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Consumer's Surplus versus Compensating Variation Revisited

by

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Abstract

Consumer's surplus and a second-order approximation to compensating variation are compared to the exact compensating variation for a subsystem of several commodities. An empirical application to the relative retail price distortions created by the U.S. dairy program is presented. In the application, a new specification for weakly integrable incomplete demand models is developed and estimated. The integrable demand model has sufficient flexibility that it is not rejected by the data. Both welfare approximations are found to introduce large errors in the measurement of deadweight losses.

Keywords: Compensating Variation, Consumer's Surplus, Demand Analysis, Economic Welfare, Integrability

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Consumer's Surplus versus Compensating Variation Revisited

Introduction

Economics involves the estimation of costs and benefits. For single good demand models, Hausman (1981) has shown that when deadweight losses are of interest, exact welfare measures can reduce measurement errors substantially relative to consumer's surplus approximations. A comparable statement has yet to be made when the focus of the analysis includes several demand equations. This issue is complicated by the fact that computing exact welfare measures for several goods requires the structure of the theory of consumer choice. The popular term for this structure is *integrability.*

It is well–known that integrability of the demands is necessary and sufficient for the existence of the exact money metrics compensating and equivalent variation. Even the approximation arguments for the use of consumer's surplus (Willig) are based on the assumption that an underlying preference function exists. This seems to suggest that integrability ought to be taken seriously by empirical demand analysts. On the other hand, it is convenient to specify demand equations as *ad hoc* functions of the prices of the goods of interest, the prices of closely related goods, income, and a small set of demographic or other shift variables. The demand functions are transformed, if necessary, to a form that is linear in the unknown parameters and the model estimated by standard least squares estimation methods.

Some of my past research has focused on the theoretical structure of incomplete demand models (LaFrance 1985, 1986, 1990; LaFrance and Hanemann). *Inter alia*, I have argued in this work that the theoretical implications of many of the common *ad hoc* demand models are so severe that this approach is not as attractive as it first appears to be. However, one question that commonly was raised during the course of this work is, "Just how important is it to estimate demand models that satisfy the rigorous and generally nonlinear restrictions of integrability?" This question is the focus of the present paper.

In the paper I briefly discuss three approaches to welfare measurement with subsystems of demand equations. The three approaches differ in their treatment of the implications of utility maximization on the demand equations. The first approach, consumer's surplus, does not impose the theoretical restrictions on the data at all. Rather, a substitute restriction is applied to ensure that a unique welfare measure is obtained. This restriction is symmetry of the cross–price derivatives of the ordinary demand functions Burt and Brewer; Chiccetti, Fisher, and Smith; LaFrance and de Gorter). The second approach, which I call *linear compensating variation*, imposes the integrability restrictions at a single point of the data, say, the sample means (Chavas; Huang and Haidacher; Safyurtlu, Johnson, and Hassan). The quantities demanded are the first–order partial derivatives of the expenditure function with respect to prices, while the Slutsky substitution terms are the second–order cross partial derivatives. Consequently, at the point of Slutsky symmetry for the demand equations, we can apply Taylor's theorem to obtain a quadratic expansion of the expenditure function with respect to prices. This allows us to approximate the exact compensating variation of a price change from the base point to second order, in line with the arguments suggested by Burt and Brewer. The third approach is *weak integrability*. This approach imposes the theoretical restrictions on the demand functions over a range of values of the data. This approach permits the recovery of the dual preference structure for the goods of interest and the calculation of the exact welfare measures for changes in the prices of those goods (LaFrance and Hanemann). Its main drawback is that the integrability restrictions are nonlinear in the parameters.

I also undertake a comparison of the relative merits, or lack thereof, of the three different approaches. The comparison utilizes a system of five demand equations for the per capita consumption of U.S. dairy products over the period 1950 through 1985. The functional form for the consumer's surplus and linear compensating variation models is linear in the variables and in the parameters, which is a common choice for multiproduct welfare analyses (Burt and Brewer; Chicchetti, Fisher, and Smith; Freebairn and Rausser; and LaFrance and de Gorter). The restrictions for both of these models are also linear in the parameters, which is convenient for estimation. The linear model is convenient for one other reason. It permits the empirical application of a newly discovered weakly integrable incomplete demand model that has the same number of parameters with comparable restrictions as the consumer's surplus model, although the integrable model is nonlinear in the parameters (LaFrance 1990).

The results of the empirical comparison are consistent with, but stronger than, the results of previous work involving single demand equations. Because per capita demands for dairy products are very price and income inelastic, the lion's share of the consumer welfare effects of the retail price distortions due to the dairy program are income transfers away from consumers. These transfers are common to all of the welfare measures compared in the study. As a result, the magnitude of the total change in consumer welfare is similar across the different measures (Willig). However, the normative economic evaluation of policies focuses on the deadweight loss to consumers (Hausman). With this metric, the linear consumer's surplus deadweight loss is less than 17.5 percent of the compensating variation measure. The linearized compensating variation deadweight loss is nearly twice the size of the exact measure and has the wrong sign!

The differences between the linear approximations and the theoretically correct measure are due in part to the fact that linear estimation methods cannot impose curvature restrictions on the ordinary or compensated cross–price effects. To isolate this effect, a fourth model was estimated. This model was restricted to have a symmetric, negative semidefinite matrix of ordinary cross price effects. This restriction improved the performance of the consumer's surplus approximation considerably. The new estimate of deadweight loss is 115 percent of the compensating variation measure. But there are two good reasons not to use this approximation. First, although 15 percent does not appear to be large, it is an unnecessary error. Second, the parameter restrictions for the concave consumer's surplus model are nonlinear and as difficult to impose as integrability.

