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PRICE STABILIZATION
UNDER
RATIONAL EXPECTATIONS

by

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Price control

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I. Introduction

Price stabilization programs have become an integral part of domestic agricultural policy in most of the industrialized countries of the world. One objective of these programs is to limit price instability by setting support levels at the long run expected level. It is important to consider how alternative stabilization schemes, used to set the support price, affect producer expectations.

When producers are aware of policies that affect the future path of prices adaptive expectations, based on past prices, is no longer appropriate (Fisher). Under the rational expectations hypothesis (Muth), producers will incorporate all of the available information into their price forecast. The parameters of the producer decision rules will be functions of the objective function, the stochastic processes generating the exogenous variables and the stabilization policy (Fisher).

Based on the "Lucas Critique" (Lucas), this implies that the reduced form parameters of the system will not be invariant to different policies. The fact that producers are rational and account for government policy in the formation of their expectations has been, for the most part, ignored in the formation of agricultural stabilization programs. The purpose of this paper will be to determine if price stabilization programs influence the stochastic process generating prices under rational expectations.

The remainder of the paper is divided into three sections. Section II provides the theoretical justification of how price stabilization programs affect the autoregression generating prices. The analysis is extended in Section III to a simple model which provides examples of the impact of two types of price stabilization on the convergence and variance of price. The final section is the conclusions of the paper.

II. Theoretical Considerations:

In order to motivate what follows, we will begin our analysis with the properties of a first order autoregressive stochastic process. Suppose P_t (written in deviation form) is a random variable that is generated according to the stationary, first order autoregressive stochastic process:

$$P_t = bP_{t-1} + e_t \tag{1}$$

where $-1 < b < 1$ are parameters and e_t is white noise.[1] The variance of P_t is denoted $VAR(P_t) = \sigma^2/(1-b^2)$, where σ^2 is the variance of e_t. The parameter b not only affects the variance of P_t, but also affects the rate at which P_t converges to its steady state value. In general, the higher the absolute value of b, the higher the variance of P_t and the slower the rate of convergence of P_t to its steady state value.

In the context of this paper, the autoregressive process given by equation (1) is of interest because it is the solution to a variant of Muth's (1961) model of market equilibrium under storage and rational expectations. Muth's model (written in deviation form) is used to represent the commodity market as

Demand: $Q^d_t = -\beta P_t \quad \beta>0$ (2)

Supply: $Q^s_t = \delta E_{t-1}P_t+U_t \quad \delta>0$ and U_t is a random error (3)

Stocks Demand: $S_t = \alpha(E_tP_{t+1}-P_t) \quad \alpha>0$ (4)

Market Clearing: $Q^s_t = Q^d_t+(S_t-S_{t-1})$ (5)

The solution to prices under the Muth model, derived in Appendix A, is

$$P_t = \lambda P_{t-1} + \frac{1}{\alpha\lambda}U_t \tag{6}$$

where λ is the smallest value chosen to satisfy the quadratic expression

$$\lambda + \frac{1}{\lambda} = \frac{\delta + \beta}{\alpha} + 2.$$

The variance of P_t, say VAR $P(\lambda)$, under the Muth model is therefore

$$\text{VAR } P(\lambda) = \frac{\sigma_U^2}{\alpha^2\lambda^2\ (1-\lambda^2)}, \tag{7}$$

where σ_U^2 is the variance of U_t.

Now suppose the government chooses to stabilize prices to producers. In this paper, two types of price stabilization will be considered: (1) producers receive the unconditional mean price and (2) producers receive a two year moving average in prices. The unconditional mean price is consistent with the Waugh-Oi-Massel framework used extensively in the theoretical literature on price stabilization. The two year moving average price stabilization program[2] is similar to Canadian price stabilization programs. The rest of this section will consider the impact of these two policies on equation (6) in turn.

Consider the impact of guaranteeing producers the unconditional mean price. In this case, equation (3) will change because producers will now receive the mean price. Therefore, equation (3) would become $Q_t^s = \delta E(P) = 0$ where $E(P)$ is the unconditional mean of price. The unconditional mean is zero because the system is written in deviation form.

