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HYSTERESIS AND ASSET FIXITY UNDER UNCERTAINTY

by

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Abstract

The fixed asset theory, when viewed as an investment/disinvestment theory, implies a simple two-parameter control-limit decision rule. The theory is extended to incorporate the stochastic nature of farm revenue. The results show that ongoing uncertainty leads to a widening of the range of inaction because there is a positive value of waiting. The effects of sunk costs, or the divergence of acquisition costs from salvage values, on the degree of investment/disinvestment irreversibility become more pronounced when uncertainty is present.

Key words: asset fixity, sunk cost, uncertainty, acquisition costs, salvage values, hysteresis, stochastic optimal control.

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I. Introduction

Much of the economic literature concerns the nature of "hysteresis" – a band of inactivity and caution before reorienting assets in response to relative price changes. For example, in production economics the classic theory of asset fixity proposed by Joseph W. Willet, Glenn L. Johnson, Clark Edwards and others maintains that an asset is fixed "if it ain't worth varying". Essential to this fixed asset theory is the serious recognition of the divergence of acquisition costs from salvage values in the factor market. Emphasis is placed on the operational definition of fixed assets. According to this theory, if an asset's "value in use" is bounded by off-farm opportunities for acquisition and salvage, the asset is regarded as fixed. Asset fixity is then used to explain the inelastic segment of the product supply response curve.

An argument very similar to asset fixity is developed in labor economics, but emphasis is much more put on a firm's investment/disinvestment nature of decision making. For example, Gary S. Becker considers a firm's different decisions to lay off or hire employees with and without specific training. Becker states that "For sunk costs are sunk, and there is no incentive to lay off employees whose marginal product is greater than wages, no matter how unwise it was, in retrospect, to invest in their training. Thus workers with specific training seem less likely to be laid off as a consequence of a decline in demand than are untrained or even generally trained workers" (1962, p.22).

In an article entitled "Labor as a Quasi-Fixed Factor" Walter Y. Oi formalizes this idea with a model incorporating the divergence of labor's marginal value product from the wage rate. He argues that sunk costs, e.g., hiring costs and on-the-job training expenses, drive a wedge between the marginal value product and the wage rate and form a buffer absorbing short-run fluctuations in a factor's marginal value product. Since the relative magnitude of this wage, measured by the degree of fixity, differs among occupations or grades of labor, short-run changes in product demands thus lead to differential shifts in labor demands.

One obvious deficiency of fixed asset theory is its total neglect of the stochastic nature of output prices or revenue. That is, an once-and-for-all price change is generally assumed. However, both Becker and Oi move one step further to consider the effects of uncertainty on a firm's decision to lay off or hire workers and provide an insightful explanation of the short-run behavior of labor markets. They argue that there is an incentive not to lay off workers with specific training when their marginal product is only *temporarily* below wages, and the larger a firm's investment the greater the incentive not to lay off such workers. "Thus both the quit and layoff rate of specifically trained workers would be relatively low and fluctuate relatively less during business cycles" (Becker 1962, p.23).

Recently the theme on investment inertia, hysteresis or asset fixity in an environment of ongoing uncertainty is carried forward by Robert McDonald and Daniel Siegel (1986), Avinash Dixit (1989a, 1989b). The main idea comes from the theory of option pricing in financial economics. According to this theory, an asset which is presently laid up can be regarded as an option on becoming active by incurring the exercise price, namely the sunk cost of investment. Likewise, an asset which is in operation is an option on becoming idle by paying any lay-up cost. Solving these two linked option pricing problems yields the values of the two assets, and the rules for exercising the options, namely the output prices that trigger investment and disinvestment.

The main purpose of this paper is to highlight the investment/disinvestment nature of asset fixity and investigate the possible effect of uncertainty on the degree of an asset's fixity. The next section shows that embedded in the fixed asset theory is a simple two-parameter control-limit decision rule. The third section models the farmer's decision whether to invest or disinvest as an optimal stochastic control problem. The fourth section provides some qualitative results. The last section provides a summary and some concluding comments.

