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PRICING COMMODITY OPTIONS WHEN THE UNDERLYING FUTURES PRICE EXHIBITS TIME-VARYING VOLATILITY

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Abstract

This paper outlines a model for pricing options when the underlying futures price exhibits time-varying volatility. Futures price movements are characterized using a GARCH model. In an empirical application, the GARCH option pricing model predicts market premiums significantly better than the standard Black model, which assumes volatility is constant.

PRICING COMMODITY OPTIONS WHEN THE UNDERLYING FUTURES PRICE EXHIBITS TIME-VARYING VOLATILITY

There is now considerable evidence that proportional changes in commodity futures prices are not independent draws from an identical normal distribution. Gordon finds evidence of excess kurtosis and time-varying volatility in commodity futures price movements, suggesting systematic deviations from normality. He modeled these phenomena using the stable Paretian family of distributions. Baillie and Myers find that a generalized autoregressive conditional heteroscedastic (GARCH) model, assuming a student *t* density for the conditional distribution of price changes, does a good job of modeling the excess kurtosis and volatility changes in commodity futures price movements. The advantage of the GARCH specification is that convenient assumptions about the conditional density of price changes, such as the normal or student *t*, lead to a rich model which allows for time-varying volatility and excess kurtosis in the unconditional distribution of price changes.

Despite this growing body of evidence, the standard model used to price commodity options continues to assume that proportional changes in the underlying futures price are identically independently distributed (i.i.d.) and normal (Black).[1] Empirical tests comparing prices predicted by Black's model to actual market option prices generally conclude that the model does a poor job of pricing deep in-the-money and deep out-of-the-money options; although near-the-money options are often predicted well (e.g. Hauser and Neff). These failures of the model could be explained by the inappropriate distributional assumption on commodity futures prices.

This paper outlines methods for pricing commodity options when time-varying volatility in the underlying futures price follows a GARCH process. There is no closed form solution to

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[1] A commodity call (put) option gives the buyer the right, but not the obligation, to buy (sell) a specified futures contract at the maturity date on the option for a predetermined fixed price, called the strike price. These instruments can be used to manage risks and speculate on commodity price changes.

the option pricing problem but the procedure is easy to implement using monte carlo methods and a simulation model. We also derive some closed form approximations to the simulation model. Because they account more carefully for the true distributional properties of futures price changes, it might be expected that GARCH option pricing models will outperform Black's model in predicting actual market option prices. This proposition is tested by applying alternative models to the problem of pricing options for soybean futures on the Chicago Board of Trade. It is found that the GARCH option pricing model predicts actual prices significantly better than Black's model using historical volatilities.

Option Pricing Models

Cox and Ross have developed a simple way of deriving the Black-Scholes option pricing formula on which Black's model for commodity options is based. They note that the Black-Scholes model provides an option pricing formula which is preference free. Therefore, if an equilibrium option price can be derived assuming one particular preference structure, then it must be a solution for any preference structure which permits equilibrium. This suggests solving the problem for the preference structure which is the most tractable, that of risk neutrality.

In a risk-neutral world, two important restrictions would hold. First, the option will be priced according to its expected value at maturity, discounted back to the current period at the risk-free rate. For commodity call options this implies

(1) $$P_t = e^{r_t(t-T)} E_t\{\max[0,\ F_T - K]\}$$

where P_t is the price at time t of an option maturing at T; r_t is the current risk-free interest rate; E_t is expectation conditional on information available at time t; F_T is the price of the futures contract at maturity; and K is the strike price on the option. If the futures price is above the strike price at maturity then the call will have a value equal to the difference between the two prices, otherwise it will be worthless. The second restriction is that the current futures price is an unbiased predictor of the futures price at maturity, $F_t = E_t(F_T)$. If these restrictions did not hold in a risk-neutral world then there would be unexploited (expected) profit opportunities.