Resolving the market clearing identity given by equation (5) results in the following optimal price autoregression, derived in Appendix A,

$$P_t = \hat{\lambda}\, P_{t-1} + \frac{1}{\alpha\hat{\lambda}}\, U_t, \tag{8}$$

where $\hat{\lambda}$ is now chosen to be the smallest value satisfying the quadratic expression

$$\hat{\lambda} + \frac{1}{\hat{\lambda}} = \frac{\beta}{\alpha} + 2.$$

Comparing the optimal price autoregressions under the mean price stabilization program and under no stabilization, one can see that the restrictions are different. Specifically, since $\frac{\beta}{\alpha} + 2 < \frac{\beta + \delta}{\alpha} + 2$, it is unambiguously true that $\hat{\lambda} > \lambda$. This means that convergence to long run equilibrium is slower under the mean price stabilization program than under no stabilization.

The effect of this stabilization policy on the variance of price under the mean price stabilization program is uncertain.

Denote the variance of price under the mean price stabilization program as VAR $P(\hat{\lambda})$, then

$$\text{VAR } P(\hat{\lambda}) = \frac{\sigma_u^2}{\alpha^2 \hat{\lambda}^2 (1-\hat{\lambda}^2)}. \tag{9}$$

Therefore, the variance in prices under the mean price stabilization program versus no stabilization

$$\begin{matrix}\text{increases} \\ \text{remains the same} \\ \text{decreases}\end{matrix} \quad \text{as} \quad \frac{\lambda^2(1-\lambda^2)}{\hat{\lambda}^2(1-\hat{\lambda}^2)} \quad \begin{matrix} > \\ = \\ < \end{matrix} \; 1. \tag{10}$$

Now consider the policy where government chooses to stabilize prices on the basis of a two year moving average in prices. As in the previous case, this will change the prices rational producers expect to receive. The supply equation given by equation (3) now becomes

$$Q^S{}_t = \tfrac{1}{2}\delta(E_{t-1}P_t + P_{t-1}) + U_t. \tag{11}$$

Resolving the system by substituting equation (11) into the market clearing condition (5) gives the following optimal price autoregression, derived in Appendix A,

$$P_t = \lambda^* P_{t-1} + \frac{1}{\alpha\lambda^*} U_t, \tag{12}$$

where λ^* is now chosen to be the smallest value satisfying the quadratic expression

$$\lambda^* + \frac{\phi_4}{\lambda^*} = \phi_3,$$

where

$$\phi_3 = \frac{(\frac{1}{2}\delta+\beta)}{\alpha} + 2 \text{ and}$$

$$\phi_4 = 1 - \frac{1}{2}\,\frac{\delta}{\alpha}.$$

The variance of P_t now becomes

$$\text{VAR P}(\lambda^*) = \frac{\sigma_U^2}{\alpha^2\lambda^{*2}\ (1-\lambda^{*2})}, \qquad (13)$$

where VAR $P(\lambda^*)$ is the variance of prices under the two year moving average price stabilization program. In general, the speed of convergence is

$$\text{remains}\ \begin{matrix}\text{slower}\\ \text{the same}\\ \text{faster}\end{matrix} \quad \text{as} \quad \lambda^* \begin{matrix}>\\=\\<\end{matrix} \lambda$$

and the variance of prices

$$\text{remains}\ \begin{matrix}\text{increases}\\ \text{the same}\\ \text{decreases}\end{matrix} \ \text{as} \quad \frac{\lambda^2(1-\lambda^2)}{\lambda^{*2}(1-\lambda^{*2})} \begin{matrix}>\\=\\<\end{matrix} 1.$$

Comparing equation (6) with equations (8) and (12), and equation (7) with equation (9) and (13), we can see that neither the rate of convergence of prices nor the variance in prices is independent of the stabilization program. Thus a critical feature of price stabilization programs is their effects on the price autoregression of the system.

