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FLEXIBLE SPECIFICATION OF MIXED DEMAND SYSTEMS

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FLEXIBLE SPECIFICATION OF MIXED DEMAND SYSTEMS

Empirical studies in applied demand analysis using systems of demand equations typically rely on one of two assumptions: either prices are assumed predetermined or quantities are assumed predetermined. The first of these assumptions leads to quantity dependent or direct demand systems, such as the (direct) translog (Christensen, Jorgenson, and Lau), the almost ideal demand system (Deaton and Muellbauer), or the (direct) differential or Rotterdam model (Theil). This is the usual representation of preferences that arises in the case of the consumer, who is typically taken as making optimal consumption decisions for given prices and income. Its use at the aggregate level is equivalent to assuming that supplies are perfectly elastic and that demands adjust to clear the market. This condition may hold for aggregate (market) data when one is modeling the demand of tradable goods in the case of a small open economy, or when prices are administratively set (e.g., public utilities). The alternative of assuming that quantities are predetermined, and that prices adjust to clear the market, leads to price dependent or inverse demand systems, such as the (inverse) translog (Christensen, Jorgenson, and Lau) or the (inverse) Rotterdam (Barten and Bettendorf). This approach may be useful when analyzing the demand for perishable products defined over a short period of time

In addition to the two polar cases of direct and inverse demand functions, there is another class of models that allows one to sidestep the task of estimating both demand and supply functions in a simultaneous equations framework. This is the case of the "mixed demand" functions (Chavas), where the price of some goods are predetermined, such that their respective quantities demanded adjust to clear the market, while for the remaining set of goods it is the quantity supplied which is predetermined and prices must adjust to clear the market. Despite its obvious potential for applications, stemming from the fact of being a combination of the two polar cases discussed above, the mixed demand approach has been virtually ignored in empirical applications.

A possible reason for the lack of empirical applications of mixed demand systems is the fact that to characterize their properties knowledge of both direct and indirect utility functions is required. This means that commonly used flexible functional forms (FFFs), such as those underlying the translog and the almost ideal demand systems, cannot be used to specify a mixed demand system because these FFFs do not have a closed form dual representation. A flexible mixed demand system can nonetheless be specified by approximating the mixed demand equations directly by a differential approach, leading to a Rotterdam mixed demand system, and this is the strategy followed in this paper.[1] The crucial issue in this context is the specification of cross-equation symmetry restrictions. For this purpose this paper develops a new and simple approach to the derivation of Slutsky relationships for mixed demands which uses the concept of virtual or shadow prices of the related area of rationed demand (Gorman, Neary and Roberts). The proposed mixed Rotterdam specification is illustrated with an application to Canadian meat demand, which seems particularly suited to the requirement of a mixed system because of the institutional feature of supply management in the poultry sector.

The paper is organized as follows. The theory of mixed demand functions is discussed, followed by the specification of our differential mixed demand system. The data used in the application is then described, and the estimation results are presented and discussed. The main points of the paper are then summarized in the concluding section.

Theory of mixed demands

Let $x_A=(x_1,x_2, \ldots, x_m)$ denote the vector of commodity that are chosen optimally and let $x_B=(x_{m+1},x_{m+2}, \ldots, x_n)$ denote the vector of commodity in fixed quantity but whose price is optimally determined. If p_i denotes the nominal price of good i, and y is income, then $\pi_i \equiv p_i/y$ is the corresponding normalized price, and π_A and π_B denote the vector of normalized prices for the two subsets of goods. Mixed demands are then derived from the constrained optimization problem (Samuelson, Chavas):

$$(1) \qquad \max_{x_A,\pi_B} \{U(x_A,x_B) - V(\pi_A,\pi_B) \mid \pi_A \cdot x_A + x_B \cdot \pi_B = 1\}$$

where U(.,.) and V(.,.) are the direct and indirect utility functions, quasi-concave and quasi-convex in their respective arguments. The solution to this problem gives the marshallian mixed demands $x_A(x_B,\pi_A,1)$ and $\pi_B(x_B,\pi_A,1)$. Clearly, at the optimum, $U(x_A(x_B,\pi_A,1),x_B) = V(\pi_A,\pi_B(x_B,\pi_A,1)) \equiv \tilde{V}(x_B,\pi_A,1)$, where $\tilde{V}$ is the mixed utility function.

