theta} dF} \geq 0. \tag{22}$$

Equation 22 establishes that electricity will never be marginally priced below cost.

PROPOSITION 3 : The usual right end point condition for nonlinear price schedules holds :$P(\bar{A})=c$ and $R(\bar{A})=k$.

The right end point condition has a long history. The requirement that there be no distortion at the top of the schedule has been demonstrated for nonlinear income tax models by Mirrlees (1971) and Seade (1977), for models of product quality by Rosen and Mussa (1978) and for nonlinear price models by Spence (1977), Goldman, Leland & Sibley (1984) and Mirman & Sibley (1980). The result seems to hold in a wide variety of settings: in product markets and in labor markets, and for welfare maximizing and profit maximizing decision makers. Willig (1979) has shown that in product markets this condition is necessary for Pareto optimality.

By contrast, there have been very few models in which this condition has been violated by optimally chosen price structures. Exceptions have been Ordover & Panzar (1980 & 1982), Srinagesh (1986 & 1990), Srinagesh and Bradburd (1989), Srinagesh, Bradburd & Koo (1990), and Oren, Smith & Wilson (1985). In the first two papers the monopolist sells a product that is an input used by other firms, and general equilibrium repurcussions in the secondary output market drive a wedge in the usual end point condition. In Srinagesh (1986), rate of return regulation distorts the input choice of the firm and creates a wedge between private and social marginal cost. In Srinagesh (1990), the distortion arises because the monopolist is constrained to use linear prices on some of the goods he sells. The papers by Srinagesh & Bradburd (1989), and Srinagesh, Bradburd & Koo (1990) yield nonstandard assumptions because the consumer heterogeneity violates the usual monotonicity assumption. In the last paper, no explanation is offered for the failure of the right end point condition, but it appears that the cross elasticity of demand between output and capacity may have a role to play. We return to this point later in our discussion of mixed linear-nonlinear pricing.

PROPOSITION 4 : If $W_{\theta t}=0$ and if θ is uniformly distributed, then $R(A) \geq k$.

We have not been able to rule out completely the possibility that capacity will be marginally priced below cost in our model. However, we have established one set of conditions under which pricing below cost will be ruled out. While the conditions postulated by this proposition may appear somewhat restrictive, it is worth noting that the proposition places no (additional) restrictions on any derivatives of W other than $W_{\theta t}$, or on $f(t)$. Furthermore, our conditions are sufficient for prices to exceed cost, they are not necessary.

4. MIXED LINEAR-NONLINEAR PRICING

In this section we study a special case of the more general nonlinear pricing model introduced earlier. In particular, we assume that electricity continues to be sold according to a nonlinear price schedule, $p(q)$, but fuses are now assumed to be sold at the linear price, r. This price structure derives its interest not from the observation that all price structures are of this particular form, but from the implications of mixed linear-nonlinear pricing for the relationship between prices and costs. Specifically, it will be shown that the right end point condition generally fails in this case and that there is generally pricing below cost. This suggests that if the possibility of resale or repeat purchase for some components of the monopolist's output rules out the use of fully nonlinear pricing, then we should expect to find marginal prices below marginal cost. Mirrlees (1976) formulated and solved the problem of mixed linear-nonlinear taxation. His model was very general and the solution opaque. Our more restrictive assumptions allow us to obtain a clearer characterization of the optimal prices.

The details of the model are fairly similar to those of the more general case studied in the first two sections of this paper. Here we present the more interesting results and discuss their significance: longer derivations are relegated to Appendix 3. The two first order conditions for the firm's problem are:

$$\partial E\pi/\partial p = \int_{t^*}^{\tilde{t}}\int_{\theta^*}^{\bar{\theta}} dG\,dF + \int_{\tilde{t}}^{\bar{t}}\int_{\tilde{\theta}}^{\bar{\theta}} dG\,dF - [p(q)-c]\cdot\left\{\int_{t^*}^{\tilde{t}} g(\theta^*)\partial\theta^*/\partial p\,dF + \int_{\tilde{t}}^{\bar{t}} g(\tilde{\theta})\partial\tilde{\theta}/\partial p\,dF\right\}$$

$$-[r-k]\cdot\partial\tilde{\theta}/\partial p\cdot g(\tilde{\theta}) = 0. \qquad (23)$$

and

$$\partial E\pi/\partial r=\int_0^{\bar{A}}\{[p(q)-c]\cdot[-\int_{t}^{\bar{t}}dFg(\tilde{\theta})\partial\tilde{\theta}/\partial r]-[r-k]\cdot[g(\tilde{\theta})\partial\tilde{\theta}/\partial r]+\int_{\tilde{\theta}}^{\bar{\theta}}dG\}dq=0.$$

PROPOSITION 5 : The firm always selects a positive fuse price, $r > 0$.

