Inflation and welfare in long-run equilibrium with firm dynamics

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Inflation and welfare in long-run equilibrium with firm dynamics*

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Abstract

We analyze the welfare cost of inflation in a model with cash-in-advance constraints and an endogenous distribution of establishments' productivities. Inflation distorts aggregate productivity through firm entry dynamics. The model is calibrated to the United States economy and the long-run equilibrium properties are compared at low and high inflation. We find that increasing the annual inflation rate by 10 percentage points above the average rate in the U.S. would result in a fall in average productivity of roughly 1.3 percent. This decrease in productivity is not innocuous: it is responsible for about one half of the welfare cost of inflation.

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1 Introduction

Whether the adoption of monetary policy rules that reduce inflation and interest rates leads to important welfare gains is a central question in monetary economics[1]. Calculations often suggest that the effects of changes in the inflation rate on capital accumulation are modest. However, if international differences in income per capita are explained by differences in the accumulation of productive factors and by differences in the efficiency in the employment of these factors, then the welfare cost of inflation will be high if it discourages the accumulation of factors of production or if it leads to less efficiency in their use. The first possibility has been extensively examined in the literature however the latter has been neglected. In this paper we begin the exploration of this second possibility.

Measures of the welfare cost of inflation are usually derived by comparing steady states levels of aggregate consumption at different rates of money growth within the framework of monetary equilibrium growth models. Money is often introduced by means of cash-in-advance constraints which require agents to hold money balances to facilitate transactions. Cooley and Hansen (1989) show that when the neoclassical growth model is augmented with this structure, the relative price of consumption with respect to leisure increases as the long-run rate of monetary growth increases. Consequently agents substitute away from labor, which induces employment and output to drop. Stockman (1981) shows that, when the cash-in-advance constraint also applies to investment goods, a similar effect operates through lower capital accumulation. At moderate inflation rates, these models produce welfare costs equivalent to slightly less than one percent of real income; for example, Cooley and Hansen (1989) report that, in steady state, a 10 percent inflation rate results in a welfare cost of about 0.4 percent of income relative to an optimal monetary policy.

However, in these earlier models average productivity is exogenous and only the accumulation of factors of production matters to determine income. Gomme (1993), De Gregorio (1993) and Jones and Manuelli (1995) extend the work on the effects of monetary policy to models of endogenous growth and find the welfare cost of inflation to be either of the same magnitude or an order of magnitude smaller. But their work assumes a single representative firm and abstract from heterogeneity in production units. If, however, the allocation of aggregate resources across uses is important in understanding cross-country

[1]See Lucas (2000).

differences in per capita incomes, then it is not only the level of factor accumulation that matters, but also how these factors are allocated across heterogeneous production units[2]. Since large differences in income per capita cannot be accounted for simply by differences in the accumulation of production factors, to answer the question of whether the welfare cost of inflation is important we should consider a framework where the allocation of factors across establishments with different productivity levels is potentially affected by money[3]. In the context of a general equilibrium monetary economy model, Dotsey and Ireland (1996), persuasively argue that the inflation tax may distort a variety of marginal decisions and that various small distortions may combine to yield substantial estimates of the welfare cost of inflation. Thus, to confidently examine whether an economy is better off at low levels of inflation in the framework of monetary equilibrium growth models, average productivity should be endogenous and potentially affected by the monetary growth rate.

In this paper, we investigate what is the impact of higher rates of monetary growth on the real economy including output, consumption, investment, hours worked and productivity in a model where the productivity distribution of incumbent establishments is endogenous. For this purpose, we build a model characterized with cash-in-advance constraints on consumption and investment goods, and in addition we assume that liquidity constraints also apply to the creation of new establishments. Because efficiency in the use of the factors of production is an important channel influencing output, the model considers establishment heterogeneity along the lines of Hopenhayn (1992), Hopenhayn and Rogerson (1993) and Melitz (2003). In this framework, we are able to analyze the effect of long-run monetary growth on output per worker and we confirm the finding of previous literature that monetary growth has a negative impact on output in a cash-in-advance economy. In addition to discouraging investment and labor supply, we find that an increase in the long-run rate of money growth increases the cost of creating new establishments.

[2]There is substantial evidence of the importance of capital and labor allocation across establishments as a determinant of aggregate productivity. Studies document that about half of overall productivity growth in U.S. manufacturing can be attributed to factor reallocation from low productivity to high productivity establishments for different time periods. See for instance Baily et al. (1992), Bartelsman and Doms (2000) and Foster et al. (2008), among others.

[3]Indeed, the prevailing view in development accounting is that cross-country differences in income per capita are mostly explained by differences in Total Factor Productivity. See King and Levine (1994), Klenow and Rodriguez-Clare (1997), Prescott (1998), Hall and Jones (1999) and Caselli (2005).

As a result, incumbent establishments' profits must increase so as to encourage industry entry. This occurs through a fall in the equilibrium wage rate. The fall in wages allows new establishments with low productivity to stay in the industry leading to a reallocation of the factors of production toward less efficient establishments. This adjustment in the size distribution of production plants lowers average productivity in the economy.

We calibrate the model to the U.S. economy and find that increasing the annual inflation rate by 10 percentage points above the average rate in the U.S. would result in a fall in average productivity of about 1.3 percent. Quantitatively, it may be responsible for almost 1/2 of the effect of inflation on welfare. We consider several alternative calibrations to the benchmark economy, revealing the importance of the assumptions made regarding the returns to scale and the dispersion of productivities across establishments. We show that, when money distorts establishments' entry dynamics, the welfare cost from a monetary policy that increases inflation rates from 2 percent to 12 percent may represent roughly 5 percent of aggregate consumption, confirming results by Atkeson et al. (1996) on the importance of heterogeneity and decreasing returns to scale for interpreting cross-country differences in macroeconomic outcomes.

In work which is also related to this paper, Wu and Zhang (2001) examine the effects of anticipated inflation in a framework characterized by monopolistic competition and a well defined industry structure. In their paper, firms' mark-ups are affected by the rate of inflation. They find that at higher rates of inflation the number of firms is less and their size is smaller. The resulting welfare cost of inflation is larger than the conventional estimates. In our paper, the welfare cost of inflation is also higher than those obtained in conventional models. However, in our paper markets are competitive and the higher welfare cost is associated with the change in the productivity distribution of incumbent establishments. Thus, firm entry is affected by anticipated inflation and the model also predicts that the number of productive establishments as well as their average size is lower at high rates of inflation.

Given the abundance of empirical evidence indicating the importance of producers' heterogeneity and selection-based productivity growth, it is hardly surprising that an influential literature has developed, which examines the reallocation effects of policy distortions. In an article mentioned earlier, Hopenhayn and Rogerson (1993) consider the effect on average productivity and welfare of employment protection in a setting characterized with

firm entry and exit dynamics. They find that a tax on job destruction results in a decrease in average productivity of over 2 percent. In a related paper Veracierto (2001) extends Hopenhayn and Rogerson's analysis of firing taxes by introducing a flexible form of capital and considering transition dynamics. Veracierto finds that firing taxes equal to one year of wages have large long-run effects: they decrease steady state output, capital, consumption, and wages by 7.84 percent and steady state employment by 6.62 percent. With the purpose of studying the role of international trade, Melitz (2003) shows how aggregate industry productivity growth caused by reallocations across heterogeneous establishments contribute to additional welfare gains from trade liberalization.

The role of policy distortions in environments with industry dynamics has also influenced the literature on development. For instance, Restuccia and Rogerson (2008) consider policy distortions that lead to reallocation of resources across heterogeneous firms. Their aim is to examine whether policies that leave aggregate relative prices unchanged but distort the prices faced by different producers can explain cross-country differences in per capita incomes. In their benchmark model they find that the reallocation of resources implied by such policies can lead to decreases in output and productivity in the range of 30 to 50 percent, even though the underlying range of available technologies across establishments is the same in all policy configurations. Samaniego (2006) proposes a model of plant dynamics to analyze the effects of policies that affect establishments differently depending on the stage of their life-cycle, notably subsidies to failing plants. He finds that these subsidies may increase aggregate productivity. Guner et al. (2008) find that policies that distort the size-distribution of incumbent establishments may lead to substantial output and productivity falls. Finally, Alfaro et al. (2008) investigate, using plant-level data for several countries, whether differences in the allocation of resources across heterogeneous plants are a significant determinant of cross-country differences in income per worker. They find that allowing for firm heterogeneity improves the model ability to explain differences in productivity across countries. Our paper introduces firm heterogeneity and industry dynamics into a monetary growth model and considers the distortions introduced by the inflation tax, when money holdings are required to create new establishments.

Another important literature examines the welfare cost of inflation in the context of monetary search models. Influential papers include Rocheteau and Wright (2005), and Lagos and Rocheteau (2005). The debate over the welfare costs of inflation in this framework

is still very much ongoing. Under the canonical search model of money with ex-post bilateral Nash bargaining between a buyer and a seller, higher anticipated inflation decreases search effort, the frequency of trades, and aggregate output leading to welfare losses. However, the welfare losses are small under competitive search and may be negative under price-taking behavior.

The remainder of the paper is organized as follows. In section 2 we lay out the details of our model and describe the stationary competitive equilibrium. In Section 3 we investigate the qualitative effect of changes in the monetary growth rate on the endogenous real aggregates and the size distribution of productive establishments. Section 4 discusses the procedure for calibrating our model and section 5 presents our model-based quantitative findings. Finally, section 6 concludes.

2 The model

We consider a cash-in-advance production economy, which exhibits establishment level heterogeneity as studied by Hopenhayn (1992) and Hopenhayn and Rogerson (1993). Establishments have access to a decreasing returns to scale technology, pay a fixed cost to remain in operation each period and are subject to entry and exit. In what follows we first describe the problem of the household confronted with a cash-in-advance constraint, next we describe the production side in more detail and finally characterize the stationary competitive equilibrium.

2.1 The household

There is an infinitely-lived representative household with preferences over streams of consumption and leisure at each date described by the utility function

$$U = \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + A \ln L_t \right),$$

where C_t is consumption at date t, L_t is leisure and $\beta \in (0,1)$ is the discount factor. The representative agent is endowed with one unit of productive time each period and has $K_0 > 0$ units of capital at date 0. She owns three types of assets: capital, cash, and production establishments. The mass of (incumbent) establishments at time t is denoted by H_t.

The timing of the household decision problem resembles the one in Stockman (1981). The household enters period t with nominal money balances equal to m_{t-1} that are carried over from the previous period and in addition receives a lump-sum transfer equal to gM_{t-1} (in nominal terms), where M_t is the per capita money supply in period t. Thus, the money stock follows the law of motion

$$M_t = (1+g)\, M_{t-1}.$$

Output has three purposes: (i) it can serve as a *consumption good*; (ii) as an *investment good* which increases the stock of capital owned by the household; (iii) as a *marketing good* which has to be purchased in order to create new establishments. Households are required to use their previously acquired money balances to purchase goods. Because we want to compare situations when the constraint applies to some types of good but not to others, we introduce three parameters that we denote by θ_i with $i = c, k, h$. When $\theta_c = 1$ the cash-in-advance constraint applies to the consumption good, when $\theta_k = 1$ purchases of the investment good are constrained and when $\theta_h = 1$ the constraint applies to the marketing good needed to create a new establishment. When $\theta_i = 0$ $(i = c, k, h)$ the constraint does not apply to the specific good and this good is said to be a *credit good* in the Lucas and Stokey (1987) sense. Hence, the constraint reads as

$$\theta_c C_t + \theta_k X_t + \theta_h \kappa E_t \leq \frac{m_{t-1} + gM_{t-1}}{p_t}, \tag{1}$$

where p_t is the price level at time t, X_t is investment, given by

$$X_t = K_{t+1} - (1-\delta)\, K_t \tag{2}$$

and κ is the quantity of marketing good that has to be purchased to create each new establishment and constitutes a sunk cost. E_t is the mass of new establishments created.

The representative household must choose consumption, investment, leisure, nominal money holdings and the mass of new establishments subject to the cash-in-advance constraint (1) and the budget constraint

$$C_t + X_t + \kappa E_t + \frac{m_t}{p_t} \leq w_t\,(1 - L_t) + r_t K_t + \bar{z}_t H_t + (m_{t-1} + gM_{t-1})\,/p_t, \tag{3}$$

where w_t is the wage rate, r_t the interest rate and $\bar{z}_t$ are average dividends across incumbent establishments.