The mixed demand functions $x_A(x_B,\pi_A,1)$ and $\pi_B(x_B,\pi_A,1)$ satisfy the typical restrictions of consumer theory. First of all, they satisfy the adding up condition $\pi_A \cdot x_A + x_B \cdot \pi_B = 1$ implied by the budget constraint. Secondly, the homogeneity condition implies that $x_A(x_B,\pi_A,1)$ is homogeneous of degree zero in nominal prices and income, i.e., $x_A(x_B,\pi_A,1) = x_A(x_B,p_A,y)$. Similarly, the optimal nominal prices for group B are homogeneous of degree one in (p_A,y), implying that $\pi_B(x_B,\pi_A,1)$ are homogeneous of degree zero in (p_A,y). Hence $p_B = y \cdot \pi_B(.) \equiv p_B(x_B,p_A,y)$. It also follows that the mixed utility function is homogeneous of degree zero in p_A and y, which means that $\tilde{V}(x_B,\pi_A,1) = \tilde{V}(x_B,p_A,y)$.

The symmetry restrictions can be illustrated in terms of the compensated mixed demand functions. As shown by Chavas, the compensated mixed demands are the same as the compensated demands under rationing, although mixed demands should be carefully distinguished from rationed demands (in the latter some markets do not clear). Compensated rationed demand can be characterized in terms of the restricted cost function $C(x_B,\pi_A,u)$ defined as (Gorman, Deaton):

$$(2) \qquad C(x_B,p_A,u) = \underset{x_A}{\mathrm{Min}} \{ \; p_A \cdot x_A \;\; | \;\; U(x_A,x_B) = u \; \}$$

$C(.)$ is homogeneous of degree one and concave in p_A and convex and decreasing in x_B. From the derivative property, the partial derivatives of $C(.)$ with respect to p_A give the compensated mixed demands for goods in the A group, that is the solutions to problem (2). Moreover, the partial derivatives with respect to x_B give (the negative of) the compensated shadow or virtual prices of group B goods. These shadow prices are the compensated price dependent demand functions of x_B. Hence:

$$(3.1) \qquad \partial C/\partial p_i = x_i^c(x_B,p_A,u) \qquad i \in A$$

$$(3.2) \qquad \partial C/\partial x_k = - \; p_k^c(x_B,p_A,u) \qquad k \in B$$

The compensated demand functions $x_i^c(x_B,p_A,u)$ are homogeneous of degree zero in p_A while the compensated price dependent demand functions $p_k^c(x_B,p_A,u)$ are homogeneous of degree one in p_A. Curvature and symmetry conditions imply that the matrix of partial derivatives $[\partial x_A/\partial p_A]$ is symmetric and negative semi-definite; the matrix of partial derivatives $[\partial p_B/\partial x_B]$ is symmetric and negative definite; and $\partial x_i^c/\partial x_k = \partial^2 C/\partial p_i \partial x_k = \partial^2 C/\partial x_k \partial p_i = -\partial p_k^c/\partial p_i$ for all $i \in A, k \in B$. These three conditions imply that the Hessian of the restricted cost function is skew symmetric.

