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Multi-Period Rangeland Investment Analysis with Safety-First Constraints and Uncertain Data

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Abstract

Impacts of crested wheatgrass seeding investments on ranch equity and income indicators are examined. Expected ending net worth is maximized subject to stochastic forage production and prices using multiperiod nonlinear programming with safety-first income constraints. Seedings are found to be beneficial by stabilizing forage supplies and permitting herd expansion.

Keywords: Stochastic programming, risk, investment analysis, range economics

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Parameter uncertainty effects agricultural production and marketing decisions and, consequently, farm financial performance. Much of the literature dealing with stochastic events in agriculture concentrates on short run decisions, often ignoring the long term consequences of decisions on farm financial performance. Although appropriate for many decisions, short run perspectives may be inappropriate for many investment decisions.

This paper considers the long term consequences of a rangeland investment designed to increase springtime forage supplies, a traditional source of uncertainty to the Western cattle producer. Crested wheatgrass is a common replacement for less productive range species for improving the quantity and quality of spring forage (Dewey and Asay). Production increases resulting from the establishment of crested wheatgrass for a spring forage supply are well documented (Hart et al.).

The investment model described here considers parameter uncertainty from both exogenous and endogenous sources. Forage production levels are uncertain, arising from stochastic annual precipitation levels. Annual forage supplies are modeled as chance constraints, based on alternative cumulative probability values of a hyperbolic tangent distribution function describing forage yield. Financial performance indicators such as net ranch income and annual changes in net worth result from values of the choice variables and random prices. Safety-first criteria constrain ranch income values using lower partial moment procedures developed by Atwood.

The following model represents the investment decision:

[1a] $\text{Max } e^{-rT} E(NW_T)$

subject to

[1b] $A_1 X \leq b$

[1c] $A_2 X \leq b$

[1d] $YX - F = 0$

[1e] $Y_1 X - t + d \geq 0$

[1f] $\tau d - Q = 0$

[1g] $t - \Gamma Q \geq g$

The objective function is the maximization of discounted expected ending net worth. Constraints [1b] are deterministic constraints relating resource usage to known supplies, as well as transfer functions characteristic of a dynamic model. Constraints [1c] constrain annual forage supplies on both native range and crested wheatgrass by b, a random forage production level based on annual precipitation. These forage supply constraints are expressed as deterministic constraints in the chance constrained sense (Charnes and Cooper). Thus, for a given precipitation level, forage production will exceed b $(1-\alpha)$ percent of the time, where α is the probability that resource supplies will be less than or equal to b. Y is a matrix of financial coefficients relating choice variables X to annual profitability and solvency measures.

Constraints [1e] to [1g] incorporate Atwood's sufficiency conditions. Y_1 is a submatrix of Y relating X to annual net ranch income (NRI) under each state of nature. T (t) is an endogenously selected reference level of income used to compute the lower partial moments of annual NRI. D is a vector of negative deviations from target levels of annual NRI. τ is the probability associated with the state of nature corresponding to each element of D. Q is the average value of the negative deviations over all states. The final constraint [1g] relates the reference level of NRI, t, to target levels g. This constraint ensures that the safety first relationship, $Pr(NRI \leq g) \leq 1/\Gamma$, is satisfied.

Behavioral Assumptions of the Model

Results described in this paper result from maximizing expected ending net worth, without explicit consideration of the distribution of this random variable. However, employing safety first constraints on annual distributions of NRI make the model consistent with the class of mean-risk models shown by Fishburn to result in solutions that will be stochastically efficient. Using the linear negative deviation from target measures (constraint [1e]) assures the results will be second degree stochastically efficient (Atwood).

Data

Cost and production data are based on Cooperative Extension budgets for Elko County, Nevada (Myer and Hackett). The first year breeding herd consists of 507 mature cows, 76 replacement heifers, and 25 bulls. All factor prices are in 1985 dollars. Animal costs, excluding feed and fixed costs, are $94.44 per cow (USDA). Variable costs of removing sagebrush/native range and planting crested wheatgrass are $23.16 per acre (Sonneman et al.). Variable feed costs include hay production ($45.01 per acre), hay purchase ($65 per ton), and BLM grazing fees ($1.35 per animal unit month). Beginning balance sheet data for the ranch are in Table 1.

