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Factor Demands in the U.S. Food-Manufacturing Industry

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This paper analyzes the demand for labor, capital, and energy in the U.S. food-manufacturing industry using Allen and Morishima elasticities of substitution. The demand for capital is more elastic than for labor and energy, and these production factors are substitutable, especially between capital and labor.

Key words: conditional factor demands, elasticity of substitution.

Food - processed

Some U.S. food-processing technologies require heavy use of energy and capital equipment, while labor costs relative to value of shipments are smaller than for other types of manufacturing. In past years, the food manufacturers faced a steady increase in the price ratio between labor and capital and a sharp increase in the price of fuel and energy during the 1970s but a continued decrease since 1981. To make decisions in resource allocation, food manufacturers need information on the nature of industrial demand for factor inputs.

Numerous studies have analyzed demand for production factors in various sectors of an economy. Examples include Berndt and Christensen, Berndt and Wood, and Fuss in the general manufacturing sector, and Binswanger, Ray, Lopez, and Shoemaker in the agricultural sector. However, little attention has been given to factor demands in the U.S. food-manufacturing industry. The objective of this study is to analyze the demand for nonfood inputs such as labor, capital, and energy in U.S. food manufacturing. Agricultural or marine raw products and packaging and containers are not considered, partly because of little perceived interdependent relationships between materials and nonfood inputs and partly because of difficulty in measuring quantity and price.

In addition to the Allen partial elasticity of substitution (*AES*), a major focus is on measuring the Morishima elasticity of substitution (*MES*) to explain factor demands and their interdependent relationships. Because *MES* is not frequently used by applied economists, a brief explanation of *MES* and its linkage to cost function is given followed by an application to the U.S. food-manufacturing industry.

Conceptual Framework

Consider a production technology $F(X)$ for a vector of n-factors X. The corresponding cost function is defined as the minimum cost of attaining product Q at a vector of factor prices W as

$$(1) \qquad C(W, Q) = \min_{X} [W'X; F(X) = Q].$$

This cost function is concave and linear homogenous in W.

Denoting $C_i(W, Q)$ and $C_{ij}(W, Q)$, respectively, as the first- and second-order partial derivatives of the cost function with respect to factor prices, one can apply Shephard's lemma and derive a conditional factor demand for the ith factor X_i^* as a function of W and Q:

$$(2) \qquad X_i^*(W, Q) = C_i(W, Q).$$

This conditional factor demand function is homogenous of degree zero in factor prices. Furthermore, the (constant-output) cross-price elasticity, E_{ij}, for ith factor with respect to jth factor price is then obtained as

$$(3) \qquad E_{ij} = W_j C_{ij}(W, Q)/C_i(W, Q).$$

Hicks defined the elasticity of substitution for a two-factor production function as the ratio of factors in response to a change in their relative prices. Later Allen (p. 504) extended the definition to account for the adjustments for more than two factors and defined "partial elasticity

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of substitution" between ith and jth factors as below:

$$(4) \quad AES_{ij} = \left\{ \left[\sum_{i=1}^{n} X_i^*(W, Q)F_i \right] \Big/ [X_i^*(W, Q)X_j^*(W, Q)] \right\} (F_{ij}^*/F),$$

where F_i is the marginal product of ith factor, F is a determinant of the Hessian matrix (its element denoted as F_{ij}) bordered by marginal products, and F_{ij}^* is a determinant of the cofactor of F_{ij} in the matrix of F.

Although AES has been widely used, its applicability to the demand for production factors is rather limited. The definition of AES deviates from Hicks' definition for two-factor production and does not explain factor substitution explicitly. Besides, AES is not a measure of the curvature of the isoquant, it provides no information about relative factor shares, and it cannot be interpreted as the marginal rate of substitution (Blackorby and Russell).

An alternative measure of factor substitution known as the Morishima elasticity of substitution is defined as a logarithmic derivative of a quantity ratio in factors with respect to a ratio of its factor prices:

$$(5) \quad MES_{ij} = -\partial \ln[X_i^*(W, Q)/X_j^*(W, Q)]/\partial \ln[W_i/W_j].$$

MES measures the percentage change in the ratio of a pair of factors in response to a change in their relative prices. It is a natural generalization of the Hicksian two-variable elasticity.

Because the conditional factor demand function (2) is homogenous of degree zero in factor prices, the demand function is invariant by dividing through the prices with a particular price, say W_j; that is,

$$(6) \quad X_i^*(W, Q) = X_i'(W_1/W_j, \ldots, W_{j-1}/W_j, W_{j+1}/W_j, \ldots, W_n/W_j, Q).$$

The differentiation can be carried through the chain rule by differentiating the $X_i'(.)$ directly with respect to the variable expressed in terms of W_i/W_j. The result is a workable form of MES expressed as

$$(7) \quad MES_{ij} = E_{ji} - E_{ii}.$$

This derivation is more convenient and straightforward than the expression derived in Blackorby and Russell.