We assume that the gross growth rate of money, $1+g$, always exceeds the discount factor, β, which is a sufficient condition for (1) to always bind in equilibrium and existence of a stationary equilibrium[4]. We sometimes denote real money balances by $\mu_t = \frac{m_t}{p_t}$.

2.2 Production establishments

Once a new establishment is created at t, its idiosyncratic productivity $s \in S$ is revealed as drawn from a distribution $F(s)$ and remains constant over time until the establishment exits the industry. At $t+1$ the establishment starts production. Incumbent establishments produce output by renting labor and capital. The production function of an establishment with idiosyncratic productivity s at time t is

$$y_{s,t} = s n_{s,t}^{\alpha} k_{s,t}^{\nu} - \eta, \tag{4}$$

where $n_{s,t}$ and $k_{s,t}$ are labor and capital employed, η is a fixed operating cost, $\alpha \in (0,1)$, $\nu \in (0,1)$ and $\nu + \alpha < 1$. The flow profits of an incumbent establishment are given by

$$z_{s,t} = \max_{n_{s,t}, k_{s,t}} \left\{ s n_{s,t}^{\alpha} k_{s,t}^{\nu} - w_t n_{s,t} - r_t k_{s,t} - \eta \right\}, \tag{5}$$

where w_t is the wage rate and r_t is the interest rate.

Establishments exit both because of exogenous exit shocks and endogenous decisions. In particular, in any given period after production takes place, each establishment faces a constant probability of death equal to λ. Moreover, an establishment decides to leave the industry if its discounted profits are negative. Given that we only analyze the stationary equilibrium of the economy and idiosyncratic productivities are constant over time, it turns out that the only moment when an establishment decides to leave the industry is upon entry. This is because profits are constant over time in the stationary equilibrium. Consequently, establishments choose to exit when

$$z_s < 0.$$

We denote by s^* the idiosyncratic productivity threshold below which establishments choose to exit. Specifically, s^* is such that $z_{s^*} = 0$.

[4]It can be shown that the existence of a steady state requires $1 + g \geq \beta$. See Abel (1985).

Given the first order conditions which solve the problem of incumbent firms (5) the labor demand by an establishment with productivity s is

$$n_{s,t} = s^{\sigma} \left(\frac{\alpha}{w_t}\right)^{(1-\nu)\sigma} \left(\frac{\nu}{r_t}\right)^{\nu\sigma} \tag{6}$$

and the demand for capital reads

$$k_{s,t} = s^{\sigma} \left(\frac{\alpha}{w_t}\right)^{\alpha\sigma} \left(\frac{\nu}{r_t}\right)^{(1-\alpha)\sigma}, \tag{7}$$

where $\sigma = (1-\alpha-\nu)^{-1}$. Replacing the factor demands into the profit function yields

$$z_{s,t} = \Omega \frac{s^{\sigma}}{w_t^{\alpha\sigma} r_t^{\nu\sigma}} - \eta, \tag{8}$$

where $\Omega = \alpha^{\alpha\sigma}\nu^{\nu\sigma} - \alpha^{(1-\nu)\sigma}\nu^{\nu\sigma} - \alpha^{\alpha\sigma}\nu^{(1-\alpha)\sigma}$.

Let $h(s;t)$ denote the mass of incumbent establishments with productivity level s at time t. The motion equation for $h(s;t)$ is given by

$$h(s;t+1) = (1-\lambda)h(s;t) + E_t dF(s) I[s \geq s^*], \tag{9}$$

where I is an indicator function that takes value one if the expression in brackets is true and zero otherwise. With $H_t = \int_{s\in S} h(s;t)ds$ denoting the mass of incumbent establishments. Consequently, the mass of entrants reads

$$E_t = \frac{H_{t+1} - (1-\lambda)\, H_t}{1 - F\left(s^*\right)}. \tag{10}$$

Finally, following Melitz (2003), it is useful to define average productivity as

$$\bar{s}_t = \left\{ \int_{s\geq s^*} s^{\sigma} \frac{dF(s)}{1-F(s^*)} \right\}^{\frac{1}{\sigma}}. \tag{11}$$

Hence, with knowledge of s^* one can identify $\bar{s}_t$. From equation (8), this implies that average dividends read as

$$\bar{z}_t = \int_{s\geq s^*} z_{s,t} \frac{dF(s)}{1-F(s^*)} ds = \Omega \frac{\bar{s}_t^{\sigma}}{w_t^{\alpha\sigma} r_t^{\nu\sigma}} - \eta. \tag{12}$$

2.3 Household optimal behavior

The Bellman equation characterizing household's optimal behavior reads as

$$V\left(m_{t-1}, K_t, H_t\right) = \max_{C_t, L_t, m_t, K_{t+1}, H_{t+1}} \left\{ \ln C_t + A \ln L_t + \beta V\left(m_t, K_{t+1}, H_{t+1}\right) \right\}, \tag{13}$$

and is subject to the cash-in-advance constraint (1) and the budget constraint (3).

Let ϕ_t and γ_t be the Kuhn-Tucker multipliers for the constraints (1) and (3), respectively. The first-order conditions which characterize the solution to the problem of the household are

$$\frac{1}{C_t} - \theta_c \phi_t - \gamma_t = 0, \tag{14}$$

$$\frac{A}{L_t} - \gamma_t w_t = 0 \tag{15}$$

$$\beta V_1 \left(m_t, K_{t+1}, H_{t+1}\right) - \frac{\gamma_t}{p_t} = 0, \tag{16}$$

$$\beta V_2 \left(m_t, K_{t+1}, H_{t+1}\right) - \theta_k \phi_t - \gamma_t = 0, \tag{17}$$

$$\beta V_3 \left(m_t, K_{t+1}, H_{t+1}\right) - \frac{\kappa}{1 - F(s_t^*)} (\theta_h \phi_t + \gamma_t) = 0, \tag{18}$$

plus the budget constraint and the complementary slackness condition associated with the budget constraint. Moreover, by the envelope theorem, the shadow values of money, capital and the mass of establishments are respectively

$$V_1 \left(m_{t-1}, K_t, H_t\right) = \frac{\phi_t + \gamma_t}{p_t}, \tag{19}$$

$$V_2 \left(m_{t-1}, K_t, H_t\right) = (1 - \delta) \left(\theta_k \phi_t + \gamma_t\right) + \gamma_t r_t. \tag{20}$$

and

$$V_3 \left(m_{t-1}, K_t, H_t\right) = \frac{1 - \lambda}{1 - F(s_t^*)} \kappa \left(\theta_h \phi_t + \gamma_t\right) + \gamma_t \bar{z}_t. \tag{21}$$

Combining (19), (20) and (21) and the first-order conditions (16), (17) and (18) yields the three Euler equations

$$\beta \frac{\phi_{t+1} + \gamma_{t+1}}{p_{t+1}} - \frac{\gamma_t}{p_t} = 0, \tag{22}$$

$$\beta \left(1 - \delta\right) \left(\theta_k \phi_{t+1} + \gamma_{t+1}\right) + \beta \gamma_{t+1} r_{t+1} - \theta_k \phi_t - \gamma_t = 0 \tag{23}$$

and

$$\beta \frac{1 - \lambda}{1 - F(s_{t+1}^*)} \kappa \left(\theta_h \phi_{t+1} + \gamma_{t+1}\right) + \beta \gamma_{t+1} \bar{z}_{t+1} - \kappa \frac{\theta_k \phi_t + \gamma_t}{1 - F(s_t^*)} = 0. \tag{24}$$

Equations (14) and (22)-(24), combined with the intra-temporal first-order condition (15) and the budget constraint (3) characterize the solution to the household problem.

2.4 Market clearing

Market clearing conditions for labor and capital are given, respectively, by

$$N_t = \int_{s \in S} n_{s,t} h(s;t) ds \tag{25}$$

and

$$K_t = \int_{s \in S} k_{s,t} h(s;t) ds. \tag{26}$$

Market clearing in the money market requires

$$m_t = M_t. \tag{27}$$

Finally, the economy's feasibility constraint reads

$$C_t + X_t + \kappa E_t = Y_t, \tag{28}$$

where $Y_t \equiv \int_{s \in S} y_{s,t} h(s;t) ds$.

2.5 Stationary equilibrium

We consider the steady-state competitive equilibrium of the model. In a steady-state equilibrium, all rental rates and real aggregates are constant over time. Moreover, the gross rate of inflation $\Pi \equiv \frac{P_{t+1}}{P_t}$ is also constant, equal to the gross rate of monetary growth $1 + g$. Thus, we henceforth ignore all time subscripts to simplify the notation.

We now illustrate three effects of inflation related to the three cash-in-advance constraints of the economy.

Since the shadow values ϕ and γ are each positive and constant in the steady-state[5], from equations (14), (15) and (22), consumption and leisure in the steady-state equilibrium satisfy the condition

$$\frac{L}{C} = \frac{A}{w} \left[1 + \theta_c \left(\frac{1+g}{\beta} - 1 \right) \right]. \tag{29}$$

Equation (29) suggests that, when the cash-in-advance constraint applies to consumption, an increase in inflation raises the cost of consumption relative to leisure. This result corresponds to the effect examined in Cooley and Hansen (1989).

[5]See Stockman (1981).

Given equations (22) and (23), the representative household problem yields the stationary equilibrium rental rate of capital, given by

$$r = \left(\frac{1}{\beta} - 1 + \delta\right)\left[1 + \theta_k\left(\frac{1+g}{\beta} - 1\right)\right] \tag{30}$$

Equation (30) shows that the rental cost of capital is increasing in the rate of anticipated inflation when the cash-in-advance constraint applies to the investment good. It also suggests the following mechanism. When the cash-in-advance constraint applies to investment, inflation increases the cost of holding money balances, which reduces capital accumulation. As a result, at higher inflation, the rental cost of capital is higher. This result is due to Stockman (1981).

Finally, from equations (22) and (24) a free-entry condition reads

$$\kappa\left[1 + \theta_h\left(\frac{1+g}{\beta} - 1\right)\right] = [1 - F(s^*)]\,\frac{\beta\bar{z}}{1-\beta(1-\lambda)}. \tag{31}$$

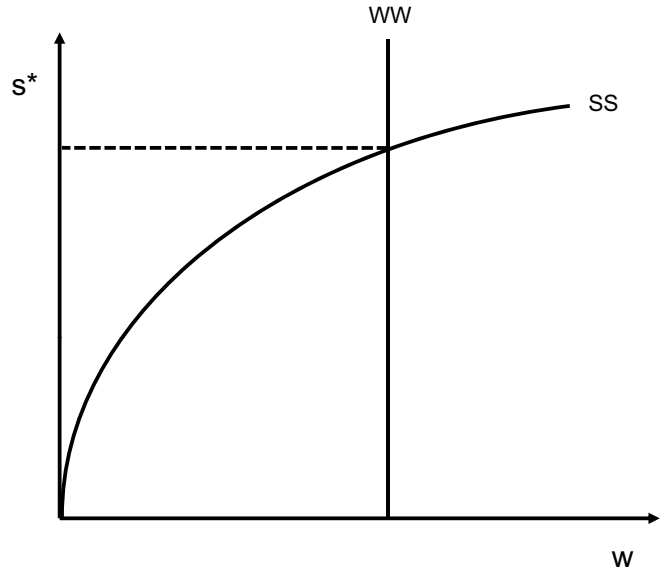
Equation (31) states that in equilibrium the sunk cost that has to be paid to create a new establishment (the left-hand side of (31)) has to be equal to the expected discounted profits from creating this establishment (the right-hand side of (31)). The rate of discount of profits depends on the household discount factor β and the probability λ that the new establishment dies in future periods. The probability $[1 - F(s^*)]$ also appears on the right-hand side of (31) because one has to account for the probability of successful entry when evaluating discounted profits.

Equation (31) characterizes the mechanism by which money growth affects the establishments entry decision. When the cash-in-advance constraint applies to the marketing good, an increase in inflation makes entry more costly. The next Section shows that this has an effect on average productivity too.