Proof: $\lim_{r\to 0}\partial E\pi/\partial r = \int_0^{\bar{A}}[k\cdot g(\tilde{\theta})\partial\tilde{\theta}/\partial r+\int_{\tilde{\theta}}^{\bar{\theta}}dG]dq$ which is strictly positive because $\partial\tilde{\theta}/\partial r>0$ and $\tilde{\theta}<\bar{\theta}$ for all $q<\bar{A}$. The Kuhn- Tucker condition for the choice of r requires that $\partial E\pi/\partial r\leq 0$ if the optimal choice of r is zero; hence we can conclude that r is positive.

Corollary 1: If capacity costs are zero (for example during off peak periods) then capacity will be sold at a profit. If the underlying problem is well-behaved so that the solution is continuous in the parameters, then for given values of the other parameters there will exist an interval of low capacity costs $[0,k']$ for which the price charged for a unit fuse will exceed the cost of capacity to the firm. The implication of this result for pricing below cost is brought out by the next proposition.

PROPOSITION 6 : If $r > k$ then $p(\bar{A}) < c$.

Proof: Take the limit of (22) as $q\to A$:

$$-[p(\bar{A})-c]\cdot[\int_{t(\bar{\theta})}^{\bar{t}} g(\bar{\theta})\partial\theta^*/\partial p\cdot dF+\int_{t(\bar{\theta})}^{\bar{t}} g(\bar{\theta})\partial\tilde{\theta}/\partial p dF]-[r-k]\cdot g(\bar{\theta})\partial\tilde{\theta}/\partial p=0.$$

Propositions 1 and 5 guarantee that $t(\theta)<\bar{t}$; and $\partial\theta^*/\partial p$ and $\partial\tilde{\theta}/\partial r$ are both positive. Thus, $p(\bar{A})-c$ and $r-k$ both have strictly negative coefficients in the above equation and they must either be of opposite sign or both be zero. We have shown that when $k=0$, $r-k>0$. In this case, Proposition 6 guarantees that electricity will be priced marginally below cost in a neighborhood of the right end point of the price schedule. More generally, when k is sufficiently close to zero we will see electricity priced below cost.

Nothing in the model rules out cross-subsidization in the other direction, with $r<k$ and $p(\bar{A})>c$. While we have not established sufficient conditions for this form of cross subsidy, numerical simulation suggests that as we increase k, holding all other parameters fixed, we move to a knife edge where $r=k$, and then to another range of parameter values where $r<k$. In this range Proposition 6 guarantees that $P(\bar{A})>c$.

The economic intuition underlying our result on pricing below cost can be found in the loss leader problem first studied by Allen (1971). An increase in the price of fuses induces consumers to install smaller fuses; this reduces their rationing temperature and in turn reduces their expected demand for electricity. An increase in the price of electricity reduces the option value of the fuse, which depends on the difference between the willingness to pay for electricity and its marginal price. Thus, in our model electricity and fuses are complementary goods. Allen showed that one of the conditions under which a monopolist selling complementary goods would price one below cost was if the cross elasticity of demand was large relative to the own elasticities. This is relevant to the range of models we have studied. In the first model where both goods were sold according to a nonlinear price, complementarities at the level of the market demand curves were not significant. Changing $P(q)$ ($R(q)$) over an interval dq only affected the demand for fuses (electricity) of those consumers whose fuse sizes (ex post demands) were in that particular dq interval. In the mixed linear-nonlinear price model, however, a change in the price of fuses affects the ex post demands of all consumers by changing their rationing temperature. The complementarities at the level of market demand are significantly greater and the loss leader phenomenon appears as a robust characteristic of the profit maximizing price structure. Further, Allen argued that the good that would be subsidized would be the one that had the relatively smaller volume of sales at perfectly competitive prices. Proposition 6 is certainly in agreement with this conclusion: when k is low, and the demand for fuses relatively high, the firm will use electricity as the loss leader. What is different in our model is that pricing below cost is the norm, whereas in the non- discriminating model of Allen the loss leader example was an unexpected exception.

