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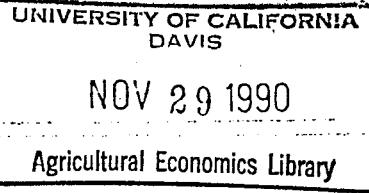
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Productivity - Mathematical Models

## International Comparisons of Agricultural Productivity Development and Usefulness



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The simultaneous developments in duality theory, in the use of flexible functional forms in economic research, and in the theoretical linkages between index numbers and production technologies have augmented the tools to address productivity measurement issues.<sup>1</sup> As a result there have been many studies of agricultural productivity growth in the U.S. and other countries in the past two decades; international comparisons, however, have received considerably less attention.<sup>2</sup> While from a methodological point of view there is no reason to distinguish intertemporal from international productivity comparisons, the latter types of comparisons involve additional data-related problems. First, reliable information from many countries may be difficult to obtain. Second, each country's data must be measured in the same units. A third issue is the need for comparability of input and output groups. These groups are often less comparable among countries than over time within one country.

In this paper we provide an overview and critique of methods for comparing agricultural productivity and efficiency among countries that integrate economic production theory with empirical practices and discuss the relationship of these measures to competitiveness. We conclude by relating the methodological needs to ongoing efforts to construct an internationally consistent data set.

### Framework for Productivity Analysis

Productivity research encompasses analysis of both changes in rates of growth and changes in levels. In theory, the distinction between growth rates and levels is trivial since one may be derived from the other. In practice, the uses have been quite different. Productivity growth is used as an indicator of how much more output can be obtained from

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a given set of resources by a given production unit, i.e., a firm, sector, or country, and is often used with time-series data. Productivity levels are often used to compare different production units at a point in time.

Measures of productivity are related to the production technology. They all involve a comparison of the output level produced relative to the inputs used. Output-based indexes measure productivity differences as differences in maximum output conditional on a given level of inputs; input-based indexes define productivity differences as differences in minimum input requirements conditional on a given level of output. Specific productivity indexes imply some assumptions about the underlying production technology.<sup>3</sup> Indexes that use only an incomplete list of inputs or measure output as value-added can be interpreted as implying restrictions on the technology in the latter case (see Capalbo and Denny, 1986) and implicit assumptions about the role of other inputs in the former case.

There are two basic approaches to productivity measurement: parametric or statistical procedures and accounting or index number methods. Parametric procedures rely on the estimation of the production technology. To illustrate, suppose the production function for each sector is known or can be estimated:  $y_i = f_i(k, l)$  where  $i$  indexes the sector,  $y$  denotes output, and  $k$  and  $l$  denote capital and labor, respectively. Select any input bundle  $(k_0, l_0)$  and calculate  $y_{i0} = f_i(k_0, l_0)$  for all countries. The sector in a particular country  $i$  will be more efficient than in country  $j$  if  $y_{i0} > y_{j0}$ . The proportional difference in output levels between any two sectors may be defined as the relative efficiency level and an index can be constructed by defining a base sector. In general, the comparative ranking is not independent of the choice of the input bundle  $(k_0, l_0)$ . This is desirable since it allows some countries to be more efficient than others for some input bundles and not for others. Otherwise, country  $i$ 's production function would need to lie everywhere above country  $j$ 's.

To use cost functions for comparing efficiency, a similar procedure is employed. Assuming that the cost functions,  $g_i(w_k, w_l, y)$  for each country are known, choose a

particular set of input prices,  $w$ , and output level,  $y$ , and calculate  $c_{io} = g_i(w_{k0}, w_{l0}, y)$  for each country. The lowest cost sector is the most efficient. The remarks made with respect to the nonuniqueness of the ranking to choice of the input bundle also pertain here to the choice of the input prices and output level.

If the data needed to estimate production or cost functions are available, making productivity comparisons utilizing the parametric methodology is straightforward. However, since all of the required information is rarely available, an alternative position for making productivity comparisons utilizes accounting methods which extract a measure of relative efficiency without knowing the complete production technology. The economic rationale for using the accounting approach to measure productivity differences among countries or sectors is found in a series of papers by Caves, Christensen and Diewert (CCD) (1982a,b), and Denny and Fuss (DF) (1983a,b). These papers are, in part, a response to justify the methodology used by Jorgenson and Nishimizu (1978) to compare U.S. and Japanese aggregate productivity levels.

The 'preferred' accounting method assumes that the production technology can be approximated by a second-order function.<sup>4</sup> In particular, a second-order approximation in the logarithms of the outputs and inputs will be used in the case of the production function, and in the logarithms of the input prices, output levels and cost in the case of the cost function. The second order approximation assumption is utilized for several reasons: data limitations are likely to make higher order approximations infeasible, and more importantly, use of a quadratic function as the second order approximation allows direct linkage to the economic measures of productivity and efficiency. If the production function can be approximated by a quadratic function, then the use of Diewert's quadratic lemma implies that the difference in the logarithm of output between two sectors can be expressed exactly as a weighted sum of the differences in the logarithms of the inputs and

a term which can be interpreted as the difference in the productivity level between countries.