III. An Application to the Barley Market in Western Canada:

We estimate the system of equations given by equations (2) through (4) for barley in the prairie region of Canada using data from 1942 to 1983. Data on the supply and disposition of Canadian barley over the period 1942 to 1982 was obtained from the Grain Trade of Canada and the Cereals and Oilseeds Review, both of which are publications of Statistics Canada. In the model domestic use includes barley exports in order to maintain the assumption of autarky. Data on the price of barley, wheat and livestock was obtained from the Statistical Handbook of Saskatchewan Agriculture and Food. These prices should provide an adequate representation of the Canadian market for the purposes of this analysis. In each case, the price is deflated by an index of gross national product.

The strategy used to estimate the parameters of the model exploits the assumption of rational expectations. The ex-post realizations of the prices are used as proxies for the expectational price variables in equations (2) through (4). The use of ex-post proxies for expectational price variables means that an instrumental variable technique is needed because these regressors are not independent of the error terms (Eckstein). On the other hand, any variable in agents' information sets is uncorrelated with the error terms of each equation. These variables provide a ready set of instruments that can be used to derive consistent estimates of the parameters (Hansen and Sargent). Another issue concerns serial correlation since serially correlated errors cannot, in general, be ruled out (Hansen or Gallant).

Hansen's generalized method of moments (GMM) estimator can be used to obtain consistent estimates of the parameters under the assumption that the regressors of the model are correlated with the error terms and the errors themselves are serially correlated. The actual algorithm that is used to estimate the parameters is outlined in Gallant.

In order to identify the model it is necessary to specify a demand and supply shifter. The price of livestock (PL) is used in the demand equation as barley is a major input in the feeding of cattle. The price of wheat (PW) is used in the supply equation as it is a substitute in the production decision. The four instruments used in the estimation procedure are prices for livestock, barley, and wheat each lagged one period and the difference in barley price lagged one period.

The sum of squared errors (SSE) is distributed as a Chi square with 4 degrees of freedom (Hansen). This Chi-square value is a test of the overall model (e.g. Gallant). In this case, the over-identifying restrictions on the model are not rejected.

Table 1 presents the estimated parameter values used to calculate the eigenvalues of the system.[3] In each case there is one stable and real eigenvalue, the other eigenvalue being less than one. Taking the inverse of the stable eigenvalue yields the price coefficient of the first order price autoregression, Table 2. As $\hat{\lambda} = .533$ is greater than $\lambda = .377$ convergence is slower under mean price stabilization. However, price variance declines since the ratio of variances is less than one, Table 2. In the case of the two year moving average, convergence is faster as λ^* $= .283$ is less than $\lambda' = .377$. Price variance increases as indicated by a value of 1.65 for the ratio of price variances.

There is a trade off between the rate of convergence and variance of price in the prairie barley market. Producers may prefer the lower price variance in lieu of more rapid convergence to the long run expected price. The weights assigned to each of these components will be a function of the risk preferences of producers.

IV. CONCLUSIONS

Under the rational expectations hypothesis, producers use the information contained in the stabilization program to form expectations which will influence the stochastic process generating prices. By using a simple model it was possible to isolate the effects of these rules on the stochastic process generating prices.

The results indicate the choice of stabilization program affects the speed of convergence of prices to the long run expected price as well as the variance in prices. Mean price stabilization will decrease the rate of convergence to the long run expected price for every commodity. It is unclear how convergence will be affected under a two year moving average in prices. In both cases the effect on price variance will be dependent on the particular parameters of the system. In the empirical example, the price variance decreased under the mean price scheme and increased under the two year moving average in prices. The two year moving average in prices program increased the rate of convergence to the long run expected price.

How stabilization programs influence the expectations of rational producers and the stochastic processes affecting prices should be considered in the formation of stabilization programs. By examining the eigenvalues of a system of prices, under any proposal, it is possible to show the impacts of different stabilization schemes on both the convergence and instability of prices.

Endnotes

1. The first moment of prices, the unconditional mean, is unaffected by any of the stabilization schemes considered in this paper.