Proposition 6 has a history of sorts. Oi (1971) was the first to show that if consumers' demand curves crossed one another the profit maximizing two part tariff may require that the service be sold at a loss and profit be taken entirely through the entry fee. Ng & Weisser (1974) generalized this result. Littlechild (1975), in his study of two part tariffs with consumption externalities, suggested that it sometimes paid the firm to set an entry fee below the cost of a connection and to generate profit only on the service sold to consumers. Schmalensee (1981) refers to these as a policy of "giving away the razor and making money on the blades" and based on casual empiricism, suggested that they are the norm (p 457). Averch & Johnson (1969) argued that a partially regulated monopolist would subsidize his unregulated products and make up the losses on the regulated market where an above market rate of return was guaranteed. More recently Sherman and Visscher (1982) and Srinagesh (1986) have shown that a monopolist selling a single good according to a nonlinear price will select a marginal price less than marginal cost if he faces rate of return regulation. Oren, Smith & Wilson suggest that such pricing below cost can occur even in the absence of regulation and strategic considerations if the description of goods, the economic environment and the pricing structure is made rich enough. Furthermore their result is robust in that it holds for a whole class of examples. Proposition 6 is robust too; it holds for a range of values of k and does not impose any functional restrictions on the demand or density functions.

While the role of complementarity in the provision of loss leaders has been long recognized, its function in the context of nonlinear pricing models is of more recent vintage. Schmalensee was the first to note that the complementarity of the right to buy the good and the purchase of the good itself was probably responsible for the widespread practice of pricing below cost. The additional insight offered by the results we have derived is that the restrictions on the admissible price functions have important implications for the degree of complementarity and therefore for pricing below cost. What is surprising is that as we go from linear to partly nonlinear prices, pricing below cost becomes the norm, but when we move to more fully nonlinear prices (Section 2) pricing below cost disappears.

The intuition underlying Oren, Smith & Wilson's result is less clear but seems to mesh with our results. The cost function of the seller, the utility function of the purchasers and the price function all conform in their model: they are all additive measures on the real plane. They point out that this implies that there is a limit to gains from substitution and "direct substitution between points differing in both dimensions is excluded" (p 552). In light of our results, it seems likely that this fundamental complementarity they build into preferences is responsible for pricing below cost.

5. CONCLUSIONS

Firms adopt a variety of marketing strategies to influence consumer behavior. This paper has been concerned with one such strategy: the use of selective incentives embodied in a nonlinear pricing scheme. We have developed a general model of public utility pricing under risk in which the price structure is used to sell the good and also allocate scarce capacity among consumers. For the more general price structure examined by us, we derived the profit maximizing price rule and provided some conditions that would result in pricing above cost. For the special case in which capacity is priced linearly, we found that pricing below cost was the rule and not the exception. We suggested that the underlying complementarity between the service and the derived demand for the capacity to consume the service was responsible for this result. Further, we have suggested that this complementarity is stronger when the form of the price structure is constrained. To the extent that storage or the possibility of resale of some components of a monopolist's output constrain his ability to use fully nonlinear prices we would expect some real price structures to display prices below cost.

On the demand side, a necessary extension is to allow for intraperiod variation in demand and noncoincident peaks across customers. The current assumption requires that network capacity is equal to the sum of all individual demands for capacity. This is clearly not required if diffferent individuals use their fuses fully in different intervals. In this case, capacity will be more like a quasi-public good, and the optimal pricing rule will change.

On the supply side, the most interesting area for future research would be to include a consideration of supply side uncertainties. Black outs, brown outs and selective load shedding are rationing strategies that are sometimes triggered by excessive demand when all consumers turn on their air conditioners at the same time. Often, however, unforeseen accidents (floods, thunderstorms, fire) knock out part of a utility's capacity leaving it unable to meet all demand. If we allow for supply side uncertainty of this kind, self rationing cannot be used to allocate scarce capacity in all states of nature; it must be supplemented by a rationing rule that will be put into effect when a supply shock reduces capacity below the rated fuse sizes of all consumers. The interaction between this rationing rule and the self rationing scheme should prove to be a fertile area for research.

Second, It would be useful to extend the analysis of nonlinear prices to oligopoly. Many of the services to which nonlinear pricing models are applicable are characterized by intense rivalry among a few firms. The market for long distance calls is an example.

