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ECONOMETRIC TESTS OF FIRM DECISION MAKING UNDER UNCERTAINTY: OPTIMAL OUTPUT AND HEDGING DECISIONS

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Abstract

The competitive firm under price uncertainty which hedges and faces basic risk is examined. Assuming constant absolute risk aversion, reciprocity conditions linking optimal output, hedging, and input decisions and leading to testable econometric restrictions are derived. The theoretical model is empirically tested with data from a large California feedlot.

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Comparative statics of the competitive firm under price uncertainty were first developed by Sandmo (1971) and Batra and Ullah (1974). More recently, Pope (1980) and Chavas and Pope (1985) extended these results by deriving the corresponding symmetry and homogeneity properties. These latter two papers have particular importance in econometric modeling because symmetry or reciprocity results are equality restrictions which can be easily imposed in empirical research and, hence, increase the degrees of freedom and allow the theory to be tested. However, while theoretical works on various related models have become quite common in the literature, few studies have actually empirically tested these models. Notable exceptions include Antle (1987), Brown and Snow (1990), Chavas and Holt (1990), and Antonovitz and Roe (1986) which empirically test various models of firm optimization under uncertainty.

Holthausen (1979) and Feder, Just, and Schmitz (1980) introduced hedging into the model of the firm under price uncertainty. This work was expanded to include basis risk by Batlin (1983) and Paroush and Wolf (1989), and the corresponding comparative statics were derived. This paper will further extend this work by deriving reciprocity conditions which link optimal output, hedging, and input decisions and lead to testable econometric restrictions. A second contribution will be to empirically test this model with data from a large cattle feedlot located in the San Joaquin Valley of California.

Symmetry Conditions Under Risk Averse Decision Making

The model is based on a competitive firm maximizing the expected utility of uncertain wealth, denoted by $EU(w + \pi)$ where nonstochastic initial wealth is w and profit, which is stochastic, is denoted by π. For a producer hedging in a futures market in which basis risk is present, the decision making problem can be specified as follows. In the first time period, inputs are purchased at known prices r_1, and the producer sells f units of output at the known futures price p_1^f. In the second period when production is complete, output is sold at the prevailing spot price in the region. The producer offsets the position in the futures market by buying back f units of output at futures price p_2^f. When making decisions in the first period, the future spot price $\tilde{p}$ and the second period futures price $\tilde{p}_2^f$ are unknown variables. The optimal output and hedging decisions are determined by maximizing expected utility of wealth defined by

$$EU(\tilde{\pi}) = EU[w + \tilde{p}Q(X_1) + (p_1^f - \tilde{p}_2^f)f - r_1x_1 - K] \tag{1}$$

where the $Q(X_1)$ represents the production function which is nonstochastic, K is fixed costs, and r_1 and x_1 are vectors of input prices and quantities, respectively. The decision maker may hedge but does not speculate in the futures markets, or

$$Q(X_1) - f \geq 0. \tag{2}$$

The primal-dual method proposed by Silberberg (1974) is used to derive symmetry conditions which result in cross-equation restrictions linking shifts in optimal output and hedging decisions in response to changes in exogenous parameters. A subset of the symmetry terms from the primal-dual matrix is identified by equations (3) - (5):[1]

$$-E\left[U_{\pi\pi}f\pi_x' x_{\bar{p}} + U_\pi f_{\bar{p}} + U_{\pi\pi}f\pi_f f_{\bar{p}}\right] = E\left[U_\pi Q_{\bar{p}_2^f} + U_{\pi\pi}Q\pi_x' x_{\bar{p}_2^f} + U_{\pi\pi}Q\pi_f f_{\bar{p}_2^f}\right] \tag{3}$$

$$E\left[U_\pi Q_w + U_{\pi\pi}Q\pi_x' x_w + U_{\pi\pi}Q\pi_f f_w\right] = E\left[U_{\pi\pi}\pi_x' x_{\bar{p}} + U_{\pi\pi}\pi_f f_{\bar{p}}\right] \tag{4}$$

$$-E\left[U_{\pi\pi}f\pi_x' x_w + U_\pi' f_w + U_{\pi\pi}f\pi_f f_w\right] = E\left[U_{\pi\pi}\pi_x' x_{\bar{p}_2^f} + U_{\pi\pi}\pi_f f_{\bar{p}_2^f}\right] \tag{5}$$

where the expected values of $\tilde{p}$ and $\tilde{p}_2^f$ in the first period are denoted by $\bar{p}$ and $\bar{p}_2^f$,

