MODELING CYCLICAL CATTLE PRICES IN A MONTE CARLO SETTING

by

Larry W. VanTassell, J. Richard Conner, and James W. Richardson*

Presented as a selected paper at the AAEA Annual Meetings, August 4-8, 1990, Vancouver, British Columbia.

*Authors are respectively, Assistant Professor, Department of Agricultural Economics and Rural Sociology, University of Tennessee; Professor, Department of Agricultural Economics, Texas A&M University; and Professor, Department of Agricultural Economics, Texas A&M University.

MODELING CYCLICAL CATTLE PRICES IN A MONTE CARLO SETTING

A methodology for simulating harmonic regressions is presented that allows for the stochastic simulation of the harmonic regressions when various orders of autocorrelation are present. Statistical properties of the historical correlations are respectably maintained in an empirical example using ten livestock classes which exhibit first and twelfth order autocorrelation.

#8452

distribution each year by a specified factor. Richardson and Nixon used a similar technique to generate correlated random prices for seven cattle classes in FLIPSIM. Users of FLIPSIM specify seasonal average prices for each year of the planning horizon. Triangular or empirical distributed random deviates which are correlated among the livestock classes are then drawn and added to the specified price trends.

Gutierrez was one of the first to incorporate the cattle price cycle into a simulation analysis. Modal steer prices were estimated from a harmonic regression model. The stochastic component was represented as a random deviate drawn from a triangular probability distribution around the modal value. Prices for other cattle classes were determined from the stochastic steer price by using a specified price adjustment factor depending upon the position in the cycle. Little and Ray also used a harmonic regression model to specify prices for feeder steers, but utilized a normal random error term to add the stochastics. Little and Ray then determined fat cattle prices from a simple linear relationship between feeder and slaughter cattle.

Several inadequacies exist in the modeling efforts previously mentioned. First, either a cyclical component was excluded in the simulation, or the cyclical component was estimated for only one class of livestock. Second, when the cyclical component was estimated, the presence of autocorrelation was not accounted for in the estimation or the simulation.

The objectives of this paper are to estimate harmonic price regressions for several classes of cattle while incorporating autoregressive characteristics inherent in the estimated models. A method will then be introduced which allows for the correlation of random normal deviates between classes of livestock and the autoregressive parameters.

#8452

MODELING CYCLICAL CATTLE PRICES IN A MONTE CARLO SETTING

Simulation has become a popular tool for determining the feasibility of long-term investments. One of the major advantages of utilizing simulation analysis for these studies is simulation's ability to deal with dynamics and stochastics too complex to be represented by more rigid mathematical models such as MOTAD or quadratic programming. For simulation analysis to be reliable, the major sources of risk need to be quantified and combined in such a way as to adequately describe the interacting relationships which exist between the uncertain environment and the productive processes of a farming operation. Much of the uncertainty in farm income can be attributed directly to market risk or the fluctuation in cattle prices. A proper accounting of price fluctuations is essential to determining the economic feasibility of long-term investments.

Cycles in cattle inventories provide much of the impetus, especially in the long-run, behind fluctuation in beef cattle prices (Brunk; Uvacek). The cattle cycle may be an important factor in determining the feasibility of long-term management practices. For example, while economic benefits from many investments may last ten, twenty, or more years, the ability of these investments to cash flow is of particular importance. A few years of strong prices directly after implementation may greatly increase the feasibility of long-term investments. The positioning of investments relative to the position of the cattle cycle may therefore prove important.

Cattle prices have been represented by a myriad of methods in previous simulation studies. Beck et al. made cattle prices stochastic by specifying triangular distributions for each class of stock. Long-term trends in livestock prices were incorporated by shifting the parameters of the

Estimation of the Harmonic Price Equations

Harmonic regression models have been used to project a potential path for cattle prices (Franzmann and Walker; Gutierrez; Little and Ray). These models have accounted for seasonal variation, cyclical variation, and a long-term linear trend in each livestock class. The general model as adapted from Franzmann and Walker is:

(1) $P_{it} = B_o + B_1 2\pi t + B_2 Sin(2\pi t/L1) + B_3 Cos(2\pi t/L1) + B_4 Sin(2\pi t/L2) + B_5 Cos(2\pi t/L2) + \mu$

where P_{it} is the predicted price for the i^{th} class of cattle in time period t, B's are parameter estimated, L is the specified cycle length and μ is the residual error term. Franzmann and Walker incorporated a 12 month seasonal component for L1, and a 120 month cycle for L2.

