



**AgEcon** SEARCH  
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

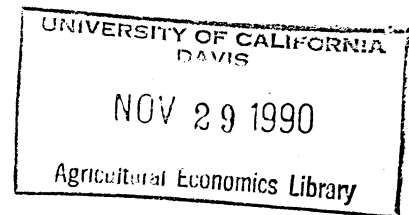
**Help ensure our sustainability.**

Give to AgEcon Search

AgEcon Search  
<http://ageconsearch.umn.edu>  
[aesearch@umn.edu](mailto:aesearch@umn.edu)

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

8717



PRODUCTION FUNCTIONS OF WESTERN IRRIGATED CROPS

Noel R. Gollehon, Michael R. Moore, and Donald H. Negri

ABSTRACT

For two functional forms, this paper reports econometric estimates of production functions for thirteen major irrigated crops using a common dataset and a consistent specification of the variables across crops. Estimates of the output elasticity of irrigation water are uniformly highly inelastic for each crop.

Gollehon and Moore are an agricultural economist and economist, respectively, with the Water Branch, Resources and Technology Division, Economic Research Service, USDA. Negri, now at Willamette University, was formerly an economist with the Resources and Technology Division. The authors are indebted to the Agricultural Division, Bureau of the Census, for allowing the use of the primary data. The sample design, data collection, and processing were conducted by the Bureau of the Census. The authors alone take responsibility of the economic analysis and results. The views expressed are the authors' and do not necessarily represent policies or views of the U.S. Department of Agriculture.

AAEA

1990

Irrigation research

## PRODUCTION FUNCTIONS OF WESTERN IRRIGATED CROPS

Relationships among crop yield, water application, and other inputs to irrigated production is an important research topic stemming from the greater input intensity of irrigated agriculture relative to rainfed agriculture. Farm producers demand information on irrigation water productivity because, as a controlled input, tradeoffs in the timing and volume of water applications affect production and profit. Historically, the public sector demanded similar information for planning purposes in support of the Bureau of Reclamation's responsibility to develop western water resources.

Although research on production functions for irrigated crops remains important for producers, the policy purpose has changed from supporting water resource development to informing decision makers on water management options. With increased competition for western surface water supplies and sustained mining of ground water reserves, irrigated producers in most regions will have the incentive (either through regulations or water markets) to consume less water. Information on the output elasticity of irrigation water for major crops and the substitutability between irrigation water and other inputs, consequently, provides an empirical basis for assessing the ramifications of irrigation water conservation on agricultural output.

Data availability repeatedly has limited study of production functions for irrigated crops. Although sound agronomically, crop-specific response functions estimated from field-experiment data were difficult to link to full economic production functions. Field-experiment data, in most cases, fails to provide tradeoffs between irrigation water and irrigation technology or between irrigation water and land. This has been the case for both traditional (Yaron; Hexem and Heady), and more recently, von Liebig response

function estimation (Ackello-Ogut, et al.; Grimm, et al.; Paris and Knapp). A second type of production function research focused less on agronomic relations and more on labor, machinery, and other inputs as substitutes for irrigated land and water (Ruttan; Brown and Beattie; Madariaga and McConnell). This approach relies on farm-level data (rather than crop-specific data) that inevitably is aggregated to a county level. With farm-level information only, revenue becomes the dependent variable to form a common measure across crops. This prevents an understanding of the underlying crop-specific relationships between output and inputs.

This paper estimates production functions for a comprehensive set of thirteen irrigated crops with both Cobb-Douglas and quadratic specifications. The paper focuses on aggregate crop output response to irrigation water and irrigation water management, while correcting for other factors with a consistently defined set of explanatory variables. Estimated water input--crop output elasticities provide empirical evidence of output changes that would follow policy-induced changes in water input.

