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MARKET-LEVEL MEASURES OF THE VALUE OF WEATHER INFORMATION: CONCEPTUAL AND EMPIRICAL CONSIDERATIONS

by

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ABSTRACT

Assuming risk-neutral producers with rational expectations, ex-ante market-level measures of the value of weather information to both consumers and producers are derived. Methods to obtain empirical estimates of these ex-ante measures from observed data are derived and discussed.

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MARKET-LEVEL MEASURES OF THE VALUE OF WEATHER INFORMATION: CONCEPTUAL AND EMPIRICAL CONSIDERATIONS

Although the impact of weather forecasts on the production and input decisions of individual producers has been widely studied (e.g. Baquet, Halter, and Conklin (1976); Byerlee and Anderson (1969); Hashemi and Decker (1969); and Brown, Katz, and Murphy (1986)), the effects of this information on market-level prices and quantities has been the focus of very few studies. In Lave's (1963) analysis of the California raisin industry, he suggested that additional rainfall protection resulting from better weather information would increase raisin supply leading to lower producer profits due to inelastic demand. However, Lave did not consider how optimal producer decisions might change if they knew that better weather information might lead to lower prices. Babcock (1990) was the first to examine how optimal producer decisions are affected by better weather information when they know that market equilibrium prices and quantities will also be affected by this information.

The work in this paper builds upon and extends Babcock's work in a number of ways. Perfectly competitive, risk-neutral producers with rational expectations about output price are also assumed. However, the distribution of weather as well as weather forecasts may have any discrete or continuous distribution. Ex-ante estimates of the value of weather information to both consumers and producers are derived. In addition, this work focuses on how to obtain empirical estimates of these ex-ante measures of the value of information from actual observed variables such as price and output. The derivation of the model also resulted in reciprocity conditions which could be used to empirically test the validity of the model and its underlying assumptions.

The Model

Perfectly competitive risk-neutral producers are assumed. Without loss of generality, supply is assumed to come from a single representative producer. The model will first be formulated in terms of expected output and prices so that ex-ante measures of the value of information can be determined. In a later section in the paper, the model will be reformulated in terms of easily observed realized output and prices making it amenable to empirical implementation. More specifically, the representative producer's production function is assumed to have multiplicative risk

$$\tilde{Q} = Q\tilde{\theta} \qquad \text{where } E\tilde{\theta} = 1. \tag{1}$$

The producer's output, $\tilde{Q}$, is a random variable which includes a nonstochastic choice variable, Q (the choice of inputs), multiplied by a stochastic variable, $\tilde{\theta}$, representing weather. An example of this specification would be a rain at harvest which leads to spoilage as a constant fraction of the crop. As suggested by Newbery and Stiglitz (1981) and consistent with Babcock's formulation, the representative producer's cost function is assumed to depend on the choice variable, Q. In addition, a quadratic cost function will be assumed,

$$C(Q) = d + eQ + fQ^2. \tag{2}$$

Market level demand is assumed to be linear and, without loss of generality, nonstochastic

$$P^R = a - bQ^R \tag{3}$$

where Q^R and P^R are realized output and price, respectively. That is, if θ^R is the actual weather that occurred, realized output is given by $Q^R = Q\theta^R$.

Let $\tilde{\theta}_s$ represent the producer's subjective distribution of weather. A number of different alternative subjective distributions will be considered in the paper, and their implications on market equilibrium will be discussed. First, however, the producer's objective function and market equilibrium will be stated in terms of this general subjective distribution. Even though the demand function itself is nonstochastic, the price the producer will receive is a random variable since quantity supplied is stochastic and can be expressed as

$$\tilde{P} = a - b\tilde{Q} = a - bQ\tilde{\theta}_s. \tag{4}$$

The representative producer's profit is a random variable and is given by

$$\tilde{\pi} = \tilde{P} \cdot \tilde{Q} - C(Q). \tag{5}$$

Because risk neutrality is assumed, the producer will maximize expected profits over the choice variable Q:

$$\max_{Q} E\tilde{\pi} = E\{[a - bQ\tilde{\theta}_s]Q\tilde{\theta}_s\} - d - eQ - fQ^2. \tag{6}$$

