8634

1990

HOW MISLEADING CAN ALLEN ELASTICITIES OF SUBSTITUTION BE?

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AAEA 1990

#8634

ABSTRACT

The Allen elasticity of substitution has recently been shown to be an incorrect measure of substitutability when there are more than two inputs. This paper presents an empirical comparison of the Morishima and Allen elasticities based on a translog cost function estimated using data for aggregate southeastern U.S. Agriculture. The results indicate substantial differences in the two measures.

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Introduction

One of the most fundamental measures in the theory of production is the elasticity of substitution. Introduced by Hicks over 50 years ago, its original intent was to help explain changes in the distribution of income between labor and capital with a scalar measure. As such, the elasticity of substitution was defined as the percentage change in the capital labor ratio given a percentage change in the marginal rate of substitution, or equivalently, given a percentage change in the relative prices of capital and labor.

As evidenced by the debate in the 1970s and early 1980s on whether capital and energy were substitutes or complements (Berndt and Wood; Fuss; Griffin and Gregory; Griffin), interest in attempting to estimate the elasticity of substitution has not waned. However, with the increased computational ability, data availability and the popularization of duality and flexible functions, empirical; models have progressed from specifying output as function of only two inputs (labor and capital) to more general specifications that include additional factors of production (e.g. labor, capital, energy and materials).

When the number of inputs considered increases beyond two, there arise a number of competing measures for the elasticity of substitution between any two inputs (Mundlak; Nadiri). The differences in these elasticity measures stem from differing assumptions concerning what happens to the "other" inputs in the model. Of these competing definitions, partial elasticity of substitution developed by Allen (AES) has become the measure of choice in virtually all empirical studies.

In a recent note in the American Economic Review, Charles Blackorby and Robert Russell (1989) demonstrated that when there are more than two inputs, the AES:

> "(i) is <u>not</u> a measure of the 'ease' of substitution or curvature of the isoquant, (ii) provides <u>no</u> information about relative factor shares (the purpose for which the elasticity of substitution was originally defined), and (iii) <u>cannot</u> be interpreted as a (logarithmic) derivative of a quantity ratio with respect to a price ratio (or the marginal rate of substitution)" (p. 883).

Given the widespread use of the AES both in the classroom and in empirical work, this is indeed a very disturbing statement.

In place of the AES, Blackorby and Russell (1989) propose the Morishima elasticity of substitution (MES) discovered independently by Morishima, and Blackorby and Russell (1975). In contrast to the AES, the MES provides a valid measure of the ease of substitution and can be shown to be the logarithmic derivative of an input ratio taken with respect to the input price ratio or marginal rate of substitution when the production process contains more than two inputs. Thus, the MES is fully consistent with Hicks' original notion of the elasticity of substitution.

The purpose of this paper, is to provide an empirical comparison of the AES and MES obtained from a translog cost function estimated using aggregate data for the Southeastern U.S. agricultural sector. While the results of this analysis do not form a basis from which general inferences concerning the relationship of the AES and MES may be rendered, they do provide a glimpse at how misleading the AES can be.

In the next section, the basic expressions for the AES and MES are presented and some of the distinguishing characteristics of the MES are listed. In the third section, the empirical model and results are presented. The final section contains some concluding comments.

The Allen and Morishima Elasticities of Substitution

Although the AES can be derived from a primal production function, it is usually defined using the dual cost function. Denoting the cost function as C(y,p) where y is output and p is a n-dimensional vector of inputs prices taken as exogenous to the firm, the AES between inputs i and j is given by

$$(1) \qquad AES(i,j) = \frac{C(y,p)C_{ij}(y,p)}{C_i(y,p)C_j(y,p)}$$

where C_i and C_{ij} denote the first and second partial derivatives of the cost function with respect to the ith and jth prices. Applying Shephard's Lemma and rearranging yields the expression

$$(2) \qquad AES(i,j) = \frac{E_{ij}}{S_j}$$

where E_{ij} is the Hicksian cross-price elasticity of demand and S_j is the cost share of the jth input. As is well known, the AES is symmetric in that AES(i,j) = AES(j,i).

The MES, as noted by Blackorby and Russell (1989) is derived by direct extension of the elasticity of substitution for the case of two inputs proposed by Hicks. Recognizing that demand for the ith input (x_i) can be obtained by Shephard's lemma $x_i = C_i(y,p)$, the MES is obtained by differentiating the expression

$$\ln[C_i(y,p)/C_j(y,p)]$$

with respect to logarithm of the price ratio p_i/p_j. As noted by Blackorby and Russell (1989), a change in the price ratio may be achieved by varying either p_i, p_j or both. If attention is restricted to varying the price ratio by only varying either p_i or by varying p_j, then the MES obtained by varying relative prices through a variation in p_i may be expressed as

$$(3) \qquad MES(i,j) = \frac{p_i C_{ij}(y,p)}{C_j(y,p)} - \frac{p_i C_{ii}(y,p)}{C_i(y,p)} .$$

Invoking Shephards Lemma allows (3)to be written as

$$(4) \qquad MES(i,j) = E_{ji} - E_{ii}$$

where E_{ji} and E_{ii} denote the Hicksian own and cross elasticities of demand. In a similar fashion the MES may be derived by changing relative input prices through a variation in p_j. This yields the MES

$$(5) \qquad MES(j,i) = E_{ij} - E_{jj}.$$

As can be seen, in contrast to the AES, the MES is an asymmetric measure.