If autocorrelation is present, the model can be estimated via GLS as:

(2) $P_{it} = B_o + B_1 2\pi t + B_2 Sin(2\pi t/L1) + B_3 Cos(2\pi t/L1) + B_4 Sin(2\pi t/L2) + B_5 Cos(2\pi t/L2) + \mu_t$

with

(3) $\mu_t = \rho_1 \mu_{t-1} + \rho_{12} \mu_{t-12} + \zeta_t$

where ρ_1 is a first order autoregressive process and ρ_{12} is a twelfth order autoregressive process. The predictor is then defined as (Johnson and Johnson):

(4) $P_{n+1} = b_o + b_1 2\pi t + b_2 Sin(2\pi t/L1) + b_3 Cos(2\pi t/L1) + b_4 Sin(2\pi t/L2) + b_5 Cos(2\pi t/L2) + \rho_1 \mu_n + \rho_{12} \mu_{n-11}$.

Correlation of Autoregressive Equations

When simulating a series of equations that have autoregressive parameters, the problem of correlating the generated random error terms between equations can present a problem. Without the autoregressive components, the traditional method of correlating the random errors between classes would follow the well know method introduced by Clements et al. and Richardson and Condra where a correlation matrix (P) is estimated from the error terms of equation 1 and factored into a unique upper triangular matrix R so that (using matrix notation):

$$[3]\ P = RR'.$$

The elements for matrix R can be found by factoring the P matrix by the "square-root" method, i.e., taking the square-root of the matrix. The R matrix (an n x n upper triangular matrix) is then multiplied by an n x 1 vector of independent standard normal deviates (W) to obtain an n x 1 vector of correlated standard normal deviates (C) as:

$$[4]\ C = RW.$$

This procedure is repeated with as many vectors of W as needed to generate the desired number of random values. By doing this, the historical correlations between the livestock classes can be represented in a simulation model. A shortcoming, however, of the procedure is that it does not allow for autocorrelation of random variables.

The procedure to generate autocorrelated random variables utilizes the techniques above to correlate the random deviates except that the appropriate w is solved for to assure identity and correlation in the historical error terms. For example, assume equations are estimated for heifer and steer calves which exhibit a first order autoregressive process. A 4 x 4 correlation matrix could be obtained from the residuals of the two equations along with the lagged residuals of each equation. Using the traditional method (Clements et al.), this correlation matrix would then be factored into a unique upper right triangular matrix via the "square root method" and multiplied by a vector of random normal deviates W to obtain the vector of correlated random normal deviates C as:

$$[6] \quad \begin{vmatrix} c_{1t} \\ c_{2t} \\ c_{3t} \\ c_{4t} \end{vmatrix} = \begin{vmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ & r_{22} & r_{23} & r_{24} \\ & & r_{33} & r_{34} \\ & & & r_{44} \end{vmatrix} * \begin{vmatrix} w_{1t} \\ w_{2t} \\ w_{3t} \\ w_{4t} \end{vmatrix}$$

where t equals the current year. If c_{1t} and c_{2t} represent the correlated deviates for heifer and steer prices respectively, and c_{3t} and c_{4t} represent the correlated deviates for heifer and steer prices lagged 1 period, the values generated in period t for c_{1t} and c_{2t} by definition must equal the values generated for c_{3t} and c_{4t} in period t + 1. In the present format this will not be the case and given the fact that all four deviates must be generated in the same process to assure proper correlation, a modification in the procedure must be made. This modification entails the solving for the appropriate w_{3t} and w_{4t} to assure identity throughout the process. In our example, in period t=1:

$$[7] \quad c_{21} = \sum_{j=1}^{4} (r_{2j} * w_{j1}).$$

In period t=2, c_{42} would be calculated as:

$$[8] \quad c_{42} = \sum_{j=1}^{4} (r_{4j} * w_{j2}).$$

Because c_{42} by definition must equal c_{21} we can substitute c_{21} for c_{42} to get:

$$[9] \quad c_{21} = \sum_{j=1}^{4} (r_{4j} * w_{j2})$$

$$= r_{44} * w_{42}.$$

The random normal deviate w_{42} needed to assure that c_{42} is defined to equal c_{21} can then be defined as:

$$[10] \quad w_{42} = c_{21}/r_{44}.$$

Using this same logic, c_{11} can be made equal to c_{32} by solving for w_{32} as:

$$[11] \quad c_{11} = \sum_{j=1}^{4} (r_{3j} * w_{j2})$$

$$= (r_{33} * w_{32}) + (r_{34} * w_{42}),$$

with:

$$[12] \quad w_{32} = (c_{11} - r_{34} * w_{42}) / r_{33}.$$

Therefore, instead of all normal deviates (w) being randomly drawn, w_{3t} and w_{4t} are calculated conditional upon the correlated deviates obtained for c_{1t-1} and c_{2t-1}. At the beginning of each iteration (year 1), initial values can either be given to w_{42} and w_{32} to start each iteration at the same point (e.g., w_{42} and w_{32} could be given the values of the actual deviates in the last year data was available), or values for w_{42} and w_{32} can be generated stochastically.

An Empirical Example

Cattle price cycles were estimated for 400-500, 500-600, and 600-700 pound steer and heifer calves along with prices for cull cows, replacement heifers, cow-calf pairs, and replacement bulls. Data were collected from the North Central Texas Auction markets (Texas Department of Agriculture) for cow-calf pair and heifer prices; from Amarillo Direct Sales Reports (U.S. Department of Agriculture) for steer and heifer calves along with utility cows; and from auction summary reports for registered Hereford bulls (American Hereford Journal). Monthly prices from January 1972 through December 1988 were used.

GLS equations for the ten livestock classes are shown in Table 1. As was hypothesized, autocorrelation was present in each of the equations. A first and twelfth order autoregressive process was found to be significant is each equation except for bull prices where only a twelfth order process was significant. Figure 1 contains a graph of the historical and predicted price of 400-500 pound heifers from 1972 through 1988, and Figure 2 contains a graph of the projected trend for 400-500, 500-600, and 600-700 pound steers. The projected trend portrays some of historical relationships that have existed among the different weight classes of steers at the various stages of the cattle cycle (U.S. Department of Agriculture, 1983). For example, a smaller price spread exists during the bottom of the cycle than at the top.

The prediction equations were stochastically simulated over a ten year planning horizon for twenty iterations using correlated random normal deviates developed from a 30 x 30 correlation matrix developed from the harmonic function residuals, the residuals lagged 1 month, and the residuals lagged 12 months. Correlation coefficients were computed for the simulated price series and upon visual inspection appeared to be close to the historical correlations. A statistical procedure for testing the equality of the covariance matrix described by Morrison (1967, pp. 152-153) was used to test the null hypothesis that the sample covariance matrix is equal to the observed covariance matrix. The test reveal that at a 5 percent level of significance, the null hypothesis was rejected, therefore conclude that the null hypothesis is indeed tenable.

Summary and Conclusions

A proper accounting of price fluctuations is essential in determining the economic feasibility of long-term investments via a simulation analysis. This is especially true for cattle prices, where seasonal variation, cyclical variation, and long-term linear trends are present. The problem of simulating harmonic regressions which account for this variation is complicated when more than one livestock price class is simulated and autoregressive errors are present. The methodology extended in this paper allows for the stochastic simulation of the harmonic regressions while respectably maintaining the statistical properties observed in the historical series. While this methodology should allow for a more accurate representation of cattle prices in simulation studies. It can also be extended to provide intertemperal correlation of yields and other production variables. Further research is needed though, to investigate ways historically error terms may be better generated in a Monte Carlo simulation so as to better maintain all statistical properties.

Table 1. Generalized Least Squares Estimates of the Cyclical Trend Cattle Price Model.[a,b]

$$H4 = 36.36 + 0.03LT + 1.82S1 - 1.34C1 - 10.29S2 + 7.76C2 + \zeta_t$$
(12.0) (7.7) (3.9) (-2.9) (-4.2) (3.3)

$$R^2 = .97 \quad \sigma = 2.54 \quad \rho_1 = -0.97 \quad \rho_{12} = 0.09$$

$$H5 = 35.22 + 0.03LT + 1.67S1 - 0.52C1 - 9.49S2 + 7.65C2 + \zeta_t$$
(14.1) (8.9) (3.6) (-1.1) (-4.8) (4.0)

$$R^2 = .97 \quad \sigma = 2.57 \quad \rho_1 = -0.96 \quad \rho_{12} = 0.10$$

$$H6 = 34.27 + 0.03LT + 1.48S1 - 0.33C1 - 8.49S2 + 7.58C2 + \zeta_t$$
(15.6) (10.1) (3.4) (-0.8) (-4.9) (4.5)

