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Random Weather Shocks and Biased Estimates of Excess Demand

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1990

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Abstract

In an excess demand system where random weather shocks enter multiplicatively rather than additively, estimates of the underlying parameters will be in-efficient under the "small country" assumption and biased in the "large country" case. A Monte Carlo simulation demonstrates that the magnitude of the bias is potentially very large.

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1. Introduction

Random weather disturbances have caused severe shortfalls in U.S. and world production of the major field crops at least twice in the past ten years. Since excess demand for a traded commodity is partly determined by domestic supply, estimation of the relation implicitly assumes the form of any supply disturbance.

If the random weather disturbance is symmetrically distributed about its' mean value, and enters as an additive disturbance to planned production, than standard estimation techniques such as Ordinary Least Squares (OLS) are appropriate. The intuition being that the weather disturbance is subsumed in the error term, and that if the shock is thought of as the deviation about planned production, the term will have mean zero. That is, the weather disturbance will "wash out" over the sample period.

A more reasonable way in which to model the supply disturbance for many field crops is to assume that the random supply disturbance enters multiplicatively, rather than additively. This assumption is particularly well suited to agricultural production since we would expect that any weather induced shock would lead to a deviation of realized production about planned production which is proprtional to planned production. As an example, we would expect a drought to have a considerably larger effect on the level of realized production if 100 million acres were planted rather than 1 million acres.

This paper will demonstrate that if the random weather disturbance is not explicitly specified, OLS will obtain in-efficient estimates in a system of excess demand for

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a traded good in the "small country" case where price is assumed exogenous, and biased estimates in the "large country" case where price is determined within the system.

2. The "small country" case.

First, the "small country" case in which price is exogenous will be examined. For the purpose of this paper, a very simple model of excess demand for traded goods will be assumed. In order to focus on the supply disturbance problem, weather will be the only model mis-specification considered in evaluating the estimation rules.

2.1 The Economic Model.

A standard model of excess demand for a good which is both produced and traded by the region will be used.

Let

$$ED = QD - QS_r$$

$$QD_t = a - bP_t + \varepsilon_{1t}$$

$$QS_{pt} = c + dP_t + \varepsilon_{2t}$$

So

$$ED_t = (a - bP_t + \varepsilon_{1t}) - ((c + dP + \varepsilon_{2t})(1 + \omega_t) \tag{2}$$

Which can be rearranged as follows:

$$ED_t = \alpha - \beta P_t + \nu_t \tag{3}$$

where

$\alpha = a - c$

$\beta = b + d$

$\mu_t = \varepsilon_{1t} - \varepsilon_{2t}$

$\nu_t = c\,\omega_t + d\,\omega_t P_t + \varepsilon_2 \omega_t + \mu_t$

Equation (3) is the reduced form specification from which the elasticity of excess demand can be estimated.

The sampling properties of the OLS estimator can be easily examined within the framework of unobservable variables as found in Judge et.al. (1985). Since the random weather disturbance ω_t are assumed i.i.d. with mean zero and constant variance σ_ω^2 , and independent of price, OLS will obtain unbiased estimates of the reduced form coefficients, however the estimates will be in-efficient. That is, for any finite sample, the OLS estimator will have a larger variance compared to the estimator where the multiplicative nature of the weather disturbance has been explicitly accounted for.

The lack of efficiency can most easily be seen by examining the variance of the error term in equation (3):

$$Var(\nu_t) = \sigma_\omega^2(c^2 + 2cdP_t + d^2P_t{}^2) + \sigma_\mu^2$$

Since the variance of the equation error in period (t) depends on prices in period (t), it is heteroscadastic and thus the OLS estimator is not best linear unbiased (Judge et. al.). The OLS estimator is consistent even when weather is not explicitly accounted for. That is, as the sample size gets very large, the variance of the estimator does decrease. However given the small sample sizes often used in empirical work, the consistent and unbiased properties may pale compared to the inefficiency of the estimated parameters.