2. A longer moving average of prices results in a higher order autoregressive process which makes the analysis complicated and complex. It is unclear to the authors how a longer moving average in prices could be solved under the assumption of rational expectations.

3. Strictly speaking, the existence of shifters in the estimated barley demand and supply functions and the existence of a serially correlated error structure means that the first order autoregressive process for prices given in the text is not consistent with the estimated model (see, for example, Sargent, 1979). We have not solved for the correct solution in prices, including the shifters as forcing variables and the serially correlated error structure, in order to maintain the simplicity of the theoretical model. Isolating only the own price effects of the stabilization program is also more consistent with the literature on stabilization.

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Appendix A

The purpose of this appendix is to derive the various reduced form solutions given in the text in greater detail. The Muth model of market clearing, using rational expectations (see Sargent) is

$$Q_t^d = -\beta P_t \tag{A1}$$

$$Q_8^s = \delta E_{t-1} P_t + U_t \tag{A2}$$

$$S_t = \alpha (E_t P_{t+1} - P_t) \tag{A3}$$

$$Q_t^s = Q_t^d + S_t - S_{t-1} \tag{A4}$$

Substituting equations (A1) through (A3) into the market clearing condition given by (A4) and rearranging, taking projections against information at time t-1, one gets

$$E_{t-1} P_{t+1} - \phi_1 E_{t-1} P_t + P_{t-1} = \frac{1}{\alpha} E_{t-1} U_t, \tag{A5}$$

where $\phi_1 = \left[\frac{\beta + \delta}{\alpha} + 2\right]$.

Expressing equation (A5) in lag operator notation and multiplying by B results in the following optimal autoregression for prices

$$[1 - \phi_1 B + B^2] E_{t-1} P_t = \frac{1}{\alpha} E_{t-1} U_{t-1}, \tag{A6}$$

where B is the lag operator. The values denoted λ_1 and λ_2 for this model are found by solving the quadratic form

$$[1 - \phi_1 B + B^2] = (1 - \lambda_1 B)(1 - \lambda_2 B),$$

so that

$$\lambda_1 + \lambda_2 = \phi_1 \quad \text{and}$$

$$\lambda_1\lambda_2 = 1.$$

From $\lambda_1\lambda_2 = 1$, we find $\lambda_2 = 1/\lambda_1$. Therefore, the two roots appear as a reciprocal pair. Setting $\lambda_1 = \lambda$, the first equality becomes

$$\lambda + \frac{1}{\lambda} = \phi_1. \tag{A7}$$

Given the definition of ϕ_1, the smaller eigenvalue, say λ, necessarily lies between zero and one and the larger root $(=1/\lambda)$ is greater than one. We solve the stable eigenvalue backward and the unstable eigenvalue forward (e.g., Sargent), that is

$$E_{t-1}P_t = \lambda P_{t-1} - \frac{\lambda B^{-1}}{1-\lambda B^{-1}} \frac{1}{\alpha} E_{t-1}U_{t-1}. \tag{A8}$$

The solution to equation (A8) is therefore

$$E_{t-1}P_t = \lambda P_{t-1} - \frac{\lambda}{\alpha} \sum_{i=0}^{\infty} \lambda^i E_{t-1}U_{t+i} \tag{A9}$$

Since, by assumption U_t is a white noise process, the $E_{t-1}U_{t+i} = 0$ for all i, so that equation (A9) reduces to

$$E_{t-1}P_t = \lambda P_{t-1}. \tag{A10}$$

Since equation (A10) holds for P_t, it must also hold for P_{t+1}, so that

$$E_{t-1}P_{t+1} = \lambda P_t \tag{A11}$$

Substituting equation (A10) and (A11) into equation (A5) and rearranging, one gets

$$P_t = \lambda P_{t-1} + \frac{1}{\alpha\lambda} U_t. \tag{A12}$$

Next, consider the effect of a price stabilization scheme that stabilizes prices to producers at the unconditional mean of prices, $\overline{P}$. In this case $E_{t-1}P_t = E_{t-1}(\overline{P}) = 0$ in the supply equation given by equation (A2), so that equation (A5) becomes

$$E_{t-1}P_{t+1} - \phi_2 E_{t-1}P_t + P_{t-1} = \frac{1}{\alpha} U_t. \tag{A13}$$

where $\phi_2 = (\frac{\beta}{\alpha} + 2)$. Now the eigenvalues are chosen to satisfy the restrictions

