A Unanimous Consent Solution to the Supply of Public Goods:

Getting PPI Rules from a PI Process

Michael C. Farmer

Department of Agricultural Economics and Rural Sociology
The Ohio State University
2120 Fyffe Road
Columbus, Ohio 43210-1099

AAEA 1991

Funding for this research was provided by a grant from the National Science Foundation. NSF Grant Order: BBS-8710153

The author thanks Alan Randall for stimulating my interest in the topic and for providing an opportunity to work on this subject.

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ABSTRACT

I model a cooperative bargain for the supply of non-rival goods. The model departs from cooperative games generally by accepting a second best framework and core reducing behavior
by the implementation problem. The solution admits the Kaldor-Hicks hypothetical consent efficiency rules as decision rules to a unanimous consent game.

INTRODUCTION

The rationale typically offered for benefit cost rules in the public arena depends on utilitarian ethics and rational planning institutions. However, alternative rationales are occasionally offered. Also Randall (1984,1985) has suggested, but not demonstrated formally, that a society of rational individualists would chose to invoke a benefit cost rule but always subject to exceptions; individualists would want rules that restrain the rent-seeking activities of others, while leaving the doors open to their own rent-seeking efforts.

Here I develop a formal cooperative game that generates the result that Randall described. The game has several properties which give it appealing plausibility. In an environment with nonrival goods, where Pareto optimal transfers are prohibitively costly, individualists would agree to impose a benefit cost rule but would permit exceptions in order to retain (buy) the continuing allegiance of the most dissatisfied members of society.

BACKGROUND

The difficulty of identifying a principled method for determining decision rules for public investments is deeply rooted in the history of economic thought.

Perhaps the most persuasive paradigm regarding this issue was formally introduced by Erik Lindahl in 1919 (Lindahl 1919). The salient features of his model, known today as the Lindahl Equilibrium, extended the accepted notions of the 19th century marginalists regarding competitive market

equilibria to the joint determination of the level of production and the level of taxation assessed to each individual (the price) for public projects.

This decision model of public expenditure suggests a result that upholds the equi-marginal principle of consumer choice, generating a result that is Pareto Optimal. Also referred to as the benefits received principle (Tresch,1985) because no individual is taxed in excess of her marginal willingness to pay, any movement from this level of production once achieved will make at least one person worse off. The optimal properties of this thought experiment have been rigorously upheld. Foley (Foley, 1970) has shown the Lindahl Equilibrium to be an optimal result when we model public goods as private exchange markets; and with costless transfer this point is in the core. Yet we argue below that the result is not robust with respect to other, more plausible models of inter-personal public expenditure decision processes.

It is not trivial to note that these markets are not private exchange markets. The implementation problem associated with a rigorous application of the benefits received principle has pre-empted any widespread application of the Lindahl solution. The difficulty of efficiently estimating the aggregate demand for a public good[6] is well documented; yet the financing side of the Lindahl solution requires individuals to identify themselves at least as a type of consumer for every public good in order to assess their tax prices. The problem of soliciting preference revelation at this level of detail is at

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[6] The discussion here relates generally to both non-rival and non-exclusive goods. The model presented later, however, is restricted to non-rival goods. The model can be generalized to include externality issues or natural monopolies while maintaining the essential results regarding the choice of benefit-cost rules.

best an expensive exercise and at worst strategically impossible given reasonable constraints on government.[7]

So instead of a joint, simultaneous quantity/price solution for each and every project, the single identification of the quantity suggested by the Lindahl solution has emerged as a de facto decision rule. The benefits received principle and actual Pareto Improvements(PI) have given way of necessity to the Kaldor-Hicks recommendations of Potential Pareto Improvements(PPI), or the normal practice of benefit-cost decisions where compensatory
transfers do not actually transpire. With careful attention to any strong income effects, this PPI quantity solution could at least maximize social surplus.

Yet to committed individualists as Wicksell, this was simply not enough. If we require our public institutions and decision rules at least to be potentially explained by a unanimity process as the litmus test of legitimacy, then surely the prima facie case for benefit-cost rules is fairly weak.