To make these restrictions operational, it is necessary to relate the compensated mixed demand functions $x_i^c(x_B,p_A,u)$ and $p_k^c(x_B,p_A,u)$ to the marshallian mixed demand functions $x_i(x_B,p_A,y)$ and $p_k(x_B,p_A,y)$. Chavas defines alternative compensation equations that make this possible, but a simpler and perhaps more useful approach is possible. As mentioned above, what distinguishes mixed demands from rationed demands is the fact that there is no disequilibrium in the case of mixed demands as the price of commodities that are in fixed supply adjusts to clear the market. Hence, these markets must clear at the shadow or virtual price level. It follows that, in the mixed demand case, the total cost of achieving the utility level u when (p_A,x_B) are given is:

$$(4) \qquad \tilde{C}(x_B,p_A,u) \equiv C(x_B,p_A,u) + p_B^c(x_B,p_A,u)\cdot x_B$$

where $p_B^c = -\,\partial C/\partial x_B$.[2] Equation (4) defines the mixed cost function $\tilde{C}(x_B,p_A,u)$, which measures the expenditure that must be incurred to achieve the utility level u given that p_A is the price vector for the commodity vector x_A, the commodity vector x_B must be consumed, <u>and</u> the prices that must be paid for the commodities in fixed supply are their shadow prices.

The mixed cost function allows one to relate compensated and marshallian mixed demand functions via the identities:

$$(5.1) \qquad x_i^c(x_B,p_A,u) \equiv x_i(x_B,p_A,\tilde{C}(x_B,p_A,u))$$

$$(5.2) \qquad p_k^c(x_B,p_A,u) \equiv p_k(x_B,p_A,\tilde{C}(x_B,p_A,u))$$

These identities can be used to derive useful Slutsky relationships. To this end, first note that from (4) and the derivative property results in (3) one obtains:

$$(6.1) \qquad \partial\tilde{C}/\partial p_i = x_i^c + \sum_{k=m+1}^{n}(\partial p_k^c/\partial p_i)x_k$$

$$(6.2) \qquad \partial\tilde{C}/\partial x_k = \sum_{s=m+1}^{n}(\partial p_s^c/\partial x_k)x_s$$

Differentiating the identities in (5), and using the results in (6), yields the following Slutsky equations:

(7.1) $\partial x_i/\partial p_j = \partial x_i^c/\partial p_j - (\partial x_i/\partial y)[x_j^c + \sum_{k=m+1}^{n}(\partial p_k^c/\partial p_j)x_k]$

(7.2) $\partial x_i/\partial x_k = \partial x_i^c/\partial x_k - (\partial x_i/\partial y)[\sum_{s=m+1}^{n}(\partial p_s^c/\partial x_k)x_s]$

(7.3) $\partial p_k/\partial p_i = \partial p_k^c/\partial p_i - (\partial p_k/\partial y)[x_i^c + \sum_{s=m+1}^{n}(\partial p_s^c/\partial p_i)x_s]$

(7.4) $\partial p_k/\partial x_s = \partial p_k^c/\partial x_s - (\partial p_k/\partial y)[\sum_{r=m+1}^{n}(\partial p_r^c/\partial x_s)x_r]$

Because symmetry restrictions are expressed in terms of the properties of the compensated functions $x_i^c(x_B,p_A,u)$ and $p_k^c(x_B,p_A,u)$ while for the purpose of estimation it is the ordinary mixed demand functions $x_A(x_B,p_A,,y)$ and $p_B(x_B,p_A,y)$ that must be specified, the Slutsky relationships in (7) provide the vehicle by which symmetry restrictions can be incorporated into a flexible mixed demand system in a theoretically consistent fashion.

A Rotterdam Mixed Demand System

The maximization problem (1) from which mixed demands are derived illustrates that knowledge of both the direct and the indirect utility functions is required to derive mixed demand functions. This means that commonly used FFFs cannot be employed for empirical purposes because they typically do not have a closed form dual. For example, if one were to specify the direct utility function in terms of the Translog form used by Christensen, Jorgenson, and Lau, there would be no closed form dual functional form that could consistently and simultaneously represent the indirect utility function.[3] The alternative of specifying a FFF for the mixed utility function $\tilde{V}$ also does not appear very useful, as can be verified by applying the derivative property to the following identity relating the mixed cost function to the mixed utility function (and recalling (6)):

$$(8)\qquad \tilde{V}(x_B, p_A, \tilde{C}(x_B, p_A, u)) \equiv u$$

A better approach is to approximate mixed demands directly by a differential (Rotterdam) demand system, and using the theory derived above to impose the theoretical restrictions.