3. The "large country" case.

Applied empirical studies often aggregate world trade into a small subset of regions due to computational costs and data constraints. The simplest and possibly most studied case is the two region model in which the world is divided into the exporter and the importer. This is commonly called the large country case due to the assumption that the importer is large in the sense that its' decisions effect price, that is to say that price is endogenous.

As Orcutt described in 1950, this system must be specified as a set of simultaneous equations. For the purposes of this paper a very simple model will be assumed in order to make the problem at hand more transparent. As in the small country case as described earlier, weather will be assumed to enter the excess demand relationship multiplicativly. Since price is assumed to be endogenous, an excess supply relation must also be specified.

3.1 The Statistical Model.

Excess demand (ED) is specified as the difference between domestic demand and realized supply. Domestic demand is determined by price (P) and an exogenous demand shifter (Z_1), which is assumed to be independent of the weather disturbance. Domestic supply is a function of price and weather.

It is assumed that any random or omitted variables are independent of the exogenous demand shifter (Z) and the weather disturbance. Given this assuption, the standard additive error term can be dropped for clarity without effecting the problem at hand.

$$ED_t = (a_1 - b_1P_t + c_1Z_{1t}) - ((d_1 + e_1P_t\ )(1 + \omega_t)$$

Or.

$$ED_t = \alpha_1 - \beta_1P_t + c_1Z_{1t} - d_1\omega_t - e_1P_t\omega_t \qquad (1')$$

Where

$\alpha_1 = a_1 - d_1$

$\beta_1 = b_1 + e_1$

Excess supply (ES) is the difference between the exporters' domestic supply and domestic demand. The exporters' domestic supply is a linear function of prices and the weather disturbance. For simplicity, both regions are effected by the same weather disturbance. The qualitative results of this section will be uneffected by this simplifying assumption.

Domestic demand is again specified as being determined by price and an exogenous identifying variable (Z_2). Let:

$$ES_t = (d_2 + e_2P_t)\ (1{+}\omega_t) - (a_2 - b_2P_t + c_2Z_{2t})$$

Or.

$$ES_t = \alpha_2 + \beta_2 P_t - c_2 Z_{2t} + d_2 \omega_t + e_2 P_t \omega_t \qquad (2')$$

$\alpha_2 = d_2 - a_2$

$\beta_2 = e_2 + b_2$

Notice that equations (1') and (2') can be written as.

$$ED_t = (\alpha_1 - d_1\omega_t) - (\beta_1 + e_1\omega_t)P_t + c_1 Z_{1t}$$

$$ES_t = (\alpha_2 - d_2\omega_t) - (\beta_2 + e_2\omega_t)P_t + c_2 Z_{2t}$$

Intuitively we can think of the structural relations in terms of random coefficients. Notice that the intercepts and coefficients are not fixed, but are determined by the random weather disturbance ω . Intuition aside, the sampling properties of the reduced form parameters must be derived since random coefficients are not sufficient for bias in OLS estimates. Note that random coefficients did not cause bias in the "small country" case.

3.2 Analysis of Sampling Properties.

In order to analyze the problem the reduced form equations must be solved for. As specified, two reduced form equations exist, P^* and Q^*. To solve for the reduced form expressions excess demand is equated to excess supply.

$$P_t^* = \frac{\alpha_1-\alpha_2}{\beta_1+\beta_2+e_1\omega_t+e_2\omega_t} + \frac{c_1 Z_{1t}}{\beta_1+\beta_2+e_1\omega_t+e_2\omega_t} + \frac{c_2 Z_{2t}}{\beta_1+\beta_2+e_1\omega_t+e_2\omega_t} + \frac{(d_1+d_2)\omega_t}{\beta_1+\beta_2+e_1\omega_t+e_2\omega_t}$$

or

$$P_t^* = \frac{\alpha_1-\alpha_2}{\beta_1+\beta_2} W_t + \frac{c_1}{\beta_1+\beta_2} W_t Z_{1t} + \frac{c_2}{\beta_1+\beta_2} W_t Z_{2t} + (d_1+d_2)\omega W$$