Welfare Measurement with Incomplete Demand Systems

Throughout the paper I employ notation similar to that in my previous theoretical work on incomplete demand systems, which is summarized here. Let $x = [x_1,...,x_n]'$ be the vector of the goods of interest and $p = [p_1,...,p_n]'$ the corresponding price vector; let $z = [z_1,...,z_m]'$ be the vector of all other commodities and $q = [q_1,...,q_m]'$ the corresponding price vector; let $s = [s_1,...,s_k]'$ be a vector of demographic or other demand shifters; and let income be y. We estimate the n demands for x,

(1) $$x = h(p,q,y,s),$$

but we neither observe nor estimate the demands for z. It is assumed throughout that all prices and income are deflated by a linear homogeneous function of the prices of the other goods. Thus, (p,q,y) are interpreted as "real" prices and income.

In the empirical application, x is the per capita annual consumption of the following five dairy products: fresh milk and cream; butter; cheese; frozen dairy products; and other dairy products (evaporated and nonfat dry milk). The deflator for all prices and income is the consumer price index for nonfood items and the income measure is per capita disposable income. The "prices" of other goods included in the empirical demand equations are the consumer price indices for nonalcoholic beverages (coffee, tea, cocoa, and carbonated soft drinks), for fats and oils (margarine, salad dressings and cooking oils, and lard), and for meats, poultry, fish and eggs. The demographic shift variables are the mean, variance, and skewness of the age distribution of the U.S. population.

Consider a change in the prices of x from p_0 to p_1 with (q,y,s) held constant. Specifically, we are interested in the economic welfare costs for U.S. consumers of the retail price effects of the Federal dairy program. The consumer's surplus measure of the effects of this price change, cs, is defined by the line integral

(2) $$cs = -\int_{p_0}^{p_1} h(p,q,y,s)'dp.$$

This measure is uniquely defined for any monotonic path from p_0 to p_1 if and only if the ordinary cross–price derivatives are symmetric,

(3) $$\partial h^i(p,q,y,s)/\partial p_j = \partial h^j(p,q,y,s)/\partial p_i, \quad i,j = 1, \ldots n.$$

For the linear demand model employed in the next section

(4) $$h(p,q,y,s) = \alpha + A_q q + A_s s + Bp + \gamma y,$$

where α is an $n{\times}1$ vector, A_q is an $n{\times}m$ matrix, A_s is and $n{\times}k$ matrix, B is an $n{\times}n$ matrix, and γ is an $n{\times}1$ vector of parameters, consumer's surplus is well–defined if and only if B is symmetric, $B = B'$. When the demands for x are weakly integrable, the relationship between consumer's surplus and exact welfare measures are examined in detail in LaFrance and Hanemann.

Without integrability, however, it is unclear what relationships, if any, exist between consumer's surplus and other potential welfare measures. As noted in the introduction, the attractions of consumer's surplus, especially in terms of the linear demand model (4), are that the empirical model and implied parameter restrictions are easy to estimate and impose and the consumer's surplus line integral is easy to calculate; it is a simple quadratic form in prices.

Burt and Brewer argued that consumer's surplus provides a second–order approximation to the expenditure function, e(p,q,u,s), in p–space. This is strictly true only if the income effects for the demands for x are zero, since in general the Slutsky matrix, that is, the n×n matrix of compensated substitution effects for x,

$$S \equiv \partial h/\partial p' + \partial h/\partial y h', \tag{5}$$

is the Hessian matrix of the expenditure function with respect to p. However, the idea of a quadratic approximation to indirect preferences as represented by the expenditure function is appealing. The reason is that if (5) is imposed on the estimation problem at the point p_0, then Taylor's theorem implies that a second–order approximation to the expenditure function exists around that point in the form

$$e(p,q,u,s) \approx e(p_0,q,u,s) + \frac{\partial e(p_0,q,u,s)}{\partial p'}(p-p_0) + \tfrac{1}{2}(p-p_0)'\frac{\partial^2 e(p_0,q,u,s)}{\partial p\partial p'}(p-p_0). \tag{6}$$

Hotelling's lemma implies that $x_0 = \partial e(p_0,q,u,s)/\partial p$. On the other hand, the duality between the expenditure function and the indirect utility function implies that

$$\partial e(p_0,q,u,s)/\partial p \equiv h(p,q,e(p_0,q,u,s),s). \tag{7}$$

Therefore, the Slutsky matrix evaluated at the point (p_0,q,u,s),

$$S_0 = \frac{\partial h(p_0,q,e(p_0,q,u,s),s)}{\partial p'} + \frac{\partial h(p_0,q,e(p_0,q,u,s),s)}{\partial y}h(p_0,q,e(p_0,q,u,s),s)', \tag{8}$$

is the n×n matrix of second–order terms for the quadratic expansion.

The exact compensating variation for the price change, cv, is defined by

$$v(p_0,q,y,s) = v(p_1,q,y{-}cv,s), \tag{9}$$

where v(p,q,y,s) is the indirect utility function. Setting both sides of (9) equal to u and solving for the expenditure function provides an equivalent expression for compensating variation in terms of the expenditure function

(10) $$cv = e(p_0,q,u,s) - e(p_1,q,u,s).$$

Combining (6) through (10), we obtain a second–order approximation to compensating variation, cv, as

(11) $$cv \approx -\left[x_0'(p_1-p_0) + \tfrac{1}{2}(p_1-p_0)'S_0(p_1-p_0)\right].$$

For the linear demand model (4), the local symmetry conditions are linear in the parameters,

(12) $$B + \gamma x_0' = B' + x_0\gamma'.$$

This implies that the cv approximation (11) can be obtained conveniently with standard linear estimation methods subject to linear constraints. This is the rationale for calling this measure *linear compensating variation.* Equation (11) is a precise statement of the quadratic approximation to the expenditure function in p–space advocated by Burt and Brewer.