The assumption of a perfectly competitive market implies that each individual producer has no effect on market price. Assuming rational expectations means that all producers act as if they know the underlying parameters of the market-level supply and demand functions. Thus, in the maximization problem specified above, the producer does not think he can influence market price although he knows the parameters a, b, d, e, and f and that the collective actions of all producers in the market will affect the price. Hence, differentiating equation (6) with respect to Q,

$$\frac{\partial E\tilde{\pi}}{\partial Q} = E\{[a - bQ\tilde{\theta}_s]\theta_s\} - e - 2fQ = 0. \tag{7}$$

Letting h = 2f, taking expectations, rearranging, and denoting the optimal choice of inputs under $\tilde{\theta}_s$ as Q_s,

$$aE\tilde{\theta}_s - bQ_s = e + [h + b(E\tilde{\theta}_s^2 - 1)]Q_s. \tag{8}$$

Solving for optimal input choice:

$$Q_s = \frac{aE\tilde{\theta}_s - e}{h + bE\tilde{\theta}_s^2} . \qquad (9)$$

Ex-Ante Value of Weather Information

The actual underlying distribution of weather will be conceptualized as climatology, $\tilde{\theta}_c$. It is not unreasonable to conceive that producers can obtain (or be given) very good estimates of the moments of $\tilde{\theta}_c$ from long-run frequencies. If the producer's subjective distribution is given by climatology and assuming the multiplicative specification of risk, $E\tilde{\theta}_s = E\tilde{\theta}_c = 1$. Also, expected output is given by $E\tilde{Q}_c = QE\tilde{\theta}_c = Q_c$. Since some variability in climate will be assumed, $\sigma_c^2 = E\tilde{\theta}_c^2 - (E\tilde{\theta}_c)^2 > 0$ and $E\tilde{\theta}_s^2 = E\tilde{\theta}_c^2 > 1$. Using equations (8) and (9), expected market equilibrium and output when producers use climatology can be obtained:

$$a - bQ_c = e + [h + b(E\tilde{\theta}_c^2 - 1)]Q_c \qquad (10)$$

$$Q_c = \frac{a - e}{h + bE\tilde{\theta}_c^2} . \qquad (11)$$

Expected producer profits are given by

$$E\tilde{\pi}_c = E\{[a - bQ_c\tilde{\theta}_c]Q_c\tilde{\theta}_c\} - d - eQ_c - fQ_c^2. \qquad (12)$$

Market equilibrium and expected output under climatology are illustrated in Figure 1. The right hand side of equation (10) will be referred to as the "supply determining" equation since its intersection with expected demand determines expected output, Q_c. Expected consumer surplus is given by triangle A while expected producer profits are given by the area B + C + E + F. It is interesting to note that the risk neutral producer with rational expectations does not produce where expected marginal cost equals expected demand. Also, expected output declines if average climate remains the same but climate becomes more variable, since $E\tilde{\theta}_c^2$ increases causing the slope of the supply determining equation to become steeper.

Next, the ex-ante value of perfect information to both producers and consumers will be examined. Suppose that the producer had perfect information that θ^R would occur, and let Q_p^R represent the corresponding realized output. Uncertainty is eliminated from the decision and the producer maximizes profits

$$\max_{Q_p^R} \pi_p = (a - bQ_p^R)Q_p^R - d - e\frac{Q_p^R}{\theta^R} - f\left(\frac{Q_p^R}{\theta^R}\right)^2 . \tag{13}$$

The corresponding first order condition is

$$\frac{\partial \pi}{\partial Q_p^R} = a - bQ_p^R - \frac{e}{\theta^R} - h\frac{Q_p^R}{\theta^{R^2}} = 0. \tag{14}$$

Market equilibrium and realized output are given by

$$a - bQ_p^R = \frac{e}{\theta^R} + \frac{hQ_p^R}{\theta^{R^2}} \tag{15}$$

$$Q_p^R = \frac{a\theta^{R^2} - e\theta^R}{h + b\theta^{R^2}} . \tag{16}$$

Knowing that the producer will receive perfect information but before the information is actually obtained, output is a random variable represented by

$$\tilde{Q}_p = \frac{a\tilde{\theta}_c^2 - e\tilde{\theta}_c}{h + b\tilde{\theta}_c^2} . \tag{17}$$

