MODELING SOIL EROSION CONTROL POLICY: A MULTI-LEVEL DYNAMIC ANALYSIS

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INTRODUCTION

While the importance of topsoil and the problem of topsoil loss have long been recognized, more than four decades of federal programs have done little to stay the erosion of our farmland (McConnell; Napier and Forster; Rasmussen; Walker). One-third of U.S. cropland topsoil has been lost in the last 200 years (Walker), and sheet and rill erosion on U.S. cropland continues at a rate of 1.6 billion tons annually (USDA/SCS, 1990).

Increased public awareness of these issues continues to put pressure on policymakers to solve our environmental problems while maintaining an abundant, inexpensive food supply. Policy debates, past and present, have focused primarily on physical actions to be taken rather than upon policy goals and objectives (Robinson).

Yet the use of physical goals of soil loss is often challenged by economists who argue that it is not the physical loss from the farm that is important, but the costs incurred by loss of topsoil and offsite damages from agriculturally-generated sedimentation (Robinson). Of the two, offsite sediment damages are far greater than onsite productivity damages (McConnell; Swanson, 1979).

Sediment is the largest polluter of ponds, streams, rivers and reservoirs (Clark, et al.; Miller and Everett; Wade and Heady; Water Resources Council). Sediment trapped in ditches and lakes reduces water holding capacity and increases the likelihood of flooding. It increases dredging costs of rivers and harbors, fills reservoirs, damages wildlife habitats and diminishes recreational enjoyment of water resources. The annual offsite cost of erosion in the United States is estimated to be $6.2 billion (Clark, et al.).

Recently, more attention has been paid to combining the economic and physical components of agricultural pollution problems. Kenneth Robinson states "a shift away from emphasizing physical targets to the use of economic criteria (or some combination of the two) probably would lead to greater returns to society from the dollars currently invested in conservation activities" (p. 153).

This study focuses on combining economic and physical relationships to develop a workable policy model to optimize both agricultural revenues and environmental quality. The specific objective of this study is to develop a soil conservation policy model, incorporating the objectives of both producers and policymakers, that: 1) estimates the economic transfers between agricultural producers and society resulting from agricultural production and its associated sediment-based externalities, and 2) estimates a socially optimal level of taxation, given varying levels of perceived importance to society of agricultural income versus environmental quality.

A multi-level optimal control model is developed that optimizes the producer's dynamic problem in the first stage, then uses the producer's optimal decision paths in the second stage as components of the policymakers' dynamic problem. The second stage problem utilizes weights indicating the relative importance of the policymaker's competing goals. Those weights are used to construct a dynamic policy frontier demonstrating the relationships between policy goals and the resulting optimal solutions. The model used is an adaptation of the multi-level programming model developed by Candler and Norton and extended by Sylvia and Anderson.

DEVELOPING A SOIL EROSION POLICY MODEL

A basic premise of this work is that several specific elements are necessary for the development of a soil erosion policy model. Those elements include modeling (1) the interaction of producer and policymaker (Anderson and Lee; Candler, et al.; Candler and Norton; Sharp and Bromley; Sylvia and Anderson), (2) the dynamic nature of the sedimentation process (Ervin, et al.; Rausser, 1980; Swanson, 1982), and (3) the recognition of multiple, competing goals of the policymaker (Rausser and Yassour; Sylvia and Anderson; Van Kooten, et al.).

Interaction of Producer and Policymaker

While a production model may examine the optimal choices of a single decision-making producer, a model of policy regulation must consider both the producer and the policymaker. Anderson and Lee point out that research on regulatory policy designed to correct open-access market failure usually examines only the optimal level of pollutant released into a watershed. Models are built with production of the pollutant as the control variable. Yet the policymaker does not have direct control over production of pollutant. He has control only over the regulatory instrument itself. Thus, the control variable for the producer is production, but the control variable for the policymaker is the governing instrument.

A recently developed modeling approach that explicitly recognizes this dichotomy is multilevel programming. Developed by Candler and Norton in 1977, multi-level programming is intended to optimize a system of "two separate decision makers in hierarchical relationship, each with his own objective function and control over distinct but interacting variables" (Candler, et al.).

This concept would appear to hold significant potential for applied policy analysis, yet Candler, et al. found less than satisfactory results in attempting to solve analytical or numerical problems. This was later accomplished by Sylvia and Anderson, who developed an analytical system for net-pen aquaculture development. While no empirical work was performed, the framework was established for the practical use of multi-level optimal control models.

The Dynamic Nature of Sedimentation

Ervin, et al. state "conservation programs . . . should be evaluated relative to some hypothetical 'optimal' erosion control solution which maximizes net social benefits" (p.274). Since the sedimentation process is inherently dynamic in nature, such an "optimal solution" would necessarily be derived from a model which explicitly recognizes soil as both stock and flow resource.

In referring to the on-farm problem, Rausser (1980) advances that soil conservation is a problem in capital theory whose operational implementation is the management of soil resources over time. It is this aspect of the soil loss process, the interrelationships of both stock and flow resources over time, that demands an explicitly dynamic methodology such as optimal control theory.

Multiple Policymaker Goals

Even more than the producer, a policymaker must balance many competing goals in policy implementation. The policymaker is influenced in specific implementation decisions by political pressures, social welfare, and his own preference function. Reichelderfer and Boggess state that the correct specification of program decision makers' objectives is critical. Rausser (1982) attempted to achieve this by specifying a weighted political preference function. The model incorporated a weighting scheme to represent the relative importance of the various objectives of an agricultural policymaker. Similarly, Sylvia and Anderson applied weights to various policy goals in a dynamic model of net-pen aquaculture. The relative values of the weights were then allowed to vary, providing a set of policy frontiers over time.

THE THEORETICAL MODEL

The Producer's Dynamic Model

The farmer produces crops and erosion as he seeks to maximize the time stream of discounted profits. The erosion results from tillage of the soil for planting and from harvesting. Erosion production is measured as some multiple of production and the eroded soil is eliminated through runoff into streams and ditches carried by rain. The erosion affects the productivity of the farm production and negatively impacts waterways downstream as sediment. While potential instruments for controlling erosion by the farmer might include controls such as tillage practices, site placement, and field size and slope, it is assumed that such factors are fixed and that the only existing instrument is production. Variations in the other factors are assumed to be accounted for in the cost equation. The only instrument available to the policymaker to affect the farmer's decisions is a per unit tax[1] on erosion.