To operationalize the option pricing formula, a distributional assumption must be made on futures price changes so that the conditional expectation in (1) can be evaluated. Black's assumption is that period-to-period changes in the logarithm of futures prices are i.i.d. normal:

$$\text{(2)} \qquad \Delta f_t = \mu + \epsilon_t \, ; \qquad \epsilon_t \sim N(0, \sigma^2)$$

where $\Delta f_t = \ln F_t - \ln F_{t-1}$; and σ^2 is a (constant) one-period variance. Using this distributional assumption, and the unbiased futures restriction, then (1) can be evaluated to give the standard formula derived by Black to price commodity options (Rubinstein). This formula is usually operationalized by estimating σ^2 with a moving sample variance of past price changes (say over the most recent 30 days).

One of the major problems with Black's formula is that proportional futures price changes appear to violate i.i.d. normality. In particular, price changes exhibit excess kurtosis and time-varying volatility (Gordon, Baillie and Myers). Suppose we generalize the probability model for futures price movements to

$$\text{(3)} \qquad \Delta f_t = \mu + \epsilon_t \, ; \qquad \epsilon_t | \Omega_{t-1} \sim t(0, h_t, \nu) \, ; \qquad h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1}$$

where Ω_{t-1} is a set of information available in period $t-1$; h_t is the conditional variance of futures price changes; and $t(0,h_t,v)$ is the student t distribution with variance h_t and v degrees of freedom. This is a GARCH(1,1) model first introduced by Bollerslev and used successfully by Baillie and Myers to model the excess kurtosis and time-varying volatility in commodity futures price movements.

Suppose that options can still be priced using the risk-neutral valuation expression (1). This is not an innoculous assumption because when futures price volatilities are time-varying and stochastic (as under the GARCH model) then it will not generally be possible to construct a (continuously adjusted) portfolio of options and underlying futures which is risk free. The reason is that stochastic volatility adds an additional source of risk which is not generally diversifiable (Hull and White; Johnson and Shanno). Thus, the arbitrage argument which forms the foundation of the Black-Scholes valuation approach breaks down.

Despite this, however, there are good reasons for assuming that options are valued as they would be in a risk-neutral world (risk-neutral valuation) even when volatilities are stochastic. First, Brennan has developed a set of conditions under which risk-neutral valuation occurs in equilibrium even when agents are risk averse and cannot embed the option in a risk-free portfolio. While somewhat restrictive, these conditions do suggest that risk-neutral valuation can be applicable even in the case of stochastic volatility. Second, risk-neutral valuation may be an adequate approximation, particularly when a significant number of risk-neutral, or nearly risk-neutral, agents are operating in the market. Third, the alternative to a risk-neutral valuation is an option pricing formula which depends on the risk preferences of all of the agents operating in the market. Because information on individual risk preferences is generally unknown, such a formula would be of little practical use. For these reasons we proceed using a risk-neutral approach to valuation.

Unfortunately there is no closed form solution to the integration implied by the risk-neutral valuation formula (1) under the GARCH model (3). The problem is that the GARCH model implies that the logarithm of the futures price at maturity, f_T, equals the logarithm of the initial futures price, f_t, plus a sum of weakly dependent and heterogeneously distributed GARCH innovations, $\{\epsilon_{t+i}\}_{i=1}^{T-t}$. Thus, not only is the resulting distribution for f_T not normal but it can be shown that it has no closed form solution (Engle). In the absence of a closed form solution there are two ways to proceed in developing an option pricing model: monte carlo simulation and closed form approximations to the true distribution of f_T. Each will be discussed in turn.

A Monte Carlo Approach

The monte carlo approach involves evaluating the expected option price at maturity via numerical methods. Suppose that the GARCH model (3) has been estimated using data available at time t. Then make m random draws from the t-distribution with $\hat{\nu}$ degrees of freedom, where m is the number of periods to maturity of the option and ν is estimated from the GARCH model. Using the GARCH model and these random draws, the initial futures price F_t can be simulated forward to generate *one* realization of the terminal futures price at maturity.[2] Denote this realization F_T^i.