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[1] For notational convenience, subscripts are used to denote derivatives. For example, $U_\pi = \partial U/\partial\pi$, $\pi_x' = (\partial\pi/\partial x_1, \ldots, \partial\pi/\partial x_n)$, etc.

respectively. Substituting equation (4) into the left hand side of equation (3) and equation (5) into the right hand side of equation (3) yields

$$E\left[U_\pi Q_{\bar{p}_2^f} + Q(U_{\pi\pi} f\pi_x' x_w + U_\pi f_w + U_{\pi\pi} f\pi_f f_w) \right] =$$

$$E\left[U_\pi F_{\bar{p}} + f(U_\pi Q_w + U_{\pi\pi} Q\pi_x' x_w + U_{\pi\pi} Q\pi_f f_w) \right] \tag{6}$$

or,

$$Q_{\bar{p}_2^f} + fQ_w = -f_{\bar{p}} + Qf_w . \tag{7}$$

Different subsets of symmetry terms from the primal-dual matrix (which will not be stated here for the sake of brevity) can be used to derive the following two reciprocity results:

$$Q_{p_1^f} + Qf_w = f_{\bar{p}} + fQ_w \tag{8}$$

$$-f_{p_1^f} - ff_w = f_{\bar{p}_2^f} - ff_w . \tag{9}$$

If it is further assumed that the producer's utility function is characterized by constant absolute risk aversion (CARA), the shifts in wealth do not affect output and hedging decisions (i.e., $Q_w = f_w = 0$).

The key role played by the assumption of CARA is demonstrated by Chari, Jagannathan, and Jones (1990) for evaluating producer welfareusing futures markets and by Lapan, Moschini, and Hanson (1991) in deriving comparative statics results for decision making in futures and options markets. Then by combining equations (7) and (8) and assuming CARA, the following empirically testable cross-equation restrictions result:

$$Q_{\bar{p}_2^f} = -f_{\bar{p}} = Q_{p_1^f} \tag{10}$$

$$-f_{p_1^f} = f_{\bar{p}_2^f} . \tag{11}$$

The Empirical Model

Using the indirect expected utility function, Just, Hueth, and Schmitz (1982) derived results analogous to Hotelling's lemma for the competitive firm under price uncertainty. Similar results can easily be shown to hold for this model where optimal output and hedging decisions are expressed as functions of derivatives of the indirect expected utility function:

$$Q^*(x_1) = -\frac{\partial EU(\pi^*)/\partial \bar{p}}{\partial EU(\pi^*)/\partial K} \tag{12}$$

$$f^* = \frac{\partial EU(\pi^*)/\partial \bar{p}_2^f}{\partial EU(\pi^*)/\partial K} . \tag{13}$$

Let indirect expected utility, V, be a function of its parameters given by the vector $\vec{w} = (\bar{p},\ \sigma_s^2,\ p_1^f,\ \bar{p}_2^f,\ \sigma_f^2,\ \sigma_{sf},\ r_i,\ K)$ where all elements have been normalized around their means and σ_s^2, σ_f^2, σ_{sf}, represent the variances and covariance of spot and futures prices, respectively. Then, approximating V by a second-order Taylor series expansion,

$$V(w) = \alpha_o + \sum_{i=1}^{8} \alpha_i w_i + \frac{1}{2} \sum_{i=1}^{8} \sum_{j=1}^{8} \beta_{ij} w_i w_j \tag{14}$$

where symmetry requires that $\beta_{ij} = \beta_{ji}$.