where

$$W_t = \frac{\beta_1 + \beta_2}{\beta_1 + \beta_2 + e_1\omega_t + e_2\omega_t}$$

$$P_t^* = \pi_{p1} W_t + \pi_{p2} W_t Z_1 - \pi_{p3} W_t Z_{2t} + \omega_t W_t k_1$$

$$\pi_{p1} = \frac{\alpha_1-\alpha_2}{\beta_1-\beta_2}$$

$$\pi_{p2} = \frac{c_1}{\beta_1-\beta_2}$$

$$\pi_{p3} = \frac{c_2}{\beta_1 - \beta_2}$$

$$k_1 = d_1 + d_2$$

P^* is now in terms of the reduced form parameters, a random exogenous and unobservable variable (W), and the exogenous observable (Z). If the reduced form parameters are now defined by the vector Π_p ,the reduced form equation P^* can be written in matrix notation as.

$$P^* = WZ\Pi_p + W\varpi k_1 \qquad (3')$$

Where P^* is a T by 1 vector, W is a T by T matrix with the W_t on the diagonal and zeros elsewhere. Z is a T by 3 matrix , Π_p is a 3 by 1 coefficient vector, ϖ is a T by 1 vector of the ω_t and k_1 is a scalar constant.

The second reduced form equation Q^* is solved by substituting the expression for P^* into the structural equation. Following the same steps as in the reduced form for price, Q^* can be represented as.

$$Q^* = WZ\Pi_q + W\omega ZC + W\omega\Omega + \varpi d_2 \qquad (4')$$

With the dimensions of W, Z, and Π as above, and ω is a T by T matrix with the ω_t on the diagonal and zeros elsewhere. Ω is a T by 1 vector with the individual elements being a linear function of the structural parameters and the ω_t

The system can be interpreted as follows. In looking at the reduced form equations (from which the structural parameters will be identified) we can see that if the weather disturbance is not observed, the true identifying matrix WZ is measured with error. That is, the true identifying matrix is WZ, but we only observe Z. Notice that when the ω_t are zero, the W_t equal 1 and the matrix W is the identity matrix. However since E(1/x) does not equal 1/E(x) the expected value of the W_t is not likely to equal one even though the expected value of the ω_t is zero. Futher more the measurement error is proportional to the observed right-hand side variables.

It is important to remember that we are working with the reduced form equations, not the structural equations. Even though we have assumed that weather is independent of all observable exogenous variables, the measurement error in the reduced form relations is correlated with the underlying exogenous R.H.S. variables. If weather is not explicitly accounted for, that is, if the true structural relations as shown in equations (1') and (2') are not correctly specified, estimates of the reduced form and thus structural parameters will be biased.

To see that the problem of weather induced bias can be delt with, notice what happens when the ω_t and $\omega_t P_t$ which enter the true structural relations are treated as observable exogenous variables. If the ω_t and $\omega_t P_t$ are observable, the estimable reduced form equation is;

$$P^* = \pi_{p1} + \pi_{p2} Z_1 + \pi_{p3} Z_2 - \pi_{p4} \omega_t - \pi_{p5} P_t \omega_t$$

In this simple model no error term remains, however as long as any omitted variables which might enter the error term in practice are independent of weather, the reduced form parameters will be unbiased. This result holds for the reduced form relations Q^* .

Since the expected value of W is unknown, the finite sampling properties cannot be determined easily. It can be shown that if weather is not specified, that the estimated reduced form parameters will not converge to the true parameters, even as the sample goes to infinity. A simple demonstration of the asymptotic properties is left to the appendix.

4. A Monte Carlo Simulation

The sign and magnitude of weather induced bias can not be determined analytically since the bias is a function of unobservable parameters and realized exogenous variables. If values are assigned to these parameters (the underlying structural parameters of demand and supply), a simulation model can be used to yield information concerning the direction and magnitude of the bias.