The "producer" in this model is assumed to be a homogeneous aggregation of all individual producers within the producing area.

The producer's formal problem is:

(1) $$\text{Maximize} \int_{t=0}^{T=\infty} e^{-rt}[pY(t) - C(Y(t)) - \tau E(t)]\,dt$$

(2) subject to: $\dot{E} = F(E(t), Y(t))$,

and the usual set of non-negativity and convexity conditions, where: p is the revenue per unit of production, Y is the number of units produced, C is a convex production cost function, τ is a per unit erosion tax, E is the erosion stock, t is time, r is the private instantaneous annual rate of discount, and F represents the erosion state equation.

Using an optimal control framework, the current-value Hamiltonian is:

(3) $$H^{cv} = [pY(t) - C(Y(t)) - \tau E(t)] + \lambda F(E(t), Y(t)).$$

The necessary conditions are:

(4) $$\frac{\partial H}{\partial Y} = (p - C'(Y)) + \lambda F'(Y) = 0$$ (optimality condition)

(5) $$-\frac{\partial H}{\partial E} = \dot{\lambda} - r\lambda = \tau - \lambda F'(E)$$ (adjoint equation)

(6) $$\frac{\partial H}{\partial \lambda} = F(E, Y) = \dot{E}$$ (equation of motion).

The optimality condition indicates that the short-run profits must just equal the incremental erosion costs to the farmer. The adjoint equation relates the change in the marginal cost of erosion to the tax rate, the private rate of discount, and the erosion stock. The equation of motion indicates that the physical/economic optimum must equate the production and natural decay of erosion. It is implicitly assumed that the transversality and complementary slackness conditions hold as follows: $\lambda(T) \geq 0$ and $\lambda(T)E(T) = 0$.

The state equation, showing the change in the erosion stock over time, is assumed to be represented by the explicit equation:

(7) $$F(E(t),Y(t)) = \dot{E} = -hE(t) + kY(t), \text{ subject to } 0<h,k<1,$$

where h is the coefficient of decay of the erosion stock and k is the erosion production coefficient. Thus, the net change in erosion stock in a given period is the erosion generated from production less the eroded soil carried out of the study area by the action of wind or water.

The producer's cost function is assumed to take the form:

(8) $$C(Y(t)) = q + dY(t) + gY^2(t), \text{ subject to } q\geq 0;\ d,g>0.$$

This form is consistent with microeconomic theory and previous empirical analysis (Ervin, et al.; Henderson and Quandt; Sylvia and Anderson).

The Pontryagin conditions can be manipulated into a dynamic plane equation system of two first-order differential equations in $\dot{Y}$ and $\dot{E}$:

(9) $$\dot{Y} = Y(r+h) + \frac{(d-p)(r+h) + \tau k}{2g},$$

(10) $$\dot{E} = -hE(t) + kY(t), \text{ subject to } 0<h,k<1.$$

To evaluate the motion of this system, we must first identify those locations in which the system is stable, that is, the steady states associated with these equations. At equilibrium, the change in Y over time, $\dot{Y}$, becomes zero. Thus setting equation (9) equal to zero and solving for Y will result in the equilibrium level of output, Y^*:

(11) $$Y^* = \frac{(p-d)}{2g} - \frac{\tau k}{2g(r+h)}.$$

The equation of motion (6) will also be zero at steady state. By substituting (11) into (7) and setting it equal to zero, the equilibrium erosion stock, E^*, may be found:

(12) $$E^* = \frac{k}{h}Y^* = \frac{k(p-d)}{2gh} - \frac{\tau k^2}{2gh(r+h)}.$$

Since it is presumed that the system is not at equilibrium, we are particularly interested in the stability of the equilibrium and the motion that takes place elsewhere in the plane. We can determine that motion, and the nature of the steady state, with equations (7) and (9). We take the derivative of each with respect to Y,[2] and determine the sign of the result:

(13) $$\frac{\partial \dot{Y}}{\partial Y} = r + h \quad > 0,$$

(14) $$\frac{\partial \dot{E}}{\partial Y} = k \quad > 0,$$

and plot the motion that occurs around the steady state.

The two isoclines divide the plane into four quadrants as shown in Figure 1, and the trajectories within each quadrant define a saddle-point equilibrium. In order to meet the transversality conditions, where $T=\infty$, the producer must approach the equilibrium at Y^*, E^* as $t \to \infty$. Since point A is a saddle-point, the only approach path is along the convergent separatrix where $\dot{Y}=0$. The producer's profit maximizing strategy is to adjust output to Y^*. Erosion stock will then either increase or decrease toward equilibrium at E^*, depending upon whether the initial condition of E is to the right or to the left of $\dot{E}=0$.

The Policymaker's Dynamic Model

Once the optimal producer's behavior is determined, the policymaker's dynamic problem may be solved. Assume that the policymaker wishes to maximize the stream of discounted net revenue from agricultural production, water quality, and taxes on erosion. The producer may use the producer response information derived from the producer's model to determine the optimal tax policy. The policymaker's formal problem is:

(15) $$\text{Maximize} \int_{t=0}^{T=\infty} e^{-\delta t}[apY(t) - b\gamma E(t) + c\tau(t)E(t)]\, dt$$

(16) subject to: $$\dot{E} = F(E(t), Y(t))$$

(17) $$\dot{Y} = G(Y(t), \tau(t))$$

$$a+b+c=1;\ 0 \le a, b < 1;\ 0 < c \le 1;$$

and the usual set of non-negativity and convexity conditions where:

a, b, c = the relative weights the policymaker attaches to agricultural revenue, water quality, and tax revenue, respectively; G = the state equation for Y resulting from the producer's movement along the optimal path; γ = the unit cost of the erosion stock, E ; and δ = the social instantaneous annual rate of discount.