$$R^2 = .97 \quad \sigma = 2.40 \quad \rho_1 = -0.94 \quad \rho_{12} = 0.10$$

$$S4 = 44.29 + 0.04LT + 1.79S1 - 1.51C1 - 11.96S2 + 9.15C2 + \zeta_t$$
(13.8) (8.0) (3.2) (-2.7) (-4.7) (3.7)

$$R^2 = .98 \quad \sigma = 3.02 \quad \rho_1 = -0.95 \quad \rho_{12} = 0.09$$

$$S5 = 41.81 + 0.03LT + 1.66S1 - 0.50C1 - 10.21S2 + 8.67C2 + \zeta_t$$
(14.1) (8.9) (3.6) (-1.1) (-4.8) (4.0)

$$R^2 = .98 \quad \sigma = 2.68 \quad \rho_1 = -0.94 \quad \rho_{12} = 0.10$$

$$S6 = 40.10 + 0.03LT + 1.38S1 + 0.18C1 - 9.13S2 + 8.33C2 + \zeta_t$$
(17.7) (10.0) (2.7) (0.4) (-5.2) (4.8)

$$R^2 = .97 \quad \sigma = 2.87 \quad \rho_1 = -0.91 \quad \rho_{12} = 0.10$$

$$UT = 26.22 + 0.02LT + 1.85S1 - 1.65C1 - 5.30S2 + 4.85C2 + \zeta_t$$
(18.3) (8.5) (5.3) (-4.8) (-4.8) (4.5)

$$R^2 = .96 \quad \sigma = 1.92 \quad \rho_1 = -0.90 \quad \rho_{12} = 0.10$$

$$RP = 32.12 + 0.02LT + 1.85S1 - 0.25C1 - 9.02S2 + 6.33C2 + \zeta_t$$
(14.8) (6.2) (2.8) (-0.4) (-5.6) (4.0)

$$R^2 = .91 \quad \sigma = 3.66 \quad \rho_1 = -0.83 \quad \rho_{12} = 0.09$$

$$PR = 328.48 + 0.30LT + 6.32S1 - 21.99C1 - 91.35S2 + 93.97C2 + \zeta_t$$
(7.9) (5.4) (0.7) (-2.5) (-3.1) (3.3)

$$R^2 = .93 \quad \sigma = 44.58 \quad \rho_1 = -0.83 \quad \rho_{12} = 0.09$$

$$BL = 806.16 + 1.09LT - 132.34S1 - 72.41C1 - 21.14S2 + 162.53C2 + \zeta_t$$
(8.4) (8.5) (-2.0) (-1.1) (-0.4) (2.7)

$$R^2 = .54 \quad \sigma = 420.03 \quad \rho_{12} = -0.40$$

[a] t-values for each parameter are in parenthesis below parameter estimates.

b σ = standard deviation; ρ_1 = first order autocorrelation; ρ_{12} = twelfth order autocorrelation; H4, H5, H6, S4, S5, and S6 = heifer (H) and steer (S) price per cwt. for 400-500(4), 500-600(5), and 600-700(6) pound calves, respectively; UT = price per cwt. for utility cows; RP = price per cwt. for replacement heifers; PR = price per pair for cow-calf pairs; BL = price per head for replacement herd sires; LT = $2\pi t$; S1 = $Sin(2\pi t/L1)$; C1 = $Cos(2\pi t/L1)$; S2 = $Sin(2\pi t/L2)$; C2 = $Cos(2\pi t/L2)$; L1 = seasonal component of 12 months; L2 = cycle length of 120 months; t = time trend (i.e., 1, 2, 3 ...).