#### Production Function Specification

The traits of the dataset used in the research motivated many of the fundamental modelling decisions. The primary dataset is composed of cross-sectional data from the 1984 Farm and Ranch Irrigation Survey (hereafter FRIS) (U.S. Department of Commerce). The core variables are crop-specific observations of output per acre, irrigation water application per acre, land, and irrigation technology. Three modelling decisions follow from this context. First, we choose the primal rather than the dual approach because: (1) the data on yield, irrigation water, and land are the best data available in terms of observation numbers and crop coverage, and (2) cross-sectional price data

contain little variation, thus making a dual approach less appropriate. Second, we estimate per acre production functions rather than converting yield and water to total output and input to estimate standard production functions.<sup>1</sup> Third, we do not estimate von Liebig response functions because of their detailed field-level data requirements.

Assuming input nonjointness in multicrop production on a farm, crop-specific production functions can be specified and estimated (Just, et al). The per acre production function for the Cobb-Douglas specification is

$$(1) \quad y = Ax_1^\alpha x_2^\beta x_3^\gamma x_4^\delta e^{\sum_{i=1}^n \rho_i z_i + e_1}$$

where:  $y$  is output per acre;  $A$  is a constant;  $x_1$  is irrigation water application per acre (acre-inches);  $x_2$  is rainfall per acre (inches);  $x_3$  is cooling degree days per acre (degree-days);  $x_4$  is land (acres);  $e$  is the exponential function;  $z_i$  ( $i=1, \dots, n$ ) are a series of dummy variables for water management, farm characteristics, climate, and soil quality; and  $e_1$  is an error term. Because the functions are on a per acre basis, the exponent on land measures returns-to-scale rather than the conventional output elasticity of land.<sup>2</sup>

The specification of the quadratic function is

$$(2) \quad y = a + \sum_{i=1}^3 b_i x_i + \sum_{i=1}^3 c_i x_i^2 + \sum_{i=1}^n d_i z_i + e_2$$

where:  $a$  is a constant;  $e_2$  is an error term; and the remaining variables are defined as before. Land is not an argument in the function because the quadratic specification imposes constant returns-to-scale.

## Data and Variables

The primary dataset is composed of 8,009 responses to the 1984 FRIS from the seventeen western states. The survey emphasizes irrigation related decisions, and contains no information on labor, machinery, and human capital.

Crop-specific data for thirteen crops (alfalfa hay, barley, corn silage, cotton, dry beans, grain corn, grain sorghum, other hay, potatoes, rice, soybeans, sugar beets, and wheat) provided by the survey respondents include: output per acre, irrigation water per acre, land, and irrigation technology. The output, water (IRRWATER), and land data enter the analysis directly as variables. Gravity systems are used as the base irrigation technology, with sprinkler technology (SPRKLRTech) and other technologies (drip, trickle, or subirrigation) forming two dummy variables.

Farm-level data from FRIS includes data on irrigation and general farm characteristics. Water management information based on the method of deciding when to apply water forms a set of water management dummy variables with both higher (HIGHMGMT) and lower (LOWMGMT) management measured relative to a base. Surface water as the sole supply source is specified as a dummy variable (SURFACE). Responses on whether irrigation was discontinued long enough to affect yields is the final irrigation-related dummy variable (DSCNTN).

Dummy variables for four general farm characteristics include: whether the farm has a relatively large acreage of dryland crop production; whether the farm is a small irrigated operation; the crop's share of total irrigated cropped area on the farm; and the organizational type of farm ownership.

Several weather, climate, and soil quality variables are merged with the FRIS data to improve the explanation of crop yields by providing information on the physical environment. They are county-level data. Weather variables

include rainfall and cooling degree days within the 1984 growing season, the number of days that rain exceeds one inch, and the number of days when temperature exceeds 90 degrees. Rainfall measures water available for plant growth in addition to irrigation water, while cooling degree days measures energy availability. These variables are modelled as primary input variables (as in Madariaga and McConnell) rather than as dummy variables.

Climate variables include average rainfall and average cooling degree days modeled as dummy variables to minimize multicollinearity problems with weather variables. The climate variables serve as proxies for unobserved producer decisions (e.g., seed variety or tillage practices) affected by climate but made prior to the observation of weather in the production year.

The soil quality variables, cropland classification and soil texture, are dummy variables bracketing a base condition. The third soil indicator variable, slope, represents the gradient in percentage terms.