To obtain an estimate of the expected terminal option price at maturity, $\hat{P}_T$, repeat this procedure n times to get n sample values of F_T^i. Then use[3]

[2] A detailed outline of the simulation method is available from the authors on request.

[3] In the empirical results which follow we set $n = 1000$ but improve the precision of the estimate by applying the control variate method (Boyle). Details are available from the authors on request.

(4) $$\hat{P}_T = \frac{1}{n}\sum_{i=1}^{n} \max[0, F_T^i - K].$$

The current option price is obtained by simply discounting $\hat{P}_T$ back to t at the risk-free rate of interest, $P_t = e^{r_t(t-T)}\hat{P}_T$. The result can be compared with the actual market price for the relevant option at the relevant date, and/or to option prices generated using Black's model.

Closed Form Approximations

We examine two simple closed form approximations to the expected option price under the GARCH simulation model. The first involves taking the one-period-ahead conditional variance, $\hat{h}_{t+1}$, estimated with the GARCH model using data up until period t, and assuming that all subsequent price changes are i.i.d. normal with constant variance $\hat{h}_{t+1}$. This leads to the standard Black formula with $\hat{h}_{t+1}$ replacing the σ^2 estimated with historical volatilities. This approximation is easy to implement once the GARCH model has been estimated. However, it has the obvious disadvantage of assuming (incorrectly) that, even though there has been time-varying volatility in the past, future conditional variances will be constant up until the maturity date on the option.

The second approximation involves using the GARCH model to forecast future conditional variances of price innovations over the time to maturity of the option. From results for predicting conditional variances given in Baillie and Bollerslev, this leads to a formula for estimating the conditional variance of f_T in the GARCH(1,1) model of

(5) $$Var(f_T|f_t) = \frac{\hat{\omega}}{(1 - \hat{\alpha} - \hat{\beta})}[T - t - \sum_{j=1}^{T-t}(\hat{\alpha} + \hat{\beta})^{j-1}] + \hat{h}_{t+1}\sum_{j=1}^{T-t}(\hat{\alpha} + \hat{\beta})^{j-1}.$$

Thus, if the distribution of f_T conditional on f_t is approximately normal with variance given by (5) then it can be shown (e.g. Rubinstein) that the option pricing formula is

$$(6) \qquad P_t = e^{r_t(t-T)}\left\{ F_t C\left(\frac{\ln(F_t/K) + 0.5\,Var(f_T|f_t)}{\sqrt{Var(f_T|f_t)}} \right) - KC\left(\frac{\ln(F_t/K) - 0.5\,Var(f_T|f_t)}{\sqrt{Var(f_T|f_t)}} \right) \right\}$$

where $C(\cdot)$ is the cumulative distribution function for the standard normal.

This approximation is straightforward to compute and has the advantage of taking time-varying volatility directly into account. Nevertheless, the assumption of normality is generally violated in practice because the actual distribution of f_T conditional on f_t has excess kurtosis compared to the normal. Thus, it is expected that this approximation will not predict market prices as well as the monte carlo simulation which takes excess kurtosis in futures price changes into account.

Empirical Results

Four alternative option pricing models have been outlined: (a) the standard Black model with historical 30 day volatilities (Black's model); (b) the GARCH option pricing model estimated via monte carlo simulation (GARCH model); (c) Black's model with a GARCH volatility estimate (GARCH Approximation I); and (d) Black's model with a predicted GARCH volatility path (GARCH Approximation II). The performance of each of these models is now compared by applying them to the task of pricing options for soybean futures on the Chicago Board of Trade.

Daily closing prices for soybean futures with a July delivery date were obtained from the Chicago Board of Trade database. The sample consists of day-to-day changes in the logarithm of July futures prices between July 1987 and July 1990, a total of 758 observations. When each July contract matures, the data switch to the next July contract maturing in the following year. A dummy variable is introduced into the GARCH model to account for the effect of switching contracts at annual intervals. All data are for July contracts because the aim is to price options written on July soybeans.