According to equation (7), the effect of a variation in the factor price ratio W_i/W_j can be divided into two components: (a) the effect on $X_j^*(W, Q)$ given by the cross-price elasticity E_{ji}, and (b) the effect on $X_i^*(W, Q)$ given by E_{ii}. One property of MES is asymmetry in that the effects of change in W_i/W_j and W_j/W_i upon their corresponding adjustments to the ratio of factor demands need not be the same. MES can also provide complete comparative static information about relative factor cost shares in response to a change in factor prices expressed as

$$(8) \quad \partial \ln[W_i X_i^*(W, Q)/W_j X_j^*(W, Q)] / \partial \ln[W_i/W_j] = 1 - MES_{ij}.$$

The relative cost share is decreasing (increasing) if the MES is greater (less) than one.

Thus far, factor demand relationships are expressed in terms of an unknown cost function. Some empirical applications use a translog cost function expressed as

$$(9) \quad \ln C = \sum_{i=1}^{n} \alpha_i \ln W_i + 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \ln W_i \ln W_j + \sum_{i=1}^{n} \beta_{iq} \ln Q \ln W_i + \alpha_q \ln Q + 1/2\, \beta_{qq} (\ln Q)^2 + \sum_{i=1}^{n} \delta_i \ln W_i T + \delta_q \ln QT + \mu_1 T + \mu_2 T^2,$$

where variables C is cost, Q is production, W_i is the ith factor price, and T is a time trend to represent the level of technical progress.

To ease potential estimation problems such as lack of degrees of freedom, the cost function is not estimated directly; rather, a set of the factor cost share equations is estimated. A typical cost share equation for ith factor, say S_i, derived by applying Shephard's lemma to the cost function is expressed as

$$(10) \quad S_i = \alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln W_j + \beta_{iq} \ln Q + \delta_i T \qquad i = 1, 2, \ldots, n.$$

Moreover, in order to satisfy the properties of neoclassical production theory and the adding-up property of cost shares, the following parameter restrictions are required:

$$\sum_{i=1}^{n} \alpha_i = 1,\ \beta_{ij} = \beta_{ji} (i, j = 1, 2, \ldots, n), \text{ and } \sum_{j=1}^{n} \beta_{ij} = \sum_{i=1}^{n} \beta_{ij} = \sum_{i=1}^{n} \beta_{iq} = \sum_{i=1}^{n} \delta_i = 0.$$

Given the estimated cost parameters, one can derive *MES*, *AES*, and the price elasticities of conditional factor demands as follows:

$$(11)\quad MES_{ij} = (\beta_{ij} + S_i S_j)/S_j - (\beta_{ii} + S_i^2 - S_i)/S_i \; (i \neq j),$$

$$(12)\quad AES_{ii} = (\beta_{ii} + S_i^2 - S_i)/S_i^2,$$

$$(13)\quad AES_{ij} = (\beta_{ij} + S_i S_j)/S_i S_j \; (i \neq j),$$

$$(14)\quad E_{ii} = (\beta_{ii} + S_i^2 - S_i)/S_i, \text{ and}$$

$$(15)\quad E_{ij} = (\beta_{ij} + S_i S_j)/S_i \; (i \neq j).$$

U.S. Food-Manufacturing Application: Data

To investigate the demand for labor, capital, and energy in the U.S. food-manufacturing industry, data are required on the unit price and total cost of each factor as well as on the value and quantity of industrial production. Most of the data are compiled from the *Annual Survey of Manufactures* (ASM), SIC code 20 (Food and Kindred Products) for 1971–86. Prior to this period, data for each sector of food manufacturing experienced serious definitional changes. The aggregate food sector includes the following nine categories: (1) meat products, (2) dairy products, (3) preserved fruits and vegetables, (4) grain mill products, (5) bakery products, (6) sugar and confectionery products, (7) fats and oils, (8) beverages, and (9) miscellaneous food and kindred products.

The value of food-manufacturing production is defined as the value of shipments adjusted for inventory changes of finished products. The aggregate price of the food-manufacturing product is defined as the Laspeyres price index for the producer price indexes of each food category (compiled from the *Producer Price Index,* PPI), using 1982 shipment values as weights. The aggregate price index is used to deflate the value of production to obtain an approximate quantity measure.