The $pY(t)$ term is the increase in the value of the objective functional due to agricultural production, $-\gamma E(t)$ is the decrease in the value of the objective functional due to erosion damages, and $\tau(t)E(t)$ is the increase in the value of the objective functional due to tax revenues.

Using the explicit form in (7) and the value of Y^* in (11) yields the Hamiltonian:

(18) $$H = ap\left[\frac{p-d}{2g} - \frac{\tau k}{2g(r+h)}\right] - b\gamma E + c\tau E + \mu\left[-hE + k\left(\frac{p-d}{2g} - \frac{\tau k}{2g(r+h)}\right)\right].$$

The necessary conditions are:

(19) $$\frac{\partial H}{\partial \tau} = -\frac{apk}{2g(r+h)} + cE - \frac{\mu k^2}{2g(r+h)} = 0$$

(20) $$-\frac{\partial H}{\partial E} = b\gamma - c\tau + \mu h = \dot{\mu} - \delta\mu$$

(21) $$\frac{\partial H}{\partial \mu} = \dot{E} = -hE + k\left[\frac{p-d}{2g} - \frac{\tau k}{2g(r+h)}\right].$$

An examination of the Hamiltonian demonstrates that the system is linear in the control. Thus, by Miller, the solution is of the "bang-bang" type and the following tax rates derived from (19) would prove optimal:

$$\text{If } \frac{apk}{2g(r+h)} - cE \begin{cases} > \dfrac{\mu k^2}{2g(r+h)} & \text{then } \tau^{min} \\ = \dfrac{\mu k^2}{2g(r+h)} & \text{then } \tau^{*} \\ < \dfrac{\mu k^2}{2g(r+h)} & \text{then } \tau^{max} \end{cases}$$

The interpretation of this solution is straightforward. $\frac{apk}{2g(r+h)}$ is the amount by which agricultural revenues are decreased in the current period, cE is the amount by which tax revenues are increased in the current period, and $\frac{\mu k^2}{2g(r+h)}$ is the amount by which the discounted stream of future erosion damage decreases, all when the tax rate is increased by one unit. Thus, when $\frac{apk}{2g(r+h)} - cE > \frac{\mu k^2}{2g(r+h)}$, the value of agricultural production and tax revenue is more valuable to the policymaker than the cost of erosion damage resulting from production. Given this, the policymaker wants to increase production to Y^* and he implements tax rate τ^{min}. When $\frac{apk}{2g(r+h)} - cE < \frac{\mu k^2}{2g(r+h)}$, the policymaker values clean water more highly than current production and he implements tax rate τ^{max} to decrease agricultural erosion.

In a manner similar to that of the producer problem, the Pontryagin conditions can be manipulated to yield the optimal tax rate, τ^*:

(22) $$\tau^* = \frac{(h+\delta)(p-d)(r+h)}{k(2h+\delta)} - \frac{aph(h+\delta)}{kc(2h+\delta)} + \frac{b\gamma h}{c(2h+\delta)}.$$

The optimal tax rate equation can be thought of as a tax response function of the policymaker's, or society's, values of agricultural production versus water quality. A greater value placed on agricultural revenues (*a*) by the policymaker results in a lower equilibrium tax rate. Conversely, a greater value placed on water quality (*b*) results in a higher equilibrium tax rate. The relationship between the value for tax revenues (*c*) and the tax rate, however, depends upon the elasticity of the tax revenue supply curve which is determined by the relative values of *h* and *k* (Sylvia and Anderson).

THE EMPIRICAL MODEL

The model is applied to a three-county region in West Tennessee consisting of Obion, Weakley, and Gibson counties. These three counties are located in the area categorized by the Soil Conservation Service as Major Land Resource Area 134 (USDA). With elevations ranging from twenty-five to one hundred meters (USDA), this region lies between the Mississippi bottomlands and the Cumberland Plateau.

The majority of the land in the area is used for agriculture, with corn, soybeans and wheat being the primary crops. These three commodities represent over seventy percent of the agricultural production of the area (Mundy and Gray). Agricultural practices and production enterprises are consistent throughout the three counties (Mundy and Gray), as are the hydrological characteristics (Water Resources Council). Precipitation is abundant in the area, averaging about fifty inches per year, and yields of major crops are above state averages (Mundy and Gray).

There are twelve primary soil types in the watershed. The soils are mainly from deep or moderately deep loess with Loring, Falaya and Collins accounting for more than fifty percent of the total land area. The region is subject to the highest average soil erosion rates in the nation (Moore and Klaine).

Economic Coefficients

The economic data required to apply the model include the cost equation, the commodity price, the unit cost of offsite sediment damages and the private and social discount rates. The producer's cost equation is not directly available. A linear programming/linear regression model is developed to estimate these equations. Each of the twelve soil types is associated with a cost of production and a yield potential. The model selects those combinations which meet the exogenously specified revenue constraint at the least cost.

Once developed, the basic model is run multiple times using an increasing revenue requirement. The resulting minimum cost levels are then associated with the corresponding levels of output, through linear regression, to obtain continuous equations representing the relationship between cost and output. The model is run for each major commodity in the area--corn, soybeans and wheat--assuming homogeneous production of the commodity being modeled in each run.

One problem involved in modeling an entire watershed is the variation in tillage practices. Practices vary from one farmer to another and over time. To account for this while still maintaining a parsimonious model, each commodity model is run under two different tillage conditions, conventional tillage and conservation tillage. This creates a bound on the results and allows for an analysis of the sensitivity of the results to tillage practices.

Six models result from the system explained above and are referred to in this study as the primary model configurations. Specifically, the primary model configurations are: conventional tillage corn; conservation tillage corn; conventional tillage soybeans; conservation tillage soybeans; conventional tillage wheat; and conservation tillage wheat.

All coefficients derived from the regression are significant at a .95 or .90 level of confidence, except the quadratic term of the conservation tillage wheat model (Table 1). The Y and Y^2 terms in the regression correspond to the d and g terms in the optimal control model respectively.