Expected output and profit are given by

$$E\tilde{Q}_p = E\left(\frac{a\tilde{\theta}_c^2 - e\tilde{\theta}_c}{h + b\tilde{\theta}_c^2}\right) \tag{18}$$

$$E\tilde{\pi}_p = E\left\{(a - b\tilde{Q}_p)\tilde{Q}_p - d - e\frac{\tilde{Q}_p}{\tilde{\theta}_c} - f\left[\frac{\tilde{Q}_p}{\tilde{\theta}_c}\right]^2\right\}. \tag{19}$$

To compare expected output with perfect information and with climatology, it is necessary to compare equations (11) and (18). However, in general, it is not possible to make this comparison without specific knowledge of the probability density function of the random weather variable, $\tilde{\theta}_c$. It is also apparent from examining equation (19) that a simple graphical illustration of producer profits or surplus would be difficult. However, if a specific value of $E\tilde{Q}_p$ can be determined, consumer surplus can be easily illustrated graphically. The case when expected output with perfect information is greater than with climatology is illustrated in Figure 1 where consumer surplus increases by the trapezoid B + C + D. Changes in producer surplus can be determined by comparing $E\tilde{\pi}_c$ and $E\tilde{\pi}_p$ in expressions (12) and (19).

Lastly, suppose that producers could receive a weather forecast, say γ_i, which is more accurate than climatology. Furthermore, although not critical to the analysis, it will be assumed that producers use this forecast to update their prior distribution (assumed to be climatology) in a Bayesian manner. Denote this distribution by $f(\tilde{\theta}|\gamma_i)$. Letting Q_{γ_i} represent the optimal input choice with forecast γ_i and using equations (8) and (9), Q_{γ_i} and market equilibrium can be determined as

$$aE(\tilde{\theta}|\gamma_i) - bQ_{\gamma_i} = e + [h + b(E(\tilde{\theta}^2|\gamma_i) - 1)]Q_{\gamma_i} \tag{20}$$

$$Q_{\gamma_i} = \frac{aE(\tilde{\theta}|\gamma_i) - e}{h + bE(\tilde{\theta}^2|\gamma_i)} \tag{21}$$

Expected output and profit after forecast γ_i is received are given by

$$E\tilde{Q}_{\gamma_i} = Q_{\gamma_i} E(\tilde{\theta}|\gamma_i) \tag{22}$$

$$E\tilde{\pi}_{\gamma_i} = \int f(\tilde{\theta}|\gamma_i)[a - bQ_{\gamma_i}\tilde{\theta}]Q_{\gamma_i}\tilde{\theta}d\tilde{\theta} - d - eQ_{\gamma_i} - fQ_{\gamma_i}^2. \tag{23}$$

These equations, however, are ex-post in the sense that they represent equilibrium values after a particular forecast has been received. To find the ex-ante value of the forecast information, let $g(\gamma)$ represent the probability density function of the weather forecasts, $\tilde{\gamma}$. (These forecasts are often described as "signals" providing additional information about $\tilde{\theta}$. See Antonovitz and Roe (1988) for further discussion.) Expected output and profit with forecast information but before a particular forecast has been received are given by

$$E\tilde{Q}_F = \int g(\gamma)Q_\gamma E(\tilde{\theta}|\gamma)d\gamma \tag{24}$$

$$E\tilde{\pi}_F = \int g(\gamma)E\tilde{\pi}_\gamma d\gamma. \tag{25}$$

Once again, without specific information about $f(\tilde{\theta}|\gamma)$ and $g(\gamma)$, it would be difficult to assess the magnitude of $E\tilde{Q}_F$ in comparison to Q_c or $E\tilde{Q}_p$ or to graphically illustrate producer surplus.

Empirical Considerations

Thus far, the analysis has been described in terms of expected prices and outputs. However, these are difficult or impossible to observe variables. Hence, if the parameters of the demand and supply equations are to be estimated, the model must be reformulated in terms of observed or realized outputs, prices, weather, and other relevant factors.