Oren, Smith & Wilson (1986) have taken some steps in this direction by characterizing the Nash equilibria for a variety of strategies allowed the firms. Saidi & Srinagesh (1981) provided an example, in the context of trade policy, in which a monopolist (the government) sold imports in a domestic market that was also served by a competitive fringe of small domestic firms that reacted passively to the government's choice of a nonlinear price on imports. Much remains to be done.

It would also be useful to examine the role of the more common forms of regulation on price structures. It is hoped that this paper is a first step towards exploring these issues.

FOOTNOTES

1. There is now a large literature on nonlinear pricing. Early work looked at special cases such as two block (Gabor 1955) and two part tariffs (Oi 1971), or both (Leland & Meyer 1976). Murphy (1977) studied multipart tariffs. Spence (1977) was the first to consider price a continuous function of quantity purchased; the technique he used had been pioneered by Mirrlees (1971) in his study of optimal taxation. Spence (1980) developed an alternative discrete framework for deriving nonlinear prices; the same framework was used by Guesnerie & Seade (1982) in their analysis of optimal taxes. Goldman, Leland & Sibley (1984) introduced the simplifications that are at the core of this paper. A comprehensive list of references can be found in a recent book by Brown & Sibley (1986) which surveys this area. There is an overlap between the literatures on nonlinear prices, optimal taxes, insurance schemes and the principal agent literature. In this paper we do not attempt to develop the most general model that could be simultaneously applied to all the above-mentioned fields. Our more narrow focus is on the issues of particular interest to public utility pricing.

The literature on rationing schemes stems from a seminal contribution by Brown & Johnson (1969). They considered the joint choice of a welfare maximizing utility that chose capacity and price before uncertainty was resolved, given that the good would be allocated to those with the highest willingness to pay if random demand exceeded scarce capacity. Visscher (1971) examined implications of alternative rationing rules for the price- capacity choice. More recently, Panzar & Sibley (1978) have examined the properties of self-rationing schemes.

2. The properties of the price schedule are endogenous to the model. By restricting our attention to a convenient subset of price schedules we may well preclude consideration of the true, globally optimal price schedule. The first order approach (discussed in the principal agent context by Rogerson (1985)) restricts attention to the price schedules for which the consumer's behavior is fully characterized by his first order condition. Rogerson provides conditions for this assumption to be innocuous for one kind of discrete problem. Goldman, Leland & Sibley show that the restriction is innocuous for one kind of nonlinear pricing model. Most other authors note that the exact conditions for the restriction to be innocuous are not known, and pass on (e.g. OSW, p 553 and Holmstrom (1979) fn 11). We follow the latter route.

3. We adopt the convention (implicit in Panzar-Sibley) that A and q are measured in comparable units. Thus if A is measured in kilowatts and if the period is one week, then q is measured in kilowatt weeks. Only then will statements such as A r q make sense.

APPENDIX 1

In this appendix we present the derivation of the first order conditions to the firm's problem. The expression for the firm's expected profit, with irrelevant arguments omitted, is:

$$E\pi(\varepsilon,\mu) = \int_0^{\bar{A}} \{[P(q)+\varepsilon\cdot\eta(q)-c]\cdot[\int_{\tilde{t}(\varepsilon,\mu)}^{\bar{t}} dF \int_{\tilde{\theta}(\varepsilon,\mu)}^{\bar{\theta}} dG + \int_{t^*(\varepsilon)}^{\tilde{t}(\varepsilon,\mu)} \int_{\theta^*(\varepsilon)}^{\bar{\theta}} dGdF]+$$

$$[R(q)+\mu\cdot\psi(q)-k]\cdot \int_{\tilde{\theta}(\varepsilon,\mu)}^{\bar{\theta}} dG\}dq \qquad \text{(A.1)}$$