### Econometric Results

The alternative forms of the per acre production functions in equations (1) and (2) are estimated using ordinary least squares, with the Cobb-Douglas function estimated in a linear-in-logarithms form. The largest number of parameters are estimated for the quadratic alfalfa hay equation and the other hay equation, 31 in each equation, and the fewest are estimated for the Cobb-Douglas rice equation, 17. The number of estimated parameters changes by crop because the definitions of the weather, climate, and soil quality variables remain identical across crops. Thus, crops produced in diverse physical conditions (e.g., alfalfa) have parameters estimated for many conditions, while crops not produced in diverse physical conditions have fewer.

The results appear satisfactory for most crops given the large number of parameters estimated and the cross-sectional nature of the data.<sup>3</sup> The performance varies by crop. With the Cobb-Douglas specification, significant parameters (at the 5% level) range from 19 of 27 estimated parameters for wheat and 18 of 26 for grain corn to 3 of 17 for rice and 4 of 22 for soybeans. The number of observations of each crop, which varies from 142 for rice to 3,516 for alfalfa, partly explains the performance range of the  $t$  statistics. Even crops with unsatisfactory results, like soybeans, provide information on the relationship between irrigation water and yield. The quadratic specification, in general, exhibits similar variety and performance across crops (Table 1).

Similarly, the adjusted  $R^2$ s in the Cobb-Douglas case range from .60 for cotton and .54 for rice to .10 for soybeans and .09 for dry beans. With the quadratic form, adjusted  $R^2$ s are similar to the Cobb-Douglas results<sup>4</sup>.

#### Irrigation Water

As a determinant of crop output, irrigation water performs strongly regardless of the functional form, with most  $t$  statistics exceeding significance at the .01 level (Table 1). The cases of insignificance are few: barley, in both specifications, despite 1,168 observations; dry beans in the Cobb-Douglas specification; and rice and sugar beets in the quadratic specification. With rice and sugar beets, diagnostics indicated collinearity between irrigation water and irrigation water squared. An added-variables test on the significance of the two variables considered jointly, in fact, shows significance at the .05 level and .10 level in the rice and sugar beets equations, respectively.

The cross-sectional data used in this analysis provided results surprisingly consistent with expectations. Irrigation water in all functions (with quadratic rice as an exception), whether statistically significant or not, provided concave functions exhibiting diminishing marginal product of irrigation water. However, our quadratic functions show high yield-maximizing water application rates relative to estimates in the literature. For example, a comparable basis exists for five crops as reported in Grimm, et al., p.188, and our results, which show significantly higher water input levels to achieve maximum yields in four of five crops. The nature of survey data on actual producer decisions explains the difference. When behaving rationally, producers will not intentionally apply water in the range of negative marginal product, whereas field experiments consciously attempt to determine the point of yield maximization by applying irrigation water past the point of zero marginal product. Consequently, functions estimated with the FRIS data should provide good estimates of producer behavior, even if estimates of the yield maximizing point are not the most accurate.

#### Water Management

Irrigation technology may serve either to increase water application efficiency per se or to substitute for poor land quality, like sandy or sloped soil conditions (Caswell and Zilberman; Lichtenberg). Both roles should increase crop yields provided that other variables control for land quality. SPRKLRTECH, the sprinkler dummy variable, is frequently insignificant, having the expected sign and significance with only quadratic alfalfa hay and other hay. Further, with the Cobb-Douglas form, coefficients on cotton, dry beans, and grain corn are negative and significant (Table 1).

The sprinkler technology results are surprisingly weak for crop-specific data. The finding from previous research that irrigation technology substitutes for poor quality land may explain this. Sprinklers tend to be installed in the fields with relatively sandy soil or sloped topography. Consequently, sprinklers serve incidentally as a measure of field-level land quality. This is a finer level of geographic detail than the county-level soil quality variables used in this estimation. In other words, the two functions of irrigation technology--substituting for water versus substituting for land quality--cannot be isolated accurately with the current dataset. Matching field-level data on land quality would improve the analysis.