The proxy used in this study for the real social instantaneous annual rate of discount is the nominal rate on AAA corporate bonds for June 1987 less the percentage change in price from June 1986-1987 (Federal Reserve Board). This rate, $\delta = 0.05$, is assumed to be a good average of the productivity of low-risk investments in the capital markets. The assumed private instantaneous annual rate of discount is based upon the annual long-term fixed deposit rates offered by commercial banks in the area. The private discount rate of $r = 0.10$ is the median of rates quoted by several banks operating in Memphis, Tennessee.

The unit cost of offsite sediment damages is drawn from Alexander and English which identifies offsite sediment damage costs for river basins across the United States. In this paper, the authors used an interregional sedimentation model to estimate the impact of the Conservation Reserve Program on sedimentation and on the offsite costs of sediment damage. The authors combined the offsite erosion damage costs reported by Clark, et al. and the erosion levels reported in the 1982 National Resources Inventory (NRI) to provide a base from which cost per ton estimates are generated. Erosion data were taken from the NRI, aggregated to a producing area level, and multiplied by appropriate sediment delivery ratios. The resulting estimates of suspended and deposited sediment were aggregated into regional estimates, and the total sediment costs were divided by this result. The unit cost of offsite sediment damages for this area is $6.38 per ton.

The price of each commodity is obtained from *Tennessee Agriculture, 1990* (Tennessee Department of Agriculture), reflecting 1988 values. The 1988 values are used to maintain consistency with the cost figures. The prices used are: $2.75 per bushel for corn, $7.65 per bushel for soybeans, and $3.40 per bushel for wheat. All prices and costs reflect 1988 values.

Physical Coefficients

The physical data required are the coefficients of the equation of motion: the sediment growth coefficient and the sediment decay coefficient. The sediment growth coefficient is developed using the Erosion Productivity Impact Calculator (EPIC), a plant growth/erosion simulator.

Seventy-two runs of the EPIC model are performed, incorporating every combination of the primary model configurations with each of the twelve soil types. The production practices used are the same as those used to develop the production cost values in the linear programming models. A fifty year simulation is run and the average annual erosion is extracted for each of the seventy-two models. This results in twelve erosion levels for each primary model configuration. A weighted average of these 12 values is calculated, based upon the relative percentages of soil types. Since the model is based upon the erosion that reaches the water as sediment, not raw erosion, these six erosion levels are multiplied by the threshold sediment delivery ratio for the watershed. The threshold sediment delivery ratio is defined as that proportion of average annual total erosion which is delivered to a significant stream (Alexander and English). "A 'significant' stream begins where flow becomes adequate for a beneficial use such as a municipal water supply, water-based recreation, a fishery, or domestic use."[3] The sediment growth coefficient for each primary model configuration is shown in Table 2.

The sediment decay coefficient is much more difficult to estimate than the sediment growth coefficient. A literature search revealed no studies in which sediment movement as defined in this model have been measured. Discussions with soil scientists[4] served largely to confirm the difficulty in estimating such a figure. Some generalizations can be made, however. One is that sediment decay as described in this model can be expected to be extremely slow. Further, the level of sediment decay will be very minor as compared with new sediment contained in both agricultural and non-agricultural runoff. While this decay could be expected to vary between 0 and 25 percent annually, depending upon the specific hydrology of an area, most values will fall in the lower extreme of the range. An estimate of .01 is selected for this coefficient. This estimate, while somewhat arbitrary, is consistent with the available knowledge at this time.

OPTIMAL DYNAMIC STRATEGIES

The Optimal Tax Function

Substituting the coefficients from Table 2 into equation (22) yields the following optimal tax functions in terms of *a*, *b*, and *c*:

(23) $\tau^{*}=2.604-\frac{.404a}{c}+\frac{.911b}{c}$ Conventional Corn;

(24) $\tau^{*}=2.917-\frac{.483a}{c}+\frac{.911b}{c}$ Conservation Corn;

(25) $\tau^{*}=2.462-\frac{.340a}{c}+\frac{.911b}{c}$ Conventional Soybeans;

(26) $\tau^{*}=2.974-\frac{.431a}{c}+\frac{.911b}{c}$ Conservation Soybeans;

(27) $\tau^{*}=1.017-\frac{.562a}{c}+\frac{.911b}{c}$ Conventional Wheat;

(28) $\tau^{*}=.467-\frac{.775a}{c}+\frac{.911b}{c}$ Conservation Wheat.

There is no *a priori* determination of unique values for ***a***, ***b***, and ***c***. Rather, these variables represent the range of relative weights that society can place on the different benefits represented in the model. As indicated following equation (17), these values are restricted such that $a+b+c=1;\ 0\leq a,b<1;\ 0<c\leq 1$. It is assumed for the empirical analysis that tax revenue (***c***) is not the primary purpose of such a policy and is not of primary concern to society relative to agricultural production and environmental quality. However, it is also clear from equations (23) through (28) that the value of ***c*** cannot be zero. Thus the value of ***c*** is initially held to .1.

Table 3 shows the value of τ^* for each primary model configuration. With ***c*** assumed to have a value of .1, the values of ***a*** and ***b*** can each vary between 0 and .9. Since both agricultural production and environmental quality, in some amount, are necessary for survival, the case in which either ***a*** or ***b*** are zero is omitted and the coefficients are allowed to vary between .1 and .8. The actual value of such coefficients is not likely to approach either extreme due to the importance to society of both commodities. In several cases, the value of the optimal tax becomes negative as the weights move toward favoring agriculture, indicating that a subsidy to agriculture may be the optimal path for society.

These optimal tax values may be thought of as the value of externalities imposed upon society, weighted by the value society places on the competing goods of agricultural production and environmental quality. A non-weighted value may be obtained by solving the system again without the ***a***, ***b***, or ***c*** coefficients:

$$(29) \qquad \tau=\frac{(h+\delta)(p-d)(r+h)}{k(2h+\delta)}-\frac{ph(h+\delta)}{k(2h+\delta)}+\frac{\gamma h}{(2h+\delta)}.$$

The non-weighted equations result in the values shown in Table 4, which represent the actual transfer of value from society to agricultural producers for each primary model configuration. These economic transfer values are just over three dollars per ton for corn and soybeans, and are substantially lower for wheat.