The ranch utilizes approximately 78,000 acres of public rangeland. Range forage production is adequate to supply initial herd requirements under average precipitation levels. In low rainfall years, when range forage production is less than herd requirements, animals are removed from the public range early and fed hay. Fed hay is priced at market rates. Unfed hay is sold. Although annual hay production is a choice variable, cow numbers are limited by the amount of hay produced for a four and a half month winter feeding period.

Rainfall Data

Sample autocorrelation and partial autocorrelation functions were calculated for annual rainfall data from the Elko, Nevada reporting station for the period 1930 through 1984. No significant year to year correlation structure was found in the series.

Goodness of fit tests failed to reject the hypothesis that the annual data fit the gamma distribution. Maximum likelihood estimators were calculated for the distribution's two parameters ($\alpha = 11.28$ and $\beta = 0.86$). Annual rainfall values for the model were generated using IMSL.

Range Forage Production

Sneva and Hyder report annual precipitation and forage production observations as percentages of normal for various range sites representative of the Intermountain West. These 95 paired observations were used to estimate an empirical distribution of forage and precipitation using procedures reported in Taylor.

OLS estimates of coefficients served as starting points for maximum likelihood estimators. OLS regression related

$$[2]\quad u = .5 \ln \{\frac{F(Y,R)}{1 - F(Y,R)}\} = P(Y,R) = b_0 + b_1 Y + b_2 Y^2 + b_3 R$$

where F(Y,R) is the cumulative probability distribution for the sorted observations of forage yield (Y) and rainfall (R).

First order conditions characterizing the maximum value of the maximum log likelihood function were then solved simultaneously using GAMS/MINOS. The resulting empirical distribution was then

$$[3]\qquad F(Y,R) = 0.5 + 0.5 \tanh [P(Y,R)]$$

where tanh is the hyperbolic tangent function.[1]

Forage yield in the model was thus stochastic, represented by the distribution function [3]. Precipitation levels generated by IMSL were used to represent state specific values of R in P(Y,R).

Different values of F(Y,R) of each year's distribution of yields were used to provide chance constrained right hand side values for forage production. Values were derived by solving explicitly for Y for different values of αe

$$[4]\quad \alpha = F(Y,R) = 0.5 + 0.5 \tanh (P(Y,R))$$

$$= 0.5 + 0.5 \{(e^{u} - e^{-u})/(e^{u} + e^{-u})\}$$

1 The hyperbolic tangent of u is $\tanh u = (e^{u}-e^{-u})/(e^{u}+e^{-u})$.

yields, by inverting,

[5] $u = 0.5\ (\ln(\alpha) - \ln(1-\alpha))$

Since u is quadratic[2], Y, expressed as percentage of average forage production, can be solved for any specified values of α and R:

[6] $$Y = \frac{-b_1 \pm \sqrt{[b_1^2 - 4b_2(b_0 - \alpha + b_3R)]}}{2b_2}$$

where Y will be the two roots of the quadratic expression. No difficulty was encountered in choosing the "more reasonable" root in the empirical application.

Stochastic rainfall and, consequently, forage production was assumed to determine number of days of grazing available on the range. Animal growth on crested wheat (native range) was fixed at 2.31 and 1.91 (1.45 and 1.21) pounds per day for steer and heifer calves, respectively (Williams).

Additional assumptions were incorporated corresponding to crested wheat. To reduce model size, any investment occurred in year one. Two years of rest followed seeding. Investment costs exceeding surplus cash available after satisfying net ranch income target levels came from increased non-real estate borrowing.

Cattle Price Generator

Two considerations motivated the choice of procedures used to generate prices for the simulation model. First, prices display a cyclical pattern over time. Autocorrelation and partial

2 Coefficient values resulting from the reduced gradient procedures of MINOS were $b_0 = -0.871$, $b_1 = 0.068$, $b_2 = -1.094E-4$, and $b_3 = -0.047$.

autocorrelation functions supported the existence of this time trend for steer, heifer, bull, and cow price series used in the model. Second, prices for the four animal types are highly correlated. Generation of prices for the model thus relied on both time series techniques and on correlation techniques to preserve the relationship among the four price sets.