Hence, inflation may have three effects, depending on the structure of the cash-in-advance constraint. It may affect labor supply, capital accumulation and the productivity distribution of incumbent establishments. Each effect contributes to lowering the level of output. This allows us in the next Section to state a Proposition on the real effects of inflation. Before doing this, we go through the remaining relations characterizing the equilibrium.

In the stationary competitive equilibrium the optimal exit rule by incumbent establishments requires $z_{s^*} = 0$. This yields a solution for the productivity threshold, given

Figure 1: Determination of s^* and w in the stationary economy



by

$$s^* = w^{\alpha} r^{\nu} \left(\frac{\eta}{\Omega} \right)^{1-\alpha-\nu}. \tag{32}$$

Since the equilibrium interest rate is determined by (30), the exit condition characterizes a relation between the wage rate and the productivity threshold which is represented by the SS locus in Figure 1.

In turn, the expected value of entry, i.e. the right-hand side of the free-entry condition (31) is locally independent of s^* by the envelope theorem (see Appendix A for proof). Consequently, the equilibrium wage rate is independent of s^*, as illustrated by the WW locus in Figure 1. Hence, in an equilibrium with production the free-entry condition determines the wage rate.

Finally, solving for the fixed point of (9) and integrating over productivity levels yields

$$H = E \int_{s \in S} \frac{I[s \geq s^*]}{\lambda} dF(s), \tag{33}$$

which, combined with the resource constraint (28), gives a solution for the mass of incumbent establishments, completing the characterization of the stationary competitive equilibrium. Specifically, the stationary competitive equilibrium is defined as follows[6]:

[6]It is shown in the appendix B that the equilibrium exists and is unique.

Definition 1. *A stationary competitive equilibrium is a wage rate w, a rental rate of capital r, an aggregate distribution of establishments $h(s)$, a mass of entry E, a household value function $V(m, K, H)$, an establishment profit function z_s, a productivity threshold s^*, policy functions for incumbent establishments n_s and k_s and aggregate levels of consumption C, employment N, capital K and real money balances μ, such that:*

i. The household optimizes: equations (29), (30) *and* (31)*;*

ii. Establishments optimize: equations (6), (7) *and* (32)*;*

iii. Markets clear: equations (25), (26), (27) *and* (28)*;*

iv. $h(s)$ is an invariant distribution, i.e. a fixed point of (9)*.*

To summarize, the model is solved as follows. First, the rental cost of capital is pinned down by equation (30). Then, given the value of r, one can solve for the values of the wage rate w and the productivity threshold s^* from (31) and (32). One can consequently characterize fully the stationary distribution of capital, employment, profits and output with equations (4), (6), (7) and (8) across incumbent firms. Finally, the feasibility constraint (28), together with the other market-clearing conditions and the first-order condition for leisure (29), allow to determine the mass of incumbents H and all the aggregates of the economy such as investment, consumption, output, the stock of capital and employment[7].

3 The real effects of inflation

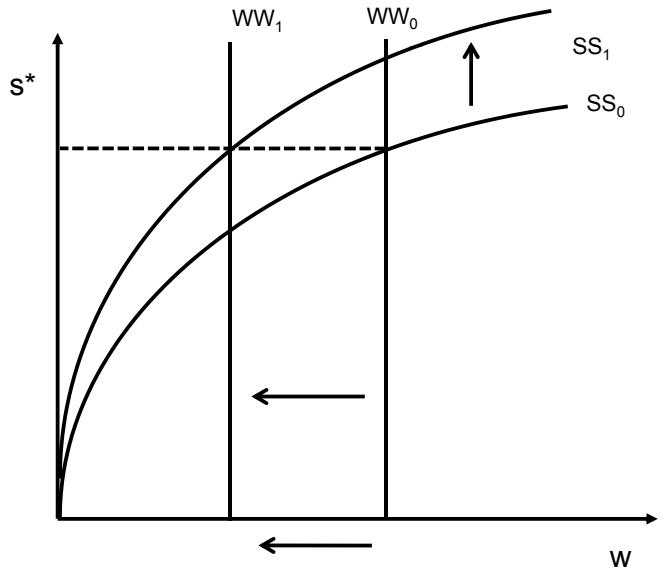
We now investigate the relation between inflation, the equilibrium aggregates K and N, and the size distribution of productive establishments, characterized by s^*. Proposition 1 summarizes our main result

PROPOSITION 1. *Consider the stationary competitive equilibrium as defined earlier.*

i. If $\theta_c = \theta_k = \theta_h = 0$, an increase in the inflation rate Π has no effect on the economy.

[7]In the Appendix E, we present all the equations that characterize the stationary equilibrium for the particular restriction that we impose on the distribution F. See also Section 4, where we describe the calibration procedure.

Figure 2: Effect of an increase in the monetary growth rate g on s^* and w when $\theta_k = 1$ and $\theta_h = 0$



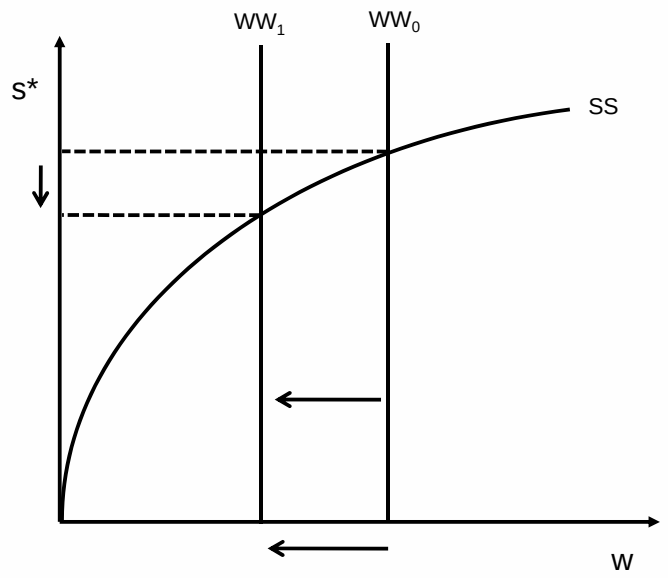
ii. *If $\theta_c = 1$ and $\theta_k = \theta_h = 0$, an increase in the inflation rate Π is associated with a fall in the equilibrium capital stock K and a fall in the employment rate N. However, the productivity threshold, s^*, does not change.*

iii. *If $\theta_k = 1$ and $\theta_c = \theta_h = 0$, an increase in the inflation rate Π is associated with a fall in the equilibrium capital stock K and a fall in the employment rate N. However, the productivity threshold, s^*, does not change.*

iv. *If $\theta_h = 1$ and $\theta_c = \theta_k = 0$, an increase in the inflation rate Π is associated with a fall in the equilibrium capital stock K, a fall in the employment rate N and a fall in the productivity threshold, s^*.*

In what follows we discuss some aspect related to Proposition 1, however, the detailed proof is developed in the Appendix D. When $\theta_i = 0$ for all i, all goods are *credit goods* and therefore money growth has no real effects. When consumption is a *cash good* condition (29) is affected by money growth. At high rates of inflation, the marginal utility of leisure must fall with respect to the product of the wage rate and the marginal utility of consumption, leading the household to supply less labor. Lower hours worked leads to lower output and therefore lower consumption and capital stock. The rental cost of capital, determined

Figure 3: Effect of an increase in the monetary growth rate g on s^* and w when $\theta_h = 1$ and $\theta_k = 0$



by (30), remains the same and, therefore both the SS relation and the WW relation, in Figure 1, are unaffected. Thus the wage rate and average productivity are unaffected.

When $\theta_k = 1$, i.e. investment is a *cash good*, condition (30) is affected. At high rates of inflation the return on capital must increase as individuals are less willing to invest. The increase in the rental cost of capital lowers profits for the same wage rate and therefore the probability of a successful entry decreases at each wage rate (i.e. the SS locus in Figure 2 shifts upward). However the probability of successful entry must remain unchanged in equilibrium since the cost of creating a new establishment (the left-hand side of equation (31)) has not changed. Thus, for there to be an equilibrium with entry, the wage rate must fall sufficiently for the free entry condition to be satisfied. The WW locus in Figure 1 shifts left. At high rates of inflation the wage rate is lower and the average productivity and the probability of successful entry are unaffected, as illustrated by Figure 2.

When the marketing good is a *cash good*, $\theta_h = 1$, the cash-in-advance constraint increases the cost of creating new establishments and the comparative static is the same as the one corresponding to an increase in the sunk cost, illustrated in Figure 3. In particular, consider the comparative statics of moving from a stationary equilibrium with a low rate

of monetary growth to an equilibrium with a high rate of monetary growth. For there to be an equilibrium with entry, firms' expected value of entry must increase. Since the rental cost of capital remains unchanged, firms are not willing to enter the industry unless the wage rate falls. Accordingly the WW locus has to shift to the left which translates into a movement along the SS curve. This in turn leads to a lower productivity threshold.

4 Calibration

In this section we describe the model calibration procedure. Since we consider different model specifications – corresponding to different values for θ_i, $i = c, k, h$ – the calibration of some parameters changes across specification. When this happens, we report the interval range of values taken by the parameter (see Table 1).

In order to solve our model we need to specify a distribution for the establishments' productivity draws $F(s)$. Following Helpman et al. (2004), we assume a Pareto distribution for F with lower bound s_0 and shape parameter $\varepsilon > \sigma$, i.e. $F(s) = 1 - \left(\frac{s_0}{s}\right)^{\varepsilon}$. The shape parameter is an index of the dispersion of productivity draws: dispersion decreases as ε increases, and the productivity draws are increasingly concentrated toward the lower bound s_0. This assumption has two advantages: it generates a distribution of idiosyncratic productivities among incumbent establishments that fits microeconomic data quite well[8] and delivers close-form solutions for the endogenous aggregates[9]. Specifically, the distribution of productivities among incumbent establishments, which is the distribution F truncated at $s = s^*$, is also Pareto with lower bound s^* and shape parameter ε.

We calibrate the model to data for the United States. The length of each period is one year. The growth rate of the money supply g is chosen to be 2.43 percent which matches the average annual rate of inflation in the U.S. between 1988 and 2007, reported in the World Economic Indicators database. For labor and capital income shares, α and ν respectively, empirical evidence concerning establishment level returns to scale, reported by Atkeson and Kehoe (2005) suggests the relation $\alpha + \nu = 0.85$. In particular, these authors consider this choice to be consistent with the evidence in Atkenson et al. (1996). The separate identification of α and ν is done according to the income shares of labor and capital. Based

[8]See Axtell (2001) and Cabral and Mata (2003).

[9]See the Appendix E for the complete description of the model solution.

Table 1: Parameters: summary

Notation	Value	Parameter
g	0.0243	Monetary growth rate
α	0.6095	Labor income share
ν	0.2405	Capital income share
δ	0.0956	Depreciation rate of capital
β	[0.9722 , 0.9775]	Household's discount factor
ε	7.2655	Pareto distribution shape parameter
λ	0.0696	Failure rate of incumbent establishments
s_0	1	Pareto distribution lower bound
κ	1	Sunk entry cost
η	[0.0103 , 0.0116]	Fixed operating cost
A	[2.3803 , 2.5472]	Disutility of labor

Note: The calibration of β, η and A varies according to the model specification and, in particular, according to the value taken by θ_c, θ_k and θ_c. Thus, we report the range of values taken by β, η and A.

on Gomme and Rupert (2007) we assign 28.3 percent to capital and the remainder to labor, yielding $\alpha = 0.6095$ and $\nu = 0.2405$.

The annual depreciation rate δ is chosen to be 9.56 percent based on evidence from the BEA as reported in Gomme and Rupert (2007). In particular, Gomme and Rupert (2007) distinguish between capital depreciation of market structures and capital depreciation of equipment and software. The 9.56 percent correspond to the weighted average of the depreciation rate of each component according to their share in GDP. Given the depreciation rate, the rental cost of capital r is chosen to match the investment-output ratio, given by $\frac{X}{Y} = \frac{\delta}{\nu} r$. Notice that the investment-output ratio is calculated with output net of government expenditure. The implied rental cost of capital is 12.42 percent. In turn, this implies $\beta = 0.9775$ when $\theta_k = 1$ and $\beta = 0.9722$ when $\theta_k = 0$.

Following Ghironi and Melitz (2005), we choose the shape parameter of the F distribution in order to match the standard deviation of log U. S. plant sales, which in our case is also output and is reported to be 1.67 in Bernard et al. (2003). Since in our model, this standard deviation is $\frac{1}{\varepsilon - \sigma}$, this implies that the value for ε is 7.27.