II. The "Simple" Decision Rule

In a historical review Alan E. Baquet has indicated that the theory of asset fixity is actually a theory of investment and disinvestment. This point has been further emphasized by Glenn L. Johnson in his concluding paragraph.

> If I were rewriting *The Overproduction Trap* and related literature, I would retitle what has become known as "asset fixity theory" as "investment/disinvestment theory." This would be sensible since investment and disinvestment, as the inverse of asset fixity, always have been implied by the theory. This is perhaps more important than fixity. (1982, p. 775)

When viewed as an investment/disinvestment theory, the decision rule embedded in the fixed asset theory is equivalent to the two-parameter control-limit policy in the theory of inventory (see Karlin 1958, and Scarf 1959). For concreteness, let us consider the example in Edwards although his objective is to know how much a 1954 Ford pickup truck is worth to its farmer owner and to provide an operational definition of an fixed asset using a theory of valuation. Assume the farmer can buy an identical truck as that already on farm for $500 from the used truck market. Moreover, he/she finds that the dealer who retails the Ford pickup for $500 buys it wholesale for $375.

Suppose using the services of the truck adds $600 per year to farm revenue. However, it costs $480 per year for gas, oil, and repairs to use the truck. It costs an additional $50 per year for licenses and insurance to make the truck available for use. Moreover, it is assumed that the truck has five years of remaining useful life and a junk value of $100 at the end of that time. Given these data, Edwards computes the acquisition cost and salvage value, denoted as X_H and X_L, respectively, as follows:

$$X_H = 480 + 50 + \frac{1}{5}(500 - 100) = 610,$$
$$X_L = 480 + 50 + \frac{1}{5}(375 - 100) = 585.$$

Since the current value in use $X(t) = \$600$ is bounded by off-farm opportunity costs for acquiring or disposing of similar truck, the pickup truck in the example is regarded as fixed. That is, if the truck were currently owned by the farmer, he/she would not be willing to buy another identical one because the acquisition cost $X_H = \$610$ is higher than use value $X(t) = \$600$. Neither would he/she salvage the truck because the truck is worth more in use $X(t) = \$600$ on the farm than its salvage value $X_L = \$585$. On the other hand, if the farmer did not own any truck right now, he/she would not be willing to have one for $X(t) < X_H$ at this moment. Therefore, when $X_L < X(t) < X_H$, maintaining the *status quo* is the optimal policy.

The problem of deciding if or when the truck should be sold is then the problem of determining how far revenue $X(t)$ should be allowed to fall before sale is effected. Similarly, if the farm is operated in the absence of the truck, the problem of deciding if or when it should be purchased is the problem of determining how far revenue $X(t)$ should rise before the truck is purchased. As a matter of fact, these upper and lower "critical" values of revenue are the acquisition cost X_H and salvage value X_L, respectively. When the revenue $X(t)$ is assumed to follow a stationary stochastic process as that in the model to be presented later, these critical values must evidently remain fixed over time, i.e., if it was once optimal to salvage when revenue fell to X_L, then it must also be optimal any other time revenue falls to X_L (assuming, of course, that also the various cost parameters involved do not change). What this implies, then, is that the decision rule is to be specified as follows:

> If revenue $X(t)$ rises to some level of acquisition cost X_H the farmer should buy the truck; if revenue $X(t)$ falls to some level of salvage value X_L the farmer should salvage the truck (if he/she already has one on farm).

Such a decision rule is referred to as a two-parameter control-limit policy or two-bin inventory policy which is the simplest and most natural. George M. Constantinides calls it a *simple* investment policy. In a discrete-time version of the proportional transaction costs model, he proves that the optimal investment policy is simple. In the continuous-time framework, Taksar, Klass, and Assaf (1983) obtian the same result.