Price series of ten years each were generated using an estimated ARIMA structure for steer prices. Random deviates were generated from the GGNML subroutine of IMSL from a normal distribution with the same first two moments of the errors from the ARIMA estimation. These generated values were added to each year's ARIMA forecast to simulate randomness in the series. The resulting series thus retained the trend characteristics of the historical steer price data. The generated steer price series was next normalized and procedures described in Richardson and Condra were used to produce correlated and time trended series for heifer calves, cull cows, and bulls.

Experimental Procedures

Nine price and weather states were incorporated for each of the ten years in the model. Sensitivity of the model to the probability level associated with the cumulative distribution of forage (α) and to target levels of NRI, as well as probability limits $1/\Gamma$ of the safety first constraints, was tested by successive runs under different parameter values.

Model Results

Initial model runs were conducted with crested wheatgrass acreage fixed at zero to provide a basis for comparison. Results are reported in Figures 1 and 2 and in Table 2.

Native range forage production was adequate to meet the nutritional needs of the initial 507 cow herd during years of average or greater rainfall. With a forage cumulative distribution Á value of 0.5 and mean annual rainfall levels of 9.74 inches, 42 pounds per acre of consumable forage is produced.[3] This provides sufficient forage for 581 animal units (AUs) over the seven month grazing season. A more conservative value of the forage cdf (α = 0.33) reduces the right hand side forage supply to 561 AUs under average rainfall levels.

Herd size and ending net worth are sensitive to parameter values used. Decreasing α reduces the amount of forage available, thus resulting in lower cattle numbers. Increasing the probability that annual NRI must exceed a particular target under all states of nature decreases expected ending net worth. Number of cattle sold each year increases to meet the increased NRI targets. This reduces herd build up necessary to increase ending net worth in the model.

Substantial benefits result from converting unimproved range to crested wheatgrass (Table 3). For example, with target NRI equal $0 and both the safety first probability value and α equal to 0.5, 6193 acres of range are seeded. Under average rainfall,

3 20 acres/AUM is typical of unimproved range production in Nevada at 60 percent utilization. Improvements to three acres/AUM are possible with a crested wheatgrass seeding.

total forage production on the crested acres plus the remaining unimproved acreage provides grazing for 842 AUs, a large increase from the 581 AUs calculated above under average conditions.

Benefits of the investment result from several sources. First, greater forage supplies permit herd expansion (Figures 3 and 4). Benefits from herd expansion derive from greater annual net ranch incomes and from increased net worth at the end of the planning horizon. Second, increased forage reduces the number of states of nature under which animals must be removed from the range in states of below average rainfall. Cost savings result from reducing the number of states in which hay must be purchased (or fed rather than sold), as well as reductions in the tons of hay purchased per cow.

The substitution between annual net ranch income and ending net worth observed above for the unimproved range situation also is evident when seeded acreage is a choice variable (Figure 5). As the safety first constraints become more binding, under either higher specified targets or more stringent probability limits, ending net worth is reduced to satisfy annual income requirements.

Ranch cash flows are also affected by the seeding investment (Figure 6). A slight drop in cash surplus occurs in year one resulting from loss of native range grazing on the approximately 6,000 acres being seeded. The effect is magnified in year two when additional non-real estate payments resulting from increased borrowing to finance the investment begin. However, beginning in year 4 and continuing over the remainder of the period, cash surplus exceeds the solution obtained when no investment is allowed.

Summary and Conclusions

A dynamic nonlinear optimization model was developed in this paper to determine optimal rangeland investment levels and herd sizes for a Western beef cattle ranch. Expected discounted value of ending net worth was maximized, subject to probabilistic safety first constraints on annual net ranch incomes.

The benefits of an investment designed to provide a secure supply of high quality spring forage were reflected in annual financial performance indicators, as well as increased expected ranch net worth. This micro analysis seems to validate the millions of acres of rangelands that have been reseeded to crested wheatgrass.

An additional feature of the model addresses the problem encountered often in range economic analysis of inadequate data availability. Utilizing range data on rainfall and forage production, a cdf of forage production was calculated that allows one's confidence in data quality to be directly incorporated. If the quality of available data on forage production is doubtful, a lower value of α can be used. This variation on chance constraints directly incorporates an estimation of the empirical distribution function rather than, for example, requiring that uncertain resource supplies are normally distributed.

