

# This document is discoverable and free to researchers across the globe due to the work of AgEcon Search. 

## Help ensure our sustainability. Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

Papers downloaded from AgEcon Search may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.

# NONSTATIONARITY OF SOYBEAN FUTURES PRICE DISTRIBUTIONS: OPTION-BASED EVIDENCE 

by

Bruce J. herrick, Scott H. Irwin, and D. Lynn Forster
August 1990

${ }^{\circ}$ The authors are Assistant Professor of Ag-Finance, University of Illinois, and Associate Professor and Professor of Agricultural Economics, The Ohio State University, respectively.

Presented at the Annual Meeting of the American Agricultural Economics Association, August 4-8, 1990, University of British Columbia, Vancouver, B.C., Canada.

# NONSTATIONARITY OF SOYBEAN FUTURES PRICE DISTRIBUTIONS: OPTION-BASED EVIDENCE 


#### Abstract

No-Arbitrage option pricing models are used to estimate ex ante soybean futures price distributions. Volatility measures of these distributions are modeled in an endogenous-switchpoint regression as functions of price level and time-to-maturity. Results indicate volatility measures are not stationary, and exhibit regime dependent influences of time-to-maturity and price level.


## NONSTATIONARITY OF SOYBEAN FUTURES PRICE DISTRIBUTIONS: OPTION-BASED EVIDENCE

The trading of options on agricultural commodity futures has led to an increased interest in the ability of markets to reflect information about future possible prices. Option pricing models and observed market prices have often been used in efforts to recover information about an underlying futures expected price distribution. The most prevalent examples of using options market prices to recover information about an underlying security's price distribution involve the Black-Scholes or Black option pricing models and recovery of an estimate of the ex ante price volatility (standard deviation) often termed the implied volatility (IV). Then if an estimate of the mean of the future distribution is available as well, an entire two-parameter price distribution may be constructed for the underlying asset using information from observed option prices. Many studies have then sought to either explain observed biases in pricing related to implied measures from the model, or more directly, to explain changes in the implied distributions through time series models of the implied volatilities. ${ }^{1}$

* In the case of soybean futures markets, current prices may reveal a good deal of information about expected future prices. For example, the current futures price may correspond to the mean of the expected future price distribution. However, in the absence of relatively sophisticated descriptions of the price diffusion process, and a risk premium if any, the futures price may be silent about other moments of an expected future price distribution. Options payoffs, however, are contingent upon the entire range of possible outcomes for future price and as such contain an assessment of all relevant moments of a future price distribution. Thus, options may potentially be used to reveal information about future price distributions that may not be obtained from current futures prices. And, in developing good estimates of the parameters of future price distributions, it is unlikely that one will find any better estimate than the market's (Gardner).

The ex ante price distributions derived from option prices provide a unique source of information from which inferences regarding market forces and agents' actions may be assessed. Market agents learn and update their information sets with the passage of time and resolution

[^0]of uncertainty about future events. Inferences about the speed and types of adjustments in expectations and expectation formation may be gleaned from an examination of the changes in market implied future distributions. Further, the "Term Structure" of uncertainty is revealed through a comparison of various maturity options and by examining changes in a particular contract's implied distributions through time. ${ }^{2}$ Several other studies have indicated that the variance of returns is not constant but varies through time. To the extent that these distributions could be used as an informational component in other decisions, seriously incorrect conclusions could be drawn if the issues of parameter non-stationarity are not first considered.

Unfortunately, the Black model derivation relies on a set of assumptions about the constancy- of price volatility that makes term-structure of uncertainty investigations inappropriate. The no-arbitrage method employed herein suffers no such drawback and will in fact be used for a similar type of analysis.

## NO-ARBITRAGE OPTION PRICING

A widely accepted basis for asset pricing relies on the general no-arbitrage restrictions first proposed by Ross, and Cox and Ross. They show that in the absence of arbitrage opportunities there exists a supporting pricing function for all possible states in the no-arbitrage economy. Assets in the no-arbitrage economy may be valued as the expected value of their returns with the expectation taken with respect to this pricing function. That is, the pricing function bears a direct correspondence to the probability distribution of the pure contingent claims and may therefore be used to price any replicable asset in the no-arbitrage economy, simply by discounting the expected payoffs. The supporting pricing function may correspond directly to the state probabilities and may therefore provide a more direct means to recovery of an expected price distribution than the more common Black model.

European options are particularly convenient to value in the no-arbitrage framework because the payoff function is quite simple. For a call that expires at time $T$ the payoff is simply $\max \left\{\mathrm{P}_{\mathrm{T}} \mathrm{K}, 0\right\}$ and for a put, $\max \left\{\mathrm{K}-\mathrm{P}_{\mathrm{T}}, 0\right\}$, where K is the option's strike price and $\mathrm{P}_{\mathrm{T}}$ is the time T price of the underlying security. Valuing the options as the discounted expected value of their payoffs leads to the simple form of the option pricing model given in Fackler and King, Gardner and elsewhere that
${ }^{2}$ As a simple analogy, suppose the variance of an expected price distribution for three months in the future were " k " and the variance of a price for expiration four months in the future were ten times as great, then the time during which the uncertainty is greatest would be between the third and fourth months in the future.
$c_{i, t}=e^{-r(T-t)} \int_{K_{i}}^{\infty} f\left(P_{T}\right)\left(P_{T}-K_{i}\right) d P_{T}$ and $p_{j, t}=e^{-r(T \cdot t)} \int_{0}^{K_{j}} f\left(P_{T}\right)\left(K_{i}-P_{T}\right) d P_{T}$ where $c_{i, t}$ is the day $t$ value of $a$ call at strike price $K_{i}, p_{j, t}$ is the analogous put, and $f\left(P_{T}\right)$ is the appropriately adjusted probability density function for the expiration date price, $\mathrm{P}_{\mathrm{T}}$, of the underlying futures contract. The adjustments may include a possible risk premium and other market frictions.