4.1 The Sampling Experiment.

Excess demand and supply are specified as in the preceding section. Values for the structural parameters will be assumed. Equilibrium price and quantity is solved for by specifying the level of the two demand shifters (Z_1 and Z_2) and the realized weather disturbance.

4.2 Sampling Design.

The simulation model is specified as.

$$ED_t = (a_1 + b_1 P_t + c_1 Z_{1t}) - (d_1 + e_1 P_t)(1 + \omega_t)$$

$$ES_t = (d_2 + e_2 P_t)(1 + \omega_t) - (a_2 + b_2 P_t + c_2 Z_{2t})$$

Equilibrium prices and quantities can then be solved for given assumed values for the exogenous demand shifters (Z_i), the weather disturbance, and the parameters.

For the simulation, the demand shifters are taken as historical levels of U.S. and rest of world G.N.P.. Historically observed levels of G.N.P. are used to make the results relevant to applied research, as the results are in part influenced by the variance of the exogenous variables.

The ω_t are obtained from a random number generator. The distribution is specified as normal with mean zero, and constant variance σ_ω^2. The variance was obtained as an estimate of the deviation of U.S. wheat yields about a time trend. Yields are in terms of yields per planted acre. Of course this is a very simplistic and biased estimate of the true variance of ω, however the results which follow are robust with respect to the underlying variance of the random disturbance.

The weather induced bias is partly determined by the structural parameters themselves. The value of these parameters is a widely debated topic. For this study plausible values are assumed. The first experiment will use what will be called "base" parameters. No subjective value is intended for the validity of these "base" settings. They are the base only in the sense that they will serve as the numerator for the other experiments in order to determine how robust the results are to different settings of the unknown parameters.

Five experiments were run in order to determine whether the results are robust to assumptions about the underlying parameters.

Each experiment took 900 draws of sample size 20. Each draw used the same sample of U.S and rest of world G.N.P., with only realized values of the ω_t changing from draw to draw. Given the random draws of ω, 900 equilibrium price and quantity vectors were obtained.

Each *experiment* differed only in the values assumed for the structural parameters. The realized values of the ω_t were held constant across experiments (except for experiment #2) so that the different experiments would be comparable.

Sampling Results.

Given 900 draws of sample size 20 mean sample estimates of the mis-specified (omitting ω) system can be compared to the true population parameters. Since the estimates of the reduced form parameters are biased, all estimates of the structural parameters will also be biased. Only the coefficients on price and G.N.P for the excess demand equation will be reported. The coefficient on price will be called β where $\beta = b_1 + e_1$ and the coefficient on rest of world income will be called γ where $\gamma = c_1$.

For each experiment, table (1) reports the true population parameters (β_{true} , γ_{true}) and the mean estimates of of β and γ that were obtained from Three Stage Least Squares omitting ω (β^{TSLS} , γ^{TSLS}). Results from estimating the structural parameters directly with OLS are also reported. Of course these estimates will be biased, but it is interesting to compare the results to the TSLS experiments.

TABLE 1

experiment	1	2*	3	4	5
b_1	base	base	base*0.5	base	base*2.0
c_1	base	base	base*0.5	base	base
e_1	base	base	base*0.5	base	base*2.0
b_2	base	base	base	base*2.0	base*2.0
c_2	base	base	base	base	base
e_2	base	base	base	base*2.0	base*2.0
β_{true}	-2912	-2912	-1456	-2912	-5824
β^{TSLS}	-440	-993	186	-1060	-881
	(145)	(366)	(251)	(352)	(290)
β^{OLS}	-438	-439	-93	-416	-876
	(0.481)	(0.393)	(0.470)	(0.730)	(0.962)
γ_{true}	0.817	0.817	0.409	0.817	0.817
γ^{TSLS}	0.239	0.364	0.078	0.458	0.239
	(0.0357)	(0.0882)	(0.0512)	(0.0357)	(0.0360)
γ^{OLS}	0.240	0.240	0.135	0.327	0.240
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)

*: Experiment (2) was run with the "base" parameters, however each observation of the weather disturbance was multiplied by two.