Some clues to the effect on the optimal tax of a change from conventional tillage to conservation tillage planting are provided in Table 3. When society values the environment significantly higher than agriculture, the change in the optimal tax for conventional relative to conservation tillage is much smaller than when agriculture is weighted more favorably. In effect, the conservation tillage scenarios are always more sensitive to changes in ***a*** and ***b*** than the conventional scenarios. Excepting wheat, however, the optimal tax values tend to converge with the values of ***a*** and ***b***, giving similar results near the middle of the range. As shown in the Optimal Production Function section, the physical processes constrain the problem to these mid-range values such that the difference between tillage practices becomes insignificant to the problem as a whole, at least in this specific empirical application.

The Optimal Production Function

Substituting the coefficients from Table 2 and the optimal tax levels arising from equations (23) through (28) into equation (11) yields the optimal production functions facing the producers in this area:

(30) $Y = 1.713 \times 10^8 - 5.638 \times 10^7 \tau$ Conventional Corn;

(31) $Y = 1.373 \times 10^8 - 4.033 \times 10^7 \tau$ Conservation Corn;

(32) $Y = 8.113 \times 10^7 - 2.824 \times 10^7 \tau$ Conventional Soybeans;

(33) $Y = 6.667 \times 10^7 - 1.922 \times 10^7 \tau$ Conservation Soybeans;

(34) $Y = 3.182 \times 10^7 - 2.681 \times 10^7 \tau$ Conventional Wheat;

(35) $Y = 1.667 \times 10^7 - 2.998 \times 10^7 \tau$ Conservation Wheat.

Substituting the values of τ^* as shown in Table 3 into equations (30) through (35) yields the optimal production levels for each primary model configuration by tax level (Table 5). One obvious attribute of Table 5 is that, for all primary model configurations, the production level becomes negative as the social weights move toward environmental quality and away from agricultural production. For each crop, there exists a tax level at which producers would no longer find it profitable to produce the commodity in question. They would either shift production to a different crop or leave agriculture altogether. The negative values in Table 5 suggest that this level of taxation is well within the range of those taxes which would otherwise be considered socially optimal. Given the profit structure of agriculture in the study area, producers of certain commodities simply cannot afford to absorb the total cost of their production externalities and remain in business.

To examine this issue, the production level, Y, is set to zero and equation (11) is rearranged to isolate τ:

$$(36) \qquad \tau = -(p+d)\frac{(r+h)}{k}.$$

This indicates the tax level at which production of the commodity would become unprofitable and, presumably, would cease. The zero production level tax is shown for each primary model configuration in Table 4. Not surprisingly, those crops which are more profitable without a tax are able to sustain higher tax levels before becoming unprofitable. These levels may be thought of as a maximum potential tax, constrained by the physioeconomic system, regardless of the theoretical social weights applied. Similarly, there is a tax level for each commodity, less than the zero tax level, at which all of the resources of the area will be placed in use. Equation (11) is again rearranged, this time leaving the variable Y explicit in the equation:

$$(37) \qquad \tau = -(-p+d+2gY)\frac{(r+h)}{k}.$$

The average yield per acre is multiplied by total acres in the study area and substituted for Y in (37) to obtain the full production tax levels (Table 4). Any tax below these levels will not affect the production decisions of farmers. Each zero tax level and full production tax level is associated with a pair of values of a and b. This effectively sets a bound on the range of social weights thatcan affect production of agricultural products. The range of possible social weights falls between .4/.5 and .7/.2 for a/b. This suggests that severely skewed social weights are not sustainable under the conditions in the study area. It may, perhaps, be inferred from this limited application that the importance of both goods to a healthy society may preclude an extreme bias in either direction. Since price is a determinant of this range, the market itself may act exogenously as a stabilizing force.

An analysis is performed of the sensitivity of the economic transfers, the zero production tax level and the full production tax level to changes in the model coefficients. The model is extremely sensitive to changes in price. For the corn and wheat configurations, the model is solved for the economic transfer value under current loan rate levels and under current target price levels. There is no target price for soybeans, so the model results under the loan rate price level are compared to the market price used in the base analysis. The loan rates used for corn, soybeans, and wheat are $1.96 per bushel, $4.50 per bushel, and $2.44 per bushel, respectively (Loewen, et al.). The target prices used for corn and wheat are $2.75[5] per bushel and $4.00 per bushel respectively (Loewen, et al.).

For conventional corn, the economic transfer value varies from $1.95 per ton under the loan rate to $3.11 per ton under the target price. The ranges are $1.96 to $3.35 per ton for conservation corn, $-0.22 to $2.36 per ton for conventional wheat, and $-1.58 to $1.98 per ton for conservation wheat. The economic transfer values vary from $1.63 per ton under the loan rate price to $3.03 per ton under the base market price for conventional soybeans, and $1.68 to $3.45 per ton for conservation soybeans. The wide variation in model results under different price scenarios emphasizes the importance of price estimation in this model. While historical price figures are generally accurate and relatively easy to obtain, it is important to remember that price is exogenous to this model and that this model is sensitive to the price variable. Changes in price over time would require revisiting the model to update the results.

The model is moderately sensitive to changes in the linear cost coefficient, the private instantaneous rate of discount, and the sediment growth coefficient; and relatively insensitive to changes in the quadratic cost coefficient, the social instantaneous rate of discount, the unit cost of offsite sediment damages, and the sediment decay coefficient.

The Dynamic Policy Frontier

A key feature of this model is that it gives the modeler the ability to develop solutions independent of policy goals. The parameterization of relative weights, ***a***, ***b***, and ***c***, in the model provides for a range of answers as shown in the Numerical Solutions section of this chapter. An additional benefit is the ability to represent graphically the relationship between the optimal tax function and the values of ***a***, ***b***, and ***c***.

The graphical policy frontier for conventional corn is shown in Figure 2. The optimal tax rate is shown on the vertical axis and the variable ***a*** is shown on the horizontal axis. Each line in the graphs represents a constant value of ***c***. The value of ***b*** can be calculated for any given point as $b=1-a-c$, however, the scale of ***b*** changes for each value of ***c***, and is not shown explicitly.[6]

SUMMARY

The objective of this study was to develop a soil conservation policy model, incorporating the objectives of both producers and policymakers, that: 1) estimates the economic transfers between agricultural producers and society resulting from agricultural production and its associated sediment-based externalities, and 2) estimates a socially optimal level of taxation, given varying levels of perceived importance to society of agriculture versus environmental quality.