Optimal output and hedging decisions are obtained by applying the duality results in equations (12) and (13). Noting that the mean of the random output price is w_1, the mean of the random second-period futures prices is w_4, input price is w_7 and fixed costs are denoted by w_8, the empirical model for optimal output and hedging is:

$$Q^*(X_1) = \frac{\partial V/\partial \bar{p}}{\partial V/\partial K} = \frac{\alpha_1 + \sum_{j=1}^{8} \beta_{1j} w_j}{-\alpha_8 - \sum_{j=1}^{8} \beta_{8j} w_j} \tag{15}$$

$$f^* = \frac{\partial V/\partial \bar{p}_2^f}{\partial V/\partial K} = \frac{-\alpha_4 - \sum_{j=1}^{8} \beta_{4j} w_j}{-\alpha_8 - \sum_{j=1}^{8} \beta_{8j} w_j} \tag{16}$$

Conditions sufficient to ensure that CARA holds in (15) and (16) are that β_{18}, β_{88}, β_{38}, β_{48}, and β_{78} equal zero, since implies that $\partial Q^*(X_1)/\partial K = 0$ and $\partial f^*/\partial K = 0$ and that optimal input decisions will not be affected by changes in fixed costs. By symmetry, β_{81}, β_{83}, β_{84}, and β_{87} are also zero.

The empirical model with the added restrictions to ensure CARA proved intractable for econometric estimation because of failure of the model to converge. Hence, one additional assumption was needed. Separability between both the variances and covariance of the spot and futures prices and fixed costs is imposed on the indirect expected utility function. Separability implies that the parameters of the indirect expected utility function can be partitioned into separate sub-vectors such that the parameters for fixed costs do not interact with the parameters for the variances of either the output or futures prices. A necessary and sufficient condition for separability is $\beta_{28} = \beta_{38} = \beta_{68} = 0$. Thus, the functional forms for estimating optimum output and hedging decisions are specified as:

$$Q^*(X_1) = \frac{\alpha_1 + \sum_{j=1}^{7} \beta_{1j} w_j}{\alpha_8} + \epsilon_1 \tag{17}$$

$$f^* = \frac{\alpha_4 + \sum_{j=1}^{7} \beta_{4j} w_j}{\alpha_8} + \epsilon_2. \tag{18}$$

In addition, the cross-equation restrictions in (10) and (11) imply that $-\beta_{14} = \beta_{13}$, $-\beta_{44} = \beta_{43}$, and $-\beta_{14} = -\beta_{41} = \beta_{13}$. Econometric tests for misspecification error in the functional form are presented in the section on empirical results.

Econometric Specification

A detailed, unique data set was obtained from a large California feedlot which extensively used hedging to manage price risk. Information was obtained from January 1984 through March 1986. For specific pens of cattle, the feedlot records included data on the number of cattle, total purchase weight of the feeder cattle, and the total net gain of the fed cattle. Output price received, the purchase price of the feeder cattle and the input costs incurred for each pen of cattle were gathered. Input prices included feed costs, measles vaccinations and medicine charges, yardage fees, pasture charges, and other miscellaneous costs for each pen. Information on weekly hedging decisions was obtained for each pen of cattle including the number of contracts available for hedging, the number of contracts actually hedged, along with the date and futures price at which the hedge was placed.

It was assumed that each pen of cattle represented the feedlot's basic unit of decision making in developing expected utility maximizing output and hedging decisions. Records indicated that the feedlot manager chose the size of each pen rather than filling each one to capacity and that he developed a feeding schedule and hedging plan for each pen. Hence, the dependent variables used to represent output and hedging decisions were total hundredweight marketed per pen and futures contracts sold (in hundredweights) per pen. Given the size of the feedlot and the limited access to the records, a random sample of pens was chosen.

While most of the data could be gathered from feedlot records, it was still necessary to estimate expected output and futures prices along with their variances and covariance. Information used to form these expectations was based on prices readily available to the feedlot from the California Feeder's Report and the Chicago Mercantile Exchange Yearbook. Cattle in this feedlot were typically fed from 17 to 21 weeks. A biweekly (combining every two weeks) model was used to allow for a shorter and more accurate forecast horizon of 8 to 12 biweeks, depending on the pen.

Three alternative price expectations models were estimated: a bivariate autoregressive model, a multivariate time-series model, and an implicit expectations model. Because the empirical estimates of output and hedging [equations (17) and (18)] were relatively unaffected by the choice of the expectations model, only the bivariate model will be discussed here.