Differentiating with respect to ε we obtain:

$$\partial E\pi/\partial\varepsilon = \int_0^{\bar{A}} \{\eta(q)\cdot[\int_{\tilde{t}}^{\bar{t}} dF \int_{\tilde{\theta}}^{\bar{\theta}} dG + \int_{t^*}^{\tilde{t}} \int_{\theta^*}^{\bar{\theta}} dGdF]+$$

$$[P(q)-\varepsilon\cdot\eta(q)-c]\cdot\{-f(\tilde{t})\partial\tilde{t}/\partial\varepsilon \int_{\tilde{\theta}}^{\bar{\theta}} dG - g(\tilde{\theta})\partial\tilde{\theta}/\partial\varepsilon \int_{\tilde{t}}^{\bar{t}} dF -$$

$$\int_{t^*}^{\tilde{t}} g(\theta^*)\partial\theta^*/\partial\varepsilon dF - \int_{\theta^*(t^*)}^{\bar{\theta}} dGf(t^*)\partial t^*/\partial\varepsilon + \int_{\theta^*(\tilde{t})}^{\bar{\theta}} dGf(\tilde{t})\partial\tilde{t}/\partial\varepsilon\}+$$

$$[R(q)+\mu\cdot\psi(q)-k]\cdot[-g(\tilde{\theta})\partial\tilde{\theta}/\partial\varepsilon])\}dq$$

Considerable simplification is possible. First note that $\theta^*(t^*)=\bar{\theta}$, so that $\int_{\theta^*(t^*)}^{\bar{\theta}} dG=0$. Next note that $\theta^*(\tilde{t})=\tilde{\theta}$ because the smallest consumer whose ex post demand is q at temperature $\tilde{t}$ (i.e. $\theta^*(\tilde{t})$) is also the one whose fuse size is q (by definition of $\tilde{t}$ and $\tilde{\theta}$). Therefore the first, fourth and fifth terms in the second square bracket drop out. Next evaluate the derivative at $\varepsilon=\mu=0$, and replace terms such as $\partial\tilde{\theta}/\partial\varepsilon$ with the notationally equivalent $\eta(q)\partial\tilde{\theta}/\partial\varepsilon$ to obtain:

$$\partial E\pi/\partial\varepsilon\,|_{\varepsilon=\mu=0} = \int_0^{\bar{A}} \eta(q)\{\int_{\tilde{\theta}}^{\bar{\theta}} dG \int_{t}^{\bar{t}} dF + \int_{t^*}^{t}\int_{\theta^*}^{\bar{\theta}} dG dF +$$

$$[P(q)-c]\cdot[-g(\tilde{\theta})\partial\tilde{\theta}/\partial P \int_{t}^{\bar{t}} dF - \int_{t^*}^{t} g(\theta^*)\partial\theta^*/\partial P dF] + \qquad \text{A.2}$$

$$[R(q)-k]\cdot[-g(\tilde{\theta})\partial\tilde{\theta}/\partial P]\}dq.$$

The optimality of the price structure requires that A.2 be nonpositive for all permissible variations in the price schedule. For an interior optimum in the neighborhood of the optimal $P(q)$, this reduces to the requirement that the expression in curly brackets be zero. This is the necessary condition reported in the text.

The single crossing restriction gives rise to the possibility of corner solutions which we briefly note. Essentially, single crossing schedules cannot cut any demand curve from above, but are allowed to lie along an inverse demand function over a non-null interval. Over such an interval, upward variations in the price schedule are not permitted as they will result in a new price schedule that is not single crossing. If we restrict $\eta(q)$ to be non-positive in order to stay within the permissible class of variations, then it is not necessary that the expression within the curly brackets in A.2 be zero; it can be positive while the necessary condition that the change in $E\pi$ be non-positive is satisfied. Rosen & Mussa (1979), Goldman, Leland & Sibley (1985) and Brown & Sibley (1986) (pp 209-210) dealt with the discontinuities that arise in this case. We have nothing further to add on this score. Dealing with the corner solutions does not seem to add much of substantial value to the economic intuition of the problem, its main use is in the successful numerical solution of examples. We focus on the interior solution described by equation 20.

The other necessary condition can be derived in the same way.

$$\partial E\pi/\partial\mu = \int_0^{\bar{A}} \{[P(q)+\varepsilon\cdot\eta(q)-c]\cdot[-f(t)\partial t/\partial\mu \int_{\tilde{\theta}}^{\bar{\theta}} dG - \int_{t}^{\bar{t}} dF\cdot g(\tilde{\theta})\partial\tilde{\theta}/\partial\mu + \int_{\theta^*(t)}^{\bar{\theta}} dG f(t)\partial t/\partial\mu]$$

$$+\,\psi(q)\int_{\tilde{\theta}}^{\bar{\theta}} dG + [R(q)+\mu\cdot\psi(q)-k]\cdot[-g(\tilde{\theta})\partial\tilde{\theta}/\partial\mu]\}dq.$$