Totally differentiating the mixed demands $x_A(x_B, p_A, y)$ and $p_B(x_B, p_A, y)$ obtains the following differential mixed demand system in absolute prices:

$$(9.1)\qquad w_i d\ln x_i = w_i\eta_i \, d\ln y + \sum_{j=1}^{m} w_i\eta_{ij} \, d\ln p_j + \sum_{k=m+1}^{n} w_i\psi_{ik} \, d\ln x_k \quad , \quad \forall i \in A$$

$$(9.2)\qquad w_k d\ln p_k = w_k\theta_k \, d\ln y + \sum_{j=1}^{m} w_k\rho_{kj} \, d\ln p_j + \sum_{s=m+1}^{n} w_k\theta_{ks} \, d\ln x_s \quad , \quad \forall k \in B$$

where $w_i \equiv x_i(.)p_i/y$ and $w_k \equiv x_k p_k(.)/y$ are expenditure shares, and the following mixed elasticities are used:

$$(10.1)\qquad \eta_{ij} = (\partial x_i/\partial p_j)(p_j/x_i) \qquad (i,j) \in A$$

$$(10.2)\qquad \psi_{ik} = (\partial x_i/\partial x_k)(x_k/x_i) \qquad i \in A,\ k \in B$$

$$(10.3)\qquad \rho_{ki} = (\partial p_k/\partial p_i)(p_i/p_k) \qquad i \in A,\ k \in B$$

$$(10.4)\qquad \theta_{ks} = (\partial p_k/\partial x_s)(x_s/p_k) \qquad (k,s) \in B$$

$$(10.5)\qquad \eta_i = (\partial x_i/\partial y)(y/x_i) \qquad i \in A$$

$$(10.6)\qquad \theta_k = (\partial p_k/\partial y)(y/p_k) \qquad k \in B$$

As is standard in the specification of Rotterdam models, it is useful to express marshallian elasticities in terms of compensated elasticities prior to the parameterization of the demand system. To this end, the Slutsky relations can be rewritten in terms of elasticities as:

(11.1) $\eta_{ij} = \eta^c_{ij} - \eta_i(w_j + \sum_{k=m+1}^{n} w_k \rho^c_{kj})$

(11.2) $\psi_{ik} = \psi^c_{ik} - \eta_i(\sum_{s=m+1}^{n} w_s \theta^c_{sk})$

(11.3) $\rho_{ki} = \rho^c_{ki} - \theta_k(w_i + \sum_{s=m+1}^{n} w_s \rho^c_{si})$

(11.4) $\theta_{ks} = \theta^c_{ks} - \theta_k(\sum_{r=m+1}^{n} w_r \theta^c_{rs})$

where the superscripted c denotes elasticities obtained from the compensated mixed demand functions. Using the Slutsky relations in (11), and choosing the parameterization $\alpha_i = w_i\eta_i$, $\beta_k = w_k\theta_k$, $\alpha_{ij} = w_i\eta^c_{ij}$, $\beta_{ks} = w_k\theta^c_{ks}$, $\gamma_{ki} = -w_k\rho^c_{ki}$, and $\delta_{ik} = w_i\psi^c_{ik}$, yields:

(12.1) $w_i d\ln x_i = \alpha_i d\ln\tilde{y} + \sum_{j=1}^{m} [\alpha_{ij} + \alpha_i(\sum_{k=m+1}^{n} \gamma_{kj})] d\ln p_j + \sum_{k=m+1}^{n} [\delta_{ik} - \alpha_i(\sum_{s=m+1}^{n} \beta_{sk})] d\ln x_k$