The establishments death rate λ is chosen based on empirical evidence reported in

Table 2: Calibration: targets

Target	Value
U.S.average annual inflation rate (1988-2007)	0.0243
Production function returns to scale	0.85
$\frac{\nu}{\nu+\alpha}$	0.283
Investment/GDP (net of government expenditure)	0.1851
Standard deviation of log U.S.plant sales	1.67
Manufacturing establishments (1-5 years old) failure rates	0.397
Manufacturing establishments (6-10 years old) failure rates	0.303
Hours-work (rate)	0.255

Dunne et al. (1989). These authors perform an empirical investigation of establishment turnover using data on plants that first began operating in the 1967, 1972, or 1977 Census of Manufacturers, a rich source of information concerning the U.S. manufacturing sector. They report five-year exit rates among plants aged 1-5 year old (39.7 percent), 6-11 year old (30.3 percent) and older (25.5 percent). As expected, plant failure rates decline with age. We assume entering establishments do not produce in the first year but simply discover their productivity level. Thereafter, establishments choosing not to exit the industry only exit when hit by the exogenous exit-shock. Thus, we decompose the five-year failure rate of young firms (1-5 years) into two components,

$$0.397 = F(s^*) + [1 - F(s^*)]\,\mathcal{B}_{4,1-\lambda}(3)\,, \tag{34}$$

where $\mathcal{B}_{4,1-\lambda}(3)$ the cumulative probability of 3 successes associated with the binomial distribution with 4 draws and success probability $1-\lambda$. The first term on the right-hand side of (34) is the probability of an establishment drawing a low productivity level and decide to exit. The second term is the probability of an incumbent establishment dieing over the four following years. This yields an equation in s^* and λ. The value for λ is set to match the failure rate of older incumbent firms (6-11 year old), by solving

$$0.303 = \mathcal{B}_{5,1-\lambda}(4)\,. \tag{35}$$

This yields $\lambda = 0.0696$. Equipped with λ we use equation (34) to find a relation between s^* and s_0. However, s_0 can be normalized to 1 without loss of generality because it has

no impact on the endogenous exit-decision of new establishments. This yields a solution for s^*. Only the ratio $\frac{\eta}{\kappa}$ is identifiable and, hence, we normalize the sunk cost, κ, to 1 and solve for the resulting fixed operating cost η. From condition (12) and (31) the resulting value of the fixed operating cost η ranges between 0.0103 and 0.0116, depending on the model specification.

Finally, A, the parameter measuring the disutility of labor, is chosen so that the household spends 25.5 percent of its endowment of time working, based on Gomme and Rupert (2007), who interpret evidence from the American Time-use Survey. Depending on the model specification, this yields a value for A ranging between 2.3803 and 2.5472.

This completes the calibration description. Table 1 summarizes the parameter values and Table 2 the targets informing our choices.

5 Results

We use the model economy just described to examine the interaction between money and the real sector of the economy. We first compare alternative steady states, describing how the macroeconomic aggregates, including output, consumption, investment and aggregate hours, and average productivity vary with respect to a benchmark level at various rates of money growth. We then use data from OECD countries on output and capital per worker to determine the model ability to explain cross-country evidence. Finally, we use the model to measure the welfare costs of anticipated inflation under alternative model specifications.

5.1 Steady-state properties

We choose the benchmark monetary growth rate to be 2.43 percent. This value corresponds to the average inflation rate in the U.S. between 1988 and 2007, a period of relatively low and stable inflation. Accordingly, Tables 3 and 4 report the log deviation of each macroeconomic aggregate of interest and of average productivity with respect to the levels corresponding to the benchmark steady state. We will begin by interpreting the results in each table.

Table 3 corresponds to model specifications where $\theta_h = 1$ and hence the marketing good is a *cash good*. The Table includes four Panels, each corresponding to an alternative configuration of the cash-in-advance constraint. When the cash-in-advance constraint applies

Table 3: Steady states associated with various annual monetary growth rates in log-deviation from benchmark when the marketing good is a *cash good*, i.e.: $\theta_h = 1$

	Panel A: $\theta_c = 1$ and $\theta_k = 1$					Panel B: $\theta_c = 1$ and $\theta_k = 0$				
$100 \times g$	$\beta - 1$	0.00	2.43*	10	12.43	$\beta - 1$	0	2.43*	10	12.43
Output	0.073	0.038	0.000	-0.112	-0.146	0.050	0.023	0.000	-0.070	-0.091
Consumption	0.060	0.031	0.000	-0.093	-0.122	0.050	0.023	0.000	-0.070	-0.091
Investment	0.120	0.062	0.000	-0.183	-0.239	0.050	0.023	0.000	-0.070	-0.091
Hours	0.044	0.023	0.000	-0.067	-0.088	0.039	0.018	0.000	-0.054	-0.070
Productivity	0.006	0.003	0.000	-0.010	-0.013	0.007	0.003	0.000	-0.010	-0.013
	Panel C: $\theta_c = 0$ and $\theta_k = 1$					Panel D: $\theta_c = 0$ and $\theta_k = 0$				
$100 \times g$	$\beta - 1$	0.00	2.43*	10	12.43	$\beta - 1$	0	2.43*	10	12.43
Output	0.039	0.020	0.000	-0.058	-0.075	0.012	0.005	0.000	-0.016	-0.021
Consumption	0.026	0.013	0.000	-0.040	-0.052	0.012	0.005	0.000	-0.016	-0.021
Investment	0.085	0.044	0.000	-0.129	-0.168	0.012	0.005	0.000	-0.016	-0.021
Hours	0.010	0.005	0.000	-0.014	-0.017	0.000	0.000	0.000	0.000	0.000
Productivity	0.006	0.003	0.000	-0.010	-0.013	0.007	0.003	0.000	-0.010	-0.013

Notes: * average U.S. inflation rate over the 1988-2007 period. The steady states are shown in log-deviation from the benchmark model which corresponds to the economy where the monetary growth rate is given by the U.S. average inflation rate.

to the creation of new establishments, the size distribution of productive establishments moves toward lower productivity levels at higher monetary growth rates. Hence, the average productivity of incumbent establishments is lower at high rates of inflation. The bottom row of each Panel of Table 3 reports the level of average productivity at various rates of money growth. When all goods are cash goods (Panel A) productivity falls by 1.3 percent when the rate of money growth is 12.43 percent, which is exactly 10 percentage points above the average U.S. rate of inflation. Instead, by moving from the benchmark monetary rule to the optimal money growth rule[10] ($g = \beta - 1$) productivity would increase in steady state by 0.6 percent. Inspecting each panel reveals that the money growth rule affects productivity in roughly the same way for each possible configuration of the cash-in-advance constraint as long as $\theta_h = 1$. Thus, the monetary growth rate has a robust impact on average productivity, which results directly from the fact that money holdings are a requirement for the creation of new establishments.

The results regarding the other macroeconomic aggregates are of course more sensitive

[10]In the Appendix C, we show that the optimal monetary policy is achieved in the stationary equilibirum when $g = \beta - 1$.

Table 4: Steady states associated with various annual monetary growth rates in log-deviation from benchmark when the marketing good is a *credit good*, i.e.: $\theta_h = 0$

	Panel A: $\theta_c = 1$ and $\theta_k = 1$					Panel B: $\theta_c = 1$ and $\theta_k = 0$				
$100 \times g$	$\beta - 1$	0.00	2.43*	10	12.43	$\beta - 1$	0	2.43*	10	12.43
Output	0.062	0.032	0.000	-0.096	-0.125	0.039	0.018	0.000	-0.054	-0.070
Consumption	0.050	0.026	0.000	-0.077	-0.102	0.039	0.018	0.000	-0.054	-0.070
Investment	0.109	0.056	0.000	-0.167	-0.218	0.039	0.018	0.000	-0.054	-0.070
Hours	0.044	0.023	0.000	-0.067	-0.088	0.039	0.018	0.000	-0.054	-0.070
Productivity	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Panel C: $\theta_c = 0$ and $\theta_k = 1$					Panel D: $\theta_c = 0$ and $\theta_k = 0$				
$100 \times g$	$\beta - 1$	0.00	2.43*	10	12.43	$\beta - 1$	0	2.43*	10	12.43
Output	0.028	0.014	0.000	-0.042	-0.054	0.000	0.000	0.000	0.000	0.000
Consumption	0.015	0.008	0.000	-0.024	-0.031	0.000	0.000	0.000	0.000	0.000
Investment	0.075	0.038	0.000	-0.113	-0.147	0.000	0.000	0.000	0.000	0.000
Hours	0.010	0.005	0.000	-0.014	-0.017	0.000	0.000	0.000	0.000	0.000
Productivity	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: * average U.S. inflation rate over the 1988-2007 period. The steady states are shown in log-deviation from the benchmark model which corresponds to the economy where the monetary growth rate is given by the U.S. average inflation rate.

to the model specification. Examining Panel A of Table 3 again reveals that, when all goods are *cash goods*, the change in the steady state levels of investment and output associated with the optimal money growth rule with respect to the benchmark money rule are 12 percent and 7.3 percent, respectively. These adjustments are less substantial when capital is a *credit good* but consumption is a *cash good* as shown in Panel B. Specifically, investment and output increases by 5 percent if the optimal rule is adopted, instead of the rule which mimics the average rate of inflation in the United States.

The results in Panel C suggest a prominent role for the cash-in-advance constraint on the investment good. In particular, the investment associated with optimal policy is 8.5 percent larger. Finally, Panel D is of interest because it illustrates that when the liquidity constraint only applies to the *marketing good* changes in the monetary growth rate have qualitatively the same effects although these are quantitatively small. This suggests that the cash-in-advance constraint may amplify the burden of inflation when it distorts the establishments' entry decision.

Table 4 corresponds to model specifications where $\theta_h = 0$ and hence the marketing good is a *cash good*. Examining each Panel and comparing it to the corresponding Panel in Table 3 indicates that, although the variations across money growth rates are of the

same order of magnitude, they are considerably smaller when the sunk cost is not subject to the liquidity constraint. This confirms the amplification role played by the distortion on establishments' entry. In particular, Panel B in Table 4 shows that when only consumption is a cash good, moving from the benchmark money rule to the optimal money rule increases consumption by just 3.9 percent. When instead the liquidity constraint applies to the sunk entry cost (Table 3) the impact is roughly 25 percent greater. Comparing Panels C from each Table reveals that the increase in consumption associated with the adoption of the optimal money growth rule when investment is a *cash good* and consumption is a *credit good* is 70 percent higher if the cash-in-advance constraint applies to the sunk entry cost. This illustrates clearly the gains from improvements in the allocation of the factors of production. Changes in the size distribution of incumbent establishments may therefore amplify the welfare cost of inflation because of the lost efficiency in the allocation of productive factors. Panel D in Table 4 simply illustrates that the cash-in-advance constraints are the single source of money non-neutrality.