Note the subtle yet meaningful difference between this approach and the B-S model. The B-S relies on a specified diffusion process on price that would add up to a lognormal $f\left(P_{T}\right)$. While this is a sufficient condition, it is by no means necessary and is in fact probably overly restrictive. First, there is reasonable evidence that the volatility of prices does not remain constant and that there is some nonstationarity in expected price distributions (Hauser and Anderson, Kenyon et al.). Also, several studies have found that the lognormal may not be fully descriptive of empirically observed distributions (Gordon, Hall et al.). Even if the lognormal were descriptive of observed prices, the B-S implied volatility is a parameter of the diffusion process whereas in the present case, $f\left(\mathrm{P}_{\mathrm{T}}\right)$ is of the expiration date price distribution, not a descriptor of the intervening and possibly non-constant variance.

For these reasons, we rely on a three parameter, flexible distribution known variously as the Singh-Maddala (SM) or Burr-XII distribution to describe expected prices. ${ }^{3}$ The SM is one member of a three parameter family of distributions with potentially desirable characteristics for this context. Its wide range of skewness and kurtosis can be used to fit almost any given set of unimodal data (Tadikamalla). The extreme simplicity of its distribution functions and moment function again make it an appealing candidate for study. The SM cumulative distribution function (CDF) is (Hogg and Klugman):

$$
\begin{align*}
& \mathrm{F}(\mathrm{P})=1-\left(\lambda /\left(\mathrm{P}^{\tau}+\lambda\right)\right)^{\alpha} \text { for } \alpha, \lambda, \tau, \mathrm{P}>0 \text {, and thus the density, or PDF is: }  \tag{1}\\
& \mathrm{f}(\mathrm{P})=\alpha \lambda^{\alpha} \tau \mathrm{P}^{\tau-1}\left(\mathrm{P}^{\tau}+\lambda\right)^{(\alpha+1)} . \tag{2}
\end{align*}
$$

The benefit of the no-arbitrage approach in recovering probabilistic information is that the distribution needed is more relevant to market participants because it is the end of period price distribution, not a set of parameters on the price process that add up to a price distribution and so it doesn't rely on any intervening time period effects. The drawback is that a possible early-exercise adjustment may be needed if there is significant value in this right.

[^1]
## DATA AND MODEL:

The data were taken from the Time and Sales tapes from the Chicago Board of Trade and are thought to be highly accurate and free of error. All contracts since inception of trading in options on 10/31/84 and ending in August 1988 were used. Synchronous option and futures prices were collected as follows: for each type of option, all strike prices that traded during the day were arranged according to their proximity to 11:00 (to avoid opening and closing distortions). Then, for each strike, the option that traded closest to 11:00 was matched to the nearest futures price. The result each day for each contract was a complete set of the strikes for both puts and calls that traded, with each observation as near to 11:00 as possible. In the final sample, 15020 options were used to compute a total of 1715 daily distributions across 26 different soybean futures options contracts. On average, 8.76 options per day were used to derive an implied distribution for each contract, with $64 \%$ of the options being calls. The mean time difference between 11:00 and the option's trade was less than 20 minutes and the mean difference between options prices and its matched futures prices was less than 25 seconds. * To estimate the parameters of the SM, both puts and calls were used in an effort to fit the parameters to a wide range of the segments of the distributions. The following expression was solved daily for each contract for $\beta$, the vector of parameters of the SM:

$$
\begin{equation*}
\min _{\beta}\left[\sum_{i=1}^{n}\left\{\left(c_{i, t}-\mathrm{e}^{-r\left(\mathrm{~T}^{-t)}\right)} \int_{\mathrm{K}_{\mathrm{i}}}^{\infty} \mathrm{f}\left(\mathrm{P}_{\mathrm{T}} \mid \beta\right)\left(\mathrm{P}_{\mathrm{T}}-\mathrm{K}_{\mathrm{i}}\right) \mathrm{dP} \mathrm{P}_{\mathrm{T}}\right)^{2}\right\}+\sum_{\mathrm{j}=1}^{m}\left\{\left(\mathrm{p}_{\mathrm{j}, \mathrm{t}}-\mathrm{e}^{-r(\mathrm{~T} \cdot t)} \int_{0}^{\mathrm{Kj}_{\mathrm{j}}} \mathrm{f}\left(\mathrm{P}_{\mathrm{T}} \mid \beta\right)\left(\mathrm{K}_{\mathrm{j}}-\mathrm{P}_{\mathrm{T}}\right) \mathrm{d} \mathrm{P}_{\mathrm{T}}\right)^{2}\right\}\right] \tag{3}
\end{equation*}
$$

where n is the number of calls and m is the number of puts used in day t to estimate the vector $\beta .^{4}$ These non-linear least squares daily assessments of ex ante price distribution generated provide a unique set of data from which inferences regarding market forces and agents' actions may be assessed. If the market correctly aggregates uncertain price information and the options premiums accurately reflect that information via the no-arbitrage model, then changes in the distributions reflects both decreases in time to maturity (TTM) and exogenous events that manifest themselves in parameter changes. The fact that each successive day's estimate of $f\left(P_{T}\right)$ is for a sub-period of the previous observations' suggests that a time-series model might be appropriate to explain changes in volatility and changes in other aspects of the distribution. However, weekends, missing days, holidays, thin markets, and the like would render this

[^2]approach quite difficult as no consistent interval exists between "daily" observations and hence, the interpretation of the autocorrelation measures and issues of nonstationarity would be unclear. Instead, a regression framework is imposed that takes direct recognition of the potential serial dependence and inconsistent observation interval. The posited model is quite simple in keeping some comparability to earlier approaches, while allowing very general parameter variation.