For many commodities, such as spring wheat, corn, and soybeans, production or yield is affected by soil moisture at planting and precipitation and/or temperatures during the growing season (Baier (1972); Thompson (1985) and (1986)). Producers are currently using only climatological probabilities or long-run frequencies about growing season precipitation for wheat (Brown et. al. (1986)). For corn and soybeans, long-range forecasts of July and August temperatures and rainfall, which affect yield and are used in making planting decisions, may not be much different from climatology. In these and similar instances, the appropriate demand and supply-determining equations which can be estimated from observed data can be derived as follows. Let P_c^R and Q_c^R denote observed realized price and output when producers use climatology as their subjective distributions. The realized state of weather is θ^R, and $E\tilde{\theta}_c^2$ is assumed to be determined from climatology. Then noting that $Q_c^R = \theta^R Q_c$ and substituting into equation (10) and rearranging, we obtain

$$a - bQ_c^R = a\left(1 - \frac{\theta^R}{E\tilde{\theta}_c^2}\right) + \frac{e\theta^R}{E\tilde{\theta}_c^2} + \frac{h}{E\tilde{\theta}_c^2} Q_c^R. \tag{26}$$

Clearly, the left hand side is simply the demand curve and establishes realized price. The right hand side is the supply-determining equation in terms of realized variables. Hence, the appropriate equations to estimate are given by

$$P_c^R = a - bQ_c^R \tag{27}$$

$$P_c^R = a\left(1 - \frac{\theta^R}{E\tilde{\theta}_c^2}\right) + \frac{e\theta^R}{E\tilde{\theta}_c^2} + \frac{h}{E\tilde{\theta}_c^2} Q_c^R . \tag{28}$$

This system is illustrated in Figure 2. Note that the parameter "a" appears in both equations which we expect because producers are assumed to have rational expectations and, thus, include information about demand in their output decisions. More importantly, however, this could be imposed as a cross-equation restriction and potentially used to test the hypothesis that producers are indeed risk neutral with rational expectations.

Alternatively, for other commodities, short-term forecasts are available for weather variables affecting production and yield. Examples include rain near harvest for raisins (Lave) and fall frost for apples (Baquet, Halter, and Conklin). Recall that γ_i represents the weather forecast, and for notational convenience denote $E(\tilde{\theta}|\gamma_i)$ by $E(\tilde{\theta}_i)$ and $(E\tilde{\theta}^2|\gamma_i)$ by $E(\tilde{\theta}_i^2)$. Let Q_i^R and P_i^R be realized output and price with forecast γ_i. Noting that $Q_i^R = \theta^R Q_{\gamma_i}$, using equation (14), and rearranging

$$P_i^R = a - bQ_i^R \tag{29}$$

$$P_c^R = a\left(1 - \frac{\theta^R E\tilde{\theta}_i}{E\tilde{\theta}_i^2}\right) + \frac{e\theta^R}{E\tilde{\theta}_i^2} + \frac{h\theta^R}{E\tilde{\theta}_i^2} Q_i^R . \tag{30}$$

With information about forecasts, this system (also illustrated in Figure 2) can be estimated from observed data.

Lastly, the appropriate demand and supply equations if producers have perfect information are obtained from equation (15) as

$$P_p^R = a - bQ_p^R \qquad (31)$$

$$P_p^R = \frac{e}{\theta^R} + \frac{hQ_p^R}{{\theta^R}^2} \; . \qquad (32)$$

Clearly, equation (32) represents realized marginal costs for all three information structures considered here.

Although it will not be discussed in this paper, ex-post measures of forecast and/or perfect information can be estimated by using demand, supply-determining, and realized marginal cost curves illustrated in Figure 2. Policy-makers, however, are more interested in ex-ante measures of the value of information and changes in expected output. Given the appropriate estimates of the demand and supply-determining equations (depending on whether producers use climatology or forecasts), these ex-ante measures can be obtained using the estimated parameters and the theoretical derivations of these measures presented here.

Summary and Conclusions

In Babcock's pioneering work, the effects of weather information on market equilibrium prices and quantities were examined. This study uses his basic assumptions of risk-neutrality and rational expectations. Extension and additions are made by assuming a more general distribution of weather and deriving the ex-ante value of information to both consumers and producers.