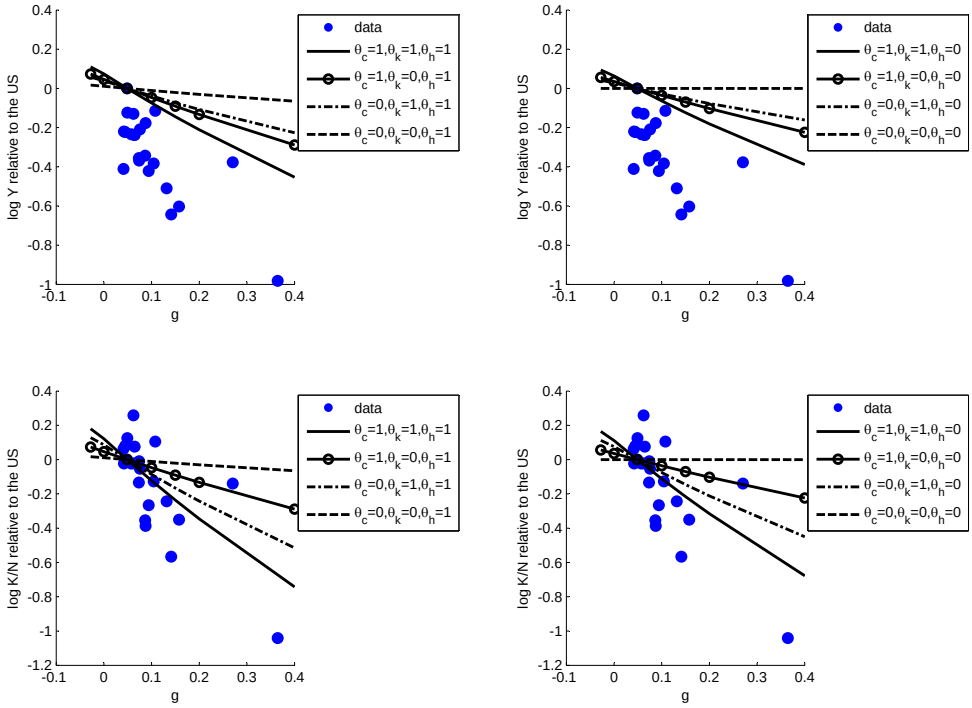
5.2 Model assessment

As just shown, our model predicts that anticipated inflation has a significant influence on the economy's steady-state. In particular, steady-state output and the capital stock fall as the growth rate of the money supply rises above the optimal level ($g = \beta - 1$). Moreover, when the cash-in-advance constraint applies to the sunk entry cost, average productivity is also predicted to fall as establishment entry decisions are distorted. Here we consider cross-section data on both output and capital per worker, and inflation rates for a sample of OECD countries[11]. The purpose is to illustrate to what extent our model is able to replicate the empirical relation between inflation and output, and inflation and capital per worker from a quantitative perspective.

Figure 4 depicts the relations between the logarithm of output per worker and inflation (upper Panels), and the logarithm of capital per worker and inflation (lower Panels), for

[11]The countries are Australia, Austria, Belgium, Canada, Denmark, Spain, Finland, France, United Kingdom, Greece, Ireland, Iceland, Italy, Japan, South Korea, Mexico, Netherlands, Norway, New Zealand, Portugal, Sweden and the United States. The inflation rate is average inflation between 1970 and 1996 and output per worker and capital per worker are 1996 aggregates.

Figure 4: Output and capital vs inflation rate across OECD countries and model fit



Note: The upper panels represent the relation between output per worker and inflation and the lower panels the relation between capital per worker and inflation in the semi-log scale. The left-hand side panels compares the data to the model specifications where $\theta_h = 1$ and the right-hand side panels compare it to model specifications where the $\theta_h = 0$. Data source: The real aggregates are frin Caselli (2005) and inflation rates are from the World Bank World Development Indicators.

different specifications of the cash-in-advance constraint. In our sample of OECD countries, the empirical correlation between inflation and the logarithm of output per worker is 0.76. The correlation between the logarithm of capital per worker and inflation is equally high. Of course, this strong correlation is not necessarily due to the causal effect of inflation on output and capital. Several causalities can justify such a negative relation. For instance, one may think that countries that are politically unstable are characterized by both low output per worker and high inflation rates. Another possible causality is encountered in Koreshkova (2006): low-income countries are characterized by a large informal sector, which can only be taxed by use of monetary policy (as opposed to fiscal policy), implying a negative correlation between income and inflation. However, it is worth addressing the question "how much of the negative relation between inflation and output per worker, and inflation and capital per worker our model is able to replicate?".

Inspection of Figure 4 suggests the model performs well when all cash-in-advance constraints apply. To give a more formal examination of this claim, we proceed as follows. With the use of ordinary least squares, we calculate the slope of a linear regression that describes the relationship between inflation and the logarithm of output per capita in our model when all cash-in-advance constraints apply, as well as its empirical counterpart. Then, we compute the ratio of the slope calculated from the simulated observations to the slope calculated from the data. This gives a quantitative assessment of the causal effect of inflation on output and the capital stock.

The regression coefficient on inflation corresponding to the linear projection of the logarithm of output per worker on inflation and an intercept is -2.10. Similarly, the regression coefficient on inflation corresponding to the linear projection of the logarithm of capital per worker on inflation and an intercept is -2.75. In turn, when the cash-in-advance constraint applies to all the three goods, the best linear fit of the relation between log output per worker and inflation implied by the model yields a coefficient on inflation of -1.32. The best linear fit of the relation between the capital-labor ratio and inflation implied by the model yields a coefficient on inflation of -2.17[12]. These estimates imply ratios that are respectively equal to 63 percent and 79 percent. These findings support the view that the causal mechanism from inflation to output and from inflation to the capital stock implied by our model explains a large share of the relation observed in the data.

5.3 Welfare costs of inflation

To obtain a measure of the welfare cost associated with inflation we proceed in the same way as in Cooley and Hansen (1989) with the single difference that we consider as a benchmark for the monetary growth rate the average rate of inflation for the U.S. instead of considering the optimal money rule. We do so, because it allows us to characterize a more immediate way what would be the benefit from adopting optimal policy and it also allows us to consider the welfare loss if inflation rates increased by 10 percentage points compared to what has

[12]These values are inside the corresponding 95 percent confidence intervals of the empirical relations. Specifically, the 95 percent confidence interval of the linear regression of log 'output per worker' on inflation is $[-2.94\,,\,-1.26]$ and the 95 percent confidence interval for the linear regression of log 'capital per worker' on inflation is $[-3.85\,,\,-1.64]$.

Table 5: Welfare costs associated with various annual growth rates of money

	$\theta_h = 1$				$\theta_h = 0$			
	$\theta_c = 1$	$\theta_c = 1$	$\theta_c = 0$	$\theta_c = 0$	$\theta_c = 1$	$\theta_c = 1$	$\theta_c = 0$	$\theta_c = 0$
$100\times$ g	$\theta_k = 1$	$\theta_k = 0$	$\theta_k = 1$	$\theta_k = 0$	$\theta_k = 1$	$\theta_k = 0$	$\theta_k = 1$	$\theta_k = 0$
$100\times\ (\beta - 1)$	2.18	1.66	1.77	1.19	1.10	0.45	0.70	0.00
0	1.14	0.77	0.91	0.54	0.59	0.23	0.37	0.00
2.43	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	-3.48	-2.33	-2.73	-1.60	-2.03	-0.85	-1.18	0.00
12.43	-4.56	-3.05	-3.57	-2.08	-2.72	-1.15	-1.57	0.00
15	-5.68	-3.80	-4.44	-2.58	-3.46	-1.49	-1.97	0.00
20	-7.78	-5.22	-6.08	-3.51	-4.91	-2.17	-2.77	0.00
40	-15.14	-10.31	-11.96	-6.81	-10.47	-5.07	-5.86	0.00

Note: * average U.S. inflation rate over the 1988-2007 period. The measure of the welfare cost of inflation is $\Delta C/C \times 100$ where ΔC is the consumption compensation needed for the representative agent to achieve the same steady state utility associated to the U.S. average rate of inflation.

been the average rate of inflation in the U.S. in the recent history[13].

To compute the welfare cost associated with variations in money growth around its benchmark value, we solve for ΔC in the equation

$$\bar{U} = \ln\left(C^* + \Delta C\right) + A \ln\left(1 - N^*\right), \tag{36}$$

where $\bar{U}$ is the level of utility attained under the benchmark rate of growth of money, $g = 2.43$, and C^* and N^* are the steady-state consumption and hours associated with the alternative money growth rule. The measure of the welfare cost of inflation used is the permanent percentage increase in consumption which makes the representative household as well of under the alternative regime as it is under the benchmark monetary policy[14]. The results of the welfare calculations are expressed as a percent of steady-state consumption ($\Delta C/C^*$), as in Cooley and Hansen (1989).

Table 5 shows our findings. The left-hand side Panel corresponds to the specifications where the cash-in-advance constraint applies to the entry sunk cost and the right-hand side Panel consider the other cases. The welfare costs of inflation we obtain are uniformly larger than the ones obtained by Cooley and Hansen (1989). Although, when the cash-in-

[13]We consider the average rate of inflation for the United States to be 2.43 percent. This corresponds to the average growth rate of prices as taken from the World Economic Indicators database in the sample period 1988-2007.

[14]The percentage increase as a fraction of income can be obtained simply by multiplying $\Delta C/C$ by the consumption/output ratio (≈ 0.815).

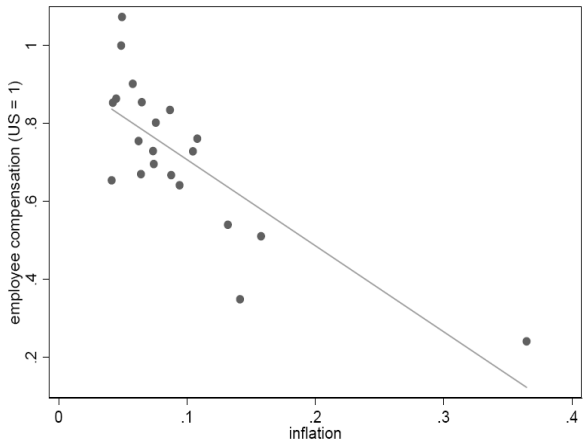
advance constraint does not apply to the sunk cost, they are roughly of the same order of magnitude.

We consider first the specification where consumption is the single *cash good* because this corresponds more closely to the Cooley and Hansen model. In this specification, the welfare cost of a 12.43 percent rate of inflation, relative to the benchmark of $g = 2.43$, is 1.15 percent of steady state consumption. The welfare gain associated with moving from the benchmark money growth rule to the optimal rule is 0.45 percent of steady state consumption. These numbers are roughly three times as large as the ones reported in Cooley and Hansen, even if average productivity is not distorted by monetary policy. When both consumption and investment are *cash goods* but the marketing good is a *credit good*, the welfare cost estimates roughly double. For example, the welfare gain associated with adopting the optimal policy becomes 1.10 percent of steady-state consumption. If only investment is a *cash good* the welfare gain is about 0.7 percent.

However, when the cash-in-advance constraint applies to the marketing good, the welfare costs of inflation increase substantially. For example, the welfare gain associated with adopting the optimal policy corresponds to 2.18 percent of steady-state consumption when all goods are cash goods (first cell of the left-hand side Panel). This number is about an order of magnitude greater than the findings in Cooley and Hansen. Moreover, is twice as large as the corresponding figure when only consumption and investment are *cash goods*, (1.10 percent). Thus, it seems that roughly 1/2 of the welfare cost of inflation is driven by the distortions to the firm entry decision. Therefore, a substantial part of the welfare losses at high rates of inflation are explained by less efficiency in the allocation of resources across incumbent establishments and not just by less accumulation of factors of production.

The wage rate is often a convenient measure of welfare. Figure 5 shows the relation in the data between labor compensation per employee in 1996 and the average rate of inflation from 1970 to 1996 for 21 OECD countries (the solid line depicts the best linear fit). Clearly there is a strong negative correlation between inflation rates and the labor compensation per employee. The correlation between these two variables is -0.79. Our model also predicts such a negative relation. For instance, when the cash-in-advance applies to the marketing good, inflation increases the effective cost of entry, which has an effect on the ability of establishments to pay wages since the expected value of entry has to increase in equilibrium. The decrease in wages in turn allows low-productivity establishments to survive, yielding a

Figure 5: Labor compensation per employee and inflation rates



Data: $US (PPP adjusted) Labor compensation per employee in 1996 – OECD source – and average inflation rate between 1970 and 1996 in OECD countries. The countries are Australia, Austria, Belgium, Canada, Denmark, Spain, Finland, France, United Kingdom, Greece, Ireland, Italy, Japan, South Korea, Mexico, Netherlands, Norway, New Zealand, Portugal, Sweden and the United States.

drop in average productivity. Figure 6 illustrates how small movements in productivity are associated with strong movements in the wage rate. Thus, even if high rates of inflation are associated with modest falls in average productivity, the welfare loss is important because the fall in the wage rate is strong. The movements in the wage rate are largely driven by the establishments' entry dynamics. Thus, having an endogenous distribution of productive establishments is important to characterize fully the welfare cost of inflation.