It is quite reasonable to expect the volatility of the implied distributions to be a function of TTM by the very nature of the variable. Resolution of uncertainty, and the collapse of $e x$ ante distributions to the realized price both evolve through time. Also, the price of the underlying variable may potentially influence volatility if there is a fairly constant coefficient of variation describing the uncertainty (consider the Constant Elasticity of Variation (Cox) model as a parallel). However, with the problems of time series modeling noted above, alternate models are pursued.

It is assumed instead that the following model holds:

$$
\begin{equation*}
\Rightarrow \quad \theta_{\mathrm{t}}=\beta_{1 \mathrm{j}}+\beta_{2 j}\left(\mathrm{P}_{\mathrm{t}}\right)+\beta_{3 \mathrm{j}}\left(\mathrm{TTM}_{\mathrm{t}}\right)+\mathrm{u}_{\mathrm{j} \mathrm{t}} \tag{4}
\end{equation*}
$$

where $\theta$ is the parameter of the distribution being investigated (in this case standard deviation), the $j$ subscript is the regime index running from 1 to $r$ implying $r-1$ switchpoints, and $P_{t}$ is the futures price on day $t$. If $r=1$, (no switchpoints) the model reduces to the common ordinary least squares. If the index $j$ depends upon some other possible stochastic variable $z_{i}$ so that the regime of influence depends upon a threshold value of some $z_{i} \geq z^{*}$ for $j$ to be in a new regime, the model is of the class of switching regimes regressions. For this purpose, the most likely regime index is obviously time, but the cut off values, $z^{*}$, at which one regime supersedes the next must be estimated as well.

Brown, Durbin and Evans (BDE) were among the first to investigate the detection of switchpoints in such a context. Because of potential non-normality problems, their test, while powerful at detecting the presence of a switch point, is not well suited at defining the precise location of that point. The BDE model make use of recursive residuals and one-period-ahead forecast errors. Intuitively, each of the one-period-ahead forecast errors should contribute to the sum of squared residuals in about the same proportion. Put another way, the cumulative sum of squared errors, given that the model is correct, should increase in proportion to the number of forecasts. Hence BDE propose the following test statistic based on the series $s_{r}$;

$$
\begin{equation*}
s_{r}=\left(\sum_{j=k+1}^{r} w_{j}^{2}\right) /\left(\sum_{j=k+1}^{T} w_{j}^{2}\right), \text { for each } r=k+1, \ldots, T \tag{5}
\end{equation*}
$$

where $w_{j}^{2}$ are the recursive residuals (see Brown et al.) and by definition $s_{k}=0$ and $s_{T}=1$, and if all forecast errors are identically distributed, the $\mathrm{s}_{\mathrm{r}}$ are shown to have a Beta distribution with mean ( $\mathrm{r}-\mathrm{k}$ )/( $\mathrm{T}-\mathrm{k})$. Departures from the uniform line of the plot of $\mathrm{s}_{\mathrm{r}}$ indicate a significant switch point is in the interval $k+1, T$. A confidence interval can be set at $(r-k) /(T-k) \pm C_{o}$ with $C_{o}$ a function of the level of significance as tabulated in BDE and elsewhere. Note that if the null of no switch holds, the $w_{t}$ are independent and the use of such residuals avoids the serial correlation and non-normality problems associated with ordinary OLS residuals, and thus allows for much more powerful tests (Hays and Upton).

In order to test for the existence of multiple regime shifts, the BDE test was applied over sequentially longer intervals until a switchpoint was located. Then the most likely point in the interval was found with the log-likelihood ratio (LLR) (see Johnston, pg. 409) by computing the test statistic for each possible switchpoint and picking the one that minimized the ratio. Next, the interval up to and including the switchpoint was excluded from the sample and the process repeated. If no new switchpoints were located, then only two regimes are said to exist. If a new switchpoint was located each possible subset of the original data, as partitioned by the switchpoints was tested for the most likely occurrence. If multiple regimes caused the first switchpoint, $\mathrm{t}^{\dot{\prime}}$, to be spurious, the data over this point to the next should not reveal the same most likely $t^{*}$. If in fact, $t^{*}$ is again located, it is not a result of multiple later regimes. The process then searches for a third switch and so on until no new switchpoints are found.

The likelihood ratio tests were used primarily to locate good starting values for the maximum likelihood method described below. F-Tests of the null of equivalent parameters across regimes were also computed but the significance is not exact because the most likely switchpoint had been found a priori so the F-test is biased toward low p-values. Table 1 reports the location and significance of the switchpoints by contract.

The most notable result is the prevalence of switches located within approximately two months prior to expiration. It is suspected that the expiration of the contract immediately preceding each contract would have a spill-over effect as activities are concentrated in what has become the new, nearby contract. However, these effects would tend to be two months apart, except for the September and August contracts which have contracts which expire one month earlier. There are calendar effects and exchange rules that could cause the range of time
between adjacent option contract expirations to vary significantly. For all contracts, there may also be a gradual change in volume over the life of the futures as different participants acquire new reasons to trade various maturity instruments. A changing volume may be associated with new information that would manifest itself as a parameter change. Also, note the high levels of significance for many of the switchpoints. In all but four of the soybean contracts examined, a switchpoint was located with significance greater than .10 (smaller p-value). The caution is simply that there are frequent structural changes that require consideration before using long time series in other economic analyses.