This work also illustrates that researchers must be careful in specifying and estimating systems of supply and demand equations in which weather is an important factor influencing production. Specification of such a system in terms of observed prices and outputs when equations are linear and production risk is multiplicative is derived here. This paper, however, illustrates a methodology which could be used if an alternative specification (such as risk aversion, non-linear demand and supply, or non-multiplicative production risk) were appropriate.

Figure 1: Expected Market Equilibrium

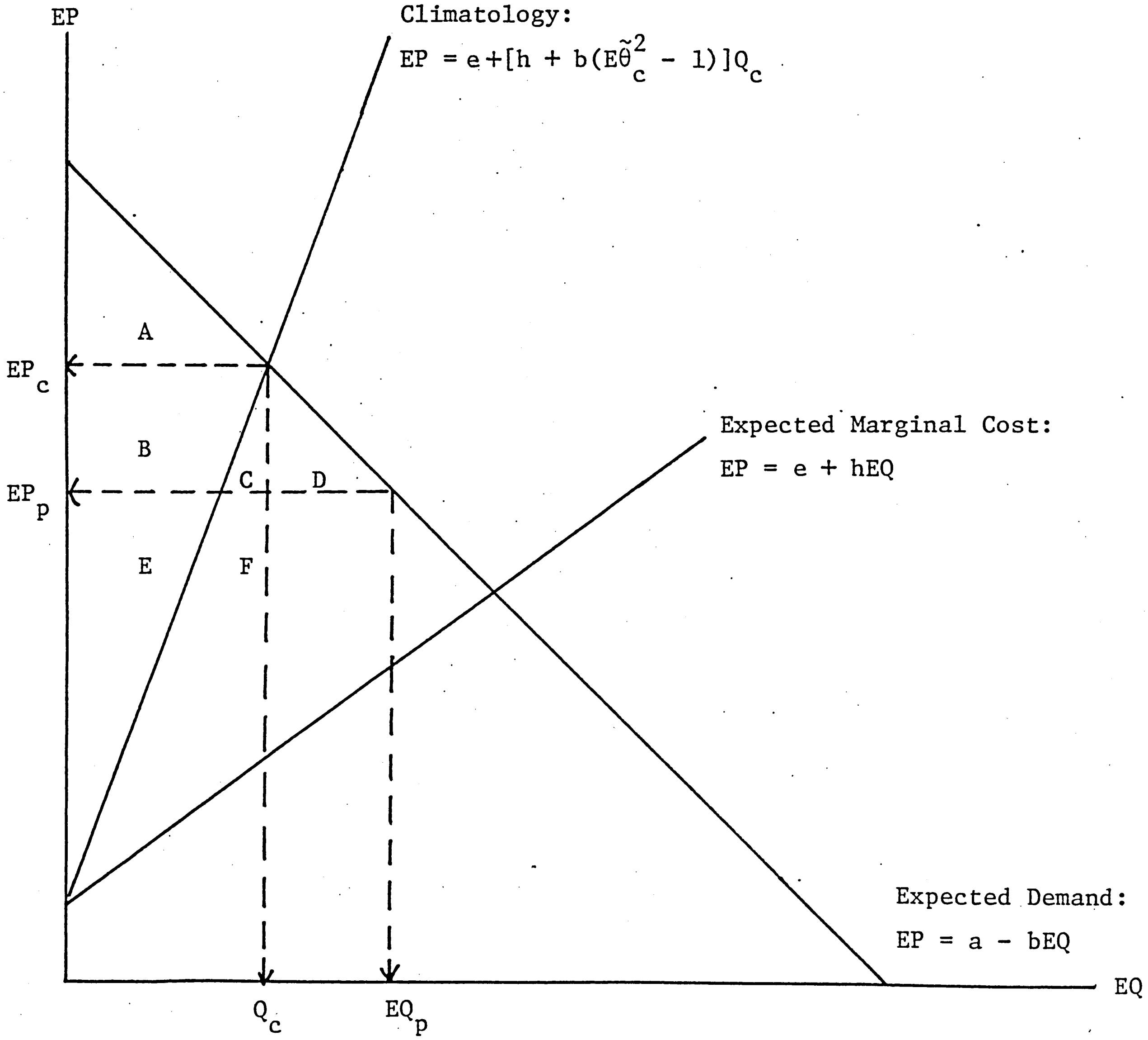
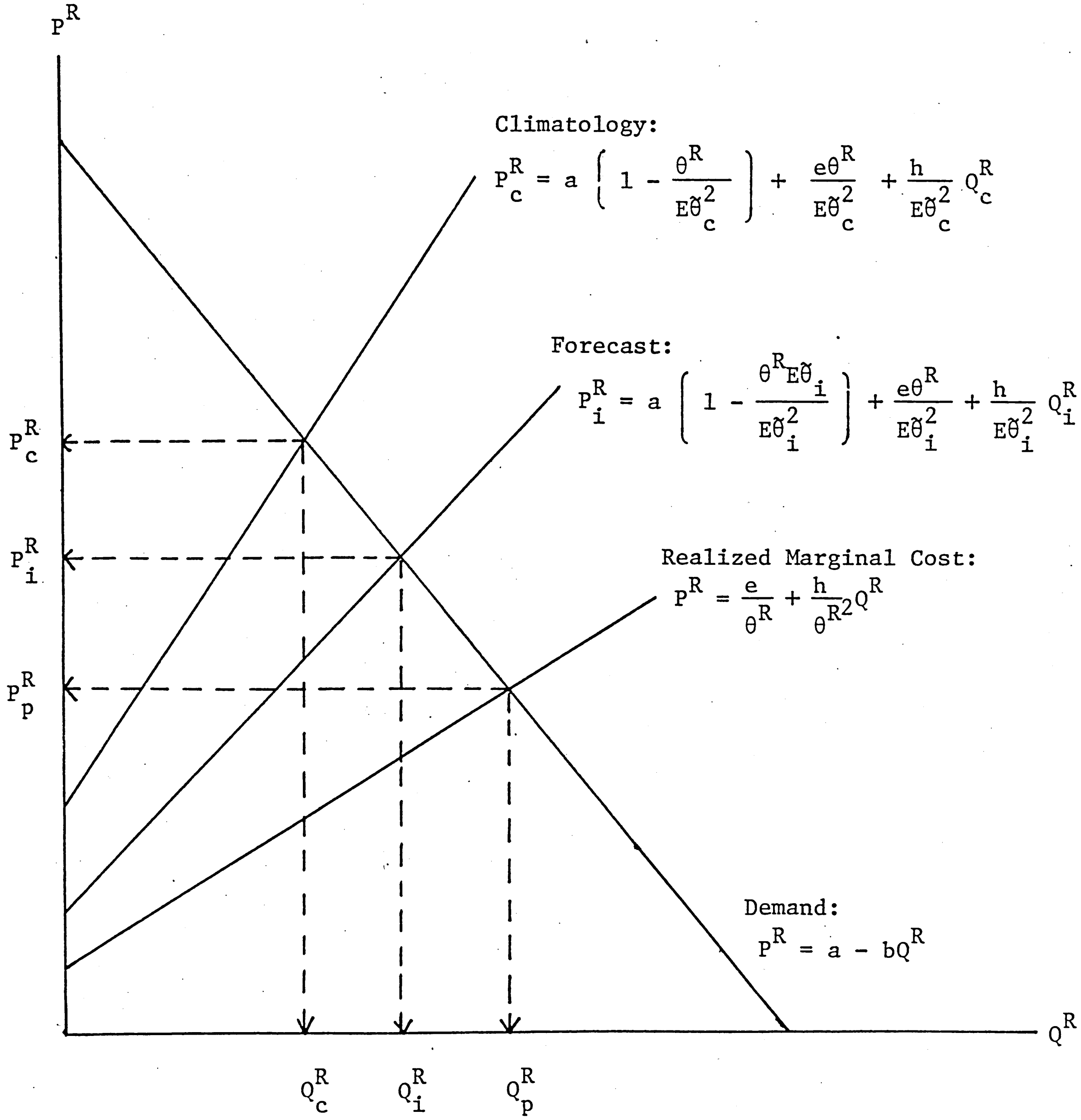


Figure 2. Observed Demand and Supply-Determining Equations



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