The general form of the bivariate autoregressive specification is

$$Z_{1t} = \alpha_1 + \sum_{m=1}^{p} \beta_{1m} Z_{1t-m} + \sum_{n=1}^{q} \gamma_{1n} Z_{1t-n} + \epsilon_1 \tag{19}$$

$$Z_{2t} = \alpha_2 + \sum_{m=1}^{p} \beta_{2m} Z_{2t-m} + \sum_{n=1}^{q} \gamma_{2n} Z_{2t-n} + \epsilon_2 \tag{20}$$

where Z_1 is the output price for slaughter cattle in California, Z_2 is the futures price for live cattle, and the specified lag lengths are noted by p and q. An empirical study by Engle and Brown (1986) demonstrated that the Hannan-Quinn (HQ) criterion yielded the most accurate multi-step predictions in comparison with alternative selection criterion. Based on the HQ criterion, a seven-period lag for output prices and an eight-period lag for futures prices were identified as the best models. Each dependent variable was regressed on lagged values of itself and lagged values of the other price variable. The lagged values of each variable were retained only if the variables were Granger-informative about the variable to be forecast. For the output price, only its own lagged values were Granger-informative at the 5 percent level. For the futures price only its own lagged values retained. Examination of the residual autocorrelations for the output and futures price series along with Q-statistic revealed no evidence of model inadequacy. The model for output and futures prices was estimated jointly as a seemingly unrelated regression.

The Pagan and Nicholls technique (1984) was used to derive the multi-step predictions for the expected output and futures prices for the period when the cattle were marketed from the feedlot. Multi-step predictions and the standard deviations of the multi-step predictions are derived, taking into account that the regression coefficients used to derive the predictions are estimated rather than known. Standard errors of the predictions and the covariance of the standard errors were also developed. The variance of the expected output price and the variance of the expected futures prices were derived from the standard errors of the predictions.

Empirical Results

This section presents the main empirical results, focussing on three main issues. First, the econometric model was tested for significant sources of specification error. Second, the restrictions implied by the model of constant absolute risk aversion are tested. Third, comparative static restrictions on firm output and hedging are examined.

An econometric test for the linear specification was based on the "rainbow test" proposed by Utts (1982). As indicated in Table 1, the linear models for output and hedging decisions revealed no evidence of misspecification error. In addition, tests for the presence of autoregressive conditional heteroscedasticity (ARCH) in the disturbances of orders one and two were performed. ARCH errors may be present since the feedlot's forecasts of mean output and futures prices and the uncertainty associated with these forecasts are updated over time and may be influenced by the magnitudes of forecast errors in preceding periods. The test statistics presented in Table 1 revealed no evidence of ARCH in the model for output and hedging decisions.

The output and hedging decisions in equations (17) and (18) were estimated using Zellner's seemingly unrelated regressions technique with the three imposed restrictions. The results of the estimated coefficients and the test statistics are presented in Table 2. Conditional on the bivariate autoregression expectations model, the restrictions implied by CARA are not rejected since χ^2 statistic does not exceed the critical value, suggesting that decisions are consistent with the hypothesis of constant absolute risk aversion. At an aggregate level, Antonovitz and Roe also found that output and hedging decisions in the livestock sector were consistent with constant absolute risk aversion.

One important implication of these findings relates to the development of welfare measures for decision making under uncertainty. Pope and Chavas defined welfare measures for producer behavior under risk aversion. In the important case of constant absolute risk aversion, producer surplus is an excellent welfare measure. Producer surplus can be used to quantify the welfare impacts of changes in expected prices and mean preserving increases in price uncertainty associated with policy changes.

The signs of the estimated coefficients are examined for consistency with the model of producer decision making under uncertainty. The comparative statics derived by Paroush and Wolf also hold for the model presented here and provide testable restrictions on the response of the amount produced to shifts in the producer's expectations of the mean and variance of the output price. An increase in the expected output price leads to an increase in the amount produced. In the econometric model for the amount produced, the estimated coefficient for the expected output price was significantly positive. The positive coefficient indicates that, as the expected output price increases, the quantity of cattle marketed from the feedlot increases. This result satisfies restrictions of the expected utility model.