At this point the switchpoints are still defined only in general terms. That is, if $t^{*}$ is the switchpoint, the tests will detect either $\beta_{t<1} \neq \beta_{\mathrm{t}>\bullet^{\circ}}$ or non-constant error variance across the regimes. In order to model the parameter variability, we must continue to admit both possibilities. If it were known that the various regimes were in fact independent, they could be so modeled. Unfortunately, that luxury is not had and the chosen model is a bit clumsier, but much more powerful. Following Goldfeld and Quandt (1973, 1976), and Quandt (1958, 1960), ând Kane and Unal $(1986,1988)$, an endogenous switchpoint regression was next used to simultaneously estimate the $\hat{\beta}_{\mathrm{j}}$, equivalent $\mathrm{t}_{\mathrm{i}}^{*}$ and an additional parameter $\sigma_{\mathrm{z}}^{*}$ indicating the gradualness of change as one regime supersedes the previous. Note the immediacy of the use of a switching regression technique in event studies. Switch points are located first and then compared with plausible events. Running the test in this order conserves degrees of freedom and helps avoid the tendency to form illusive correlations.

There are likely to be only a few relevant switchpoints if the model posited is in fact correct and is relatively stable within given regimes. However, since the number and location of the switchpoints is unknown, a set of dummies to detect the switches cannot be used in any parsimonious fashion. That is, to detect the unknown location of one switchpoint, there would need to be $(\mathrm{n}-2)^{*} 3$ sets of dummies which obviously exceeds the total degrees of freedom. A feasible problem emerges, however if a structure is imposed on the set of dummies such that a relatively small number of parameters describe an entire set of regime dummies. Then the parameters of the regression and the parameters of the dummy equations may be simultaneously estimated from the data. The model used is a variant of the "D-method". of Goldfeld and Quandt.

To allow the switch points to be endogenized, introduce transitional dummy variables, modeled as normal CDF's, $\mathrm{D}_{\mathrm{ij}}$ :

$$
\begin{equation*}
D_{\mathrm{tj}}=\int_{-\infty}^{\mathrm{z}_{\mathrm{t}}}\left[(2 \pi)\left(\sigma^{*}\right)^{2}\right]^{-1} \exp \left[-1 / 2\left\{\left(\Phi-\mathrm{z}_{\mathrm{j}}^{*}\right) /\left(\sigma^{*}\right)^{2}\right\} \mathrm{d} \Phi\right. \tag{6}
\end{equation*}
$$

where $\mathrm{j}=1, \ldots, \mathrm{k}, \ldots \mathrm{r}$ and by definition $\mathrm{D}_{10}=1$ and $\mathrm{D}_{\mathrm{tr}}=0$. Then the k th regime is multiplied by:

$$
\begin{equation*}
\tau_{\mathrm{tk}}=\prod_{\mathrm{j}=0}^{\mathrm{k}-1} \mathrm{D}_{\mathrm{tj}} \prod_{\mathrm{j}=\mathrm{k}}^{\mathrm{r}}\left(1-\mathrm{D}_{\mathrm{t}}\right) \text { for } \mathrm{t}=1, \ldots, \mathrm{~T} \text { approximating a step into and out of the regime. } \tag{7}
\end{equation*}
$$

The r regime equations are then summed to get:

$$
\begin{equation*}
\left.\sum_{\mathrm{k}=1}^{\mathrm{r}} \theta_{\mathrm{t}} \tau_{\mathrm{tk}}=\sum_{\mathrm{k}=1}^{\mathrm{r}}\left\{\beta_{1 \mathrm{k}}+\beta_{2 \mathrm{k}} \mathrm{P}_{\mathrm{t}}+\beta_{3 \mathrm{k}} \mathrm{TTM}_{\mathrm{t}}+\mathrm{e}_{\mathrm{tk}}\right)\left(\tau_{\mathrm{tk}}\right)\right\} . \tag{8}
\end{equation*}
$$

Assuming a form for the distribution of $\theta$ (i.e. normal by C.L.T.) with mean and variance of :

$$
\begin{align*}
& \mu_{t \theta}=\sum_{k=1}^{\mathrm{r}}\left\{\left(\beta_{1 \mathrm{k}}+\beta_{2 \mathrm{k}} \mathrm{P}_{\mathrm{t}}+\beta_{3 \mathrm{k}} \text { TTM }_{\mathrm{t}}\right)\left(\tau_{\mathrm{tk}}\right)\right\} \text { and }  \tag{9}\\
& \sigma_{\mathrm{t} \theta}^{2}=\sum_{\mathrm{k}=1}^{\mathrm{r}} \sigma_{\mathrm{tk}}^{2}\left(\tau_{\mathrm{tk}}^{2}\right) \tag{10}
\end{align*}
$$

where $\sigma_{\text {tk }}^{2}$ is the $\mathrm{k}^{\text {th }}$ regime error variance implies a log-likelihood function:

$$
\begin{equation*}
\mathrm{L}=-(\mathrm{T} / 2) \log 2 \pi-(1 / 2) \sum_{t=1}^{\mathrm{T}} \log \sigma_{t \theta}^{2}-(1 / 2) \sum_{t=1}^{\mathrm{T}}\left[\left\{\sum_{\mathrm{k}=1}^{\mathrm{T}} \theta_{t} \tau_{t{ }^{-}}-\mu_{t \theta}\right\}^{2}\right] / \sum_{t=1}^{\mathrm{T}} \sigma_{t \theta}^{2} \tag{11}
\end{equation*}
$$

Maximizing L.w.r.t. its unknown parameters gives the switch points, $z^{*}$; the gradualness of change, $\sigma^{*}$, and the parameters of the original regression relation. For the present purpose, the problem can be further simplified. Suppose that the $z_{i}$ are also among the regressors. And, further suppose that the regime depends only upon the magnitude of an ordered $z_{i}$. The natural candidate is of course the time index as the regimes are then ordered in time as well. As the variance of the normal CDF approaches zero, the dummy function approaches a step function at $z^{\circ}$ and the discrimination between regimes would be perfect. Although in essence, the Dmethod is simply an approximation used to make the problem estimable, certain situations would suggest that the gradualness of the mixing of regimes could also be typified by the variance of the step function. The endogenous nature of the D-method lends itself to event study as the switchpoints are taken as events and estimated rather than specified a priori.