Use $\theta^*(\bar{t})=\tilde{\theta}$ and $\partial\tilde{\theta}/\partial\mu=\psi(q)\cdot\partial\tilde{\theta}/\partial R$ to get:

$$\partial E\pi/\partial\mu\,|_{\varepsilon=\mu=0} = \int_0^{\bar{A}} \psi(q)\{[P(q)-c]\cdot[-\int_{t}^{\bar{t}} dF g(\tilde{\theta})\partial\tilde{\theta}/\partial R] + \int_{\tilde{\theta}}^{\bar{\theta}} dG + \qquad \text{A.3}$$

$$[R(q)-k]\cdot[-g(\tilde{\theta})\partial t/\partial R]\}dq.$$

Once again, in order to guarantee the optimality of P and R, we need to ensure that A.3 is zero for permissible variations. Subject to the caveat noted earlier, this requires that the term in curly braces be zero, which is the second necessary condition used in the text (equation 21).

APPENDIX 2

Proof of Proposition 2:

Equation 21 can be rewritten:

$$R(q)-k=-[P(q)-c]\cdot\int_{\tilde{t}}^{\bar{t}} dF+\frac{\int_{\tilde{\theta}}^{\bar{\theta}} dG}{g(\tilde{\theta})\cdot\partial\tilde{\theta}/\partial R}. \tag{B.1}$$

Substitute in (20), use $\dfrac{\partial\tilde{\theta}/\partial P}{\partial\tilde{\theta}/\partial R}=\int_{\tilde{t}}^{\bar{t}} dF$, and gather terms to get

$$P-c=\frac{\int_{t^*}^{\tilde{t}}\int_{\theta^*}^{\bar{\theta}} dG\,dF}{\int_{t^*}^{\tilde{t}} g(\theta^*)\partial\theta^*/\partial P\,dF}\geq 0. \tag{B.2}$$

This establishes Propsition 2.

Proof of Propostion 3:

$P(\bar{A})$ cannot be directly evaluated by taking the limit of B.2 as $q\to\bar{A}$ because the numerator and denominator tend to zero, leaving us with an indeterminate form. This happens because in the numerator $\theta^*\to\bar{\theta}$ as $q\to\bar{A}$ (only consumer $\bar{\theta}$ purchases the $\bar{A}th$ unit), while in the denominator, t^* (the temperature at which $\bar{\theta}$ buys exactly $\bar{A}$ units) tends to $\tilde{t}$, the temperature at which $\bar{\theta}$ is just rationed, because $\bar{A}$ is $\bar{\theta}'s$ fuse size. We resort to L'Hospital's Rule.

Let $N(q)$ be the numerator of B.2; fully written out it is:

$$N(q)=\int_{t^*(q,P)}^{\tilde{t}(q,P,R)}\int_{\theta^*(q,t,P)}^{\bar{\theta}} dG\,dF.$$

On differentiation:

$$dN/dq=\int_{t^*}^{\tilde{t}} -g(\theta^*(q,t,P))d\theta^*/dq\,dF+\int_{\theta^*(q,\tilde{t},P)}^{\bar{\theta}} dG f(\tilde{t})d\tilde{t}/dq-\int_{\theta^*(q,t^*,P)}^{\bar{\theta}} dG f(t^*)dt^*/dq.$$

Now $\theta^*(q,t^*,P)=\bar{\theta}$ by definition, $\theta^*(q,t,P)\to\bar{\theta}$ as $q\to\bar{A}$ and $t^*\to\tilde{t}$ as $q\to\bar{A}$. Hence $dN/dq\to 0$ as $q\to\bar{A}$.

Let $D(q)$ be the denominator of B.2. Fully written out it is:

$$D(q)=\int_{t^*(q,P)}^{\tilde{t}(q,P,R)} g(\theta^*(q,t,P))\partial\theta^*/\partial P\;dF.$$

On differentiation, we have:

$$dD/dq=\int_{t^*}^{\tilde{t}} g'(\theta^*)\,d\theta^*/dq\cdot\partial\theta^*/\partial P\,dF+\int_{t^*}^{\tilde{t}} g(\theta^*)\,\partial^2\theta^*/\partial P\,\partial q\,dF+$$

$$g(\theta^*(q,t,P))\,\partial\theta^*/\partial P\,|_{t=\tilde{t}}\,f(\tilde{t})\,d\tilde{t}/dq-g(\theta^*(q,t^*,P))\,\partial\theta^*/\partial P\,|_{t=t^*}\,dt^*/dq\cdot f(t^*).$$