The model used is an adaptation of the multi-level programming concept developed by Candler and Norton and extended by Sylvia and Anderson. A multi-level optimal control model was developed that optimizes the producer's dynamic problem in the first stage, then uses the producer's optimal decision paths in the second stage as components of the policymakers' dynamic problem. The second stage problem utilizes weights indicating the relative importance of the policymaker's competing goals. Those weights are used to construct a dynamic policy frontier demonstrating the relationships between various policy goals and the resulting optimal solution paths.

The model was applied to a three-county area in West Tennessee, composed of Obion, Weakley, and Gibson counties. The model provided information on the rates of taxation that would optimize the resources of both society and farmers given the preference of society for agricultural output versus environmental quality. These optimal tax rates represent the transfer of costs from the agricultural producer to society as a whole. The optimal tax rates were then substituted into the optimal production function for agricultural producers to determine the level of production that may be expected to result given the underlying profit structure of agricultural production and the tax rate applied.

Effective optimal taxes were determined, which would result in a range of production from zero to full production, and was identified as a bound on tax rates which would be expected to affect levels of agricultural production. The range of taxes, from full to zero production, respectively, were \$2.04 - \$3.04 per ton for conventional corn, \$2.07 - \$3.40 per ton for conservation corn, \$2.24 - \$2.87 per ton for conventional soybeans, \$2.54 - \$3.47 per ton for conservation soybeans, \$0.33 - \$1.19 per ton for conventional wheat, and \$-0.22 - \$0.56 per ton for conservation wheat. All values are in dollars per ton of sediment.

Tax rates less than the full production tax rate are expected to reduce society's cost of agricultural production without reducing output of the commodity being modeled. Tax levels in the effective range are expected to both reduce agricultural output of the commodity, to reduce offsite sediment damages from agricultural production, and to reduce the cost to society of remaining agricultural production. Tax levels greater than the zero production level are expected to result in a complete shift in production from the commodity being modeled to another enterprise. The model does not address which new enterprise that would be, nor its relative environmental impacts.

External costs of production were also calculated to indicate the absolute transfer to society, without regard to social preferences, of offsite sedimentation costs arising from agricultural production. The external costs, in dollars per ton of sediment, are \$3.11 for conventional corn, \$3.35 for conservation corn, \$3.03 for conventional soybeans, \$3.45 for conservation soybeans, \$1.37 for conventional wheat, and \$0.61 for conservation wheat.

As is characteristic of mathematical models, some concessions were made in the name of mathematical tractability. While the model is able to address both the producer's problem and the policymaker's problem, each optimal control sub-model is constrained to one state variable and one control variable. While the policymaker's model expresses a second equation of motion in equation (17), that equation satisfies the constraint by becoming zero due to assumptions in the producer's model.

The model would be rendered analytically insolvable if this constraint were violated. Such a constraint could be abandoned in a purely numerical model.

Another assumption resulting from these constraints is that of the exogenous price level. It was shown in the sensitivity analysis that the model is particularly sensitive to price, yet the model is unable to adapt to such changes endogenously. The constant price assumption causes the result shown in equation (9), that the change in the producer's optimal production over time is equal to zero. Without this assumption, the second equation of motion in equation (17) would be non-zero and the model would be analytically insolvable.

A study of this nature requires combining information from various fields of study, including economics, plant and soil science, hydrology, mathematics, and public policy. Many limitations of the model were based upon limitations in the mathematical tools applicable to applied economics. Further research into multiple-state, multiple control extensions of optimal control theory is suggested. Numerical applications are most promising in this area, but data is lacking. Thus, additional research is needed in developing an empirical data base of production costs, yields, erosion, and hydrological characteristics of a variety of watersheds across the country.

Specific research suggestions include the addition of multiple controls to both the producer and policymaker sub-models, the inclusion of price an endogenous variable in the modeling system, and the econometric estimation of agricultural cost functions. Applications and adaptations of this model may also be useful in examining a variety of policy-related issues in natural resource and environmental economics.

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Table 1. Regression results of cost equation estimation.

	Corn		Soybeans		Wheat	
	Conventional	Conservation	Conventional	Conservation	Conventional	Conservation
Constant†	0	0	0	0	0	0
Std $Error_{Reg}$	216557.2	702081.9	271129.8	375038.7	684555.5	815650.2
R^2	.99	.99	.99	.99	.99	.99
Observations	30	30	28	28	15	15
Deg. of Freedom	28	28	26	26	13	13
Y Coefficient (t-value)	1.144436 (233.42)	1.242335 (77.71)	2.61985 (137.40)	2.855822 (107.65)	2.839073 (44.22)	3.214053 (41.92)
Y^2 Coefficient (t-value)	4.7×10^{-9} (41.01)	5.5×10^{-9} (14.81)	3.1×10^{-8} (23.65)	3.6×10^{-8} (19.56)	8.8×10^{-9} (1.90)	5.7×10^{-9} (1.03)

†As the constant does not appear in the final equations, it was forced to zero in the regression.

Table 2. Model coefficient values.

Coefficient	Units	Corn Conventional	Corn Conservation	Soybeans Conventional	Soybeans Conservation	Wheat Conventional	Wheat Conservation
Linear Cost (d)	$/bu	1.14	1.24	2.62	2.85	2.84	3.21
Quadratic Cost (g)	$/bu	4.7×10^{-9}	5.5×10^{-9}	3.1×10^{-8}	3.6×10^{-8}	8.8×10^{-9}	5.7×10^{-9}
Price (p)	$/bu	2.75	2.75	7.65	7.65	3.40	3.40
Private Discount Rate (r)	$/$	.1	.1	.1	.1	.1	.1
Social Discount Rate (δ)	$/$	.05	.05	.05	.05	.05	.05
Unit Cost of Sediment (γ)	$/tn†	6.38	6.38	6.38	6.38	6.38	6.38
Sediment Decay (h)	tn/tn	.01	.01	.01	.01	.01	.01
Sediment Growth (k)	tn/bu	.5083	.0488	.1926	.1522	.0519	.0376
Average Erosion (ζ)	tn/ac	20.06	16.09	21.20	16.68	7.33	5.34
Average Yield (β)	bu/ac	120.50	115.35	38.51	38.36	49.45	49.80

†English Tons

Table 3. Optimal tax values by primary model configuration and level of social importance.