Equation (11) was estimated via maximum likelihood for various contracts which displayed regime dependence in the earlier tests of BDE and the LLR test. Starting values for the $\beta$ 's were taken as the OLS coefficients of the regression over the subperiods as delineated by the likelihood ratio test and the error variance in separate regimes were initially assumed identical. The value of $z_{i}^{*}$, or the most likely switchpoint was started at the earlier located switchpoint and not permitted to vary beyond the ends of the sample. After some experimentation, the variance of the dummy equation was set to initial values near 0 and permitted to range to $20 \%$ of the maximum time index. Greater values tended to obliterate the discrimination between regimes.

The contracts' switching regression results are given in table 2 for fifteen of the contracts that displayed regime dependence earlier. As expected, the switchpoints, $z^{*}$, correspond well to those located by the earlier methods. Also, the coefficients of the TTM and price level differ by regime indicating that there is a need to consider regime dependent influences before using these types of implied parameters in other economic contexts. Finally, the switching regression methods serve to confirm dates that may serve as events. For the present purposes, the most obvious and prevalent switches appear to be within bi-monthly intervals prior to expiration. Further classifying the causes and modeling them directly present topics for further study.

Several other interesting results are indicated. First, note that in four-fifths of the soybean contracts, the coefficient on $\mathrm{TTM}_{1}$ is greater than on $\mathrm{TTM}_{2}$, indicating that the rate of decline in volatility increases as the time to maturity decreases. The fact that the formulation actually permits the regimes to be continuously mixed, with the strength of each regimes' variables to depend on the dummy parameters, allows a great deal of smoothing of the coefficients over time. Nonetheless, there appears to be high levels of discrimination among regimes as evidenced by the low p-values on $z^{*}$ and $\mathrm{TTM}_{1}$ and TTM ${ }_{2}$. Two-thirds of the first regime price coefficients are positive and five are negative. In the second regime four-fifths of the price coefficients are positive. The preponderance of evidence is weakly in support of a positive price level effect and rather strongly in support of a multiple regime influence of TTM on volatility.

It is also interesting to consider the results from an event-study standpoint. Each of the switches admits itself as a candidate event that corresponds to a temporal change in the coefficients of the model. The mere existence of significant switchpoints renders estimates
unreliable if estimated from the entire sample. The lack of switches causes little concern, other than the loss of degrees of freedom in the current model, as similar and significant coefficients on TTM and Price could exist across regimes and the discrimination by the dummy could be very poor. In fact, an alternate interpretation suggested by Kane and Unal along these lines uses the variance of the dummy, or $\sigma^{*}$ to construct regions of time through which a specified proportion of the change between regimes takes place. Although the use of the D-Method by Goldfeld and Quandt was suggested strictly to make the problem estimable, if the resulting formulation of the model is descriptive of the structure of the process generating the observations, the interpretation of the transition parameter as $\sigma^{\circ}$ is plausible. In either case, it is interesting to allow a more general evolution of parameter change through time.

## SUMMARY

No-arbitrage pricing methods were used to derive estimates of expected future price distributions. The improved data and parameter estimation techniques of this study provide an interesting backdrop for empirical study.

- A switching regression model was used to describe the parameter variation in endogenously selected regimes. Evidence suggests that the resolution of uncertainty occurs at varying rates and may also depend on price level. The notable findings include the fact that the switches tend to occur in within two months prior to expiration and that the volatility declines at a greater rate as the time to maturity decreases. No consistent price level effects were detected, but they may in fact differ by time regime. A direct application is the generation of an ex ante benchmark for event studies. The greatest caution it points out is that inappropriate conclusions may be drawn if the issues of non-stationarity are not first considered.

Table 1.
Location and Significance of Switchpoints for Implied Soybean Futures Price Distributions

| FUTURES CONTRACT | First Switch ${ }^{\text {a }}$ Point | p-value | Second Switch ${ }^{\text {a }}$ Point | $p$-value | Comments |
| :---: | :---: | :---: | :---: | :---: | :---: |
| January 1985 | 23 | . 0600 |  |  |  |
| March 1985 | 64 | . 0123 | 50 | . 0300 |  |
| May 1985 | 46 | b | 28 | . 0002 | C |
| July 1985 | 53 | b | 32 | . 0200 |  |
| August 1985 | 24 | . 0012 |  |  |  |
| September 1985 | 18 | . 9000 |  |  |  |
| November 1985 | 56 | . 0130 |  |  | (day 28,NAN) C |
| January 1986 | 30 | . 0300 |  |  |  |
| March 1986 | 39 | b | 56 | . 1600 |  |
| May 1986 | 59 | . 0004 | 51 | . 1800 | (day 130) c |
| July 1986 | NS |  |  |  | NAN |
| August 1986 | NS |  |  |  |  |
| September 1986 | NS |  |  |  |  |
| November 1986 | 44 | . 0010 | 29 | . 0050 |  |
| January 1987 | 57 | . 1100 | 50 | . 1200 |  |
| March 1987 | 69 | . 0010 | 51 | . 0010 |  |
| May 1987 | 26 | . 0540 |  |  |  |
| July 1987 | 23 | b | 32 | . 0010 |  |
| August 1987 | 49 | . 0100 | 28 | . 0250 |  |
| September 1987 | 30 | . 0900 |  |  |  |
| November 1987 | 72 | . 0010 |  |  |  |
| January 1988 | 57 | b | 21 | . 0050 |  |
| March 1988 | 29 | b | 45 | . 0300 | (day 113) c |
| May 1988 | 93 | . 0250 | 10 | . 77.00 |  |
| July 1988 | 67 | . 1400 |  |  | (day 116) c |
| August 1988 | 31 | . 0600 |  |  |  |

a...days prior to expiration
b...p-value less than. 0001
c...other days significant at $p=.2$, or spurious switchpoints found

NS..No switchpoint located

Table 2.