(12.2) $w_k d\ln p_k = \beta_k d\ln\tilde{y} + \sum_{j=1}^{m} [-\gamma_{kj} + \beta_k(\sum_{s=m+1}^{n} \gamma_{sj})] d\ln p_j + \sum_{s=m+1}^{n} [\beta_{ks} - \beta_k(\sum_{r=m+1}^{n} \beta_{rs})] d\ln x_s$

where $d\ln\tilde{y} = [d\ln y - \sum_{i=1}^{m} w_i d\ln p_i]$ represent the change in nominal income adjusted by the change in exogenously given prices only.

Equations (12) represent the Rotterdam specification for the mixed demand system. It is linear in the variable but nonlinear in the parameters. Because of the parameterization chosen, the properties of homogeneity, symmetry, and adding-up can be set in terms of parametric restrictions. Homogeneity is satisfied when:

(13.1) $\sum_{j=1}^{m} \alpha_{ij} = 0$

(13.2) $\sum_{i=1}^{m} \gamma_{ki} = -w_k$

The adding-up conditions are:

(14.1) $\sum_{i=1}^{m} \alpha_i + \sum_{k=m+1}^{n} \beta_k = 1$

(14.2) $\sum_{i=1}^{m} \alpha_{ij} = 0$

(14.3) $\sum_{i=1}^{m} \delta_{ik} = -w_k$

and symmetry requires:

(15.1) $\alpha_{ij} = \alpha_{ji}$

(15.2) $\beta_{ks} = \beta_{sk}$

(15.3) $\gamma_{ik} = \delta_{ki}$.

An Application to Canadian Meat Demand

The mixed demand approach appears particularly suited to the Canadian meat demand case. First of all, there is virtually free trade between U.S. and Canada for beef and pork. Because Canada is a small country in the North-American market, the assumptions that beef and pork prices are exogenous to the Canadian market appears a tenable one. On the other hand, Canadian imports of poultry products are restricted by an import quota. This import quota insulates the domestic market, and the internal price formation mechanism heavily depends on the institutional setting. In particular, chicken producers are organized in provincial Marketing Boards which are coordinated by the Canadian Chicken Marketing Agency. The objective of this monopoly-like organization is to guarantee producers a favorable price, and this objective is pursued by limiting the output in the market. This "supply management" is enforced by production quotas which are first allocated to each province and then to the individual producers. Hence, for chicken it seems that equilibrium is characterized by exogenously determined supply with price adjusting to clear the market.[4]

Assuming that the meats group is weakly separable from other commodities, a mixed demand system for beef, pork, and chicken is specified. To estimate this mixed demand system, quarterly data on consumption and prices (obtained from Agriculture Canada) for beef, pork and chicken are used. These data are for the period 1980(1) to 1990(1), a period consistent with the policy setting described. The quantity data are per capita disappearance (in kilograms) of beef, pork and chicken.[5] The quantities were converted from carcass weights to retail weights using conversion factors supplied by Statistics Canada.[6] As for prices, a problem arises because Statistics Canada reports price indexes for these three commodities, but not nominal prices. To proceed, these consumer price indexes (with 1981 as base year) were converted to nominal prices using survey data obtained from Family Food Expenditure Surveys, Statistics Canada, as follows. From the data on weekly family expenditures and quantities consumed (for all classes and all provinces) for beef, pork and chicken, prices were computed for the three commodities by dividing expenditures by quantities for the years 1974, 1976, 1978, 1982, 1984 and 1986. These prices were regressed (through the origin) on the respective annual consumer price indexes. The raw moment R^2 values were over 0.99 for all the three equations and the regression coefficients were 0.052, 0.037 and 0.029. These estimated coefficients were then used in generating prices for the entire sample period.