$\hat{\lambda}_1 + \hat{\lambda}_2 = \phi_2$ and

$\hat{\lambda}_1\hat{\lambda}_2 = 1.$

Therefore, the constant price stabilization policy is similar to the case of no stabilization policy in as much as the eigenvalues are restricted to be reciprocals of one another with one stable and one unstable eigenvalue. Let the smallest eigenvalue under the constant mean price stabilization policy be $\hat{\lambda}$. The solution to the price autoregression under the mean price stabilization scheme is

$$P_t = \hat{\lambda} P_{t-1} + \frac{1}{\alpha\hat{\lambda}} U_t. \tag{A14}$$

Finally, in the case of the two year moving average in prices the supply equation becomes

$$Q_t^s = \tfrac{1}{2}\delta(E_{t-1}P_t + P_{t-1}) + U_t. \tag{A15}$$

Substituting (A15) instead of (A3) into the market clearing condition (A4) and taking projections at t-1 results in

$$E_{t-1}P_{t+1} - (\phi_3)E_{t-1}P_t + \phi_4 P_{t-1} = \frac{1}{\alpha} U_t. \tag{A16}$$

where $\phi_3 = \dfrac{(\frac{1}{2}\delta+\beta)}{\alpha} + 2$ and $\phi_4 = 1 - \frac{1}{2}\dfrac{\delta}{\alpha}$. The eigenvalues for the reduced form autoregression in prices are now chosen to satisfy the restrictions

$$\lambda_1^* + \lambda_2^* = \phi_3 \quad \text{and}$$

$$\lambda_1^*\lambda_2^* = \phi_4.$$

Unlike the other two cases, the eigenvalues for the two year moving average in prices are no longer reciprocals of one another. In this case, the eigenvalues are chosen to satisfy the expression

$$1/\lambda_1^*, 1/\lambda_2^* = \frac{\phi_3 \pm (\phi_3^2 - 4\phi_4)^{\frac{1}{2}}}{2\phi_4} \tag{A17}$$

so that one (say $1/\lambda_1^*$) is bounded to lie between negative one and positive one and the other, $1/\lambda_2^*$ is greater than one, given the definition of ϕ_3 and ϕ_4. Therefore, the solution strategy under the two year moving average in prices is the same as in the previous case of solving the stable eigenvalue backward and the unstable eigenvalue forward. This results in the following solution to the autoregression of prices

$$P_t = \lambda^* P_{t-1} + \frac{1}{\alpha\lambda^*} U_t, \tag{A18}$$

where $1/\lambda^* = 1/\lambda_2^*$ is the stable eigenvalue of the system and satisfies the quadratic expression

$$\lambda^* + \frac{\phi_4}{\lambda^*} = \phi_3.$$

Table 1: Estimated Barley Model[a]

Regressor	Q^d_t	Q^s_t	S_t
	dependent variables		
intercept	295.52 (1.61)	639.35 (5.60)	133.44 (6.69)
P_t	-32.37 (-3.02)	49.32 (1.25)	---
PL_t	2.95 (1.49)	---	---
PW_t	---	-185.27 (-2.45)	---
$(P_{t+1}-P_t)$	---	---	79.45 (1.10)
Model SSE		6.77	
degrees of freedom		4	
Chi-square (4,.05)		9.49	

a: asymptotic t-statistics in parentheses.

Table 2: Price Coefficients

	Price Coefficients[a]	Ratio of Price Variance[b]
No stabilization (λ)	.377	1
Mean price ($\hat{\lambda}$)	.533	.6
Two-year moving average (λ^*)	.283	1.65

[a] Price coefficient is inverse of stable eigenvalue.

[b] The denominator of ratio is no stabilization price variance.