## SWITCHING REGRESSION RESULTS FOR MODEL OF EXPECTED SOYBEAN FUTURES PRICE DISTRIBUTIONS

| Contract |  |  |  |  |  | $\beta_{32}$ | my | $\begin{aligned} & \text { eters } \\ & z^{\prime} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Jan 86 | $\begin{gathered} 333.454 \\ (0.000) \end{gathered}$ | $\begin{aligned} & -0.493 \\ & (0.000) \end{aligned}$ | $\begin{gathered} 1.396 \\ (0.000) \end{gathered}$ | $\begin{gathered} -94.032 \\ (0.042) \end{gathered}$ | $\begin{gathered} 0.343 \\ (0.000) \end{gathered}$ | $\begin{gathered} 0.764 \\ (0.000) \end{gathered}$ |  |  |
| March 86 | $\begin{array}{r} -245.656 \\ (0.196) \end{array}$ | $\begin{gathered} 0.589 \\ (0.100) \end{gathered}$ | $\begin{gathered} 2.327 \\ (0.000) \end{gathered}$ | $\begin{gathered} 243.311 \\ (0.000) \end{gathered}$ | $\begin{aligned} & -0.186 \\ & (0.001) \end{aligned}$ | $\begin{gathered} 0.174 \\ (0.000) \end{gathered}$ | $\begin{gathered} 7.238 \\ (0.000) \end{gathered}$ | $\begin{aligned} & 30.443 \\ & (0.000) \end{aligned}$ |
| May 86 | $\begin{array}{r} -308.930 \\ (0.004) \end{array}$ | $\begin{gathered} 0.725 \\ (0.000) \end{gathered}$ | $\begin{gathered} 0.937 \\ (0.000) \end{gathered}$ | $\begin{array}{r} -220.936 \\ (0.000) \end{array}$ | $\begin{gathered} 0.604 \\ (0.000) \end{gathered}$ | $\begin{gathered} 0.706 \\ (0.000) \end{gathered}$ |  | $\begin{aligned} & 58.362 \\ & (0.000) \end{aligned}$ |
| July 86 | $\begin{array}{r} -639.624 \\ (0.059) \end{array}$ | $\begin{gathered} 1.300 \\ (0.043) \end{gathered}$ | $\begin{gathered} 2.208 \\ (0.000) \end{gathered}$ | $\begin{array}{r} -329.536 \\ (0.000) \end{array}$ | $\begin{gathered} 0.814 \\ (0.000) \end{gathered}$ | $\begin{gathered} 0.526 \\ (0.000) \end{gathered}$ | $\begin{gathered} 9.976 \\ (0.000) \end{gathered}$ |  |
| Nov 86 | $\begin{array}{r} -267.819 \\ (0.000) \end{array}$ | $\begin{gathered} 0.696 \\ (0.000) \end{gathered}$ | $\begin{gathered} 1.338 \\ (0.000) \end{gathered}$ | $\begin{array}{r} -262.005 \\ (0.000) \end{array}$ | $\begin{gathered} 0.865 \\ (0.000) \end{gathered}$ | $\begin{gathered} 0.023 \\ (0.524) \end{gathered}$ | $\begin{gathered} 5.096 \\ (0.000) \end{gathered}$ |  |
| Jan 87 | $\begin{array}{r} -175.458 \\ (0.157) \end{array}$ | $\begin{gathered} 0.459 \\ (0.063) \end{gathered}$ | $\begin{gathered} 1.465 \\ (0.000) \end{gathered}$ | $\begin{gathered} 183.187 \\ (0.002) \end{gathered}$ | $\begin{aligned} & -0.202 \\ & (0.084) \end{aligned}$ | $\begin{gathered} 0.694 \\ (0.000) \end{gathered}$ | $\begin{gathered} 3.472 \\ (0.000) \end{gathered}$ | $\begin{aligned} & 51.457 \\ & (0.000) \end{aligned}$ |
| March 87 | $\begin{gathered} 206.990 \\ (0.303) \end{gathered}$ | $\begin{aligned} & -0.283 \\ & (0.481) \end{aligned}$ | $\begin{gathered} 0.736 \\ (0.000) \end{gathered}$ | $\begin{array}{r} -141.021 \\ (0.584) \end{array}$ | $\begin{gathered} 0.427 \\ (0.398) \end{gathered}$ | $\begin{gathered} 0.769 \\ (0.000) \end{gathered}$ | $\begin{gathered} 7.182 \\ (0.000) \end{gathered}$ | $\begin{aligned} & 64.884 \\ & (0.000) \end{aligned}$ |
| May 87 | $\begin{gathered} 314.278 \\ (0.067) \end{gathered}$ | $\begin{aligned} & -0.499 \\ & (0.133) \end{aligned}$ | $\begin{gathered} 0.420 \\ (0.135) \end{gathered}$ | $\begin{gathered} 188.478 \\ (0.000) \end{gathered}$ | $\begin{aligned} & -0.239 \\ & (0.003) \end{aligned}$ | $\begin{gathered} 0.508 \\ (0.000) \end{gathered}$ | $\begin{gathered} 1.923 \\ (0.000) \end{gathered}$ | $\begin{aligned} & 25.083 \\ & (0.000) \end{aligned}$ |