In the following section we generalize the Edwards' pickup truck example. Assume that the cost of operating the truck is C per unit of time which is kept constant for simplicity. The interest rate is r. The decision to purchase the truck incurs a lump sum cost k^+. Likewise, the decision to salvage incurs a cost k^-. We may call k^+, k^- sunk costs or asymmetric adjustment costs if $k^+ \neq k^-$. According to the fixed asset theory where the revenue $X(t)$ is deterministic, for the continuous-time case we have $X_H = C + rk^+$, $X_L = C - rk^-$, and the gap between acquision cost and salvage value is $r(k^+ + k^-)$. The simple decision rule is that don't buy the truck until the revenue is high enough such that $X(t) > C + rk^+$, and don't salvage the truck until the revenue decreases such that $X(t) < C - rk^-$.

With this general picture in mind we can now turn to a more elaborate version of asset fixity in the presence of uncertainty. We begin with an optimal stochastic control model.

III. The Model

Suppose using the services of an additional asset adds revenue $X(t)$ to farm income. For expository simplicity, factors other than this truck are constant and kept in the background. Assume that the state $X(t)$ satisfies the stochastic differential equation (taken in the Itô sense)

$$dX = \beta X \, dt + X \, dW, \tag{1}$$

where W is a normalized Weiner process and dW is an increment of Brownian motion (random walk in continuous time). That is, dW is a normalized distributed random variable whose distribution is independent of the past, which satisfies

(2) $$\mathcal{E}(dW) = 0, \qquad \mathcal{E}(dW^2) = \sigma^2 dt,$$

where $\mathcal{E}$ denotes the expectation operator. Let $\lambda^-(t)$ denote the left-hand limit of $\lambda(t)$ at the instant t. That is, $\lambda^-(t) = \lim_{s\to t, s<t} \lambda(s)$. The function $\lambda(t)$ can jump at t and $\lambda^-(t)$ may no equal $\lambda(t)$. The farmer is assumed to follow the "policy of simple form" as that discussed in previous section:

(3) $$\lambda(t) = \begin{cases} +1, & \text{if } \lambda^-(t) = 0 \text{ and } X(t) > X_H; \\ 0, & \text{if } \lambda^-(t) = 1 \text{ and } X(t) < X_L; \\ \lambda^-(t), & \text{otherwise.} \end{cases}$$

Assume that the farmer seeks to maximize the expected present value of net profits resulting from investing or disinvesting one unit of fixed asset. The objective functional is

(4) $$J(x_0, \lambda_0) = \max_{X_H, X_L} \mathcal{E}\left\{ \int_0^\infty e^{-rt}(X(t) - C)\,\lambda(t)\,dt \right\},$$

where $X(t)$ satisfies (1), and $X(0) = x_0, \lambda(0) = \lambda_0$. The operating cost is C which is kept constant. To find $J(x_0, \lambda_0)$, we introduce a new functional defined by

(5) $$J(x, \lambda, t) = \max_{X_H, X_L} \mathcal{E}\left\{ \int_t^\infty e^{-rs}(X(s) - C)\,\lambda(s)\,ds \;\middle|\; X(t) = x, \lambda^-(t) = \lambda \right\},$$

In order to derive the dynamic programming equation (DPE) which is satisfied by J, we shall compare J at the starting time t, and at a later time $t + dt$. First, in $(t, t + dt)$ the process moves from x to $x + dX$, where the random increment dX is given by (1). Therefore, (5) can be written as

$$J(x,\lambda,t) = \max_{X_H, X_L} \mathcal{E}_{dX} \Big\{ e^{-rt}(X(t) - C)\,\lambda(t)\,dt + o(dt)$$

$$\text{(6)} \qquad + J(x + dX, t + dt) \;\Big|\; X(t) = x, \lambda^{-}(t) = \lambda \Big\},$$

where $o(dt)$ is of order less than dt. A Taylor expansion of $J(x + dX, t + dt)$ around (x,t) gives

$$J(x,\lambda,t) = \max_{X_H, X_L} \mathcal{E}_{dX} \Big\{ e^{-rt}(X(t) - C)\,\lambda(t)\,dt + o(dt)$$

$$\text{(7)} \qquad + J(x,t) + J_t dt + J_x dX + \frac{1}{2} J_{xx}(dX)^2 + o(dt) \;\Big|\; X(t) = x, \lambda(t) = \lambda \Big\}.$$