Rationale for a Bargaining Model

A model of a game to jointly determine rules for public expenditure and taxation in an idealized original position where no public expenditure exists suggests several structural and behavioral characteristics.

Initially we presume agents possess well ordered preferences over the set of goods and services to be produced by a public entity. The array of these goods is finite and they are currently undersupplied by private exchange

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[7]. Multi-part schemes of preference revelation seem unlikely to emerge from an original position unless anonymity is imposed. Yet we will assume these revelation methods are allowed but are simply very costly.

markets as a result of technical features of the goods themselves (we will focus on non-rivalry). It is assumed that efficient estimates of aggregate demand, as well as the demands of broad agent types that compose the aggregate demand, for these goods are accessible without incurring costs that erode the value of collecting such information and that this information is common knowledge. Yet the informational demands of first degree price discrimination required to implement the Lindahl tax rule are relatively more costly.

A more important modelling assumption regards the strategy space of agent plays. The notion of agent by agent exchange is not considered rich enough to encompass the scope of interactions. Instead coalition formation within a global bargaining process is hypothesized to better capture social interactions in the decision process of supplying and financing public goods. Individuals are allowed as always to enter bondable contracts with one another, but combinations of agents, coalitions, may also form. These coalitions may also negotiate and enter contracts with individuals (singletons) or other n-tuple coalitions; and as such they are rightly considered agents of the game. Exchange or bargaining occurs as coalitions agree to disbursement arrangements of the total surplus generated, the value of that coalition, when the coalition members agree to form. Since all combinations of singletons are potential coalitions that can be formed at any moment, the relative value of all potential coalitions are relevant to any particular suggested disbursement of the surplus generated by the grand coalition.[8] So in determining the disbursement of the aggregate surplus, value, generated by the creation of public goods to each member of society, an

[8] The grand coalition is the unique coalition formed by the agreement of all coalitions.

individual may press her claim through the entire array of potential coalition agreements that she may enter.

Two final assumptions regarding the bargain are noted. First we are playing an exclusively second best game. Direct transfer among agents is not considered. Indeed, the cost of actually 'typing' an individual as a particular type of demander of a public good is a core motivation for this exercise. The bargain operates by a set of agreements over rules for producing and financing the creation of public goods and the strategy space is so restricted. Lastly the game is considered to be superadditive. Specifically,

$$v(S \cup T) \geq v(S) + v(T)$$

$$\text{for all } S,T \subseteq I,\ S \cap T = \{\varnothing\},$$

with v(s): value of a coalition and I: set of all singletons.
Even more we are interested in so called essential games where:

$$v(I) > \sum v(\{i\}) \text{ strictly for all } i \in \{I\}.$$

That is a contract between coalitions with no members in common can always identify a disbursement that generates actual Pareto Improvements or Pareto neutral changes to members, but the formation of the grand coalition is able to identify a disbursement that unambiguously creates a PI over the initial situation of the game with only independent singletons.

This, of course, is a cooperative game. The bargaining process is not a non-cooperative market, but rather a contractual game of consent, or 'c-game', which is offered as a more direct characterization of the social choice process.

Model of Rule Selection

There are various solution concepts to a c-game. All suggest distributions of the surplus created by the grand coalition. The more common solutions such as the Shapley power index and the Harsanyi-Zeuthen-Nash bargaining solutions can generate our result that benefit-cost rules can be a part of a unanimously accepted contract. This model uses the nucleolus (Schmeidler, 1969), developed by David Schmeidler, as its behavioral premises more closely model our construction of the game.

The nucleolus locates a distribution by treating all coalitions equally and recognizing that the optimal threat of any coalition is only its ability to defect from the grand coalition. That is, the mere act of entering into negotiations to create a social contract does not ipso facto generate new potentials for agents to injure each other - a "you're either with us or we don't care about you" (Shubik, 1982) environment exists. Therefore the solution attempts to quantify the 'attitude' any coalition has toward its share of the core surplus and measure how committed is the coalition to remain in the game.