We began estimating call option prices for July 1990 soybeans futures contracts on April 2, 1990 (three months prior to maturity). The procedure is to take data available at the date the option is being priced and first estimate the GARCH model and the historical 30 day volatility. These results are used to compute option prices using each of the four alternative models discussed above. The data set is then updated by adding the next day's observation and the process is repeated. An update of the GARCH model and historical volatilities are computed and option prices are then estimated for the next day. This updating process continues for every day up until the option expires so that we get a sequence of estimated option prices under each of the alternative models. At each step, we were careful to use only data that would be available at the date the option is being priced in order to implement each of the option pricing models. All options valued had a $6.00 strike price.

Results from the initial GARCH estimation using data from July 1987 to April 1, 1990 are shown in Table 1. Two notable features of the results are the high asymptotic *t*-ratios on the degrees of freedom and GARCH parameters, indicating the importance of excess kurtosis and time-varying volatility in soybean futures prices; and the lack of residual serial correlation in the errors or squared standardized errors, indicating an appropriate model specification.

Table 2 provides a summary of the performance of each pricing model over the three month period from the beginning of April to the end of June 1990. The mean square error, root mean square error (RMSE), and average absolute deviation (AAD) of the difference between each models predicted option value and the realized market premium are used as indicators of the model's ability to predict the market value of the option. Of the four models, the standard Black model has the most difficulty predicting the market option value with an AAD of 1.59 cents and a RMSE of 1.92 cents over the sample period. The GARCH model results in an AAD of 1.02 cents and a RMSE of 1.33 cents, clearly outperforming all of the other valuation techniques. Thus, the ability of the GARCH model to account for the true distribution properties of futures price changes appears to result in superior performance in predicting the market prices of the options.

The GARCH Approximation I produces some improvement over the standard Black model as evidenced by the lower AAD and RMSE of 1.34 and 1.69 cents, respectively. The one-period-ahead conditional variance estimate from the GARCH model therefore appears to provide a better variance forecast than the 30-day historical variance. As expected, the GARCH Approximation II results in further improvement over Black's model with an AAD of 1.13 cents and a RMSE of 1.56 cents.

Even though the GARCH Approximation II model still assumes normality, incorporating the predicted path of conditional variances implied by the GARCH model has improved the predictive power of the model over that obtainable from assuming that variances are constant over time. However, the restriction of normality clearly decreases the ability to predict option prices as evidenced by the superior performance of the monte carlo simulation method. Table 3 presents the option prices for the standard Black model, the GARCH model, and the market premium for the last three months of the July 1990 soybean futures option contract.

Conclusions

Black's standard valuation model for commodity options assumes that proportional changes in the underlying futures price are i.i.d. normal. Empirical evidence suggests that commodity futures price movements exhibit excess kurtosis and time-varying volatilities. This paper presents methods for pricing commodity options when time-varying volatility in the underlying futures price follows a GARCH process. Because the resulting valuation problem has no closed form solution, numerical methods are used to determine option prices. In addition, two closed form approximations models are derived. Empirical results show that the ability of the GARCH simulation model to better account for the true distributional properties of futures prices allows it to outperform the standard Black option pricing model in predicting market premiums. The approximation models also outperform Black's model, but are not as accurate as the GARCH simulation model.

A limitation of the study is that empirical results have examined only one option contract over a short period of time. Furthermore, while the GARCH option pricing model clearly predicts option prices better than the standard Black model, we have not yet determined the economic value of this improved performance. Future research will concentrate on relaxing these limitations.