In my first work on incomplete demand models, I found that linear demand models with nonzero income effects are weakly integrable if and only if the model parameters satisfy extreme parameter restrictions (LaFrance 1985). These restrictions imply that all of the goods with linear demands are perfect complements, which makes the linear model an unreasonable choice for exact welfare measurement. For comparison purposes, it is desirable to have an integrable model with the same number of parameters and a roughly equivalent degree of flexibility as the linear consumer's surplus model with symmetric price effects. Fortunately, I recently stumbled across just such a model by considering the integrability conditions for models that are linear in income and linear and quadratic in prices (LaFrance 1990). This model specification is

(13) $$h(p,q,y,s) = \alpha + A_q q + A_s s + Bp + \gamma(y - \alpha' p - p' A_q q - p' A_s s - \tfrac{1}{2}p' Bp).$$

The number of parameters in (13) is the same as the number in the linear demand model (4). Also, it is straightforward to show that the Slutsky substitution matrix is $S = B + \tilde{y}\gamma\gamma'$, where $\tilde{y} \equiv y - p'(\alpha+A_q q+A_s s) - \tfrac{1}{2}p'Bp$, so that symmetry of S is equivalent to symmetry of B. Thus, the integrable model (13) has the same number and form for the symmetry restrictions as the linear consumer's surplus model. However, integrability also requires the Slutsky matrix to be negative semidefinite, leading to nonlinear restrictions between the parameters. Implementation of these restrictions is discussed in the next section.

The *quasi–indirect utility function* for the demand model (13) is

$$(14)\qquad \varphi(p,q,y,s) = (y - p'(\alpha + A_q q + A_s s) - \tfrac{1}{2}p'Bp)e^{-\gamma' p}.$$

This gives the exact compensating variation for the change in prices from p_0 to p_1 as

$$(15)\quad cv = (y - p_1'(\alpha + A_q q + A_s s) - \tfrac{1}{2}p_1'Bp_1) - (y - p_0'(\alpha + A_q q + A_s s) - \tfrac{1}{2}p_0'Bp_0)e^{\gamma'(p_1 - p_0)}.$$

The next section compares the compensating variation measure (15) obtained by estimating the nonlinear demand model (13) subject to symmetry and negative semidefiniteness of the Slutsky matrix, $S = B + \bar{y}\gamma\gamma'$; consumer's surplus,

$$(16)\qquad cs = p_0'(\alpha + A_q q + A_s s + \gamma y) + \tfrac{1}{2}p_0'Bp_0 - p_1'(\alpha + A_q q + A_s s + \gamma y) - \tfrac{1}{2}p_1'Bp_1,$$

obtained by estimating the linear demand model (4) subject to symmetry of the price effects matrix, B; and linear compensating variation (11), obtained by estimating the linear demand model subject to the local Slutsky symmetry conditions (12).

An Empirical Application to the U.S. Dairy Program

For our empirical application, data on average annual retail prices for fresh whole milk, butter, cheese, ice cream, and evaporated milk were collected from several Bureau of Labor Statistics (BLS) and United States Department of Agriculture (USDA) sources. Data on the U.S. population by 10–year age groups was obtained from the 1990 *Economic Report of the President*, *Historical Statistics of the United States, Colonial Times to 1970*, and several issues of *Statistical Abstracts of the United States*. Data on the civilian unemployment rate, the average wage rate for manufacturing workers, the producer price indices for manufacturing materials and for fuels and energy, the rate of return on AAA corporate bonds, and per capita disposable income were obtained from the 1990 *Economic Report of the President*. Per capita annual consumption of U.S. dairy products was obtained from the USDA series *Food Consumption, Prices, and Expenditures*. Consumer price indices for dairy products, nonalcoholic beverages, fats and oils, meats, poultry and fish, and nonfood items were obtained from the 1978 *Handbook of Labor Statistics* and the 1978 through 1986 January issues of the BLS publication, *CPI: Detailed Report*. Space limitations preclude a more detailed discussion of the data here. A complete description of the original data and all transformations are available upon request.

From the standpoint of the effects on retail prices for dairy products, the structure of the U.S. dairy program has not changed since the 1949 Agricultural Adjustment Act. The Federal government intervenes in the dairy market in two ways. In the market for Grade A milk, that is, milk that meets the sanitary requirements to be legally sold for use in fresh milk products, Federal milk marketing orders enforce price discrimination. Processors and handlers are required by law to pay a higher price to farmers for Class 1 milk, milk that is used for fresh milk and cream, than for Class 2 milk, milk that is used for manufactured dairy products such as butter, cheese, and powdered milk. This has the effect of increasing producer revenues, stimulating supply, and leading to a surplus of Grade A milk production that "spills over" into the Grade B market, the market for milk that can be used only in the production of manufactured dairy products. In both the Grade A and Grade B markets, the Federal government supports the farm price of manufacturing milk by purchasing at the wholesale level butter, cheese, and nonfat dry milk at announced prices determined by the farm level support price for manufacturing milk (hereafter, Class 2 milk) and estimated manufacturing costs of production for those products. Over the period 1953 through 1980, the net effect of these two programs at the farm level has been to increase the farm level Class 1 price by about 17 percent and reduce the farm level Class 2 price by about 14 percent (LaFrance and de Gorter).