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Table 1. Beginning balance sheet for model ranch.

Assets		Liabilities	
Current	$10,000	Current	$23,627
Intermediate	$335,000	Intermediate	$27,718
Long-term	$341,500	Long-term	$76,855
Total	$686,500	Total	$128,200

Table 2. Solution characteristics when crested wheat acreage is fixed at 0.

Target NRI ($)	Probability Constraint $1/\Gamma$	α Value	Ending Net Worth ($)	Number of States Hay Purchased
0	0.5	0.33	660,763	13
		0.50	673,565	13
20,000	0.5	0.33	660,763	13
		0.50	673,565	13
50,000	0.5	0.33	658,456	13
		0.50	671,279	13

Table 3. Solution characteristics when crested wheat acreage is endogenous.

Target NRI ($)	Probability Constraint $1/\Gamma$	α Value	Ending Net Worth ($)	Acreage CW	Acreage Nat.	Number of States Hay Purchased
0	0.25	0.50	810,560	6193	71631	11
	0.50	0.33	796,117	6378	71446	11
		0.50	810,513	6193	71631	11
20,000	0.25	0.50	810,397	6193	71631	11
	0.50	0.33	796,117	6378	71446	11
		0.50	810,513	6193	71631	11
50,000	0.25	0.50	798,496	5994	71830	9
	0.50	0.33	795,612	6275	71549	12
		0.50	810,065	6011	71813	11
100,000	0.25	0.50	Infeasible			
	0.50	0.33	692,465	2663	75161	7
		0.50	741,126	5294	72530	5

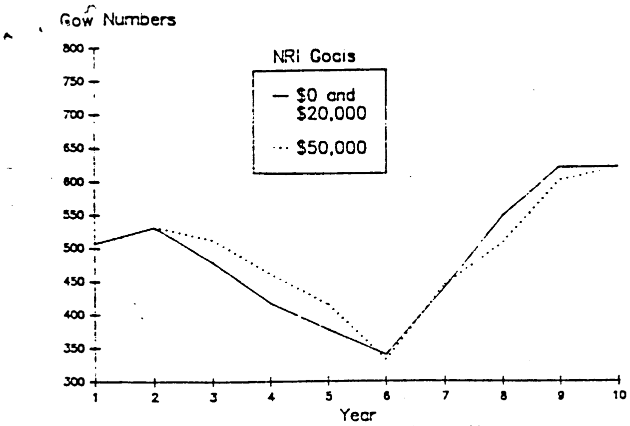

Figure 1. Cow numbers over time with Pr(NRI<g)) < 0.50, δ = 0.5, and crested wheat acreage fixed at 0.0.

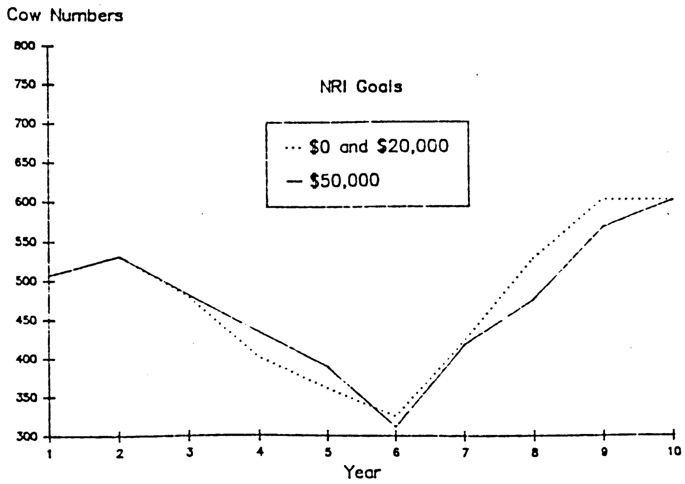

Figure 2. Cow numbers over time with Pr(NRI<g) < 0.5, δ = 0.33, and crested wheat acreage fixed at 0.0.