The other irrigation-related variables perform better. Relying on more sophisticated techniques to decide when to irrigate (HIGHMGMT) improves yields for eight of thirteen crops. Improvements range between five and twelve percent of mean yields for the eight crops. Relying on less sophisticated techniques (LOWMGMT) depresses yields for three of five crops in the Cobb-Douglas form, but only one of five crops in the quadratic form. Discontinuing irrigation for a period of the growing season (DSCNTN) depresses yields for seven of twelve crops. For the grain crops in the quadratic specification, DSCNTN reduces yields between five and nine bushels per acre. Finally, we hypothesize that, because ground water typically gives more flexibility in timing of use than surface water, relying on surface water reduces yield. Farms with surface water as their only water source (SURFACE) experience lower yields on only four crops in at least one specification, and a higher yield of wheat in both specifications. For most crops, consequently, relying solely on surface water does not constrain irrigation timing enough to influence yield.

### Water-Output Relationships and Policy Implications

The output elasticity of irrigation water provides a common measure across crops and functional forms of the effect of irrigation water on output (Table 1). For the Cobb-Douglas form, the parameter estimate on irrigation water from the regression measures a crop's output elasticity directly. For the quadratic form, the per acre function is converted to an output function and the elasticity measure is evaluated at the mean output and input levels.

Across functional forms and crops, the elasticities generally differ by very small amounts, ranging from 0.014 (barley) to 0.145 (alfalfa), and never show marked differences. They all are highly inelastic. As both functions are generated from the same data and permit declining marginal products in inputs, the small differences are not surprising.

The output elasticities generally fall in the same range as previous empirical estimates for irrigation water. Estimated with Cobb-Douglas functions, elasticities for five vegetables varied from .005 to .079 (Just, et al.) and for wheat from .041 to .241 depending on the model and the econometric technique (Antle and Hatchett). For the majority of crops, however, this paper provides new empirical evidence.

In a period of competition for existing western water supplies with no new supplies on the horizon, one broad implication of the elasticities seems clear: reductions in production associated with diminished irrigation water supply would be much smaller, relatively, than the water supply reductions. For these major irrigated crops, a 10 percent reduction in water use would induce at most a 1.5 percent reduction or less in output, ceteris paribus. For example, a 10 percent reduction in per acre water application equals 3.25 acre-inches and 2.91 acre-inches on cotton and alfalfa, respectively (Table

1). This translates, on average, into a 10.5 pounds per acre and 0.063 tons per acre decline in cotton production and alfalfa production using the quadratic elasticities. Given the mean yields of 916 pounds of cotton and 4.33 tons of alfalfa, the output reductions are relatively minor.

An economic model of multicrop profit maximization rather than a single crop production model would need to be constructed to develop a complete set of policy implications from these results.

### Conclusions

The goal of this paper is to provide empirical estimates of irrigated agriculture's response to incentives to use less water. The analysis focuses on the primal production function estimation of thirteen western irrigated crops with consistent analytical techniques. The results emphasize water technology, water management and water application rates as inputs to irrigated production.

As an explanatory variable for yield, irrigation technology was disappointing. This is probably due to the substitution of technology for poorer land quality. Irrigation management variables performed better. Notable results include: (1) more sophisticated irrigation management techniques increased yield from five to twelve percent, and (2) surface water supplied irrigators did not experience a significant yield loss. These results imply that management may substitute for more limiting inputs and that significant yield gains may not occur with improved surface water delivery flexibility at current management levels.

Estimates on water application levels as determinants of yield levels were significant, almost without exception, providing concave production functions in irrigation water. The survey nature of the data has

applicability to behavioral analysis but does not replicate functions based on experimental data. At the means, output elasticities on water indicate a very inelastic response, with a 10% reduction in water inducing at most a 1.5% reduction in yield. The inelastic nature of the water output response is not unique to this study. However, the crop coverage and consistent statistical techniques are unique.