The cost of labor is the total wage payments for production workers, while unit labor price refers to the average wage payments per production worker per hour. The cost of energy is the total cost of purchased fuels and electric energy. The producer price index for fuel and power is used to represent the price of energy.

Implicit price deflators for gross fixed nonresidential capital investment for structures and producers' durable equipment, compiled from the *Survey of Current Business* (SCB), are used as an approximate price index for capital services. The cost of capital services for equipment and structures is the sum of depreciation, rental payments, and interest on the food-manufacturing net assets. Data for depreciation charges, rental payments, and the gross book value of depreciable assets are available in ASM. Because depreciation charges for 1972–76 and 1986 are not reported, this study projects them on the basis of a log-linear regression by fitting the depreciation charges (D) as a function of beginning-of-year structure and equipment assets (K) for 1977–85:

$$\ln D = -3.3195 + \underset{(0.0292)}{1.0629} \ln K \qquad R^2 = 0.99 \quad \text{D-W} = 2.60.$$

Finally, the interest on net assets is calculated by multiplying the value of beginning-of-year assets by the Moody's Corporate Industrial Bond Rate from SCB.

Empirical Results

The cost structure of the U.S. food-manufacturing industry is characterized by estimating three cost share equations as in equation (10) for labor, capital, and energy. Because the cost shares of the three equations always sum to unity, the sum of the disturbances across these equations is zero at each observation. This implies that the covariance matrix of residuals is singular, and one of the cost share equations should be dropped from direct estimation. The remaining equations are then estimated simultaneously by applying Zeller's seemingly unrelated regression method, while the parametric constraints are incorporated.

As indicated in Berndt and Savin (p. 942), if one uses the covariance matrix of residuals obtained from applying ordinary least squares to unrestricted equations as prior information for estimation, the estimates of the parameters are the same regardless of which cost share equation is deleted. This study uses this approach, and the invariant property of estimates has been verified.

A time-trend variable as a proxy for technical progress was initially included in the model. However, because of multicollinearity, the estimated coefficients for the time trend were not statistically significant. Consequently, the time-trend variable was excluded from the model. In addition, the relatively short sample period (15 years) precluded testing for structure changes in the food-manufacturing industry.

The estimation results are reported in table 1. Most of the estimated parameters (13 out of 15) are statistically significant at the 5% level. The percentages of root-mean-square error to sample mean are about 7% or less for each equation. These estimated parameters and their covariance matrix of errors together with the observed factor cost shares at sample means are the basic information for computing the estimates of factor demand relationships contained in tables 2 and 3. The factor cost shares used in the calculation are labor (0.5376), capital (0.3393), and energy (0.1231).

Based on equations (14) and (15), the (constant-output) price elasticities of factor demands are obtained and shown in table 2. The results suggest that the demand for capital is quite elastic with an elasticity of 2.08, while the demand elasticities of labor and energy are 1.05 and 0.49, respectively. The high elasticity of demand for capital probably reflects the industry's high capital-intensive technologies. As indicated in Connor and others (p. 39), food processing is more capital intensive than such heavy industries as machinery and transportation equipment; only four major industry groups (paper, chemicals, petroleum, and primary metals) are more capital intensive than food manufacturing. The cross-price elasticities show a strong substitution relationship between capital and labor with elasticities 1.78 for the demand of capital and 1.12 for the demand of labor. Capital and energy are substitutable, and labor and energy are comple-

Table 1. Estimated Parameters of Cost Function

Equation	Constant	Factor Price of			Output Quantity	C.V.[a]
		Labor	Capital	Energy		
Share of:						
Labor	−0.77999*	−0.31504	0.42180	−0.10677	0.03897*	2.48
	(0.92447)[b]	(0.05781)	(0.06281)	(0.01201)	(0.06553)	
Capital	4.18320	0.42180	−0.48148	0.05968	−0.22115	5.15
	(1.13008)	(0.06281)	(0.07087)	(0.01533)	(0.08114)	
Energy	−2.40319	−0.10677	0.05968	0.04708	0.18218	6.81
	(0.35868)	(0.01201)	(0.01533)	(0.00608)	(0.02732)	

[a] C.V. is the percentage of root-mean-square error to sample mean. All estimates except for those marked with (*) are significant at the 5% level.
[b] Standard errors are in parentheses.