Estimation Results

Implementing the Rotterdam model requires converting the differential terms in (12) to finite logarithmic changes. Because quarterly data are used, log differences are computed between the same quarter in consecutive years rather than between two contiguous quarters. Similarly, the shares used in multiplying each of the equations are averages for the same quarters. For example, in the beef equation, the approximation for $d\log p_{bt}$ is $(\log p_{bt} - \log p_{bt-4})$, and the corresponding share is $(w_{bt-4}+w_{bt})/2$, where the subscript b indexes beef and t indexes time. Symmetry and homogeneity restrictions are maintained in the estimation. Because the homogeneity (and the adding-up) conditions entail budget shares, this restriction is imposed at the mean point. The stochastic version of the model is obtained by adding to equations (12) error terms that are assumed to be multinormally distributed and contemporaneously correlated. The resulting system is singular due to the adding-up condition, and the chicken equation is dropped in the estimation. Estimation was carried out using the nonlinear estimation procedure available in SHAZAM 6.2.

The mixed differential model is estimated with an intercept and correction for first order autocorrelation. Unreported results indicate that no significant seasonality is present, probably due to the forth-period differencing adopted.[7] The intercept in a Rotterdam model can be interpreted as the coefficient of a trend variable. Specifically, the intercept measures the share-weighted rate of change in the left-hand-side quantity (or price, in our context) which is not attributable to the effects of prices and total expenditure. Hence, it can be interpreted as an indicator of preference changes, although omitted variables that are correlated with trend (i.e. model misspecification) may also be an explanation.

The estimates of the mixed demand system are presented in Table 1. The coefficients α_{11} and β_{33} (which are weighted compensated mixed elasticities of beef and chicken, respectively) have the expected negative signs. The income coefficients are positive. The intercepts in the beef and pork equations are negative implying that the budget shares of beef and pork have been falling. Given adding-up, which requires the intercepts to add to zero when the omitted equation is considered, this also implies that the share of chicken has been rising.

The estimated mixed compensated elasticities obtained by dividing the estimated coefficients by the relevant mean shares, and their asymptotic standard errors, are reported in Table 2. The ratios of the elasticities to their respective standard errors are asymptotically normally distributed. Beef and pork are net substitutes while chicken is a substitute to both beef and pork.[8] For instance, the mixed elasticity $\psi_{PK,CK}$ of -0.113 shows that a one percentage increase in the supply of chicken causes about one-tenth of a percent decrease in the consumption of pork. The marshallian mixed elasticities, retrieved via the Slutsky relationships, are also reported in Table 2. Beef and pork are found to be gross complements. Chicken is found to be a substitute for pork but a complement to beef. The own 'quantity' elasticity of chicken is greater than one in absolute value indicating that a one percent rise in the supply of chicken would, ceteris paribus, decrease the price of chicken by more than a percent.

The mixed expenditure elasticities are close to unity for all the three commodities. For beef and pork, they indicate the usual change in consumption of the commodity due to a change in total expenditure. For chicken, however, the expenditure elasticity indicates how much more (or less) consumers (at the market level) are willing to pay for chicken when income increases by one percent. For a normal good, one would expect this elasticity to be positive, as is the case for chicken.

To compare the computed mixed elasticities with the more familiar direct elasticities estimated in other studies, the direct compensated Marshallian elasticities and the direct expenditure elasticities can be retrieved from the mixed elasticities. To this end, let ϵ_{ij} denote direct marshallian price elasticities, and in the obvious notation let ϵ_{AA}, ϵ_{AB}, ϵ_{BA}, ϵ_{BB} denote the submatrices of these elasticities for the (A,B) grouping of the mixed system. Then, following a procedure similar to that which yields the result reported in Houck, it is verified that marshallian direct and mixed elasticities are related by: $\epsilon_{AA} = [\eta - \psi\theta^{-1}\rho]$, $\epsilon_{AB} = -\psi\theta^{-1}$, $\epsilon_{BA} = \theta^{-1}\rho$, and $\epsilon_{BB} = -\theta^{-1}$, where η, ψ, θ, and ρ denote the matrices of mixed elasticities. Direct elasticities retrieved in this fashion are reported in Table 3. The own price elasticities of beef and chicken are -0.901 and -0.826, respectively, higher than that of pork which is -0.627. The expenditure elasticities of beef and pork are close to unity, while that of chicken is 0.750.[9]