The asymmetry of the MES follows from the recognition that, say a one percent change in relative input prices may be induced by a change in either the ith or jth price. When, the ith price is varied, the change in the jth input is captured through the cross price elasticity E_{ji} while the change in the ith input in captured through its own price elasticity. When the jth price is varied, the opposite occurs. Note, however, that in the case of two inputs, the MES is symmetric and equal to the AES.

Empirical Model and Results

To provide an empirical comparison of the AES and MES, a translog cost function was specified and estimated. Five aggregate inputs were considered: labor (L), capital (K), Land (A), energy (E) and materials (M). Denoting output by Y, input prices as P_i, i = L, K, A, E, M and a time trend as T, the translog cost function may be written as

$$(6) \qquad \ln C = a_0 + \sum_i a_i \ln P_i + 1/2 \sum_i \sum_j a_{ij} \ln P_i \ln P_j + \sum_i a_{iy} \ln P_{i\ln Y}$$
$$+ a_y \ln Y + 1/2 a_{yy} (\ln Y)^2 + b_t T + 1/2 b_{tt} T^2 + \sum_i b_{ti} \ln P_i T$$

where the parameter restrictions for homogeneity, $\sum_i a_i = 1$, $\sum_j a_{ij} = 0$, $i = L,K,A,E,M$, and symmetry, $a_{ij} = a_{ji}$ are imposed a priori. Logarithmic differentiation of (6) with respect to ith input price and application of Shephard's Lemma yields general expression for the share equations

$$(7) \qquad S_i = a_i + \sum_i a_{ij} \ln P_j + a_{iy} \ln Y + b_{it} T \qquad i = L,K,A,E,M$$

where S_i denotes the budget share of the ith input.

The data used in estimation spanned the 1951 to 1980 period. Implicit price indexes and expenditure data for the labor and capital inputs were obtained from Monson. Labor input was measured by the an index of total hours of farmwork. Capital was measured by an index of farm machinery. Price indexes and expenditure data for the land, materials and energy inputs were the same as used by Shumway, Saez and Gottret and were obtained from the authors.

Estimation was accomplished by imposing the appropriate parameter restrictions for homogeneity and symmetry and estimating the cost function in (6) jointly with the share equations in (7). As is well known, by virtue of the fact that the share equations sum to one, one equation must be deleted in estimation to avoid a singular covariance matrix. Hence the material equation was deleted from the system.

Joint estimation was accomplished by appending the cost function and share equations for labor, capital, land and energy with disturbance terms and using an iterated generalized least squares estimator. Under the assumption that the disturbance vectors follow a multivariate normal distribution, this estimator is equivalent to the maximum likelihood estimator. Thus, the resulting parameter estimates are invariant to which equation is deleted from the system.

The estimated parameters are presented in Table 1. Twenty two of the 27 parameters estimated (81%) had asymptotic t-values exceeding 1.96. The R-square values for the cost function and energy share equation were 0.41 and 0.75, respectively. The share equations for labor, land and machinery all had R-square value of 0.94 or higher. Based on the Durbin Watson statistics calculated for the individual equations, there was no evidence of first order autoregression.

The Allen and Morishima elasticities of substitution estimated at the mean of the data are presented in Table 2. Only 7 Of the 20 Morishima elasticities calculated exceed the corresponding Allen elasticities. Hence there is some indication the Allen elasticities may tend to overstate the ability to substitute inputs.

In many cases, the absolute difference in the two elasticity measures are rather moderate. However, in some instances the differences are substantial. Of particular note is the degree of substitutability between energy and capital. The estimated AES for energy and capital at the mean is 3.21, indicating a fair degree of substitutability between these two inputs. In contrast, the estimated MES of energy for capital 1.59 while the MES of capital for energy is estimated to be 0.80. Thus, the ability to substitute energy and capital as measured by the MES is considerably lower than that implied by the corresponding AES.

In two cases, the technical relationship between inputs is opposite for the AES and MES. The estimated Allen elasticities for land and energy, and materials and capital imply both pairs of input are net complements. In contrast, the estimated Morishima elasticities indicate that land and energy, and materials and capital are net substitutes. Although this reversal occurs between only two input pairs, it must be remembered that this accounts for twenty percent of the 10 possible pairwise input relationships being examined.

The implications of the asymmetric nature of the Morishima elasticity can be seen by returning to the relationship between capital and energy. The estimated MES of energy for capital was 1.59 whereas the MES of capital for energy was about 50% lower at 0.80. The implications of this asymmetry are of considerable importance. In essence, a one percent change in the relative price of capital and energy induced by a change in the price of energy induces a much larger change capital-energy ratio (1.59%) than does a one percent change in relative prices induced by a change in the price of capital. Note also, that the implied changes in the capital-energy ratio in either case are considerably less than implied by the estimated AES between capital and energy.