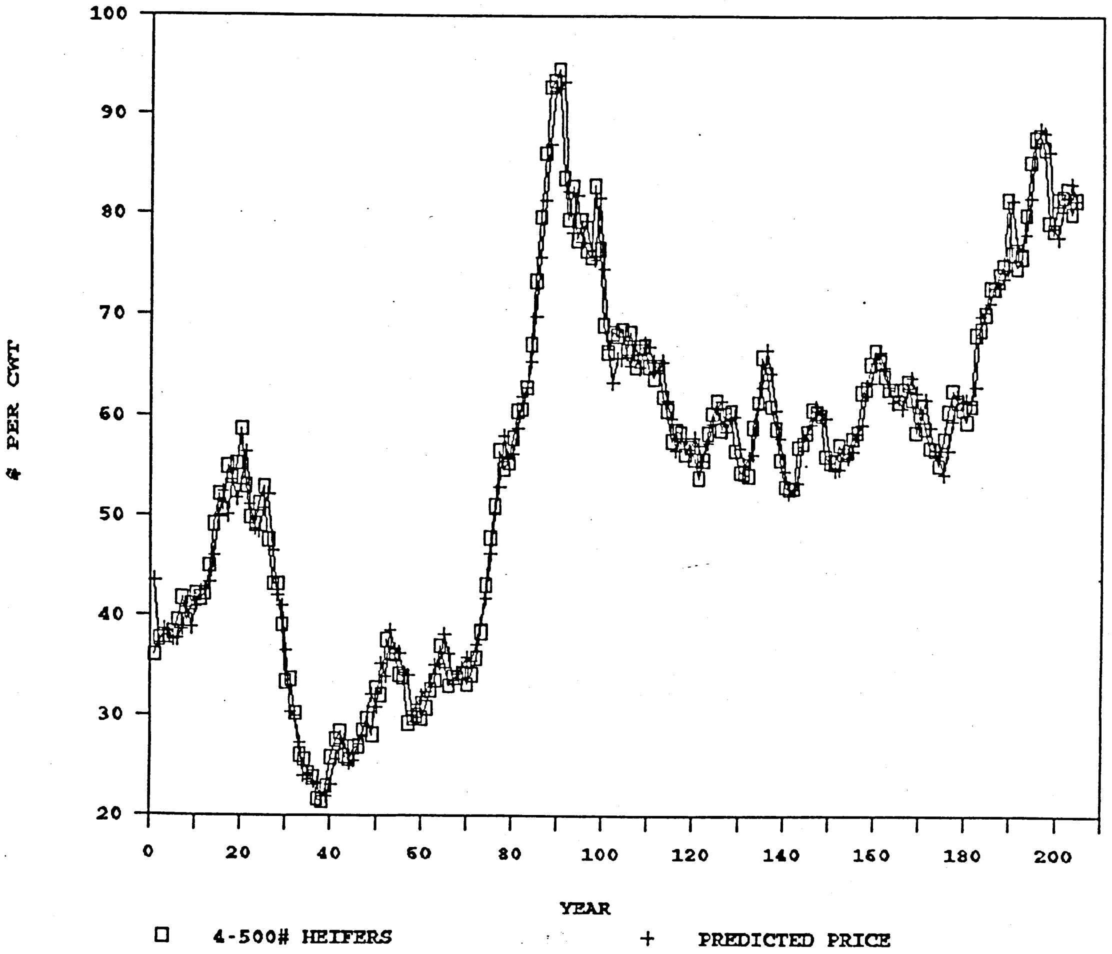

Figure 1. Historical and Predicted Prices for 400-500 Pound Heifer Calves, 1972-1988.