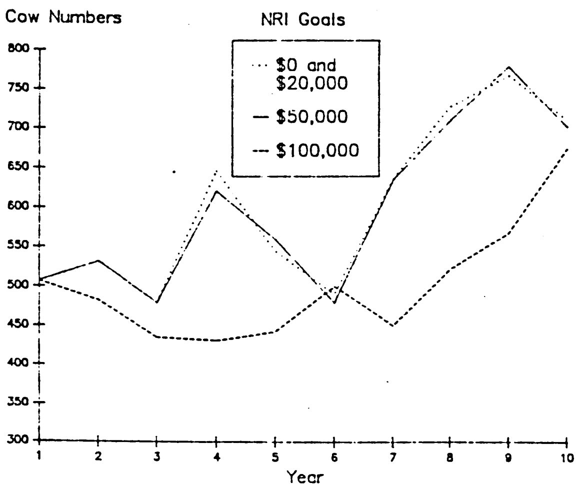

Figure 3. Cow numbers over time with Pr(NRI<g)) < 0.50, δ = 0.5, and crested wheat acreage endogenous.

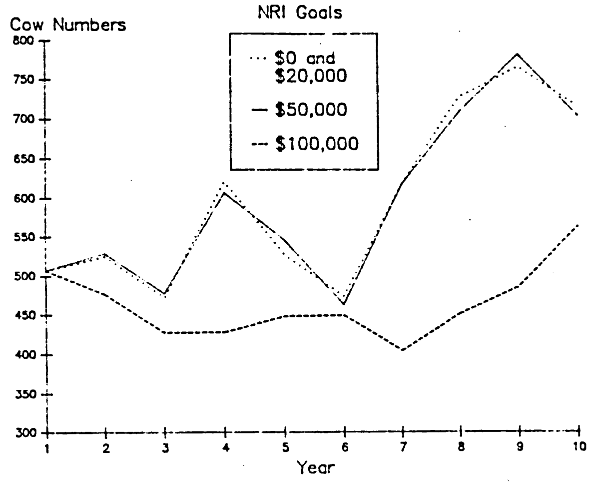

Figure 4. Cow numbers over time with Pr(NRI<g) < 0.50, alpha = 0.33, and crested wheat acreage endogenous

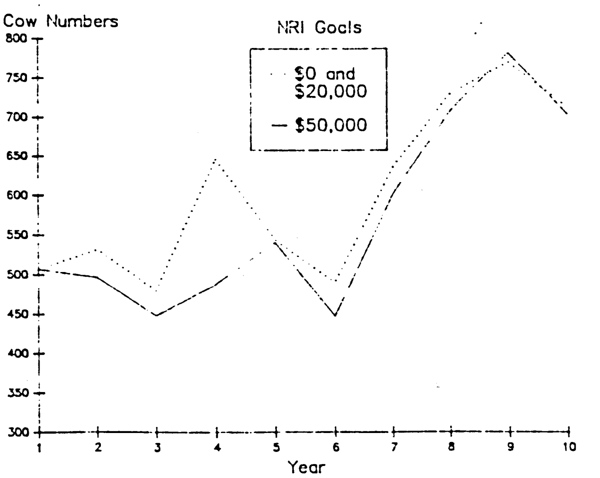

Figure 5. Cow numbers over time with Pr(NRI<g) < 0.25, alpha = 0.5, and crested wheat acreage endogenous

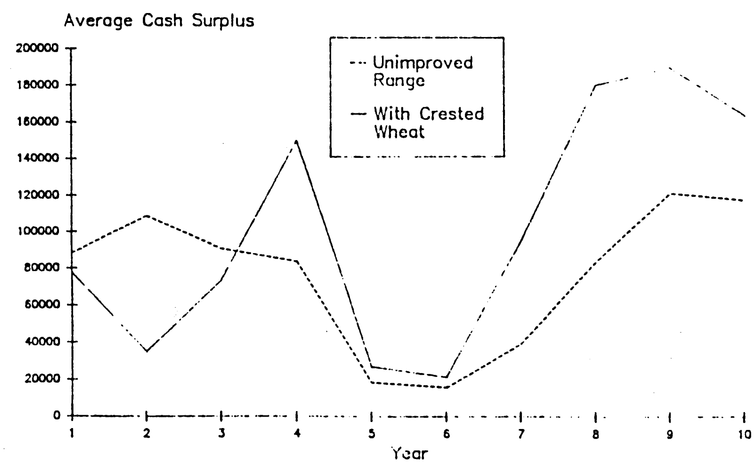

Figure 6. Average cash surplus with and without investment.