The wholesale to retail price linkage regression equations for identifying the net retail price effects of the U.S. dairy program are reported in table 1. In table 1, the retail prices of dairy products are predicted by the following factors: (a) a set of general economy variables – the civilian unemployment rate (Unem), manufacturing wage rates (Wage), producer price indices for manufacturing materials (P_{mtl}) and for fuels and energy (P_{fuel}), and the real rate of return on AAA corporate bonds (r_{bond}); (b) the mean (Avg), variance (Var), and skewness (Skew) of the age distribution of the U.S. population; (c) government dairy price variables – government purchase prices for butter (GP_{btr}), cheese (GP_{chs}), and nonfat dry milk (GP_{dm}), the average minimum Class 1 milk price (S_1), and the average support price for Class 2 milk (S_2); and (d) the consumer price indices for fats and oils (P_{fat}), nonalcoholic beverages (P_{bev}), and meats, poultry, fish, and eggs (P_{meat}). All prices are deflated by the consumer price index for nonfood items. The real rate of return on corporate bonds, r_{bond}, is constructed as follows:

$$(16) \qquad r_{bond,t} = 1 - \left[\frac{1 + i_{bond,t}}{1 + (cpi_{nf,t} - cpi_{nf,t-1})/(cpi_{nf,t} + cpi_{nf,t-1})}\right],$$

where $cpi_{nf,t}$ is the consumer price index for nonfood items in year t and $i_{bond,t}$ is the nominal rate of return on AAA corporate bonds. The consumer price index for nonfood items was used to generate all of the real price variables in order to employ a broadly defined deflator without introducing any unintended simultaneity problems between the left– and right–hand–side variables in the regression equations. Per capita disposable income is not included among the regressors for the retail price equations because of extreme multicollinearity. The condition index for the scaled matrix of variables including the above list, an intercept, and real per capita disposable income is 7445. Belsley *et. al* and Belsley provide evidence that multicollinearity leads to numerical problems with condition indices as low as 100. Regressing income on the other explanatory variables results in a nearly perfect fit ($R^2 = .9992$).

The second part of the empirical analysis links the farm–level goals of the Federal dairy program to the government purchase prices with three regression equations that predict the government purchase prices for butter, cheese, and nonfat dry milk with the farm level support price for Class 2 milk and the prices of other manufacturing inputs. The results are:

$$\underset{}{P_{btr}} = \underset{(1.56)}{11.2} - \underset{(.417)}{2.42}*Wage - \underset{(.242)}{.320}*P_{mtl} - \underset{(.188)}{.814}*P_{fuel} + \underset{(.156)}{1.11}*S_2 + \underset{(.124)}{1.53}*u_{-1} - \underset{(.124)}{.666}*u_{-2}$$

$$R^2 = .984 \qquad s = .157 \qquad dw = 1.96$$

$$P_{chs} = \underset{(.426)}{-.587} + \underset{(.0729)}{.502}*Wage + \underset{(.0970)}{.00187}*P_{mtl} - \underset{(.0430)}{.121}*P_{fuel} + \underset{(.0503)}{.954}*S_2 + \underset{(.154)}{.379}*u_{-1}$$

$$R^2 = .979 \qquad s = .0803 \qquad dw = 1.62$$

$$P_{dm} = \underset{(1.05)}{-4.53} + \underset{(.274)}{1.15}*Wage + \underset{(.180)}{.275}*P_{mtl} + \underset{(.127)}{.406}*P_{fuel} + \underset{(.108)}{.460}*S_2 + \underset{(.161)}{1.13}*u_{-1} - \underset{(.124)}{.259}*u_{-2}$$

$$R^2 = .983 \qquad s = .113 \qquad dw = 2.09$$

In this set of regression results, and all others in the paper, the numbers in parentheses are estimated asymptotic standard errors of the regression coefficients, R^2 is the correlation between the observed and predicted value of the untransformed dependent variable, "s" is the standard error of the estimate for the regression equation, and "dw" is the Durbin–Watson test statistic for serially correlated residuals.

Table 2 presents the three stage least squares regression results for the system of five per capita demands for U.S. dairy products. Results are presented for four separate models. The complete list of instruments for the three stage least squares estimation procedure is {Unem, Wage, P_{mtl}, P_{fuel}, r_{bond}, Avg, Var, Skew, GP_{btr}, GP_{chs}, GP_{dm}, S_1, S_2, P_{fat}, P_{bev}, P_{meat}, Incm}. For the first two models in table 2, estimation is by linear three stage least squares. For the last two models, the estimation procedure is nonlinear three stage least squares (Jorgenson and Laffont). Three stage least squares was used to obtain consistent estimates of the model parameters in the presence of simultaneous determination between retail prices and quantities demanded. In all of the results reported in table 2, the R^2, standard error of the estimate, and Durbin–Watson statistic for serial correlation between the error terms are calculated for the untransformed variables.

The first set of results is the linear consumer's surplus model where only the cross–price symmetry restrictions are imposed. The second set of results is the linear compensating variation model where the Slutsky symmetry conditions are imposed at the sample means. As can be seen from table 2, the problem with both of these sets of estimation results is that all of the point estimates for the income effects are positive, while only one of the own–price effects is negative. This is a serious weakness of both of these models. However, the eigenvalues for the Slutsky substitution matrix calculated at the sample means for these two models are:

Linear CS Model:	–92.44	–12.56	–3.87	9.54	13.43
Linear CV Model:	–91.57	–12.59	–3.74	9.48	13.44.

Since three of the five eigenvalues are negative in both of these models, it appears that the difficulty is due to a high degree of collinearity between the dairy product price coefficients.

In an effort to test whether or not this is indeed the case, the consumer's surplus model was re–estimated with the price effects matrix restricted to be symmetric, negative semidefinite. This was accomplished by a Choleski factorization, $B = -L'D^2L$, where L is an upper triangular 5×5 matrix with ones on the main diagonal and D is a diagonal 5×5 matrix. During the estimation of this model, three of the elements of D insisted on converging to zero. This results in a collinearity problem with the elements of L that are associated with the zero diagonals for D. Hence, the constraints $\delta_3^2 = \delta_4^2 = \delta_5^2 = 0.0001$ were imposed and a grid search over the values of ℓ_{25}, ℓ_{34}, ℓ_{35}, and ℓ_{45} was conducted. The values $\ell_{25} = 100$, $\ell_{34} = 0$, $\ell_{35} = -10$, and $\ell_{45} = 500$ resulted in a weighted sum of squared errors criterion, $Q(\beta) = 72.43$, with the following sensitivity to changes in these elements of L:

$\ell_{25} = 0$	$Q(\beta) = 73.81$	$\ell_{25} = 200$	$Q(\beta) = 72.42$
$\ell_{34} = -10$	$Q(\beta) = 72.43$	$\ell_{34} = 10$	$Q(\beta) = 72.43$
$\ell_{35} = -20$	$Q(\beta) = 72.43$	$\ell_{35} = 20$	$Q(\beta) = 72.43$
$\ell_{45} = 0$	$Q(\beta) = 72.47$	$\ell_{45} = 1000$	$Q(\beta) = 75.57$

All other parameters were estimated conditional on the fixed values for δ_3, δ_4, δ_5, ℓ_{25}, ℓ_{34}, ℓ_{35}, and ℓ_{45}. The standard errors for the elements of B were derived with Slutsky's theorem, conditional on the fixed values of the locked out parameters. A joint F–test of the symmetry and concavity restrictions with 13 degrees of freedom (10 symmetry conditions and 3 binding concavity restrictions) using the degrees of freedom corrections suggested by Laitinen, Meisner, and Judge *et. al* gives an F–statistic of F(13,115) = 1.63. The 5 percent critical value for the F(13,115) distribution is 1.80, so that we can not reject the joint hypothesis of symmetry and negative semidefiniteness for B. The nonlinear three stage least squares regression results are reported in the third part of table 3.

The final set of results in table 3 are the parameter estimates and regression statistics for the weakly integrable demand model. Two additional estimation issues had to be addressed for this model. First, it follows from equation (13) that 51 of the 55 model parameters enter each of the demand equations through the transformed income term, $\tilde{y}$. While this does not cause any difficulties from the standpoint of the asymptotic regression theory, it is important to a sample with only 36 observations. The reason is that the empirical model can (and will!) tend to fit one of the demand equations perfectly, which gives nonsense for results. To deal with this problem, I used the iterative two–stage estimation procedure discussed in LaFrance (1989, 1991), in which the current set of values for $\tilde{y}$ are generated from the previous estimates of the model parameters and $\tilde{y}$ is held fixed during the current iterative round. Consistency and asymptotic normality of the convergent solution to this procedure are shown in LaFrance (1989). To deal with the joint determination of $\tilde{y}$ during estimation of the model parameters, I treated $\tilde{y}$ as one of the endogenous variables in the three–stage nonlinear least squares estimation procedure. Fortunately, the iterative process tended to converge very quickly, requiring an average of only 5 to 6 iterations to obtain convergence within 5 significant digits of the parameters.

The second issue regarding estimation of the integrable model results from the fact that the Slutsky substitution matrix, $S = B + \tilde{y}\gamma\gamma'$, is not constant across observations of the data. This does not represent any difficulty with respect to the symmetry of B, but it is a problem with respect to the global negative semidefiniteness of S. I handled this difficulty by imposing negative definiteness on S at the sample mean of the current estimated series for $\tilde{y}$. An alternative procedure would be to require S to be negative semidefinite at the maximum value of $\tilde{y}$, thereby ensuring that the expenditure function is concave in p at all data points.

The iterative procedure is summarized as follows. At each iterative stage of the estimation process, as $\tilde{y}$ is updated also update the fixed value for the point of strict concavity, $\tilde{y}_0$. Write the symmetric, negative definite mean Slutsky matrix, $S_0 = B + \tilde{y}_0\gamma\gamma'$, in terms of a Choleski factorization $S_0 = -L'D^2L$, where L and D are defined as before. Then solve this for the price effects matrix as $B = -(L'D^2L + \tilde{y}_0\gamma\gamma')$ and estimate the elements of L, D, and γ rather than B. For a positive $\tilde{y}_0$ (the converged value is 2389.6), this transformation shows clearly that negative definiteness of the mean Slutsky matrix is a much stronger restriction than negative semidefiniteness of the price effects matrix B.

As in the case of the concave consumer's surplus model, three of the elements of D insisted on converging to zero and the constraints $\delta_2^2 = \delta_3^2 = \delta_4^2 = 0.0001$ were imposed. A grid search over ℓ_{23}, ℓ_{24}, ℓ_{25}, ℓ_{34}, ℓ_{35}, and ℓ_{45} resulted in the values $\ell_{23} = \ell_{34} = \ell_{35} = \ell_{45} = 0$, $\ell_{24} = -5$, and $\ell_{25} = 80$, with a least squares criterion of $Q(\beta) = 72.19$. There was no change in the least squares criterion to four places for the following range of values in these parameters: $\ell_{23} = -2, +2$; $\ell_{24} = -10, 0$; $\ell_{25} = 0, 160$; $\ell_{34} = -30, +30$; $\ell_{35} = -20, 20$; and $\ell_{45} = -20, +20$. As in the previous case, all other parameters were estimated conditional on the fixed values for δ_3, δ_4, δ_5, ℓ_{23}, ℓ_{24}, ℓ_{25}, ℓ_{34}, ℓ_{35}, and ℓ_{45}. The standard errors for the elements of B were derived with Slutsky's theorem, conditional on the fixed values of the locked out parameters. A joint F–test of the symmetry and concavity restrictions with 13 degrees of freedom gives an F–statistic of $F(13,115) = 1.62$, so that we can not reject the joint hypothesis of symmetry and negative definiteness for S_0.

One aspect of these empirical results worth emphasizing is the fact that the overall statistical properties of the weakly integrable demand model are as good as, indeed virtually indistinguishable from, those of the concave consumer's surplus model. Furthermore, there is very little degradation in the summary statistics for the integrable model relative to either of the linear approximations. Given the fact that the integrable model is consistent with economic theory and there is no compelling empirical evidence in the data leading us to reject it, my position is that, from a logical viewpoint, this is the clearly preferred alternative.