Table 2. Elasticities of Conditional Factor Demands and Allen Partial Elasticity of Substitution (*AES*)

Measure	Factor Price of		
	Labor	Capital	Energy
Elasticity			
Labor	−1.04849	1.12401	−0.07551
	(0.10754)	(0.11685)	(0.02235)
Capital	1.78056	−2.07953	0.29897
	(0.18510)	(0.20886)	(0.04519)
Energy	−0.32976	0.82417	−0.49441
	(0.09759)	(0.12457)	(0.04940)
AES			
Labor	−1.95048	3.31231	−0.61344
	(0.20006)	(0.34434)	(0.18155)
Capital		−6.12813	2.42872
		(0.61547)	(0.36708)
Energy	(Symmetry)		−4.01639
			(0.40133)

Note: Elasticities and their standard errors (in parentheses) are computed on the basis of equations from (12) to (15) at sample mean of cost shares. All estimates are significant at the 5% level.

mentary; however, the cross elasticities are relatively small.

The interdependencies among labor, capital, and energy are further confirmed by the Allen elasticities of substitution shown in the lower half of table 2. The elasticities signify substitution if the sign is positive and complementarity if the sign is negative. The substitution relationship between labor and capital is supported by previous studies of manufacturing industries. For example, Berndt and Christensen found that the Allen elasticities of equipment-labor and structures-labor ranged from 1.22 to 1.79 in U.S. manufacturing for 1929–68. Fuss found that the Allen elasticity between capital and labor was about 0.8 in Canadian manufacturing for 1961–71. In another study, Berndt and Wood found that the Allen elasticity between capital and labor was about 1.01 in U.S. manufacturing for 1947–71.

The Morishima elasticities of substitution calculated on the basis of equation (11) are compiled in table 3. The entries in the off-diagonal of the table reflect the adjustments of relative factors in response to a change in the ratio of relative factor prices. Their signs are all positive, implying that any pair of factors is substitutable with each other. In particular, the elasticities of factor ratios, labor-capital and capital-labor, are large, respectively, 2.83 and 3.20. The substitution between labor and energy is inconsistent with the Allen elasticity measure mainly because of different definitions; a Morishima elasticity is related to the adjustment of two factors, while a partial adjustment of one factor is allowed in an Allen elasticity.

Both *MES* and *AES* indicate a strong substitution relationship between capital and labor. In fact, there is a trend of more intensive use of capital but less of labor in past years especially in light of the steady increase in the price ratio between labor and capital since 1982. The capital input index (1982 = 100) increased from 67.50 in 1972 to a peak of 110.15 in 1984 and then slightly decreased thereafter. On the other hand, the labor input index declined from 106.57 in 1972 to 95.02 in 1986.

The interrelatedness of factor demands is also shown in the variation of industrial cost structure in response to a change in factor prices. Based on equation (8), the elasticities of each pairwise factor cost share with respect to their factor prices are shown in the lower half of table 3. As indicated before, these results are closely related to the magnitude of Morishima elasticities; the relative cost share decreases if the Morishima elasticity is greater than one and increases if it is less than one. For example, the −1.83 elasticity of labor-capital indicates a significant reduction in the cost share of labor to capital in response to relatively higher wages than capital price. On the other hand, the 0.58 elasticity of energy-labor indicates that a marginal increase of energy price would cause an increase of the cost share of energy relative to labor.

Table 3. Morishima Elasticities of Substitution (*MES*) and Effects of Factor Price on Cost Shares

	Factor Price of		
Measure	Labor	Capital	Energy
MES			
Labor		2.82905	0.71874
		(0.29153)	(0.11896)
Capital	3.20354		2.90370
	(0.32427)		(0.30093)
Energy	0.41890	0.79338	
	(0.04612)	(0.08684)	
Cost share			
Labor		−1.82905	0.28126
		(0.29153)	(0.11896)
Capital	−2.20354		−1.90370
	(0.32427)		(0.30093)
Energy	0.58110	0.20662	
	(0.04612)	(0.08684)	

Note: Elasticities and standard errors (in parentheses) are computed on the basis of equations (8) and (11) at sample mean of cost shares. All estimates are significant at the 5% level.

Concluding Comments

This study analyzes the demand for labor, capital, and energy and their interdependent relationships in the U.S. food-manufacturing industry. The results show that the demand for capital services is more highly elastic than for labor and energy. Thus, any policy measures to reduce the price of capital services, such as investment tax credits and lower interest rates, would significantly increase the demand for capital. The demand elasticities of labor, capital, and energy in response to energy price changes are relatively low. They indicate that the relatively large changes in the prices of fuel and energy experienced over the sample period did not cause much adjustment in factor utilization.

The Morishima elasticity is, in general, better than the Allen elasticity for representing the factor substitution relationship because of its capability to explicitly explain the adjustment of factor combinations in response to relative price changes. The estimated Morishima elasticities indicate that labor, capital, and energy are substitutable especially between labor and capital. This is evidenced by the recent trends in the food-manufacturing industry to substitute computers and automated machines for human operations in light of the steady increase in the labor to capital price ratio.

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