According to Paroush and Wolf, the impact of increased variability in output prices on the amount produced cannot be unambiguously signed. However, if a positive amount is produced, increases in the uncertainty of the output price reduce the amount produced. As Batlin noted, this result is identical to results presented by Sandmo and Batra and Ullah in models without hedging opportunities. In the econometric model for output decisions, the coefficient measuring the impact of output price variability was negative, although not significant. The sample information gathered from the feedlot indicates that the producer responds to an increase in the variability of output prices by decreasing the amount produced.

An increase in the expected futures price leads to a decrease in the amount produced; an increase in the current futures price has the opposite effect. The estimated coefficient for the expected futures price is negative and is in agreement with the derived comparative statics of the decision model.

The results from the estimated output equation indicate that an increase in the covariance of output and futures prices leads to an increase in the amount produced. The comparative statics developed by Paroush and Wolf examined the impact of shifts in the correlation coefficient between output and futures prices on optimal output and hedging decisions. The sign of the correlation coefficient is determined by the sign of the covariance between output and futures prices. Changes in the co-movements between output and futures prices as represented by the covariance lead to increases in the amount produced. The positive coefficient on the covariance variable in the output equation, although it is not significant, is in agreement with the derived comparative static restrictions. The negative coefficient on the covariance variable in the hedging equation, which has a very low significance level, is not consistent with the model.

Using a linear mean-variance model, Batlin showed that, as input prices increase, the amount produced declines. Comparative static restrictions on input prices can be shown to hold for risk preferences consistent with constant absolute risk aversion. The coefficients on feeder prices and interest payments on feed are negative and satisfy the required comparative statics conditions. The coefficient on the feed price was also positive, although not significant. The estimated coefficient for the medical charges and yardage fees were positive and significantly different from zero. The signs of these coefficients are inconsistent with the decision model.

Summary and Conclusions

One contribution of this paper was the derivation, under the assumption of CARA, of reciprocity results leading to testable econometric restrictions for the competitive firm under price uncertainty which hedges but faces bases risk. The second contribution was to empirically test this model with data from a large feedlot, which manages price risk through hedging.

Empirical analysis indicated that the restrictions linking optimal output and hedging decisions implied by expected utility maximization and CARA were not rejected. For the most part, comparative static results were also consistent with the expected utility model, subject to some violations. Hence, this work provides some empirical evidence to support the hypotheses that producers' decision making is characterized by expected utility maximization and CARA.

TABLE 1. Specification Tests for Output and Hedging Models

Specification Test	Rainbow Test[a]	ARCH[b] 1	ARCH[b] 2
Output Decision	.64	.672	.869
Hedging Decision	.80	.004	.028

[a] The rainbow test statistic is distributed as $F_{70,55}$. Critical value for $F_{.05,70,55}$ is 1.64.

[b] The test statistic is distributed as χ_p^2 where p is the longest lag from the ARCH specification. Critical values for test statistics: $X_{1,.05}^2 = 3.841$ and $\chi_{2,.05}^2 = 5.991$.

TABLE 2. Estimates of Output and Hedging Equations Under Constant Absolute Risk Aversion with Restrictions Imposed

Parameter	Total Output Equation Estimate	Total Output Equation t-Statistic	Hedging Equation Estimate	Hedging Equation t-Statistic
Intercept	-17.658	-3.105	15.010	2.470
Feeder Cattle Price	-8.153	-1.999	5.586	1.390
Feed Price	-0.251	-0.072	0.283	0.083
Medical Charges	0.892	2.089	-0.707	-1.728
Yardage Fees	0.214	1.869	-0.141	-1.265
Interest Charges on Cattle	-0.117	-2.008	0.036	0.630
Interest Charges on Feed	0.012	0.094	-0.076	-0.588
Expected Output Price	0.231	2.450	-0.196	-2.047
Variance of Output Price	-0.147	-0.760	0.014	0.073
Current Futures Price	0.197	2.047	0.116	0.700
Expected Futures Price	-0.197	-2.047	-0.116	0.700
Variance of Futures Price	0.124	0.452	-0.013	-0.049
Covariance Between Output and Futures Prices	0.071	0.595	-0.012	-0.097

Test of model restrictions: χ^2 value = 2.527[a]

[a] Critical value χ_3^2 at the 0.05 significance level = 7.815.

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