| July 87 | $\begin{gathered} -132.042 \\ (0.131) \end{gathered}$ | $\begin{gathered} 0.372 \\ (0.019) \end{gathered}$ | $\begin{gathered} 3.108 \\ (0.000) \end{gathered}$ | $\begin{array}{r} -125.925 \\ (0.011) \end{array}$ | $\begin{gathered} 0.486 \\ (0.000) \end{gathered}$ | $\begin{gathered} 0.080 \\ (0.157) \end{gathered}$ | $\begin{gathered} 7.675 \\ (0.000) \end{gathered}$ | $\begin{aligned} & 32.059 \\ & (0.000) \end{aligned}$ |
| Sept 87 | $\begin{array}{r} -378.198 \\ (0.005) \end{array}$ | $\begin{gathered} 0.914 \\ (0.000) \end{gathered}$ | $\begin{gathered} 1.585 \\ (0.000) \end{gathered}$ | $\begin{array}{r} -359.497 \\ (0.000) \end{array}$ | $\begin{gathered} 0.988 \\ (0.000) \end{gathered}$ | $\begin{gathered} 0.434 \\ (0.003) \end{gathered}$ | $\begin{gathered} 9.965 \\ (0.000) \end{gathered}$ | $\begin{aligned} & 40.807 \\ & (0.000) \end{aligned}$ |
| Nov 87 | $\begin{array}{r} -172.275 \\ (0.166) \end{array}$ | $\begin{gathered} 0.475 \\ (0.037) \end{gathered}$ | $\begin{gathered} 1.035 \\ (0.000) \end{gathered}$ | $\begin{gathered} -366.892 \\ (0.000) \end{gathered}$ | $\begin{gathered} 1.029 \\ (0.000) \end{gathered}$ | $\begin{gathered} 0.360 \\ (0.000) \end{gathered}$ | $\begin{gathered} 9.531 \\ (0.000) \end{gathered}$ | $\begin{aligned} & 72.116 \\ & (0.000) \end{aligned}$ |
| Jan 88 | $\begin{gathered} 191.226 \\ (0.277) \end{gathered}$ | $\begin{aligned} & -0.196 \\ & (0.514) \end{aligned}$ | $\begin{gathered} 4.491 \\ (0.000) \end{gathered}$ | $\begin{gathered} -45.968 \\ (0.407) \end{gathered}$ | $\begin{gathered} 0.317 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.348 \\ (0.000) \end{gathered}$ | $\begin{gathered} 7.557 \\ (0.000) \end{gathered}$ | $\begin{aligned} & 22.761 \\ & (0.000) \end{aligned}$ |
| March 88 | $\begin{array}{r} -152.404 \\ (0.263) \end{array}$ | $\begin{gathered} 0.389 \\ (0.078) \end{gathered}$ | $\begin{gathered} 2.137 \\ (0.000) \end{gathered}$ | $\begin{array}{r} -121: 303 \\ (0.181) \end{array}$ | $\begin{aligned} & 0.552 \\ & (0.000) \end{aligned}$ | $\begin{gathered} 0.065 \\ (0.594) \end{gathered}$ | $\begin{aligned} & 14.607 \\ & (0.000) \end{aligned}$ | $\begin{aligned} & 63.771 \\ & (0.000) \end{aligned}$ |
| May 88 | $\begin{gathered} -38.000 \\ (0.772) \end{gathered}$ | $\begin{gathered} 0.235 \\ (0.232) \end{gathered}$ | $\begin{gathered} 1.237 \\ (0.000) \end{gathered}$ | $\begin{array}{r} 323.480 \\ (0.261) \end{array}$ | $\begin{aligned} & -0.151 \\ & (0.716) \end{aligned}$ | $\begin{gathered} 0.248 \\ (0.435) \end{gathered}$ | $\begin{aligned} & 12.975 \\ & (0.000) \end{aligned}$ | $\begin{gathered} 100.262 \\ (0.983) \end{gathered}$ |
| July 88 | $\begin{gathered} 554.510 \\ (0.000) \end{gathered}$ | $\begin{aligned} & -0.372 \\ & (0.006) \end{aligned}$ | $\begin{aligned} & -2.048 \\ & (0.001) \end{aligned}$ | $\begin{array}{r} -532.197 \\ (0.000) \end{array}$ | $\begin{gathered} 1.008 \\ (0.000) \end{gathered}$ | $\begin{gathered} 1.148 \\ (0.000) \end{gathered}$ | $\begin{aligned} & 12.371 \\ & (0.000) \end{aligned}$ | $\begin{aligned} & 52.401 \\ & (0.000) \end{aligned}$ |