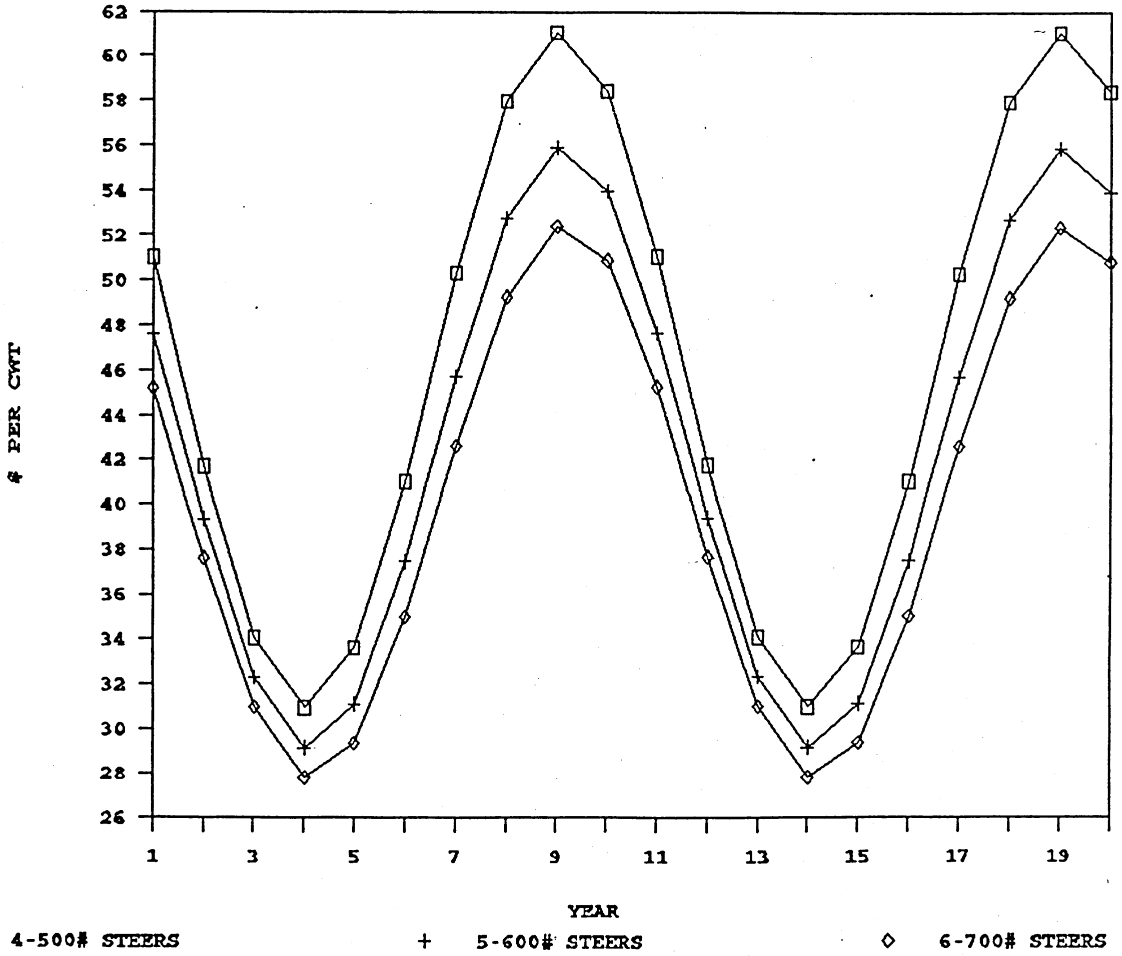

Figure 2. Simulated Trend Cycles for 400-500, 500-600, and 600-700 Pound Steers.

References

American Hereford Journal. "Yearly Auction Averages." Kansas City, MO., Various Issues, 1972-1989.

Beck, A.C., I. Harrison, and J.H. Johnston. "Using Simulation to Assess the Risks and Returns from Pasture Improvement for Beef Production in Agriculturally Underdeveloped Regions." Agricultural Systems. 8(1982):55-71.

Clements, A.M. Jr., H.P. Mapp Jr., and V.R. Eidman. "A Procedure for Correlating Events in Farm Firm Simulation Models." Oklahoma State University Agr. Exp. Sta. Bull. No. T-131, Aug. 1971.

Franzmann, J.R. and R.L. Walker. "Trend Models of Feeder, Slaughter, and Wholesale Beef Cattle Prices." American Journal of Agr. Economics. 54(1972):507-512.

Gutierrez, P.H. "The Impact of Selected Management Practices on the Economic Survivability of a Ranch Unit: An Analysis for a Southern Plains Ranch." Ph.D. dissertation, Oklahoma State University, 1985.

Law, A.M. and W.D. Kelton. Simulation Modeling and Analysis. New York: McGraw-Hill Book Co. 1982.

Little, R.D. and D.E. Ray. "An Evaluation of the Importance of the Net Operating Loss Carryback to Oklahoma Cow-calf Producers." Presented paper at the SAEA meeting, Feb. 4-7, 1990, Little Rock, AK.

Morrison, D.F. Multivariate Statistical Methods. New York: McGraw-Hill, Inc., 1967.

Richardson, J.W. and C.J. Nixon. "Description of FLIPSIM V: A General Firm Level Policy Simulation Model." Texas Agr. Exp. Sta. Bull. No. 1528, 1986.

Richardson, J.W. and G.D. Condra. "A General Procedure for Correlating Events in Simulation Models." Texas Agri. Exp. Sta., Department of Agricultural Economics (mimeo), 1978.

Texas Department of Agriculture. Texas Livestock News. Austin, TX, selected weekly issues, 1972-1988.

U.S. Department of Agriculture. Market News Branch, Livestock Division, AMS. Amarillo, TX, selected monthly issues, 1972-1988.

U.S. Department of Agriculture. Cattle Cycles: How to Profit From Them. Extension Services In Cooperation with the Extension Industry Beef Resource Committee. Misc. Pub. No. 1430, 1983, 34 pp.