For the purpose of comparison, a direct Rotterdam model (absolute price version) was also estimated. The model was estimated with an intercept and correction for first order autocorrelation. Detailed estimation results are not presented here, but they are available from the authors upon request. What is reported, in Table 4, are the compensated and Marshallian elasticities from the direct Rotterdam. The beef and pork own price (compensated and marshallian) and expenditure elasticities from the direct Rotterdam are somewhat similar to the retrieved direct elasticities, but the absolute value of the own price elasticity of chicken can be seen to be much lower in the direct demand system. The higher own-price elasticity of chicken from the mixed system is consistent with results reported by Thurman, and Shonkwiler and Taylor, who show that a least squares estimation of quantity dependent demand equations underestimates demand elasticities when prices are in fact endogenous. Hence, this analysis would suggest that the demand for chicken in the Canadian market is more elastic than suggested by previous studies.

Concluding Remarks

This paper has developed a Rotterdam specification for a mixed demand system, which is applicable when, at the market level, quantities (demanded) of some commodities are optimally determined given their prices while for the other commodities, the prices are optimally determined given their quantities. The Rotterdam specification allows a flexible representation of the mixed demand system, and in this context it overcomes some problems associated with the use of flexible functional forms. To make the Rotterdam specification operational, this paper has developed a new approach to the derivation of Slutsky relationships in a mixed demand context. The new mixed demand system proposed in this paper was illustrated with an application to the Canadian market for meats. The fact that Canada has virtually free trade for beef and pork, while its supply of chicken is restricted by supply management, suggest that a mixed demand approach is appealing in this case. Comparing the estimated elasticities derived from the mixed demand system with those of a direct Rotterdam model, it emerges that they are similar, except for the own price elasticity of chicken which is higher in the mixed demand system.

ENDNOTES

1. The Rotterdam model is obviously a flexible representation of consumer demand in that it provides a first-order approximation to an arbitrary demand system. This flexibility is usually illustrated in terms of the Rotterdam model providing an approximation in the parameter space (Theil), as opposed to the approximation in the variable space of FFFs, although the Rotterdam model can itself be viewed as an approximation in the variable space (Mountain).

2. Clearly, for $\tilde{C}$ to be defined at least one good must belong to the group A.

3. One class of preferences for which a closed form indirect utility function exists is the generalized Bergson family (Pollak). The Stone-Geary utility function, which gives rise to the Linear Expenditure System, is a member of this family.

4. The Farm Products Marketing Act of 1972 allowed the creation of marketing boards which led to the establishment of the Canadian Egg Marketing Agency in 1972, the Canadian Turkey Marketing Agency in 1973 and the Canadian Chicken Marketing Agency in 1978. Along with the Canadian Dairy Commission which existed since 1966, these agencies effectively became supply-restricting boards with considerable powers (Van Kooten). For the case of chicken, import quotas were introduced in 1979, so that supply management for the chicken industry became fully operational by the end of that year.

5. We recomputed the pork disappearance data because those supplied by Agriculture Canada, based on the methodology of Hewston and Rosien, appear affected by a methodological flaw when accounting for manufacturing and waste.

6. The conversion factor for beef was 0.74 from 1980 to 1985 and 0.73 from 1986 to 1990. The conversion factor for pork was 0.77 from 1980 to 1982 and 0.76 from 1983 to 1990.