Notes: Entries are estimated coefficients from equation (11) which correspond to the coefficients from equation (4) and parameters from the dummy equation (6). For example, $\beta_{11}$ is for the constant in regime $j, \beta_{2 j}$ is for the price level in regime $j$ and $\beta_{3 \mathrm{j}}$ is for TTM in regime $j$.
$P$-values are given in parentheses.

## REFERENCES

Anderson, R. W., "Some Determinants of the Volatility of Futures Prices", Journal of Futures Markets, 5 (1985), 331-48.

Beckers, S., "Standard Deviations Implied in Options Prices as Predictors of Future Stock Price Variability", Journal of Banking and Finance, 5 (1981), 363-382.

Black, F., The Pricing of Commodity Contracts", Journal of Financial Economics, 3 (Jan.-March, 1976), 167-79.

Black, F., and M. Scholes, "The Pricing of Options and Corporate Liabilities", Journal of Political Economy, 81 (May-June 1973), 637-659.

Brown, R. L., J. Durbin, and J.M. Evans, "Techniques for Testing the Constancy of regression Relationships over Time", Journal of the Royal Statistical Society, 2 (1975), 149-163.

Chiras, D., and S. Manaster, "The Informational Content of Option Prices and a Test of Market Efficiency", Journal of Financial Economics, 6 (June-Sept. 1978), 213-34.

Cox, J. C., and S. Ross, "A Survey of Some New Results in Financial Option Pricing Theory", Journal of Finance, 31 (1976), 383-402.

Fackler, P. F., "The Informational Content of Option Premiums", Unpublished Ph.D. Dissertation, University of Minnesota, 1986.

Fackler, P. F. and R. P. King, "Calibration of Option-Based Probability Assessments Agricultural Commodity Markets", American Journal of Agricultural Economics, 72(1990): 73-83.

Gardner, B. L., "Commodity Options for Agriculture", American Journal of Agricultural Economics, 59 (1977), 986-992.

Goldfeld, S. M., and R. E. Quandt, "The Estimation of Structural Shifts by Switching Regressions", Annals of Economic and Social Measurement, 2 (1973), 475-85.

Goldfeld, S. M., and R. E. Quandt, "Techniques for Estimating Switching Regressions", Studies in Nonlinear Estimation, Ballinger Publishers, Cambridge, Mass., 1976.

Gordon, J. D., The Distribution of Daily Changes in Commodity Futures Prices, Technical Bulletin 1702, Economic Research Service, USDA, Washington, D.C., 1985.

Grinblatt, M., and H. Johnson, "A Put Option Paradox", Journal of Financial and Quantitative Analysis, 23 (1988), 23-26.

Hall, J. A., B. W. Brorsen, and S. H. Irwin, "The Distribution of Futures Prices: A Test of the Stable Paretian and Mixture of Normals Hypothesis", Journal of Financial and Quantitative Analysis, 24 (1989), 105-116.

Harrison, J. M. and D. A. Kreps, "Martingales and Arbitrage in Multi-Period Securities Markets", Journal of Economic Theory, 20 (1979): 381-408.

Hauser, R. J., and D. K. Anderson, "Hedging with Options under Variance Uncertainty: An Illustration of Pricing New-Crop Soybeans", American Journal of Agricultural Economics, 69 (1987), 38-45.

Hays, P. A., and D. Upton, "A Shifting Regimes Approach to the Stationarity of the Market Model Parameters of Individual Securities", Journal of Financial and Quantitative Analysis, 21 (Sept. 1986), 30720.

Hogg, Robert V., and Stuart Klugman, Loss Distributions, John Wiley and Sons, New York, 1984.
Ingersoll, J. E., Theory of Financial Decision Making, Totowa, N.J.: Rowman and Littlefield, 1987.
Johnston, J., Econometric Methods, McGraw-Hill Co., New York, ed. 3, 1984.
Jordan, J. V., W. Seale, N. McCabe, and D. Kenyon, "Transactions Data Tests of The Black Model for Soybean Futures Options", The Journal of Futures Markets, 7 (1987), 73-91.

Kane, E. J., and H. Unal, "Change in Market Assessments of Deposit Institution Riskiness", Working Paper, The Ohio State University, 1987.

Kane, E. J., and H. Unal, "Modeling Structural and Temporal Variation in the Market's Valuation of banking Firms", Working Paper, The Ohio State University, 1988.

Kenyon, D., Kling, K., Jordan, J. V., W. Seale, and N. McCabe, "Factors Affecting Agricultural Futures Price Variance", The Journal of Futures Markets; 7 (1987), 73-91.

Latane, H., and R. J. Rendleman, Jr., "Standard Deviations of Stock Price Ratios Implied in Option Prices", Journal of Finance, 31 (May 1976).

Milonas, N. T., "Price Variability and the Maturity Effect in Futures Markets", The Journal of Futures Markets, 6 (1986), 443-460.

Park, H. Y., and R. S. Sears, "Estimating Stock Index Futures Volatility Through the Prices of Their Futures", The Journal of Futures Markets, 5 (1985), 223-237.

Quandt, Richard, "Tests of the Hypothesis that a Linear Regression System Obeys Two Separate Regimes", Journal of the American Statistical Association, 55 (1960), 324-330.

Quandt, Richard, The Estimation of the Parameters of a Linear Regression System Obeying Two Separate Regimes", Journal of the American Statistical Association, 53, (1958), 873-880.

Ross, S. A., "A Simple Approach to the Valuation of Risky Streams", Journal of Business, 51 (1978), 453475.

Schmalensee, R., and Trippi, R., "Common Stock Volatility Expectations Implied by Option Premia", Journal of Finance, 33 (1978), 129-47.

Shastri, K. and K. Tandon, "On Some Properties of Futures Volatilities Implied in Option Prices", Working Paper, (1987).
Singh, S. K., and G. S. Maddala, "A Function for the Size Distribution of Incomes", Econometrica, Vol. 4, No. 5, (1976): 963-70.

Tadikamalla, P. R., "A Look at the Burr and Related Distributions", International Statistical Review, 48, (1980): 337-44.


[^0]:    ${ }^{1}$ For examples see Latane and Rendleman, Schmalensee and Trippi, Chiras and Canaster, Choi and Longstaff, Park and Sears, Anderson, Milonas, Shastri and Tandon, Jordan et al., Beckers, Ball and Torous, and many others.

[^1]:    ${ }^{3}$ This distribution is among several candidates first investigated in Fackler (1986). For comparability's sake, we also used the lognormal as a candidate distribution. The results are qualitatively similar.

[^2]:    ${ }^{4}$ The Gauss programming language was used on a DTK 386 machine. The algorithms were based on similar code originally written by Paul Fackler.