Table 1
Estimation Results for the Initial GARCH Model

$$100\Delta f_t = \mu + \epsilon_t \; ; \qquad \epsilon_t \sim t(0, h_t, \nu) \; ; \qquad h_t = \omega + \delta d_t + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}$$

Parameter	Estimate	Standard Error	t-Value
$\hat{\mu}$	0.037	0.044	0.833
$\hat{\omega}$	0.024	0.014	1.761
$\hat{\delta}$	4.841	3.066	1.579
$\hat{\alpha}$	0.090	0.021	4.330
$\hat{\beta}$	0.901	0.020	44.608
$\hat{\nu}^{-1}$	0.113	0.032	3.523
Statistic	**p-Value**	**Statistic**	**p-Value**
$Q(1)$ = 1.415	0.23	$Q^2(1)$ = 1.178	0.28
$Q(5)$ = 4.272	0.51	$Q^2(5)$ = 2.503	0.78
$Q(10)$ = 15.928	0.10	$Q^2(10)$ = 4.063	0.94

Notes: d_t is a dummy variable with value 1 if the observation is the first observation on a new futures contract and zero otherwise; $Q(df)$ is a Q test for df degree autocorrelation in the residuals; and $Q^2(df)$ is a Q test for df degree autocorrelation in the squared standardized residuals (a test for residual GARCH effects not captured by the model).

Table 2
Performance of Alternative Models in Predicting Market Option Premiums

Pricing Model	Mean-Squared Error	Root Mean-Squared Error	Average Absolute Deviation
Black Model	3.67	1.92	1.59
GARCH Model	1.76	1.33	1.02
GARCH Approximation I	2.87	1.69	1.34
GARCH Approximation II	2.43	1.56	1.13

Note: The root mean-squared errors and average absolute deviation are in cents per bushel.

Table 3 Black Model, GARCH Model, and Market Prices for July 1990 Soybean Futures Options

	Option Prices				Option Prices		
Date	Black Model	GARCH Model	Market	Date	Black Model	GARCH Model	Market
4/2	15.07	20.14	18	5/14	39.22	41.34	40
4/3	15.49	19.95	17.5	5/15	30.58	33.55	30
4/4	18.15	22.50	21	5/16	30.55	31.91	30.5
4/5	20.96	24.78	23	5/17	34.82	36.84	35
4/6	20.03	23.61	23	5/18	27.12	29.20	27
4/9	18.29	21.21	20.75	5/21	20.40	22.39	19
4/10	17.75	20.64	21	5/22	19.16	19.60	18.25
4/11	18.30	20.46	21	5/23	18.69	18.65	17
4/12	15.46	17.26	18	5/24	22.86	22.96	22.25
4/16	17.56	19.55	19.5	5/25	24.91	24.45	25
4/17	20.84	22.38	23	5/29	16.70	17.01	15
4/18	18.85	20.89	21	5/30	15.79	15.26	14.63
4/19	23.04	24.35	24.75	5/31	16.58	15.75	16
4/20	26.13	27.27	28.5	6/1	13.15	12.35	10.13
4/23	26.50	27.49	29.75	6/4	8.52	8.33	5.75
4/24	21.61	23.04	23.88	6/5	8.10	6.92	6.75
4/25	31.39	33.33	31.75	6/6	8.09	6.77	7
4/26	45.10	48.14	48	6/7	11.61	10.85	12
4/27	52.21	54.54	56.5	6/8	8.90	8.57	9
4/30	49.93	52.53	51	6/11	13.26	13.30	13.25
5/1	45.42	47.74	48	6/12	12.32	11.87	13
5/2	52.79	54.79	54.25	6/13	10.85	10.05	11.5
5/3	46.04	48.24	48.5	6/14	3.52	3.95	4.5
5/4	42.98	44.48	45	6/15	1.68	1.93	2.75
5/7	51.96	53.69	54	6/18	4.22	4.54	4.13
5/8	48.07	49.62	50.5	6/19	2.33	2.36	2.13
5/9	60.04	61.43	62.25	6/20	3.22	3.10	3
5/10	60.62	61.83	62	6/21	4.73	4.77	4.38
5/11	45.93	48.96	49				

Note: Option prices are in cents per bushel for call options with a $6.00 exercise price.

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