#### FOOTNOTES.

1. The relationship between standard production functions and per-acre production functions is rarely explicitly recognized. Too frequently, researchers simply specify a per-acre function, or a yield response function, without recognizing the assumption that, by specifying the output and input data on a per-acre basis and ignoring land as an input, a constant returns-to-scale production function is implied.
2. The per-acre Cobb-Douglas function follows directly from a standard production function by dividing both sides of the equation by  $n$ . The output elasticity of land,  $u$ , can be calculated from the estimates as  $d+1-a-b-c$ . The estimated exponent on land,  $d$ , equals  $a+b+c+u-1$ . It represents the returns-to-scale with constant, decreasing, or increasing returns as  $d$  is equal to, less than, or greater than zero.
3. Due to space limits, only the water and water management variables will be discussed. Full results and an evaluation of the "correct" specification of functional form using non-nested hypothesis tests are available from the authors.
4. Not surprisingly, the disparity between adjusted  $R^2$ s reported in this paper and  $R^2$  results using experimental data is large. With experimental data, for example, Grimm, et al. report  $R^2$ s ranging from .615 to .962 for two specifications of yield response to water and nitrogen applications. Field experiments control for inputs other than water, while the FRIS survey does not contain data (much less control) for other inputs.

Table 1. Summary Statistics and Partial Production Function Results for Thirteen Western Crops.

Item	Alfalfa Hay	Barley	Corn Silage	Cotton	Dry Beans	Grain Corn	Grain Sorghum	Other Hay	Potatoes	Rice	Soybeans	Sugar Beets	Wheat
Crop Units	ton	bu.	ton	lbs.	cwt.	bu.	bu.	ton	cwt.	cwt.	bu.	ton	bu.
Observations	3516	1169	734	411	748	1485	623	1492	393	142	333	288	1923
Means:													
Output, PA	4.33	79.5	20.4	916	20.4	132.1	86	2.14	348	67.8	37.9	23.4	73.6
Irrigation Water Application, PA	29.1	20.7	24	32.5	22.6	22.1	16.9	22.9	28.2	62.5	12.1	32.9	19.1
Cobb-Douglas Estimates:													
LOG(IRRWATER)	0.138*	0.020	0.086*	0.126*	0.026	0.064*	0.115*	0.078*	0.115*	0.087*	0.094*	0.055*	0.083*
SPRKLRTech	0.071*	0.011	-0.033	-0.127*	-0.067*	-0.036	-0.050	0.160*	0.031	NA	-0.034	0.009	-0.029
HIGHMGMT	0.153*	0.111*	0.085	0.034	0.038	0.055*	0.034	0.128*	0.065*	0.024	0.123*	0.016	0.069*
LOWMGMT	-0.160*	-0.102*	NA	NA	NA	-0.060	NA	-0.101*	NA	NA	NA	NA	-0.127*
SURFACE	0.003	-0.015	-0.049*	-0.017	-0.001	-0.047*	-0.090*	-0.014	0.022	-0.023	-0.207*	-0.078*	0.047*
DSCNTN	-0.130*	-0.091*	-0.053*	-0.029	-0.070*	-0.067*	-0.115*	-0.067*	-0.058	NA	-0.026	-0.090*	-0.091*
Adj. R-Squared	0.19	0.10	0.11	0.60	0.09	0.23	0.19	0.17	0.31	0.54	0.10	0.37	0.37
Quadratic Estimates:													
IRRWATER	0.027*	0.094	-0.202*	5.69*	0.125*	0.889*	1.13*	0.009*	2.91*	-0.061	0.401*	0.116*	0.504*
(IRRWATER) <sup>2</sup>	-0.0001*	-0.001	-0.002*	-0.036*	-0.002*	-0.001*	-0.016*	-0.00001	-0.022*	0.001	-0.005	-0.001	-0.005*
SPRKLRTech	0.271*	0.666	-0.152	-84.77*	-1.25*	-0.407	-3.25	0.416*	10.98	NA	-0.107	-0.014	-1.25
HIGHMGMT	0.469*	9.66*	1.34*	24.12	0.450	7.37*	4.17*	0.233*	26.04*	2.19	3.16*	-0.001	5.04*
LOWMGMT	-0.359*	-5.00	NA	NA	NA	-7.67	NA	-0.089	NA	NA	NA	NA	-6.44
SURFACE	-0.090	-2.13	-0.642	-14.12	0.361	-5.62*	-7.59*	0.002	15.80	-0.46	-4.15*	-1.16*	2.94*
DSCNTN	-0.348*	-7.15*	-1.060*	-19.15	-1.03	-7.56*	-8.53*	-0.148*	-23.21	NA	0.959	-2.11*	-5.68*
Adj. R-Squared	0.34	0.14	0.11	0.63	0.08	0.22	0.20	0.21	0.37	0.41	0.13	0.38	0.42
Output elasticity of irrigation water:													
Cobb-Douglas	0.138	0.020	0.086	0.126	0.030	0.064	0.115	0.078	0.114	0.087	0.094	0.055	0.083
Quadratic	0.145	0.014	0.118	0.115	0.061	0.070	0.112	0.112	0.128	0.107	0.088	0.064	0.082