7. A model with seasonal dummy variables was estimated, and these dummies were found to be insignificant using a likelihood ratio procedure at the 5 percent level, indicating that the four-period differencing was also effective in removing seasonality from the data. A model with single period differencing but with an AR(4) error process and dummy variables was also estimated. This model yielded elasticities close to those of the model presented here.

8. Note that, in general, substitutability defined in terms of the 'mixed' compensated elasticities need not be equivalent to either p-substitutability defined in terms of the direct system, nor the q-substitutability defined in terms of the inverse system.

9. In fact, a likelihood ratio test at the 5 percent significance level fails to reject the null hypothesis of homotheticity at this (mean) point.

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Table 1. Estimates of the mixed demand system

Coefficient	Estimate	Standard Error	t-ratio
α_1	0.5441	0.0468	11.63
α_2	0.2879	0.0419	6.87
α_{11}	-0.0946	0.0276	-3.43
γ_{13}	-0.1485	0.0236	-6.31
β_{33}	-0.2692	0.0526	-5.12
rho	0.5236	0.1146	4.57
Intercept			
beef	-0.0083	0.0035	-2.34
pork	-0.0034	0.0033	-1.02

Note: Maximized Log-likelihood: 257.86 .
Mean shares: beef=0.492; pork=0.323; chicken=0.185.

Table 2. Mixed Elasticities at the mean

	P_{BF}	P_{PK}	X_{CK}	y
		Compensated elasticities		
X_{BF}	-0.192 (0.056)	0.192 (0.056)	-0.302 (0.048)	
X_{PK}	0.293 (0.086)	-0.293 (0.086)	-0.113 (0.073)	
P_{CK}	0.803 (0.127)	0.197 (0.127)	-1.455 (0.284)	
		Marshallian elasticities		
X_{BF}	-0.901 (0.100)	-0.205 (0.061)	-0.004 (0.073)	1.106 (0.095)
X_{PK}	-0.278 (0.131)	-0.613 (0.092)	0.127 (0.080)	0.891 (0.130)
P_{CK}	0.221 (0.183)	-0.129 (0.111)	-1.211 (0.210)	0.908 (0.195)

standard errors of the elasticities are reported in parentheses.

Table 3. Direct elasticities at the mean retrieved from the mixed system

	P_{BF}	P_{PK}	P_{CK}	y
	Compensated elasticities			
X_{BF}	-0.359 (0.086)	0.151 (0.060)	0.208 (0.055)	
X_{PK}	0.231 (0.092)	-0.308 (0.087)	0.077 (0.051)	
X_{CK}	0.552 (0.147)	0.136 (0.088)	-0.687 (0.134)	
	Marshallian elasticities			
X_{BF}	-0.901 (0.103)	-0.205 (0.063)	0.003 (0.061)	1.103 (0.105)
X_{PK}	-0.255 (0.126)	-0.627 (0.088)	-0.105 (0.060)	0.987 (0.147)
X_{CK}	0.183 (0.164)	-0.107 (0.090)	-0.826 (0.143)	0.750 (0.160)

standard errors of the estimates are reported in parenthesis

Table 4. Direct elasticities at the mean from the direct system

	P_{BF}	P_{PK}	P_{CK}	y
	Compensated direct elasticities			
X_{BF}	-0.267 (0.058)	0.174 (0.047)	0.093 (0.028)	
X_{PK}	0.265 (0.072)	-0.307 (0.074)	0.042 (0.033)	
X_{CK}	0.246 (0.079)	0.073 (0.058)	-0.319 (0.061)	
	Marshallian direct elasticities			
X_{BF}	-0.837 (0.082)	-0.200 (0.050)	-0.121 (0.033)	1.158 (0.092)
X_{PK}	-0.233 (0.111)	-0.635 (0.080)	-0.146 (0.043)	1.014 (0.147)
X_{CK}	-0.027 (0.105)	-0.107 (0.063)	-0.422 (0.064)	0.556 (0.118)

standard errors of the elasticities are reported in parenthesis.