The 1950–1985 average farm, wholesale, and retail dairy prices in 1989 dollars are:

Support Prices		Government Purchase Prices			Retail Prices				
Class 1	Class 2	Butter	Cheese	Dry Milk	Milk	Butter	Cheese	Frozen	Other
\$21.22	\$15.40	\$21.18	\$16.63	\$9.00	\$1.85	\$3.00	\$3.15	\$3.22	\$0.67

The average farm milk price without price discrimination or government purchases of manufactured products is taken from LaFrance and de Gorter as \$17.59/cwt (\$4.75 in 1967 dollars). The predicted wholesale and retail prices (also converted to 1989 dollars) obtained by setting both farm support prices at this level and the other variables at their sample means are:

Predicted Wholesale Prices			Predicted Retail Prices				
Butter	Cheese	Dry Milk	Milk	Butter	Cheese	Frozen	Other
\$26.37	\$18.74	\$9.37	\$1.70	\$2.74	\$2.96	\$3.00	\$0.67

These are the relative price changes that are used to construct our comparison of the welfare measures discussed in the previous section. All other variables on the right–hand–sides of the demand equations are set at their sample means for the comparison.

The average income transfer away from consumers is the sum of the price difference for each good times the historical average quantity demanded. In millions of 1989 dollars, this figure is $10,108. The welfare measures for the price changes from the historical average to the predicted levels, in millions of 1989 dollars, are:

	Linear CS	Linear CV	Concave CS	Integrable
Welfare Measure	$10,117	$10,102	$10,165	$10,158
Deadweight Loss	$8.73	$–95.80	$56.61	$49.99

Over 99 percent of each of the welfare measures is income transfer. This transfer is common to all of the four measures by construction. Thus, it is not surprising that all of the welfare measures are within one percent of each other. But this is precisely why Hausman's critique is so important in empirical welfare analysis. It is the relative size of the change in consumer welfare net of any income transfers that matters in cost–benefit analysis. And this is where the approximations can not stand up to the test. The linear consumer's surplus measure is 82.5 percent less than the compensating variation measure. The linear compensating variation measure has the wrong sign, undoubtedly because the linearized estimates do not impose the proper curvature on the data. Even the concave consumer's surplus measure overstates the exact compensating variation by 15 percent.

Conclusion

The concave consumer's surplus model is as difficult to estimate as the integrable model, yet does not have any direct interpretation in terms of consumer behavior. I can see no reason to perform applied welfare analysis in a manner that is not logically consistent. The *ad hoc*, short–cut method of linear consumer's surplus gives significantly different answers for deadweight loss than either concave consumer's surplus or exact welfare procedures. The imposition of concavity requirements on the consumer's surplus approach makes that procedure as "onerous" as exact welfare estimation and lacks the satisfaction of being logically and theoretically correct. So, why *not* do it right?

References

Belsley, D. A., *Conditioning Diagnostics*, New York: John Wiley & Sons, 1991.

Belsley, D. A., E. Kuh, and R. E. Welsh, *Regression Diagnostics*, New York: John Wiley and Sons, 1980.

Burt, O. and D. Brewer, "Estimation of Net Social Benefits from Outdoor Recreation," *Econometrica*, 39 (1971): 813–827.

Chavas, J–P., "Structural Change in the Demand for Meat," *American Journal of Agricultural Economics*, 65 (1983): 148–153.

Chiccetti, C., A. Fisher, and V. K. Smith, "An Econometric Evaluation of a Generalized Consumer Surplus Measure: The Mineral King Controversy," *Econometrica*, 44 (1976): 1259–1276.

Freebairn, J., and G. Rausser, "Effects of Changes in the Level of U.S. Beef Imports," *American Journal of Agricultural Economics*, 57 (1975): 676–688.

Huang, K., and R. Haidacher, "Estimation of a Composite Food Demand System for the United States," *Journal of Business and Economic Statistics*, 1 (1983): 285–291.

Jorgenson, D. W. and J. Laffont, "Efficient Estimation of Nonlinear Simultaneous Equations with Additive Disturbances," *Annals of Economics and Social Measurement*, 3 (1974): 615–640.

Judge, G. G., W. E. Griffiths, R. C. Hill, H. Lutkepohl, and T–C. Lee, *The Theory and Practice of Econometrics*, 2nd Edition, New York: John Wiley and Sons, 1985.

LaFrance, J., "Linear Demand Functions in Theory and Practice," *Journal of Economic Theory*, 37 (1985): 147–166.

LaFrance, J., "The Structure of Constant Elasticity Demand Models," *American Journal of Agricultural Economics*, 68 (1986): 543–552.

LaFrance, J., "Weak Separability and Two–Stage Budgeting: Problems of Estimation and Welfare Measurement," presented at the Australasian meeting of the Econometric Society, University of New England, Armidale (1989).

LaFrance, J., "Incomplete Demand Systems and Semilogarithmic Demand Models," *Australian Journal of Agricultural Economics*, 34 (1990): 118–131.

LaFrance, J., "When Is Expenditure 'Exogenous' in Separable Demand Models?" *Western Journal of Agricultural Economics*, 16 (1991): 49–62.

LaFrance, J. and H. de Gorter, "Regulation in a Dynamic Market: The U.S. Dairy Industry," *American Journal of Agricultural Economics*, 67 (1985): 821–832.

LaFrance, J. and W. M. Hanemann, "The Dual Structure of Incomplete Demand Systems," *American Journal of Agricultural Economics*, 71 (1989): 262–274.

Laitinen, K., "Why is Demand Homogeneity so Often Rejected," *Economics Letters*, 1 (1978): 187–191.

Meisner, J. F., "The Sad Fate of teh Asymptotic Slutsky Asymmetry Test for Large Systems," *Economics Letters*, 2 (1979): 231–233.

Safyurtlu, A., S. Johnson, and Z. Hassan, "Recent Evidence on Market Demand Systems for Food in Canada," *Canadian Journal of Agricultural Economics*, 34 (1986): 475–493.