"PA" stands for per acre and "NA" means that insufficient observations are available to estimate the variable.

\* indicates estimate is significantly different from zero at the 1% level.

+ indicates estimate is significantly different from zero at the 10% level.

# REFERENCES

- Ackello-Ogut, Christopher, Quirino Paris, and William A. Williams. "Testing a von Liebig Crop Response Function against Polynomial Specifications." Amer. J. Agr. Econ. 67(1985):873-880.
- Antle, John M., and Stephen A. Hatchett. "Dynamic Input Decisions in Econometric Production Models." Amer. J. Agr. Econ. 68(1986):939-949.
- Bernardo, Daniel J., Norman K. Whittlesey, Keith E. Saxton, and Day L. Bassett. "An Irrigation Model for Management of Limited Water Supplies." West. J. Agr. Econ. 12(1987):164-173.
- Brown, William G., and Bruce R. Beattie. "Improving Estimates of Economic Parameters by Use of Ridge Regression with Production Function Applications." Amer. J. Agr. Econ. 57(1975):21-32.
- Caswell, Margriet F., and David Zilberman. "The Effects of Well Depth and Land Quality on the Choice of Irrigation Technology." Amer. J. Agr. Econ. 68(1986):798-811.
- Chambers, Robert G., and Richard E. Just. "Estimating Multioutput Technologies." Amer. J. Agr. Econ. 71(1989):980-995.
- Hexem, Roger W., and Earl O. Heady. Water Production Functions for Irrigated Agriculture. Ames:Iowa State University Press, 1978.
- Grimm, Sadi S., Quirino Paris, and William A. Williams. "A von Liebig Model for Water and Nitrogen Crop Response." West. J. Agr. Econ. 12(1987):182-192.
- Just, Richard E., David Zilberman, and Eithan Hochman. "Estimation of Multicrop Production Functions." Amer. J. Agr. Econ. 65(1983):770-780.
- Lichtenberg, Eric. "Land Quality, Irrigation Development, and Cropping Patterns in the Northern High Plains." Amer. J. Agr. Econ. 71(1989):187-194.
- Madariaga, Bruce, and Kenneth E. McConnell. "Value of Irrigation Water in the Middle Atlantic States: An Econometric Approach." South. J. Agr. Econ. 16(1984):91-98.
- Paris, Quirino, and Keith Knapp. "Estimation of von Liebig Response Functions." Amer. J. Agr. Econ. 71(1989): 178-186.
- Ruttan, Vernon W. The Economic Demand for Irrigated Acreage--New Methodology and Some Preliminary Projections, 1954-1980. Baltimore: Johns Hopkins Press, 1954.
- U.S. Department of Commerce, Bureau of the Census. "1984 Farm and Ranch Irrigation Survey." Report AG84-SR-1. Washington, D.C.: U.S. Government Printing Office, 1986.
- Yaron, Dan. "Empirical Analysis of the Demand for Water by Israeli Agriculture." J. Farm Econ. 49(1967):461-473.