From (1) and (2), we have

$$\text{(8)} \qquad \mathcal{E}(dX) = \beta X\,dt + o(dt),$$

$$\text{(9)} \qquad \mathcal{E}(dX^2) = \sigma^2 X^2 dt + o(dt).$$

By taking the expectation over dX and $(dX)^2$ in (7), dividing by dt, and letting $dt \to 0$, we obtain (formally) the desired DPE

$$\text{(10)} \qquad 0 = J_t + \beta X J_x + \frac{1}{2}\sigma^2 X^2 J_{xx} + e^{-rt}(X(t) - C)\lambda(t).$$

Notice that in (10) the state variables are given constants at time t, that is, $X(t) = x, \lambda(t) = \lambda$. Setting $J(x,\lambda,t) = e^{-rt}V(x,\lambda)$ and substituting into DPE gives

$$\text{(11)} \qquad 0 = -rV + \beta X V_x + \frac{1}{2}\sigma^2 X^2 V_{xx} + (X(t) - C)\lambda(t).$$

Thus, the asset equilibrium condition states that the expected capital gain $\beta X V_x + \frac{1}{2}\sigma^2 X^2 V_{xx}$ plus the flow operating profit $(X(t) - C)\lambda(t)$ must be equal to the normal return rV. Clearly, we must have $X_H > C, X_L < C$ to satisfy the maximum principle. Besides, we need to consider the possible sunk cost due to regime switching. As we have assumed, switching from the status quo $\lambda_0 = 0$ to $\lambda = 1$ incurs a lump sum sunk cost k^+. Likewise, if the status quo is $\lambda_0 = 1$, then there will be a sunk cost k^- to sell the fixed asset. Let $V(X, \lambda_0 = 0)$ be the expected net present value of starting with a revenue X in the absence of the truck and following the optimal policy. Similarly define $V(X, \lambda_0 = 1)$ for the case when the investment is already engaged.

The optimal acquision cost X_H^* then must satisfy the value-matching condition

$$V(X_H^*, \lambda_0 = 0) = V(X_H^*, \lambda_0 = 1) - k^+ \tag{12}$$

and the Merton-Samuelson high-contact condition (Samuelson 1965; Merton 1973) or smooth pasting condition

$$V_x(X_H^*, \lambda_0 = 0) = V_x(X_H^*, \lambda_0 = 1). \tag{13}$$

Similarly, the optimal salvage value X_L^* that triggers the sale of a fixed asset satisfies the value-matching condition

$$V(X_L^*, \lambda_0 = 1) = V(X_L^*, \lambda_0 = 0) - k^- \tag{14}$$

and the smooth pasting condition

$$V_x(X_L^*, \lambda_0 = 1) = V_x(X_L^*, \lambda_0 = 0). \tag{15}$$

IV. Asset Fixity under Uncertainty

To get some qualitative results on the optimal levels of acquisition costs and salvage values, we define

$$\Phi(X) = V(X, \lambda_0 = 1) - V(X, \lambda_0 = 0). \tag{16}$$

By use of (11), we obtain

$$0 = -r\Phi(X) + \beta X \Phi_x(X) + \frac{1}{2}\sigma^2 X^2 \Phi_{xx}(X) + (X(t) - C). \tag{17}$$

This DPE has the general solution,

$$\Phi(X) = AX^{-\alpha_1} + BX^{\alpha_2} + \frac{X}{r-\beta} - \frac{C}{r}, \tag{18}$$

where A and B are constants to be determined, and $-\alpha_1$, α_2 are roots of the quadratic equation in ω:

$$\omega^2 - \left(1 - \frac{2\beta}{\sigma^2}\right)\omega - \frac{2r}{\sigma^2} = 0. \tag{19}$$

The value-matching and smooth pasting conditions (12-15) can be written as

$$\Phi(X_L^*) = -k^-, \quad \Phi_x(X_L^*) = 0, \quad \Phi(X_H^*) = k^+, \quad \Phi_x(X_H^*) = 0. \tag{20}$$