() standard error.

The base settings of the structural parameters roughly correspond to the elasticity at their mean value of:

b_1 : Elasticity of ROW demand w.r.t. price = -0.4

c_1 : Elasticity of ROW demand w.r.t. income = 1.0

e_1 : Elasticity of ROW supply w.r.t. price = 0.2

b_2 : Elasticity of U.S. demand w.r.t. price = -0.4

c_2 : Elasticity of U.S. demand w.r.t. income = 0.4

e_2 : Elasticity of U.S. supply w.r.t. price = 0.4

The results show that omitting the weather disturbance may lead to serious mis-specification bias in all right-hand side parameters. Notice that the mean estimate of β is less then half the true population parameter in all experiments run. It is also important to note that omitting ω leads to biased estimates of elastisities with respect to variables independent of the weather disturbance. In the experiments run, the elasticity of excess demand w.r.t. income was biased by roughly the same magnitude as the elasticity of excess demand w.r.t. price.

The results of directly estimating the structural equations (β^{OLS} and γ^{OLS}) are reported to demonstrate that an apperant "good fit" does not justify direct estimation. The applied researcher may be compelled to use direct estimation however, since the results show that estimation of the reduced form parameters (TSLS) can easily produce estimates of the wrong sign. Notice that in experiment (3) even the mean value of β was positive, while direct estimation of the structural parameters at least obtained the correct sign. Again, this analysis is not ment as a justification of direct estimation, but as a demonstration of how serious a problem mis-specification can be in applied practice.

5. Economic and Statistical Implications.

The literature on the specification and estimation of excess demand for agricultural goods covers a large body of classes of mis-specification, however no explicit treatment of the effect of multiplicative weather disturbances has been offered. Numerous studies have been undertaken which attempt to estimate the elasticity of excess demand for agricultural goods. Virtually none of these studies explicitly account for the mis-specification outlined in this paper, nor do they report the possible bias to their audience.

This paper has demonstrated for the sampling model considered, that the magnitude of the bias is potentially very large and robust to the underlying parameters. Further more, all right-hand side parameters are biased even if the exogenous variables are completly independent of the weather disturbance.

The importance of this mis-specification error is particularly relevant since applied research and analysis in fields such as trade policy, stabilization, and macroeconomic externalities depend on the magnitude of these point estimates. A suitable proxy for the weather disturbance must be specified if we are to have any confidence in estimated trade equations, and the results of studies which use estimated point estimates.

APPENDIX

Asymptotic properties of OLS on the reduced form equations: The "large country" case.

Given the reduced form equation for P^*

$$P^* = WZ\Pi_p + W\varpi k$$

$$plim\ \hat{\Pi}_p = plim\ \left[(Z'Z)^{-1}Z'(WZ\Pi_p + W\varpi k)\right]$$

$$= plim\ (Z'Z)^{-1}Z'WZ\Pi_p + plim\ (Z'Z)^{-1}Z'W\varpi k$$

$$= plim\ \frac{(Z'Z)^{-1}}{T}\ plim\ \frac{Z'WZ}{T}\Pi_p + plim\ \frac{(Z'Z)^{-1}}{T}\ plim\ \frac{Z'W\varpi}{T}k$$

And when the individual elements of the four R.H.S. expressions are examined, it is easy to show that.

$$plim\ \hat{\Pi}_p = Q_1^{-1}\ Q_2\ \Pi_p\ +\ Q_1^{-1}\ Q_3$$

Since Q_1^{-1} , Q_2 , and Q_3 are constant matrixes, the OLS estimate of $\hat{\Pi}_p$ is shown to differ from the true parameter vector even as the sample size goes to infinity.

It can be shown using the same steps as above that in the probability limit, $\hat{\Pi}_q$ does not converge to the true parameter vector.

Estimates of the structural parameters can be identified, however since the estimated reduced form parameters do not converge to their true values, the estimated structural parameters likewise will not converge to their true values.