5.4 Sensitivity to alternative parameterizations

Atkeson et al. (1996) forcefully show that the choice of the returns to scale in models with industry dynamics is an important determinant of the size of the effect of policy distortions on average productivity and welfare[15]. Therefore, in this section we consider how sensitive our estimates of the welfare costs of inflation are to changes in the returns to scale. As expected, as $\alpha + \nu$ approaches one, productivity is no longer affected by changes in the monetary growth rate and the contribution of factors reallocation to the welfare cost of inflation disappears. However, this contribution increases at a high rate, as the intensity of

[15]Moreover, it should be noted that Atkeson et al. (1996) present evidence against the hypothesis that plant production or profit functions are nearly linear. This offers support to the view that policy distortions have sizable effects.

Figure 6: Wage rate and average productivity associated with various annual growth rates of money

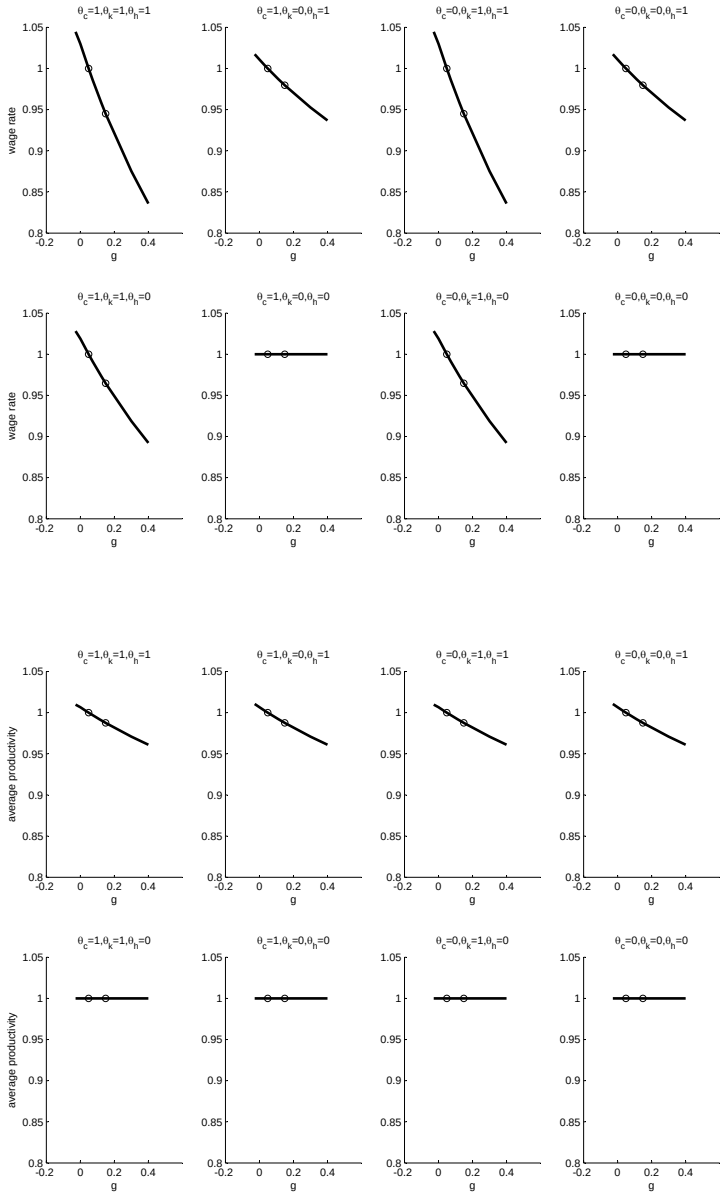


Table 6: Welfare costs corresponding to different degrees of diminishing returns to scale

		$\theta_h = 0$	$\theta_h = 1$	Share of welfare cost
$\alpha + \nu$	$100 \times \frac{\Delta \bar{s}}{\bar{s}}$	$100 \times \frac{\Delta C}{C}$	$100 \times \frac{\Delta C}{C}$	explained by fall in $\bar{s}$
0.75	-2.03	-2.17	-5.44	0.60
0.80	-1.66	-2.52	-5.04	0.50
0.85	-1.28	-2.72	-4.56	0.40
0.90	-0.89	-2.83	-4.03	0.30
0.95	-0.45	-2.87	-3.45	0.17
0.99	-0.09	-2.86	-2.98	0.04

Note: The measure of the welfare cost of inflation is $\Delta C/C \times 100$ where ΔC is the consumption compensation needed for the representative agent to achieve the same steady state utility associated to the U.S. average rate of inflation (2.43 percent), at a rate of inflation which is ten percentage points higher (12.43 percent). The average productivity $\bar{s}$ is written in log-deviation from the benchmark economy. For each model specification both consumption and investment are *cash goods*.

diminishing returns increases. Moreover, we also consider the sensitivity of our findings to changes in ε, the shape parameter of the distribution of productivity draws which controls the dispersion of incumbents' productivities.

Table 6 shows the average productivity associated to different degrees of diminishing returns to scale and the corresponding welfare cost of inflation for two different models specifications (when the marketing good is a *credit good* and when it is a *cash good*). For each model specification both consumption and investment are *cash goods*. This allows us to understand the role of productivity in explaining the welfare cost of inflation for different degrees of diminishing returns. The measure of welfare considered is the consumption compensation – as a fraction of steady state consumption under the benchmark monetary policy – needed for the representative agent to achieve the same steady state utility associated to the U.S. average rate of inflation (2.43 percent), at a rate of inflation which is ten percentage points higher (12.43 percent).

Naturally, when the returns to scale are nearly constant, $\alpha + \nu = 0.99$, the productivity is almost not affected at a higher rate of inflation. Indeed, average productivity is only 0.09 percent lower at 12.43 percent inflation, compared to what it would be at 2.43 percent inflation. Accordingly, the welfare costs of inflation are roughly the same, irrespectively of whether the cash-in-advance constraint applies to the marketing good or not. The last column of Table 6 shows how distortions to the size distribution of productive establish-

ments contribute to the welfare costs of inflation[16]. As expected, when the returns to scale are near constant this contribution is very small. However, this contribution increases fast, as the intensity of diminishing returns increases. Indeed, for the range of $\alpha + \nu$ between 0.75 and 0.90, which is likely to include the empirically relevant values, the contribution of distortions to the size distribution of incumbents is sizable, taking values between 30 and 60 percent of the total welfare cost of inflation.

As the intensity of diminishing returns increases, the share of welfare cost explained by a fall in average productivity increases (see the last column in Table 6). This happens for two reasons. First, as returns diminish faster, the distortions to the size distribution of establishments, resulting from the inflation tax, are more important and lead to significant falls in average productivity (see column 4). Thus, when the cash-in-advance constraint applies to the marketing good, i.e. $\theta_h = 1$, the welfare cost of inflation is high. However, an additional reason why the contribution of falls in average productivity to the welfare cost of inflation increases at lower values of $\alpha + \nu$ is that when the marketing good is a *credit good*, i.e. $\theta_h = 0$, the welfare cost of inflation increases as the intensity of diminishing returns to scale decreases (see column 3). This is because, when $\theta_h = 0$ the welfare cost is explained by the fall in the accumulation of factors. Thus, when $\alpha + \nu$ is low, the falls in output and welfare associated to the inflation tax are less important. Overall, for values of $\alpha + \nu$ which are empirically relevant, the contribution of distortions to the size distribution of productive establishments is substantial and the welfare costs of a 10 percentage points increase in the rate of inflation, when the cash-in-advance constraint applies to the creation of new establishments, vary between 4 and 5.5 percent of aggregate consumption.

The calibration of ε, the shape parameter of the productivity distribution $F(s)$, was based on empirical evidence, reported in Bernard et al., concerning the variability of sales across productive establishments. However, ε is admittedly a difficult parameter to choose. Thus, it is useful to examine how sensible are our findings to change in the target for the variability of firms output. Table 7 shows different welfare cost estimates as we vary the amount of establishment heterogeneity. As the dispersion of establishments' productivities increases, the fall in productivity associated to an increase in the rate of inflation, from

[16]We quantify this by computing the percentage increase in the welfare cost of inflation when the cash-in-advance constraint applies to the sunk entry cost.

Table 7: Welfare costs corresponding to different degrees of establishment heterogeneity

Volatility of output $\equiv \frac{1}{\epsilon-\sigma}$	$100 \times \frac{\Delta \bar{s}}{\bar{s}}$	$\theta_h = 0$ $100 \times \frac{\Delta C}{C}$	$\theta_h = 1$ $100 \times \frac{\Delta C}{C}$	Share of welfare cost explained by fall in $\bar{s}$
0.50	-1.07	-2.64	-4.19	0.36
1.00	-1.22	-2.70	-4.44	0.39
1.67	-1.28	-2.72	-4.56	0.40
2.00	-1.30	-2.73	-4.58	0.40
3.34	-1.34	-2.74	-4.66	0.41
5.00	-1.36	-2.75	-4.69	0.41

Note: The measure of the welfare cost of inflation is $\Delta C/C \times 100$ where ΔC is the consumption compensation needed for the representative agent to achieve the same steady state utility associated to the U.S. average rate of inflation (2.43 percent), at a rate of inflation which is ten percentage points higher (12.43 percent). The average productivity $\bar{s}$ is written in log-deviation from the benchmark economy. For each model specification both consumption and investment are *cash goods*.

2.43 percent to 12.43 percent, increases only marginally. Since the magnitudes of the productivity losses are essentially unaffected by changes in the productivity dispersion, it is not surprising to find that the magnitudes of the welfare estimates also do not change across alternative parameterizations. Therefore, we conclude that our findings are robust to changes in the variability of establishment productivity draws.

6 Conclusion

In this paper we set out to investigate whether it is important to model heterogeneity across productive establishments when quantifying the welfare cost of inflation. For this purpose, we studied a model characterized with cash-in-advance constraints on consumption and investment goods, and in addition we assume that cash-in-advance constraints also apply to the creation of new establishments. Besides discouraging investment and labor supply, an increase in the long-run rate of money growth increases the cost of creating new establishments and distorts firm entry and exit dynamics. As a result, incumbent establishments' profits must increase so as to encourage industry entry. This occurs through a fall in the equilibrium wage rate. As the equilibrium wage rate falls, less productive establishment choose to become incumbents, lowering average productivity. The adjustment in the productivity distribution of incumbent establishments is responsible for a substantial part of the welfare cost of inflation.

An important task is to investigate further the robustness of our findings. For instance, often some criticism focuses on the fact that the determination of goods as cash-purchased or credit purchased is exogenous; the consumer lacks the flexibility to optimally choose the fraction of purchases of any particular good which are payed with cash. Gillman (1993) addresses this issue within a cash-in-advance economy, by specifying an exchange function through which the consumer decides whether to use cash or costly credit to purchase a specific good. He finds that having the ability to switch to costly credit, increases the welfare costs in comparison to standard cash-in-advance economies. This is because avoiding the inflation tax means switching from fiat that uses no resources to exchange credit that uses up societal resources. Inflation acts through cash as a public tax with real proceeds returned in a lump sum fashion, while it acts through credit as a private societal tax with real proceeds destroyed. It would be interesting to study whether this result extends to our framework, with endogenous productivity. We leave this for future work.

As was mentioned earlier, Baily et al. (1992) document that about half of overall productivity growth in U.S. manufacturing in the 1980's can be attributed to factor reallocation from low productivity to high productivity establishments. It is tempting to imagine that the monetary policy tightening and resulting sustained disinflation which occurred over the same period may have contributed to the reallocation of factors and improvements in efficiency.

Appendix

A Locally vertical WW locus

The purpose of this section is to show that the WW locus is locally vertical. Hence, equilibrium wage rate w and s^* are independent. To do this, we apply the implicit function theorem to the relation (31) with the purpose of finding $\frac{dw}{ds^*}$. First, notice that the relation (31) can be re-written as

$$\kappa\left[1+\theta_h\left(\frac{1+g}{\beta}-1\right)\right]\frac{1-\beta\left(1-\lambda\right)}{\beta}+\left[1-F\left(s^*\right)\right]\eta-\frac{\Omega\int_{s^*}^{\infty}s^{\sigma}dF\left(s\right)}{w^{\alpha\sigma}r^{\nu\sigma}}=0, \tag{37}$$

which can simply be written as $\Phi\left(s^*,w\right)=0$. Moreover, by the implicit function theorem $\frac{dw}{ds^*}=-\frac{\partial\Phi(s^*,w)}{\partial s^*}\Big/\frac{\partial\Phi(s^*,w)}{\partial w}$.