The nucleolus searches the set of feasible distributions, or imputations, in the core with the object to minimize the most dissatisfied coalition. Dissatisfaction is measured by the excess distance between the value of the coalition, v(s), less the total coalition payoff, x(s), of a suggested distribution.[9] So the objective reduces to:

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[9]. Payoffs are normally expressed in terms of utility. The payoffs here are expressed as simple market surplus measures which implies constant marginal utilities with respect to the numeraire. As cooperative game theory is rooted in the potential gains of correlated equilibria over mixed strategy solutions, the problem is the same as that for non-cooperative game theory whenever pure strategies do not exist. Still some marginal utility of income differences can be incorporated into applied uses of the nucleolus, such

$$\text{Min Max } e(s)$$

$$\text{with } e(s) = v(s) - x(s) \leq 0 \quad \text{for all } s \subset I$$

(Points where the excess, e(s), is greater than zero are not reasonable). The objective is subject to a definition of the surplus, Z, the payoff, x(s), a balanced budget and both individual and group rationality. V(s) is calculated using the same demand information, but is predetermined.

Re-writing the objective:

$$\text{Max Min } [x(s) - v(s)]$$

Subject To

1. $$\mathbf{Z} = \sum_j^m \left[\int^{q_j} \sum_i^n D_{ij}(q_j)\, dq_j - \int^{q_j} C_j(q_j)\, dq_j - TC\right]$$

$j = (1,\ldots m)$ goods
$i = (1\ldots.n)$ players

D_{ij}: Demand of player i for good j
C_j : Production cost of good j

Z, surplus or value of the grand coalition, is the sum of producer and consumer surplus over m non-rival goods for the n players less implementation costs, TC where TC is:

$$\mathbf{TC} = \begin{cases} K + c(t) & \text{if } t > 0 \\ 0 & \text{if } t = 0 \end{cases} \qquad 0 \leq t \leq m$$

t is the <u>number</u> of times any of the goods are subject to an individualized tax pricing policy of which a Lindahl pricing rule is only one alternative. For simplicity, assume the default taxation rule for each agent is 1/n of the total cost.

2. $$\mathbf{X(s)} = \sum_j^m \left[\sum_{i \in s} \int^{q_j} D_{ij}(q_j) - w_{ij} C_j(q_j)\right] dq_j \quad \text{for } s \subseteq I,\ s \in S^o$$

Where x(s) is the payoff to any arbitrary coalition.
w_{ij} is the tax share assessed to player i for good j product
S^o is the coalition set with $\sum_{k=1}^n [n!/k!(n-k)!]$ elements.

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international trade agreements, if the players posses common subjective priors regarding another players intensity of gain. This of course benefits wealthier players.

3. $\sum_i^n w_{ij} = 1$ for all j

The Balanced Budget condition.

4. $\sum_i^n x(s^i) = Z$

where s^i are the n singletons contained in S°. This is the condition for group rationality, or no waste.

V(s) is given for all s in S° and is found by the same general formula for Z above as Z is merely the value for the grand coalition. Also the non-negativity conditions for production, tax shares, and all costs, as well as orderly demands are assumed.

The program searches for a solution to the optimal amount of each good to produce and the optimal tax prices to each individual for each good produced. That is the solution set, $\{q_j^*, w_{ij}^*\}$ is identified with $(m + nm)$ elements. The program will, of course, have to be discretized with respect to the goods and the tax shares and a more refined grid will create a more costly search process - relevant to an extension of this work if calculation effort itself is considered.

Three decision rules are admissible in the joint expenditure/financing solution for any good:

1. Benefit-Cost rule (BCR)

The BCR conditions imply the solution identifies the quantity where $\sum_i MB_{ij} = MC_j$ and tax prices $w_{ij} = 1/n\ MC_j$. That is, the surplus maximizing quantity is produced at the default tax rule.

2. Individualized Pricing rule (IPR)

The IPR conditions imply the solution deviates from the default tax rule, or $w_{ij} \neq 1/n\ MC_j$ for all i.