The four equations in (20) determine A, B, X_L^*, and X_H^*, completely solving the stochastic control problem. However, qualitative results may be obtained. An elaboration of (18) and (20) reveals that the function $\Phi(X)$ is concave for large values of X and convex for smaller ones; namely,

(21) $$\Phi_{xx}(X_L^*) > 0, \qquad \Phi_{xx}(X_H^*) < 0.$$

Evaluate $\Phi(X)$ at $X = X_L^*$ and use (17) and (20) to get

$$0 = rk^- + \frac{1}{2}\sigma^2 X_L^{*2}\Phi_{xx}(X_L^*) + (X_L^* - C),$$

or equivalently

(22) $$X_L^* < C - rk^-.$$

Similarly, for $X = X_H^*$

(23) $$X_H^* > C + rk^+.$$

Combining (22)-(23) gives

(24) $$X_L^* < C - rk^- < C < C + rk^+ < X_H^*,$$

or

(25) $$X_H^* - X_L^* > r(k^- + k^+).$$

When uncertainty is considered, the upper critical value rises above $C + rk^+$ and the lower critical value drops below $C - rk^-$. Therefore, what uncertainty does is to widen the gap between acquisition cost and salvage value in the deterministic case. That is, the band of inaction is enlarged from $r(k^- + k^+)$ to $X_H^* - X_L^*$. The effects of sunk cost on farmers' investment/disinvestment decision becomes more pronounced.

Intuitively, (24) implies that there is a significant value of waiting to invest or disinvest when the revenue follows a continuous-time stochastic process. The essential feature of such "hysteresis" in a farmer's investment/disinvestment decision is that he/she is faced with the mutually exclusive choice of taking an action today or at all possible times in the future where some degree of irreversibility is involved. Suppose that the current revenue $X(t)$ has dropped to the critical level of salvage value $C - rk^-$ and from there on at each point in time it will increase or decrease by a given amount with equal probabilities (a stationary *random walk*). According to the conventional fixed asset theory, the farmer should sell his/her pickup truck right away and continue to operate the farm without the truck forever after. In this case the expected net present value is zero.

But there is a value of waiting to salvage the truck. Suppose that the farmer waits one period. If at the end of this period revenue $X(t)$ has gone up, the farmer gains positive expected presented value. On the other, if revenue has dropped further down below $C - rk^-$, the farmer can decide to salvage and the expected present value is zero. By weighting these two events with the probabilities of 0.5 each, the expected present value of waiting one period is positive. Thus the farmer can do better by waiting one period unless the current revenue $X(t)$ has gone down far below $C - rk^-$ such that the expected value of waiting is wiped out. Accordingly, in the presence of ongoing uncertainty, the optimal salvage value X_L^* should be less than $C - rk^-$. Similarly, the optimal acquisition cost X_H^* should be set higher than $C + rk^+$.

V. Concluding Comments

It seems that the literature on asset fixity has suffered from an overemphasis on an operational definition or the so-called *endogenization* of fixed assets. Enormous research efforts and debates have been directed to tests whether asset fixity exists in U.S. Agriculture. To provide a deeper insight into asset fixity, a reorientation is attempted in this study: we first highlight the very investment/disinvestment nature of fixed asset theory and relate it to the simple two-parameter control-limit decision rule in the theory of inventory. We then embed this simple rule into an optimal stochastic control model where farm revenue follows a stationary continuous-time stochastic process.

The results suggest that sunk costs are essential for asset fixity and hysteresis. Moreover, the effects of sunk costs on the degree of investment/disinvestment irreversibility become more important when uncertainty is present. Thus, This research directly comes to grips with the main theme of fixed asset theory: sunk costs matter, and matter a lot. Their importance should not be neglected and/or undervalued. A reduction of differences between acquisition costs and salvage values for farm resources, as recommended by Glenn L. Johnson, is thus needed. In this sense, the theory of asset fixity shares the common theme with transaction costs economics (e.g., Oliver E. Williamson) and contestability theory (William J. Baumol, John C. Panzar, and Robert O. Willig). Further attempts should be directed to an integration of these three theories.

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