Since

$$\frac{\partial\Phi\left(s^*,w\right)}{\partial w}=\frac{\alpha\sigma\Omega}{w^{1+\alpha\sigma}r^{\nu\sigma}}\int_{s^*}^{\infty}s^{\sigma}dF\left(s\right)>0,$$

a sufficient and necessary condition for $\frac{dw}{ds^*}=0$ is simply $\frac{\partial\Phi(s^*,w)}{\partial s^*}=0$. In turn

$$\frac{\partial\Phi\left(s^*,w\right)}{\partial s^*}=f\left(s^*\right)\left(\frac{\Omega s^{*\sigma}}{w^{\alpha\sigma}r^{\nu\sigma}}-\eta\right)=0,$$

because relation (32) implies that in equilibrium $\frac{\Omega s^{*\sigma}}{w^{\alpha\sigma}r^{\nu\sigma}}=\eta$. Therefore $\frac{dw}{ds^*}=0$ and the WW locus is locally vertical.

B Existence and uniqueness of equilibrium

This Section contains a proof that the relations (31) and (32) always define a unique equilibrium[17]. The condition (31) implies a relation for average profits, given by

$$\bar{z}=\kappa\left[1+\theta_h\left(\frac{1+g}{\beta}-1\right)\right]\frac{\frac{1}{\beta}-1+\lambda}{1-F\left(s^*\right)}. \tag{38}$$

In turn, combining the relations (12) and (32) implies that average profits must satisfy the equilibrium condition given by

$$\bar{z}=\eta\left[\left(\frac{\bar{s}}{s^*}\right)^{\sigma}-1\right]. \tag{39}$$

[17] A similar argument for proving existence and uniqueness of equilibrium in this class of heterogeneous firm models can be found in Melitz (2003).

Consequently, a sufficient condition for ensuring the existence and uniqueness of s^* is that

$$j(\hat{s}) = [1 - F(\hat{s})] \left[\left(\frac{\bar{s}(\hat{s})}{\hat{s}} \right)^{\sigma} - 1 \right]$$

be monotonically decreasing from infinity to zero on $(0, \infty)$, where

$$\bar{s}(\hat{s})^{\sigma} = \frac{1}{1 - F(\hat{s})} \int_{\hat{s}}^{\infty} s^{\sigma} dF(s).$$

Define

$$\iota(\hat{s}) = \left(\frac{\bar{s}(\hat{s})}{\hat{s}} \right)^{\sigma} - 1.$$

By applying the Chain and Leibniz rules, the derivative of $\iota(\hat{s})$ with respect to $\hat{s}$ is found to be

$$\begin{aligned} \iota'(\hat{s}) &= \frac{f(\hat{s})}{1 - F(\hat{s})} \left[\left(\frac{\bar{s}(\hat{s})}{\hat{s}} \right)^{\sigma} - 1 \right] - \frac{\sigma}{\hat{s}} \left(\frac{\bar{s}(\hat{s})}{\hat{s}} \right)^{\sigma}. & (40) \\ &= \frac{\iota(\hat{s}) f(\hat{s})}{1 - F(\hat{s})} - \frac{\sigma \iota(\hat{s}) + \sigma}{\hat{s}} & (41) \end{aligned}$$

Thus, the derivative and elasticity of $j(\hat{s})$ are given by

$$\begin{aligned} j'(\hat{s}) &= -\frac{\sigma}{\hat{s}} (\iota(\hat{s}) + 1) [1 - F(\hat{s})] < 0, & (42) \\ \frac{j'(\hat{s})\hat{s}}{j(\hat{s})} &= -\sigma \left(1 + \frac{1}{\iota(\hat{s})} \right) < -\sigma. & (43) \end{aligned}$$

Since $j(\hat{s})$ is non-negative and its elasticity with respect to $\hat{s}$ is strictly negative, $j\left(\hat{S}\right)$ must be decreasing to zero as $\hat{s}$ goes to infinity. Moreover, $\lim_{\hat{s} \to 0} j(\hat{s}) = \infty$ since $\lim_{\hat{s} \to 0} \iota(\hat{s}) = \infty$. Hence, $j(\hat{s})$ is monotonically decreasing from infinity to zero on $(0, \infty)$ as needed to be proved.

C Optimal monetary policy

Here we derive the optimal rate of inflation. The proof relies on the observation that the optimal inflation rate corresponds to the case where the cash-in-advance constraint is not binding. When the cash-in-advance constraint is not binding the corresponding Lagrange multiplier is zero, i.e. $\phi_t = 0$ for all t. To derive the optimal rate of inflation we start by noticing that Equation (22) can be rewritten as

$$\phi_{t+1} = \frac{\gamma_t}{\beta} \frac{p_{t+1}}{p_t} - \gamma_{t+1}. \quad (44)$$

Hence, $\phi_{t+1} = 0$ if and only if

$$\frac{\gamma_{t+1}}{\gamma_t} = \frac{p_{t+1}/p_t}{\beta} \tag{45}$$

Given that γ_t is constant in the stationary equilibrium and positive (from equation (15)), and the growth rate of money is equal to inflation in that equilibrium, it follows that the Friedman rule applies to the stationary equilibrium of our model, that is, the optimal rate of inflation is equal to $(\beta - 1)$.

D Proof of Proposition 1

Following is a proof of Proposition 1. The case where all θ_i's are zero is trivial. In the next subsections, we analyze in more details the effect of anticipated inflation when one of the θ_i's takes value one.

D.1 Case where $\theta_c = 1$, $\theta_k = 0$ and $\theta_h = 0$

We consider first the case where $\theta_c = 1$, $\theta_k = 0$ and $\theta_h = 0$. Notice that in this context inflation does not affect the rental cost of capital in (30), nor the productivity threshold and the wage rate in (31) and (32). From (4), (6), (7) and (8), this implies that average output, employment, capital use and profits are also not affected by inflation.

To determine the effect of inflation on the other aggregates, notice that in the stationary equilibrium $X = \delta K = \delta \bar{k} H$, $\kappa E = \kappa \frac{\lambda}{1-F(s^*)} H$ and $Y = \bar{y} H$. Replace those equations and (29) in (28) to get:

$$\frac{Lw}{A\left[1 + \theta_c\left(\frac{1+g}{\beta} - 1\right)\right]} + \delta \bar{k} H + \kappa \frac{\lambda}{1 - F(s^*)} H = \bar{y} H \tag{46}$$

Given the labor-market clearing condition, we can write $L = 1 - N = 1 - \bar{n} H$. Replacing this relation in the above equation and rearranging terms leads:

$$H = \frac{w}{A\left[1 + \theta_c\left(\frac{1+g}{\beta} - 1\right)\right]} \left(\bar{y} - \delta \bar{k} - \kappa \frac{\lambda}{1 - F(s^*)} + \frac{w \bar{n}}{A\left[1 + \theta_c\left(\frac{1+g}{\beta} - 1\right)\right]} \right)^{-1} \tag{47}$$

Equation(47) shows that when $\theta_c = 1$, an increase in the anticipated rate of inflation g decreases the mass of incumbent firms H. Given that average employment, capital and output are not affected, this implies that an increase in the anticipated rate of inflation g also decreases the aggregate level of capital, employment and output.

D.2 Case where $\theta_c = 0$, $\theta_k = 1$ and $\theta_h = 0$

When $\theta_k = 1$, equation (30) shows that an increase in g increases the rental cost of capital r.

To determine the effect of inflation on the productivity threshold and the wage rate in this context we use condition (39). Replacing this relation in the free-entry condition (31), we then have

$$\kappa \left[1 + \theta_h \left(\frac{1+g}{\beta} - 1\right)\right] = [1 - F(s^*)] \frac{\beta}{1 - \beta(1-\lambda)} \eta \left[\left(\frac{\bar{s}}{s^*}\right)^{\sigma} - 1\right]. \tag{48}$$

Hence, the productivity threshold does not depend on the rental cost of capital. Following an increase in g, the negative effect of the increase in r on profits cancels out with the positive effect of a decrease in wages. This latter can be seen from equations (30), (32) and (48).

Regarding the effect of inflation on average output per establishment, remark that, from equations (4), (6) and (7), average output can be written as

$$\bar{y} = \bar{s}^{\sigma} \left(\frac{\alpha}{w}\right)^{\alpha\sigma} \left(\frac{\nu}{r}\right)^{\nu\sigma}. \tag{49}$$

By replacing (32) in the above equation, one gets

$$\bar{y} = \frac{\eta}{\Omega} \left(\frac{\bar{s}}{s^*}\right)^{\sigma} \alpha^{\alpha\sigma} \nu^{\nu\sigma}. \tag{50}$$

Hence inflation does not affect average output.

To determine the impact on average capital and employment, notice from (6) and (7) and the fact that the productivity threshold is not affected by inflation that

$$d \ln \bar{n} = -(1-\nu)\sigma d \ln w - \nu\sigma d \ln r \tag{51}$$

$$d \ln \bar{k} = -\alpha\sigma d \ln w - (1-\alpha)\sigma d \ln r \tag{52}$$

Given that

$$\alpha d \ln w = -\nu d \ln r \tag{53}$$

from equation (32) and the fact that s^* is not affected by inflation, this set of equations can be rewritten as

$$d \ln \bar{n} = \frac{\nu}{\alpha} d \ln r \tag{54}$$

$$d \ln \bar{k} = -d \ln r \tag{55}$$

Thus an increase in inflation increases the average level of employment per establishment, while it decreases average capital use.

Equation (47) is still valid if the cash-in-advance constraint only applies to investment. Consequently, if inflation increases average employment, decreases the wage rate and average capital and does not affect average output and the productivity threshold, then it decreases the mass of incumbent establishments from equation (47). Hence, aggregate output and stock of capital decrease too. But, the effect on aggregate employment is a priori ambiguous given that H decreases and $\bar{n}$ increases. To show that the effect on aggregate employment is actually negative, first notice that

$$d\ln N = d\ln \bar{n} + d\ln H. \tag{56}$$

Next, from equation (47), observe that

$$d\ln H = d\ln w - Nd\ln w - Nd\ln \bar{n} + \frac{\delta KA\left(1+\theta_c\left(\frac{1+g}{\beta}-1\right)\right)}{w} d\ln \bar{k}. \tag{57}$$

Replacing the above equation and (53) and (54) in (57)

$$d\ln N = \frac{\delta KA\left(1+\theta_c\left(\frac{1+g}{\beta}-1\right)\right)}{w} d\ln \bar{k}. \tag{58}$$

Thus, aggregate employment decreases following an increase in inflation.

D.3 Case where $\theta_c = 0$, $\theta_k = 0$ and $\theta_h = 1$

Here the rental cost of capital is not affected by inflation (see equation (30)).

To understand the effect on the productivity threshold and the wage rate, combine (8) and (32) with (31) to get

$$\kappa\left[1+\theta_h\left(\frac{1+g}{\beta}-1\right)\right] = \left[1-F(s^*)\right]\frac{\beta}{1-\beta(1-\lambda)}\eta\left[\left(\frac{\bar{s}}{s^*}\right)^{\sigma}-1\right]. \tag{59}$$

Hence an increase in inflation decreases the productivity threshold s^*.

From equation (32) it follows that the wage rate decreases too.

From (50), average output increases given s^* decreases as

$$d\ln \bar{y} = \sigma\left[d\ln \bar{s} - d\ln s^*\right]. \tag{60}$$

To determine the effect on average employment and capital, notice from (32) that

$$d\ln s^* = \alpha d\ln w. \tag{61}$$

By replacing the above equation in (6) and (7), we have

$$d\ln \bar{n} = \sigma\left[d\ln \bar{s} - \frac{1-\nu}{\alpha} d\ln s^*\right] \tag{62}$$

$$d\ln \bar{k} = \sigma\left[d\ln \bar{s} - d\ln s^*\right] \tag{63}$$

Hence, average capital increases following an increase in the rate of money growth and the impact of inflation on average employment is ambiguous.