As $q\to\bar{A}$, $t^*\to\tilde{t}$ and the first two integrals vanish. We are left with:

$$\lim_{q\to\bar{A}} dD/dq=g(\theta^*(q,t,P))\,\partial\theta^*/\partial P\,|_{\tilde{t}}\cdot f(\tilde{t})\cdot[d\tilde{t}/dq-dt^*/dq].$$

Now $\tilde{t}$ is implicitly defined by $W(q,\tilde{t},\tilde{\theta}(q,P,R))-P(q)=0$, so $d\tilde{t}/dq=-[W_q+W_\theta d\tilde{\theta}/dq-P_q]/W_t.$ On the other hand, t^* is defined by $W(q,t^*,\theta)-P(q)=0$, so $dt^*/dq=-[W_q-P_q]/W_t$. It follows that

$$\lim_{q\to\bar{A}} dD/dq=-\,g(\bar{\theta})\partial\theta^*/\partial P f(\tilde{t}(\theta)\,|_{\bar{\theta}}\{W_\theta d\theta^*/dq]/W_t\}>0.$$

Therefore, by L'Hospital's Rule we conclude that $P(q)\to c$ as $q\to\bar{A}$.

The proof that $R(\bar{A})=k$ now follows in a straightforward fashion by taking the limit of B.1 as $q\to A$.

Proof of Proposition 4.

The two first order conditions can be solved for $R-k$:

$$R-k=\frac{\int_{\tilde{\theta}}^{\bar{\theta}} dG\int_{\tilde{t}}^{\bar{t}} W_\theta dF}{g(\tilde{\theta})}-\frac{\int_{t^*}^{\tilde{t}}\int_{\theta^*}^{\bar{\theta}} dG\,dF\int_{\tilde{t}}^{\bar{t}} dF}{\int_{t^*}^{\tilde{t}} g(\theta^*)\,\partial\theta^*/\partial P\,dF}.$$

When $W_{\theta t}=0$ and $g(\theta)$ is constant, this simplifies to

$$R-k=[W_\theta/g]\left\{\int_{\tilde{\theta}}^{\bar{\theta}} dG\int_{\tilde{t}}^{\bar{t}} dF-\frac{\int_{t^*}^{\tilde{t}}\int_{\theta^*}^{\bar{\theta}} dG\,dF\int_{\tilde{t}}^{\bar{t}} dF}{\int_{t^*}^{\tilde{t}} dF}\right\}$$

We now use the fact that $\tilde{\theta}<\theta^*$, which follows because $\tilde{\theta}$ is the consumer who is just rationed at q and therefore is smaller than any consumer θ^* who purchases the *qth* unit at lower temperatures when fuses are not effective constraints. Thus if we replace θ^* by $\tilde{\theta}$, we will decrease the value of the right hand side. Further, $\tilde{\theta}$ is not a function of t and can therefore be factored out of the integral. Once we do this we are left with the result that $R-k\geq 0$.

The potential for pricing below cost is apparent in B.3. It appears possible that if $g(\theta^*)$ is small enough for some dq interval relative to W_θ and $g(\tilde{\theta})$, then the second term in B.3 will dominate, resulting in $R\leq k$. This possibility may not in fact be achievable: the second order conditions for the firm's problem and the requirement that the optimal price schedule be single crossing place complex restrictions on the density functions and the inverse demand curves. We leave open the question of whether it is possible to generate a well behaved example in which fuses are priced below cost.

APPENDIX 3

This appendix presents the details of the mixed linear- nonlinear price model of Section 3. We retain the earlier notation, with two exceptions: the prices are denoted $p(q)$ and r to differentiate them from the more general case studied earlier. The analysis of consumer demand is not different in any significant way from the earlier treatment, so we move directly to the problem of the firm. Expected profit written in our abbreviated notation, is

$$E\pi(\varepsilon,r)=\int_0^{\bar{A}}\{[p(q)+\varepsilon\cdot\eta(q)-c]\cdot[\int_{t^*(\varepsilon)}^{\tilde{t}(\varepsilon,r)}\int_{\theta^*(\varepsilon)}^{\bar{\theta}} dGdF+\int_{\tilde{t}(\varepsilon,r)}^{\bar{t}} dF\int_{\tilde{\theta}(\varepsilon,r)}^{\bar{\theta}} dG]+[r-k]\cdot\int_{\tilde{\theta}(\varepsilon,r)}^{\bar{\theta}} dG\}dq.$$