$a/b^\dagger$	Corn Conventional	Corn Conservation	Soybeans Conventional	Soybeans Conservation	Wheat Conventional	Wheat Conservation
	Dollars per Ton					
.1/.8	9.49	9.73	9.41	9.83	7.75	6.99
.2/.7	8.18	8.33	8.16	8.49	6.27	5.31
.3/.6	6.86	6.94	6.91	7.15	4.80	3.62
.4/.5	5.54	5.54	5.66	5.81	3.33	1.93
.5/.4	4.23	4.15	4.41	4.47	1.86	0.25
.6/.3	2.91	2.75	3.15	3.12	0.38	-1.44
.7/.2	1.60	1.36	1.90	1.78	-1.09	-3.13
.8/.1	0.28	-0.04	0.65	0.44	-2.56	-4.81

$^\dagger c = .1$

Table 4. External costs of agricultural production, zero production tax levels, and full production tax levels.

Commodity	Value of Economic Transfer	Zero Production Tax Level	Full Production Tax Level
		Dollars	
Conventional Corn	3.11	3.04	2.04
Conservation Corn	3.35	3.40	2.07
Conventional Soybeans	3.03	2.87	2.24
Conservation Soybeans	3.45	3.47	2.54
Conventional Wheat	1.37	1.19	0.33
Conservation Wheat	0.61	0.56	-0.22

Table 5. Optimal production levels by primary model configuration and level of social importance.

a/b	Corn Conventional	Corn Conservation	Soybeans Conventional	Soybeans Conservation	Wheat Conventional	Wheat Conservation
	Bushels					
.1/.8	-3.64×10^8	-2.55×10^8	-1.85×10^8	-1.22×10^8	-1.76×10^8	-1.93×10^8
.2/.7	-2.90×10^8	-1.99×10^8	-1.49×10^8	-9.65×10^7	-1.36×10^8	-1.42×10^8
.3/.6	-2.16×10^8	-1.43×10^8	-1.14×10^8	-7.07×10^7	-9.69×10^7	-9.19×10^7
.4/.5	-1.41×10^8	-8.63×10^7	-7.87×10^7	-4.49×10^7	-5.74×10^7	-4.13×10^7
.5/.4	-6.71×10^7	-3.00×10^7	-4.33×10^7	-1.91×10^7	-1.79×10^7	9.27×10^6
.6/.3	7.08×10^6	2.62×10^7	-7.94×10^6	6.65×10^6	2.16×10^7	5.98×10^7
.7/.2	8.13×10^7	8.25×10^7	2.74×10^7	3.25×10^7	6.11×10^7	1.10×10^8
.8/.1	1.55×10^8	1.39×10^8	6.28×10^7	5.82×10^7	1.01×10^8	1.61×10^8