We now investigate the effect of g on H. Observe that we have from (47) that

$$d\ln H = d\ln w - \frac{AY}{w} d\ln \bar{y} + \frac{AX}{w} d\ln \bar{k} - N d\ln w - N d\ln \bar{n} + \frac{AE\kappa}{w}\frac{f(s^*)s^*}{1-F(s^*)} d\ln s^*. \tag{64}$$

The above equation can be rewritten as

$$\begin{aligned} d\ln H = &\left\{\frac{AX\sigma}{w} - \frac{AY\sigma}{w} - N\sigma\right\} d\ln \bar{s} \\ &+ \left\{\frac{1-N}{\alpha} + \frac{N\sigma(1-\nu)}{\alpha} + \frac{AY\sigma}{w} - \frac{AX\sigma}{w} + \frac{AE\kappa}{w}\frac{f(s^*)s^*}{1-F(s^*)}\right\} d\ln s^*. \end{aligned}$$

Given $d\ln \bar{s} \leq d\ln s^*$, $Y \geq X$ and $\frac{1-N}{\alpha} + \frac{N\sigma(1-\nu)}{\alpha} > N\sigma$, it follows the mass of incumbents H decreases as a result of an increase in g.

The impact on aggregate employment is given by

$$\begin{aligned} d\ln N = &\left\{\frac{AX\sigma}{w} - \frac{AY\sigma}{w} + (1-N)\sigma\right\} d\ln \bar{s} \\ &+ \left\{\frac{AY\sigma}{w} - \frac{AX\sigma}{w} + \frac{AE\kappa}{w}\frac{f(s^*)s^*}{1-F(s^*)} - (1-N)\sigma\right\} d\ln s^*. \end{aligned}$$

By use of (28) and (29), this equation simplifies as

$$\begin{aligned} d\ln N = &\left\{\frac{AC\sigma}{w}\theta_c\left(\frac{1+g}{\beta} - 1\right) - \frac{AE\kappa\sigma}{w}\right\} d\ln \bar{s} \\ &+ \left\{\frac{AE\kappa}{w}\frac{f(s^*)s^*}{1-F(s^*)} - \frac{AC\sigma}{w}\theta_c\left(\frac{1+g}{\beta} - 1\right) + \frac{AE\kappa\sigma}{w}\right\} d\ln s^*. \end{aligned}$$

Hence, aggregate employment decreases following an increase in g if $\theta_c = 0$.

Notice that, from (60) and (63), the effect on average capital and average output are the same. Hence, to determine the effect on aggregate output and capital, it is sufficient

to know only one of the two effects given that they are the same. We choose to determine the effect on aggregate output:

$$d\ln Y = d\ln \bar{y} + d\ln H \tag{65}$$

This equation can be rewritten as

$$\begin{aligned} d\ln Y = &\left\{ \frac{AX\sigma}{w} - \frac{AY\sigma}{w} + (1-N)\sigma \right\} d\ln \bar{s} \\ &+ \left\{ \frac{AY\sigma}{w} - \frac{AX\sigma}{w} + \frac{AE\kappa}{w}\frac{f(s^*)s^*}{1-F(s^*)} - (1-N)\sigma + \frac{1}{\alpha} \right\} d\ln s^*. \end{aligned}$$

Given the discussion regarding the effect of g on N, by the same arguments, it follows that the effect of g on Y and K is negative as well.

E Solutions

$$r = \left(\frac{1}{\beta} - 1 + \delta\right)\left[1 + \theta_k\left(\frac{1+g}{\beta} - 1\right)\right]$$

$$w = \left(\frac{\beta\sigma/(\varepsilon-\sigma)}{\kappa\left[1+\theta_h\left(\frac{1+g}{\beta}-1\right)\right][1-\beta(1-\lambda)]}\right)^{\frac{1}{\alpha\varepsilon}} \left(\frac{s_0\Omega^{\frac{1}{\sigma}}\eta^{\frac{\sigma-\varepsilon}{\sigma\varepsilon}}}{r^{\nu}}\right)^{\frac{1}{\alpha}}$$

$$s^* = \left(\frac{\beta}{1-\beta(1-\lambda)}\frac{\sigma}{\varepsilon-\sigma}\frac{\eta}{\kappa}\frac{1}{1+\theta_h\left(\frac{1+g}{\beta}-1\right)}\right)^{\frac{1}{\varepsilon}} s_0$$

$$\bar{s} = \left(\frac{\varepsilon}{\varepsilon-\sigma}\right)^{1/\sigma} s^*$$

$$\bar{k} = \frac{\varepsilon}{\varepsilon-\sigma}\left(\frac{\alpha}{w}\right)^{\alpha\sigma}\left(\frac{\nu}{r}\right)^{(1-\alpha)\sigma} s^{*\sigma}$$

$$\bar{n} = \frac{\varepsilon}{\varepsilon-\sigma}\left(\frac{\alpha}{w}\right)^{(1-\nu)\sigma}\left(\frac{\nu}{r}\right)^{\nu\sigma} s^{*\sigma}$$

$$\bar{y} = \frac{\varepsilon}{\varepsilon-\sigma}\left(\frac{\alpha}{w}\right)^{\alpha\sigma}\left(\frac{\nu}{r}\right)^{\nu\sigma} s^{*\sigma} - \eta$$

$$\bar{z} = \Omega \frac{\varepsilon}{\varepsilon - \sigma} \frac{s^{*\sigma}}{w^{\alpha\sigma} r^{\nu\sigma}} - \eta$$

$$H = \frac{w}{A\left[1 + \theta_c(\frac{1+g}{\beta} - 1)\right]} \left(\bar{y} - \delta\bar{k} - \kappa\lambda \left(\frac{s^*}{s_0}\right)^{\varepsilon} + \frac{w\bar{n}}{A\left[1 + \theta_c(\frac{1+g}{\beta} - 1)\right]} \right)^{-1}$$

$$E = \frac{\lambda}{(s_0/s^*)^{\varepsilon}} H$$

$$K = H\bar{k}$$

$$X = \delta K$$

$$N = H\bar{n}$$

$$C = \frac{(1-N)w}{A\left[1 + \theta_c(\frac{1+g}{\beta} - 1)\right]}$$

$$Y = H\bar{y}$$

References

[1] A. B. Abel, *Dynamic behavior of capital accumulation in a cash-in-advance model*, Journal of Monetary Economics, 16 (1985), pp. 55–71.

[2] L. Alfaro, A. Charlton, and F. Kanczuk, *Plant-size distribution and cross-country income differences*, in NBER International Seminar on Macroeconomics 2008.

[3] A. Atkenson, A. Khan, and L. Ohanian, *Are data on industry evolution and gross job turnover relevant for macroeconomics?*, Carnegie-Rochester Conference Series on Public Policy, 44 (1996), pp. 215–239.

[4] A. Atkeson and P. J. Kehoe, *Modeling and measuring organization capital*, Journal of Political Economy, 113 (2005), pp. 1026–1053.

[5] R. L. Axtell, *Zipf distribution of u.s. firm sizes*, Science, 293 (2001), pp. 1818–1820.

[6] M. N. Baily, C. Hulten, D. Campbell, T. Bresnahan, and R. E. Caves, *Productivity dynamics in manufacturing plants*, Brookings Papers on Economic Activity. Microeconomics, 1992 (1992), pp. 187–267.

[7] E. J. Bartelsman and M. Doms, *Understanding productivity: Lessons from longitudinal microdata*, Journal of Economic Literature, 38 (2000), pp. 569–594.

[8] A. B. Bernard, J. Eaton, J. B. Jensen, and S. Kortum, *Plants and productivity in international trade*, American Economic Review, 93 (2003), pp. 1268–1290.

[9] L. M. B. Cabral and J. Mata, *On the evolution of the firm size distribution: Facts and theory*, American Economic Review, 93 (2003), pp. 1075–1090.

[10] F. Caselli, *Accounting for cross-country income differences*, in Handbook of Economic Growth, P. Aghion and S. Durlauf, eds.

[11] T. F. Cooley and G. D. Hansen, *The inflation tax in a real business cycle model*, American Economic Review, 79 (1989), pp. 733–48.

[12] J. De Gregorio, *Inflation, taxation, and long-run growth*, Journal of Monetary Economics, 31 (1993), pp. 271–298.

[13] M. Dotsey and P. Ireland, *The welfare cost of inflation in general equilibrium*, Journal of Monetary Economics, 37 (1996), pp. 29–47.

[14] T. Dunne, M. J. Roberts, and L. Samuelson, *The growth and failure of u.s. manufacturing plants*, The Quarterly Journal of Economics, 104 (1989), pp. 671–98.

[15] L. Foster, J. Haltiwanger, and C. Syverson, *Reallocation, firm turnover, and efficiency: Selection on productivity or profitability?*, American Economic Review, 98 (2008), pp. 394–425.

[16] F. Ghironi and M. J. Melitz, *International trade and macroeconomic dynamics with heterogeneous firms*, The Quarterly Journal of Economics, 120 (2005), pp. 865–915.

[17] M. Gillman, *The welfare cost of inflation in a cash-in-advance economy with costly credit*, Journal of Monetary Economics, 31 (1993), pp. 97–115.

[18] P. Gomme, *Money and growth revisited : Measuring the costs of inflation in an endogenous growth model*, Journal of Monetary Economics, 32 (1993), pp. 51–77.

[19] P. Gomme and P. Rupert, *Theory, measurement and calibration of macroeconomic models*, Journal of Monetary Economics, 54 (2007), pp. 460–497.

[20] N. Guner, G. Ventura, and X. Yi, *Macroeconomic implications of size-dependent policies*, Review of Economic Dynamics, 11 (2008), pp. 721–744.

[21] R. E. Hall and C. I. Jones, *Why do some countries produce so much more output per worker than others?*, The Quarterly Journal of Economics, 114 (1999), pp. 83–116.

[22] E. Helpman, M. J. Melitz, and S. R. Yeaple, *Export versus fdi with heterogeneous firms*, American Economic Review, 94 (2004), pp. 300–316.

[23] H. Hopenhayn and R. Rogerson, *Job turnover and policy evaluation: A general equilibrium analysis*, Journal of Political Economy, 101 (1993), pp. 915–38.

[24] H. A. Hopenhayn, *Entry, exit, and firm dynamics in long run equilibrium*, Econometrica, 60 (1992), pp. 1127–50.

[25] L. E. Jones and R. E. Manuelli, *Growth and the effects of inflation*, Journal of Economic Dynamics and Control, 19 (1995), pp. 1405–1428.

[26] R. G. King and R. Levine, *Capital fundamentalism, economic development, and economic growth*, Carnegie-Rochester Conference Series on Public Policy, 40 (1994), pp. 259–292.

[27] P. Klenow and A. R.-C. 1997, *The neoclassical revival in growth economics: Has it gone too far?*, in NBER Macroeconomics Annual 1997, Volume 12.

[28] T. A. Koreshkova, *A quantitative analysis of inflation as a tax on the underground economy*, Journal of Monetary Economics, 53 (2006), pp. 773–796.

[29] R. Lagos and G. Rocheteau, *Inflation, output, and welfare*, International Economic Review, 46 (2005), pp. 495–522.

[30] R. E. Lucas, *Inflation and welfare*, Econometrica, 68 (2000), pp. 247–274.

[31] R. E. Lucas and N. L. Stokey, *Money and interest in a cash-in-advance economy*, Econometrica, 55 (1987), pp. 491–513.

[32] M. J. Melitz, *The impact of trade on intra-industry reallocations and aggregate industry productivity*, Econometrica, 71 (2003), pp. 1695–1725.

[33] E. C. Prescott, *Needed: A theory of total factor productivity*, International Economic Review, 39 (1998), pp. 525–51.

[34] D. Restuccia and R. Rogerson, *Policy distortions and aggregate productivity with heterogeneous plants*, Review of Economic Dynamics, 11 (2008), pp. 707–720.

[35] G. Rocheteau and R. Wright, *Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium*, Econometrica, 73 (2005), pp. 175–202.

[36] R. M. Samaniego, *Industrial subsidies and technology adoption in general equilibrium*, Journal of Economic Dynamics and Control, 30 (2006), pp. 1589–1614.

[37] A. C. Stockman, *Anticipated inflation and the capital stock in a cash in-advance economy*, Journal of Monetary Economics, 8 (1981), pp. 387–393.

[38] M. Veracierto, *Employment flows, capital mobility, and policy analysis*, International Economic Review, 42 (2001), pp. 571–95.

[39] Y. Wu and J. Zhang, *The effects of inflation on the number of firms and firm size*, Journal of Money, Credit and Banking, 33 (2001), pp. 251–71.