There does not appear to be any general pattern regarding the asymmetry of the Morishima elasticities. There is some indication that the Morishima elasticities that involve capital or energy tend to exhibit more asymmetry than elasticities not involving these factors. However, all of the Morishima elasticities demonstrate some degree of asymmetry.

Summary and Conclusions

The Allen partial elasticity of substitution has been the empirical measure of choice for gauging the technical substitutability/complementarity of inputs for many years. However, in a recent article, Blackorby and Russell (1989) demonstrated that for technologies using more than two inputs, the AES was devoid of any meaningful economic content. In place of the AES, they suggested using the Morishima elasticity of substitution.

This purpose of this paper was to compare estimates of the Allen and Morishima elasticities of substitution obtained from a translog cost function in order to see how misleading Allen elasticities can be. While the results are not directly generalizable, they do provide a first glimpse into the magnitude by which the inferences based on the two measures may differ.

The empirical results generally suggested that the AES tend to overstate the substitutability of inputs. In some cases, as evidenced by energy and capital, the degree of overstatement can be considerable. It should, however, be noted that no attempt was made to establish whether or not the observed differences were statistically significant.

In two of the ten possible pairwise input comparisons, the AES and MES yielded conflicting inferences regarding the technical relationship between inputs. Based on the AES, material and capital, and energy and land were net complements. In contrast, the MES indicated both input pairs were behaved as net substitutes.

The MES exhibited as reasonable degree of asymmetry. The MES between input pair involving capital or energy as one of the inputs generally exhibited more asymmetry than input pairs not including them. However, no theoretical explanation for this occurrence can at present be made.

Finally, the results provide some indication of the importance of the asymmetry of the MES and the differences in values from the AES as regards policy implications. As an example, the estimated AES between energy and capital was 3.2. This indicates that energy and capital are net substitutes and that investment tax credits for investing in energy efficient farm machinery may be a viable policy to achieve some degree of energy efficiency and conservation.

The MES between energy and capital paint a much different picture. The estimated MES of capital for energy was 0.8 while the MES of energy for capital was 1.59. These estimates suggest two things. First, the technical ability to substitute energy and capital is much lower than that implied by the AES. Secondly, a change (say 1%) in the relative prices of energy and capital induced by a change in the price of energy yields a much larger response in the capital-energy ratio (1.6%) than a similar change induced by a change in the price of capital (0.8%). Thus, energy conservation policies designed to stimulate investment in energy efficient machinery by altering energy prices may be more effective than those designed to stimulate capital investment through altering the price of capital as through an investment tax credit.

Although the results of this study cannot be generalized, they do provide evidence to suggest that the differences between estimated Allen and Morishima elasticities can be substantial. In addition, the asymmetric nature of the MES have potentially significant policy implications. The extent of these implications will only be understood with additional empirical research.

Table 1. Estimated coefficients for the translog cost function.

Parametes	Estimate	t - statistic
a_O	60.529	4.9418
a_A	0.8590	5.4872
a_L	-1.6818	-5.3309
a_K	0.1630	1.8749
a_E	0.4838	4.4851
a_Y	-12.506	-4.330
a_{AA}	0.0684	-5.4163
a_{AL}	0.0347	6.2927
a_{AK}	0.0017	0.18568
a_{AE}	0.0192	-2.6112
a_{LL}	-0.0859	-6.5606
a_{LK}	-0.0109	-3.8614
a_{LE}	0.0252	6.3573
a_{KK}	0.0384	3.5055
a_{KE}	0.0146	1.9612
a_{EE}	-0.0307	-3.7014
a_{AY}	-0.0789	-3.7535
a_{LY}	0.0244	5.7079
a_{KY}	-0.0043	-0.38032
a_{EY}	-0.0525	-3.6356
a_{YY}	1.4675	4.3405
b_T	0.0410	2.1734
b_{TT}	-0.0033	-2.9053
b_{TA}	-0.0010	-0.8200
b_{TL}	-0.0001	-0.4448
b_{TK}	-0.0022	-3.1181
b_{TE}	0.0027	3.0669

Table 2: Allen and Morishima elasticities of substitution estimated at the mean of the data.

Elasticity of Substitution of for	Land	Labor	Capital	Energy	Materials
Land	-7.39832	1.51418[a] (1.49433)	1.11271 (1.42702)	-0.62959 (1.13489)	2.15137 (1.60117)
Labor	1 .51418 (1.42040)	-2.01338	0.71100 (1.09699)	1.88987 (1.57167)	1.34631 (1.35280)
Capital	1.11271 (0.60285)	0.71100 (0.56497)	-5.28092	3.20997 (0.80059)	-0.75772 (0.42648)
Energy	-0.62959 (1.32293)	1.88987 (0.94369)	3.20997 (1.59312)	-19.42929	1.54390 (1.47588)
Material	2.15137 (1.51016)	1.34631 (1.29681)	-0.75772 (0.73920)	1.54390 (1.34917)	-3.54699

[a] AES without parentheses, MES within parentheses

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