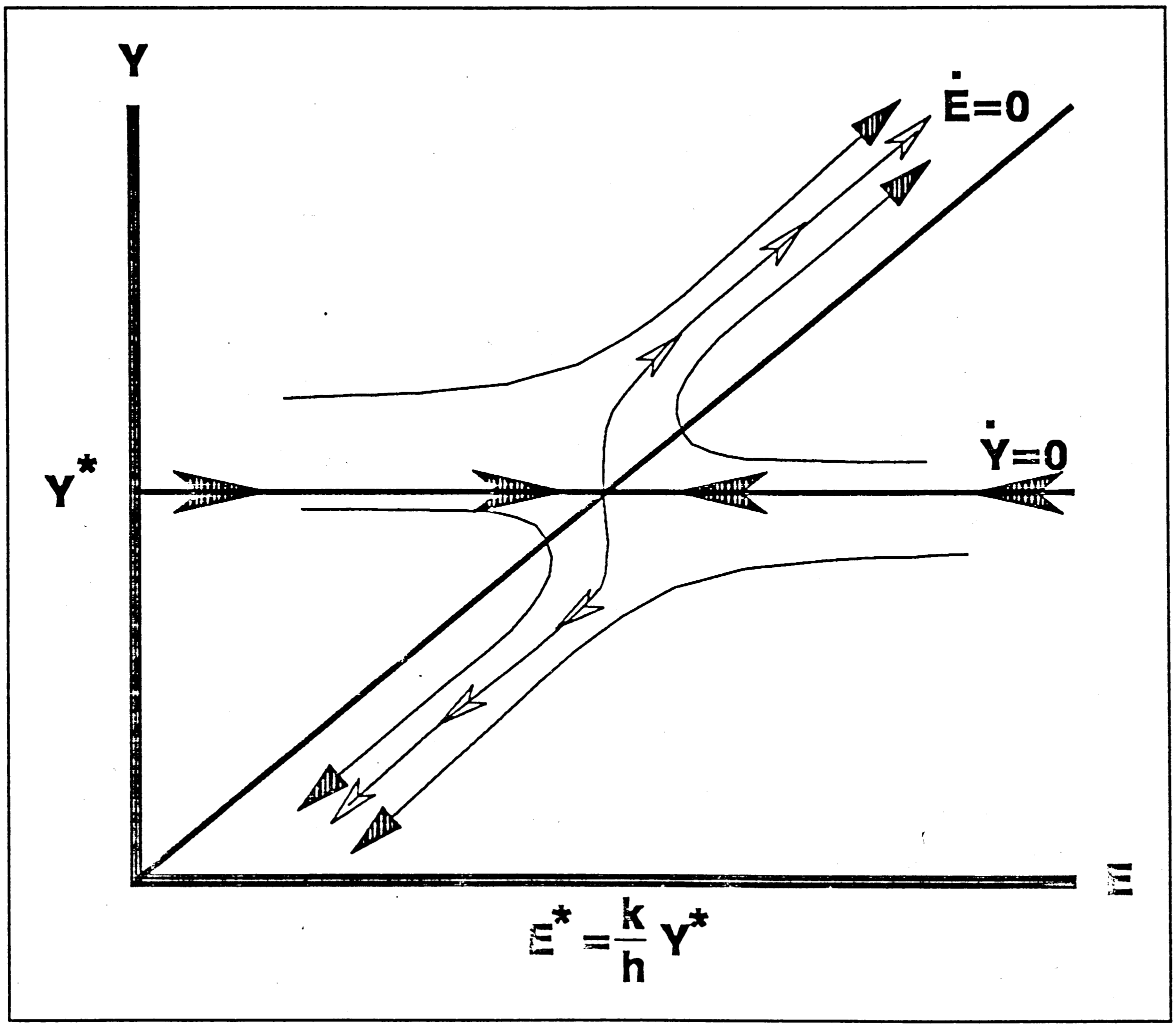

Figure 1. Phase diagram demonstrating global movement of trajectories.

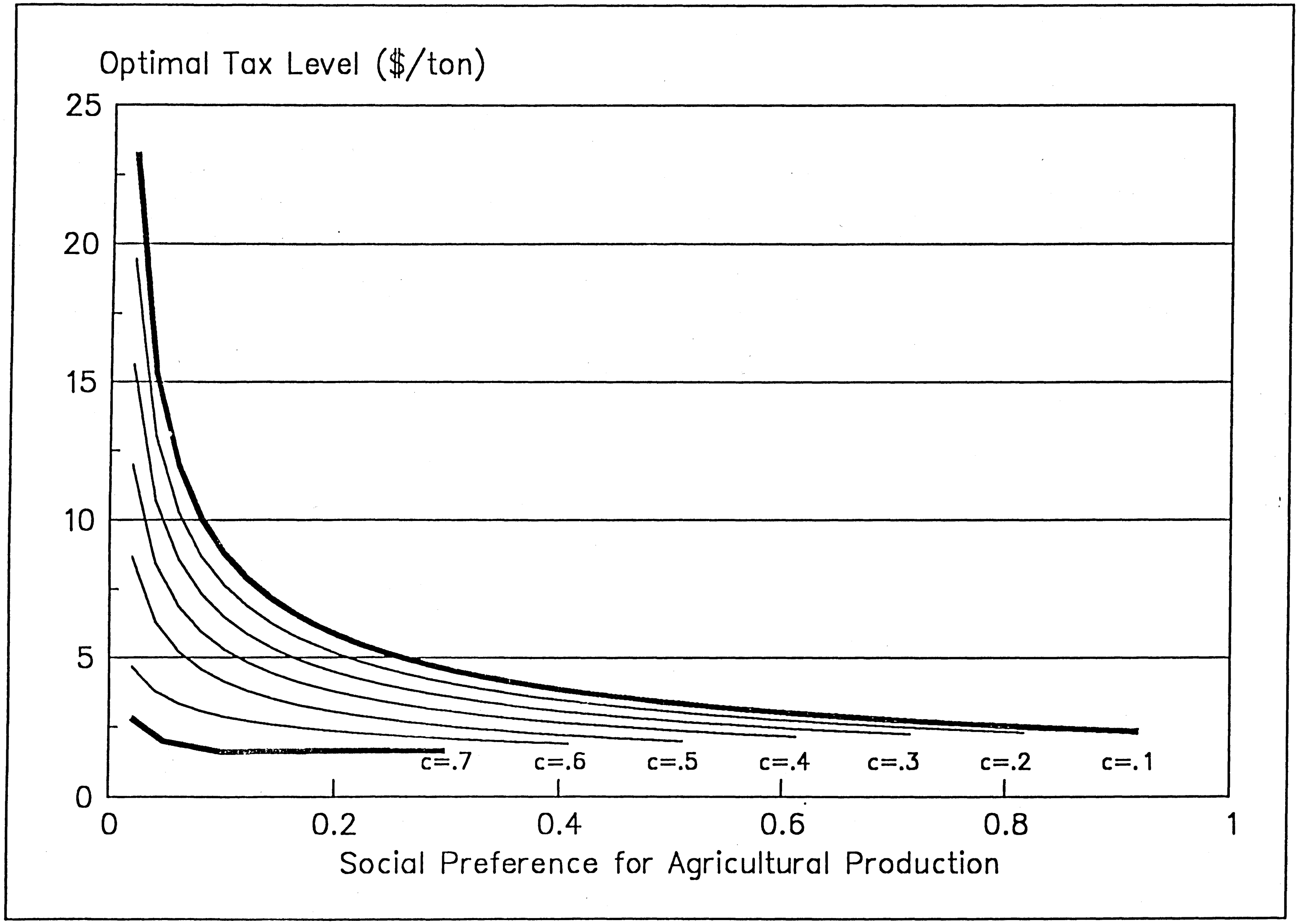

Figure 2. Dynamic policy frontier for conventional corn.

FOOTNOTES

[1]While the debate over the use of taxation as a policy instrument is important, the relative merits of either argument need not be determined to accept the usefulness of a tax-based model. The optimal tax level developed in this study could conceivably be used as the basis for a tax-based policy, yet the economic information derived from such an analysis is valuable in its own right. The optimal tax level is an economic measure of the level of externality imposed upon society by agricultural production. This optimal tax, if not imposed, may be thought of as an involuntary transfer from society as a whole to the agricultural industry.

[2]Normally, the derivative of each would be taken with respect to the other as that usually gives a more tractable result. This system is unusual, however, in that $\dot{Y}$ is not a function of E. The necessary information is still obtained by taking the derivative of $\dot{Y}$ with respect to Y, although the result is somewhat more difficult to interpret.

[3]From an unpublished Soil Conservation Service memorandum from C. Don Clark, National Sedimentation Geologist, dated December 14, 1989.

[4]Personal communication with Dr. Don Myer. USDA National Sedimentation Laboratory, Oxford Mississippi, 1991.

[5]It may be noted that $2.75 per bushel is also the price used for the corn base analysis. That value does not represent the target price, but the market price for that year.

[6]The choice of a is arbitrary. Variable b could have been explicitly shown on the horizontal axis, with a implicit.