Willig, R., "Consumer's Surplus Without Apology," *American Economic Review*, 66 (1976): 589–597.

Table 1. Wholesale to Retail Price Regressions for the U.S. Dairy Market

Retail Price	Const.	Unem Rate	Manf Wage	Mtls PPI	Fuel PPI	AAA Bond	Avg Age	Var Age	Skew Age	Govt. Purch. Prices Butter	Cheese	Nfdm	Support Prices Cl. 1	Cl. 2	Fat CPI	Bev CPI	Meat CPI	AR(1)
Milk	3.18	.00235	.121	-.0159	.0118	-.00397	-.0962	.0136	-.0129	-.0202	-.0691	.00554	.0343	.0828	-.0411	-.00412	.0309	-.249
	(.655)	(.0014)	(.0463)	(.0209)	(.0100)	(.0010)	(.0141)	(.0054)	(.0022)	(.0117)	(.0181)	(.0231)	(.0080)	(.0215)	(.0248)	(.0128)	(.0238)	(.161)
Butter	-4.31	-.00138	-.108	-.0103	-.00175	.00206	.0813	.0395	-.00238	.0925	.0527	-.0175	.0714	-.0775	.0531	-.0869	.117	.676
	(1.70)	(.0041)	(.135)	(.0645)	(.0319)	(.0033)	(.0390)	(.0160)	(.0769)	(.0346)	(.0468)	(.0604)	(.0193)	(.0596)	(.0724)	(.0353)	(.0645)	(.123)
Cheese	-3.53	-.00121	-.0585	.0613	-.00843	.00347	.102	-.00965	.0203	-.00832	-.0205	-.0433	.0677	.0893	.0690	-.0595	.0335	.610
	(3.28)	(.0079)	(.260)	(.124)	(.0607)	(.0064)	(.0747)	(.0301)	(.0145)	(.0658)	(.0908)	(.117)	(.0376)	(.115)	(.140)	(.0680)	(.125)	(.132)
Frozen	6.35	.00691	.240	-.0254	.0806	-.00008	-.114	-.0376	-.0226	.146	-.151	.206	.0155	-.138	.00427	-.0727	.139	.747
	(2.05)	(.0049)	(.163)	(.0773)	(.0387)	(.0024)	(.0472)	(.0199)	(.0295)	(.0419)	(.0557)	(.0722)	(.0228)	(.0713)	(.0869)	(.0423)	(.0769)	(.111)
Other	-.188	.00133	.0355	.0115	.0151	-.00054	.00099	.00475	-.00240	-.00094	-.0283	.00188	.0125	.0306	-.00258	.00052	-.00008	.657
	(.485)	(.0012)	(.0385)	(.0184)	(.0905)	(.0009)	(.0111)	(.0451)	(.0022)	(.00981)	(.0133)	(.0172)	(.0055)	(.0170)	(.0206)	(.0101)	(.0184)	(.126)

Regression Summary Statistics

	Milk	Butter	Cheese	Frozen	Other
R^2	.996	.990	.953	.993	.950
s	.00491	.0149	.0283	.0182	.00422
dw	2.02	1.55	1.80	1.60	1.76

Numbers in parenthesis are standard errors, "s" is the standard error of the estimate, and "dw" is the Durbin-Watson statistic.

Table 2. Three Stage Least Squares Regression Results for the U.S. Dairy Demand Model

Linear Consumer's Surplus Model

	Const.	Bev CPI	Fat CPI	Meat CPI	Avg Age	Var Age	Skew Age	Income	Retail Dairy Prices Milk	Butter	Cheese	Frozen	Other	R^2	s	dw
Milk	1832	3.97	7.29	-19.4	-36.0	-5.37	-4.27	.0299	75.7	-3.22	-16.5	-4.28	-24.7	.989	3.09	1.42
	(158)	(8.80)	(3.68)	(12.5)	(4.19)	(1.50)	(.620)	(.0081)	(41.0)	(3.76)	(5.06)	(5.01)	(16.3)			
Butter	47.1	2.71	-.490	-.282	-.401	-.529	.0350	.00011	-3.22	1.78	-11.0	3.00	3.93	.984	.235	1.73
	(16.6)	(1.00)	(.378)	(1.04)	(.476)	(.162)	(.0866)	(.0007)	(3.76)	(1.78)	(2.44)	(1.64)	(5.70)			
Cheese	-62.1	-3.39	-.0908	2.83	2.03	.673	-.241	.00146	-16.5	-11.0	5.68	-1.04	8.48	.997	.272	2.09
	(22.9)	(1.33)	(.461)	(1.16)	(.694)	(.240)	(.138)	(.0009)	(5.06)	(2.44)	(4.33)	(2.48)	(10.3)			
Frozen	28.5	2.03	-2.33	-5.43	-1.03	.317	.193	.00440	-4.28	3.00	-1.04	2.06	7.34	.979	.323	1.91
	(21.2)	(1.17)	(.517)	(1.42)	(.555)	(.199)	(.100)	(.0011)	(5.01)	(1.64)	(2.48)	(2.93)	(6.92)			
Other	225	-1.91	-.469	-1.98	-6.44	.744	-.964	.00219	-24.7	3.93	8.48	7.34	-.806	.994	.474	2.59
	(68.7)	(2.98)	(1.10)	(2.33)	(1.73)	(.603)	(.290)	(.0019)	(16.3)	(5.70)	(10.3)	(6.92)	(51.3)			