3. Inefficient Quantity rule (IQR)

The IQR conditions imply the solution deviates from the efficient quantity point, or $\sum_i MB_{ij} \neq MC_j$.

The benefit cost rule id the default rule. It is chosen unless there are overpowering reasons not to invoke it. The IPR and/or IQR may be invoked in order to encourage disgruntled members to remain in the grand coalition.

Those familiar with cooperative games will quickly note that the model does not restrict itself to core solutions that forbid core shrinkage due to bargaining; yet it retains the cooperative game theory notions of the characteristic function. One could correctly argue that the model is not a pure cooperative game. Yet this work strongly encourages a more realistic definition of a feasible core.

Any combination of these rules is clearly possible a priori. This includes uniform dominance of any one of these rules to the exclusion of the other two. However the decision process is not arbitrary. Accommodations of the marginally most dissatisfied coalition are made via the IPR or the IQR, but at a measured cost to the value of the game, or social surplus. These rules are ad hoc exceptions to the BCR, based on the incidental set of demands and the given production technologies. Benefit-Cost analysis as a decision rule is the default rule in this formulation. The principle finding is that the paradox of generating hypothetical consent rules from a unanimous consent requirement is plausibly resolved. It is also true that BCRs will not likely exhaust the scope of decision rules for public expenditure; but it seems likely that BCRs commonly would feature prominently in the final set of decision rules adopted.

CONCLUSIONS

Explicit recognition of the cooperative, bargaining strategy of the public goods game and attention to the implementation costs of individualized tax pricing yields this rather strong result for benefit cost analysis. This is not unexpected. If the creation of a legitimate state emerges out of unexploited mutual benefit to all signators, then the preservation of total surplus itself will be valued. The search for unanimity will take care to commit all of the players to the game, but the search will accommodate the marginally most dissatisfied in an expected least cost fashion. Simply put, interpersonal agreement sought to secure surplus gains to each agent will be surplus preserving. So if a BCR secures the largest aggregate surplus even with its potential to generate individual loss within a particular decision, the BCR will continue to be regarded by the players.

Beyond noting that BCRs are admissible to unanimous agreement, BCRs may quite likely predominate. A quick look at the program indicates that the two surplus reducing rules, particularly the IQR could be "just" accepted. That is very small deviations to the value maximizing quantity could be acceptable across the board in very plausible scenarios. Yet we argue this is unlikely.

An enrichment of the model could likely note that the level of detail used to search for the solution is costly. It is not just an abstract programmer's problem. There is a real cost to complex bargaining. The level of discretization of quantities or tax shares chosen, or the extent of distinction between substitute goods and the disaggregation of types of demanders are all costly choices in which decision search costs grow exponentially. The less detailed the distinctions, the more likely that BCRs will be adopted since very small incremental IQRs are not feasibly determinable.

A similar argument may be used if there is uncertainty over the array of immediate and future demands and costs for the public goods. Those expected to be the most dissatisfied may suggest particular goods that will most likely accommodate them if exempt from the BCR, deferring the actual expenditure/financing decision for these goods. Yet deferring a large set of the decisions implies that the game in its totality will be on-going with point solutions purely ephemeral. Rational pre-commitment to a large set of BCRs could demonstrably reduce the scope, and thereby the costs, of future play. This could rather commonly increase the expected payoff the all players and could be universally accepted.

In conclusion, a fairly general argument can validate the existence of a rather widespread use of rules for public expenditures making use of benefits and cost with the Kaldor-Hicks notions of hypothetical consent. The modelling conditions by which this conclusion is reached are submitted to be more plausible than the costless, exchange model embodied by the Lindahl Equilibrium. Modern literature on constitutional agreement such as the works of Nozick and Gauthier view the social contract as a package deal. Not every line of the contract is unanimously accepted without pure anonymity. Preferences are relevant but surplus maximization itself is neither the object nor result. So it is not surprising that naive market-like solutions have not been widely accepted outside the discipline while individual preference satisfaction has strongly influenced the ethics literature.

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