On differentiation we have:

$$\partial E\pi/\partial\varepsilon|_{\varepsilon=0}=\int_0^{\bar{A}}\{\eta(q)\cdot[\int_{t^*}^{\tilde{t}}\int_{\theta^*}^{\bar{\theta}} dGdF+\int_{\tilde{t}}^{\bar{t}}\int_{\tilde{\theta}}^{\bar{\theta}}dG]+$$

$$[p(q)-c]\cdot[-\int_{t^*}^{\tilde{t}} g(\theta^*)\partial\theta^*/\partial\varepsilon\, dF+\int_{\theta^*(\tilde{t})}^{\bar{\theta}} dGf(\tilde{t})\partial\tilde{t}/\partial\varepsilon-\int_{\theta^*(t^*)}^{\bar{\theta}} dGf(t^*)\partial t^*/\partial\varepsilon-$$

$$\int_{\tilde{\theta}}^{\bar{\theta}} dGf(\tilde{t})\partial\tilde{t}/\partial\varepsilon-\int_{\tilde{t}}^{\bar{t}} dFg(\tilde{\theta})\partial\tilde{\theta}/\partial\varepsilon]-[r-k]\cdot g(\tilde{\theta})\partial\tilde{\theta}/\partial\varepsilon\}dq=0.$$

As we showed earlier, $\theta^*(\tilde{t})=\tilde{\theta}$, so

$$\int_0^{\bar{A}}\eta(q)\{\int_{t^*}^{\tilde{t}}\int_{\theta^*}^{\bar{\theta}} dGdF+\int_{\tilde{t}}^{\bar{t}}dF\int_{\tilde{\theta}}^{\bar{\theta}}dG-[p-c]\cdot[\int_{t^*}^{\tilde{t}} g(\theta^*)\partial\theta^*/\partial p dF+\int_{\tilde{t}}^{\bar{t}}dFg(\tilde{\theta})\partial\tilde{\theta}/\partial p]-$$

$$[r-k]\cdot g(\tilde{\theta})\partial\tilde{\theta}/\partial p\}dq=0.$$

Interior maxima requre that the expression in curly braces be zero. This is the first necessary condition used in the text.

The other first order condition is obtained by differentiating $E\pi$ with respect to r:

$$\partial E\pi/\partial r=\int_0^{\bar{A}}\{[p-c]\cdot[\int_{\theta^*(\tilde{t})}^{\bar{\theta}} dGf(\tilde{t})\partial\tilde{t}/\partial r-g(\tilde{\theta})\partial\tilde{\theta}/\partial r\int_{\tilde{t}}^{\bar{t}}dF+\int_{\tilde{\theta}}^{\bar{\theta}}dG-[r-k]\cdot g(\tilde{\theta})\partial\tilde{\theta}/\partial r\}dq=0.$$

If we substitute $\theta^*(\tilde{t})=\tilde{\theta}$, we are left with the second necessary condition used in the text.

TABLE 1

The following functionals appear frequently as limits of integration:

1. $t^*(q,P)$, the lowest temperature at which the qth unit along the price schedule is bought by some consumer.It is defined implicitly by:

$$W(q,t^*,\bar{\theta})-P(q)=0.$$

2. $\theta^*(q,t,P)$, the smallest consumer who purchases at least q units at temperature t. It is defined implicitly by:

$$W(q,t,\theta^*)-P(q)=0.$$

3. $\hat{t}(\theta,A,P)$, the temperature at which consumer θ is just rationed if his fuse size is A, given the price schedule $P(q)$. It is defined by:

$$W(A,\hat{t},\theta)-P(q)=0.$$

4. $\tilde{\theta}(q,P,R)$, the consumer whose fuse size is q, given the price structure. It is implicitly defined by:

$$A^*(\tilde{\theta},P,R)-q=0.$$

5. $\hat{t}(q,P,R)$, the temperature at which $\tilde{\theta}$ is just rationed. It is implicitly defined by:

$$\hat{t}(q,P,R)=\hat{t}(\tilde{\theta},q,P).$$

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