Linear Compensating Variation Model

	Const.	Bev CPI	Fat CPI	Meat CPI	Avg Age	Var Age	Skew Age	Income	Retail Dairy Prices Milk	Butter	Cheese	Frozen	Other	R^2	s	dw
Milk	1832	3.85	7.26	-19.5	-36.0	-5.37	-4.27	.0301	75.9	-3.40	-16.5	-3.77	-24.7	.989	3.09	1.41
	(158)	(8.80)	(3.68)	(12.6)	(4.17)	(1.50)	(.616)	(.0031)	(41.0)	(3.77)	(5.09)	(5.07)	(16.4)			
Butter	47.1	2.71	-.491	-.281	-.400	-.528	.0352	.00010	-3.24	1.78	-11.0	3.01	3.98	.984	.235	1.73
	(16.7)	(1.00)	(.379)	(1.04)	(.475)	(.163)	(.0866)	(.0007)	(3.76)	(1.79)	(2.44)	(1.64)	(5.71)			
Cheese	-62.3	-3.39	-.0935	2.83	2.03	.674	-.240	.00146	-16.4	-11.0	5.68	-1.01	8.43	.997	.272	2.09
	(22.9)	(1.33)	(.462)	(1.16)	(.694)	(.240)	(.138)	(.0009)	(5.07)	(2.45)	(4.35)	(2.49)	(10.3)			
Frozen	28.5	2.03	-2.32	-5.43	-1.03	.317	.193	.00441	-4.24	2.98	-1.04	2.08	7.22	.979	.323	1.91
	(21.2)	(1.17)	(.518)	(1.42)	(.556)	(.199)	(.100)	(.0011)	(5.03)	(1.65)	(2.50)	(2.93)	(6.96)			
Other	226	-1.89	-.474	-1.97	-6.45	.738	-.963	.00217	-24.9	3.96	8.41	7.22	-.0322	.994	.474	2.59
	(68.8)	(2.98)	(1.10)	(2.35)	(1.73)	(.604)	(.290)	(.0019)	(16.3)	(5.71)	(10.3)	(6.93)	(51.4)			

Table 2, continued.

Concave Consumer's Surplus Model

		Bev	Fat	Meat	Avg	Var	Skew		Retail Dairy Prices							
	Const.	CPI	CPI	CPI	Age	Age	Age	Income	Milk	Butter	Cheese	Frozen	Other	R^2	s	dw
Milk	2000	8.23	7.61	-9.35	-41.7	-4.47	-4.66	.0327	-11.5	-5.40	-8.31	1.47	8.30	.988	3.32	1.13
	(161)	(8.54)	(3.87)	(12.0)	(4.07)	(1.59)	(.654)	(.0088)	(8.87)	(2.26)	(2.74)	(1.73)	(6.60)			
Butter	79.0	.927	.218	.234	-1.29	-.320	-.196	-.00035	-5.40	-2.55	-3.85	.749	3.50	.982	.246	1.28
	(13.2)	(.681)	(.332)	(1.03)	(.329)	(.142)	(.0539)	(.0007)	(2.26)	(.888)	(.933)	(.811)	(3.01)			
Cheese	-114	-.629	-1.14	1.50	3.74	.186	.153	.00162	-8.31	-3.85	-7.07	.142	12.6	.996	.292	2.07
	(15.0)	(.861)	(.374)	(1.17)	(.413)	(.196)	(.0834)	(.0008)	(2.74)	(.933)	(2.28)	(1.47)	(6.09)			
Frozen	35.1	1.11	-1.74	-5.42	-.895	.207	.151	.00343	1.47	.749	.142	-.994	4.60	.979	.324	1.91
	(16.5)	(.851)	(.388)	(1.27)	(.421)	(.179)	(.0756)	(.0009)	(1.73)	(.811)	(1.47)	(1.13)	(3.16)			
Other	132	-2.07	1.03	-3.28	-3.88	.917	-.926	.000025	8.30	3.50	12.6	4.60	-71.6	.991	.579	1.75
	(44.0)	(2.03)	(.858)	(2.34)	(1.16)	(.436)	(.209)	(.0018)	(6.60)	(3.01)	(6.09)	(3.16)	(23.0)			

Weakly Integrable Model

		Bev	Fat	Meat	Avg	Var	Skew		Retail Dairy Prices							
	Const.	CPI	CPI	CPI	Age	Age	Age	Income	Milk	Butter	Cheese	Frozen	Other	R^2	s	dw
Milk	2019	8.96	7.69	-9.03	-42.2	-4.41	-4.74	.0317	-9.33	-4.26	-6.91	.803	9.47	.988	3.34	1.12
	(171)	(8.57)	(3.87)	(11.9)	(4.33)	(1.60)	(.682)	(.0871)	(5.91)	(1.95)	(2.60)	(.968)	(5.66)			
Butter	79.5	.746	.176	.0203	-1.27	-.358	-.191	-.00029	-4.26	-2.66	-4.20	.709	5.22	.981	.253	1.22
	(13.8)	(.698)	(.328)	(1.02)	(.346)	(.133)	(.0551)	(.0007)	(1.95)	(.927)	(.660)	(.662)	(1.37)			
Cheese	-116	-.829	-1.13	1.41	3.75	.212	.144	.00178	-6.91	-4.20	-6.64	1.04	9.24	.996	.292	2.07
	(15.4)	(.826)	(.361)	(1.14)	(.424)	(.181)	(.0819)	(.0008)	(2.60)	(.660)	(1.95)	(1.05)	(3.10)			
Frozen	38.6	1.20	-1.71	-5.27	-1.08	.279	.117	.00361	.803	.709	1.04	-.395	1.67	.979	.320	1.93
	(16.6)	(.831)	(.378)	(1.20)	(.414)	(.170)	(.0709)	(.0008)	(.968)	(.662)	(1.05)	(.310)	(1.92)			
Other	133	-1.31	.931	-3.47	-3.63	.714	-.858	-.00050	9.47	5.22	9.24	1.67	-63.0	.990	.609	1.64
	(42.9)	(1.73)	(.789)	(2.34)	(.977)	(.379)	(.145)	(.0016)	(5.66)	(1.37)	(3.10)	(1.92)	(25.0)			