To illustrate, let the logarithm of the production function  $y_i = f_i(k, l)$  be approximated by a quadratic function in the logarithms of the input levels and a variable  $d$  which indexes the country,  $\log y_i = f(\log k_i, \log l_i, d)$ . This specification assumes that the production function in each country has some common elements, since  $f_i$  is replaced with  $f$ ; the presence of  $d$  allows the zero and first-order terms in the function to differ. To analyze the differences in the level of output in countries  $i$  and  $j$ , we can apply Diewert's quadratic lemma to obtain

$$\log y_i - \log y_j = \frac{1}{2} (f'_d + f'_d) (d^i - d^j) + \frac{1}{2} (f'_k + f'_k) (\log k_i - \log k_j) + \frac{1}{2} (f'_l + f'_l) (\log l_i - \log l_j) \quad (1)$$

where  $f'_z$  is the partial derivative of the production function with respect to the  $z^{\text{th}}$  argument evaluated at the  $i^{\text{th}}$  country's input vector.<sup>5</sup> The output differential in (1) is due to a spatial (country) effect, and an input effect.

Equation (1) states that we can exactly evaluate the difference in the value of the production function if we know only the first-order derivatives and the data point. Using the assumptions of constant returns to scale and competitive markets, the first order derivatives are the shares of the inputs in total cost and (1) can be rewritten as

$$\log y_i - \log y_j = \theta_{ij} + \frac{1}{2} [s_k^i - s_k^j] [\log k_i - \log k_j] + \frac{1}{2} [s_l^i - s_l^j] [\log l_i - \log l_j] \quad (2)$$

where  $s_h^i$  is the cost share of input  $h$  in country  $i$ ; and  $\theta_{ij}$  denotes the spatial effect.<sup>6</sup> Solving (2) for  $\theta_{ij}$  yields an equation which can be used to measure the differences in productivity levels. The expression  $\theta_{ij}$ , which is the Tornqvist approximation to the Divisia index, can be interpreted as the output level in country  $i$  relative to that in country  $j$  after accounting for differences in the levels of inputs used by the two sectors. It is, in essence, the geometric average of two unobservable measures: how much output sector  $j$  would produce relative to sector  $i$  at  $j$ 's observed input levels and at sector  $i$ 's observed input levels.

Using duality results,  $\theta_{ij}$  can also be interpreted in an approximate sense as a measure of relative cost efficiency. The methodology parallels the previous discussion for the production function. Let the cost function  $C_i = g_i(w_k, w_l, y)$  be approximated by a quadratic function in the logarithms of input prices and output, and d,  $\log C_i = g(\log w_k^i, \log w_l^i, y_i, d)$ . Applying Diewert's quadratic lemma we obtain an expression similar to (1), which indicates that the cost differential can be broken down into a spatial effect, an input price effect, and an output effect. Constant returns to scale and perfect competition imply that the cost differential can be expressed as

$$\log C_i - \log C_j = \rho_{ij} - (\log y_i - \log y_j) - \frac{1}{2}(s_k^i + s_k^j)(\log w_k^i - \log w_k^j) - \frac{1}{2}(s_l^i + s_l^j)(\log w_l^i - \log w_l^j) \quad (3)$$

where  $s$  denotes cost shares, and  $\rho_{ij}$  is defined as the cost efficiency differences between  $i$  and  $j$ .<sup>7</sup> Using (3), the Tornqvist approximation to the Divisia index of cost efficiency between countries  $i$  and  $j$  is given by

$$\rho_{ij} = (\log C_i - \log C_j) - (\log y_i - \log y_j) - \frac{1}{2}(s_k^i + s_k^j)(\log w_k^i - \log w_k^j) - \frac{1}{2}(s_l^i + s_l^j)(\log w_l^i - \log w_l^j) \quad (4)$$

Using (4) and (2), DF show that the Tornqvist approximations to the index of relative productivity differences and to the index of cost efficiency differences are equal (in absolute value) except for a term which is of "second-order of smallness," i.e.,  $\rho_{ij} \approx -\theta_{ij}$ .<sup>8</sup>

The CCD papers convey a similar message: under constant returns to scale, the geometric mean of two Malmquist productivity indexes is equivalent to the Tornqvist productivity index, which can be computed using information on prices and quantities only.<sup>9</sup> Furthermore, under decreasing returns to scale one can use observed data on cost and revenue to adjust the Tornqvist productivity index to reflect the scale effect (see Theorems 3 and 4 in CCD, 1982a).

### Limitations and Usefulness

Our comments in this section pertain primarily to the index number approach to productivity level comparisons. In assessing the merits, one is reminded that the approach is designed to approximate the spatial and/or time derivatives of the production technology using only data on prices and quantities, and thus some limitations should be expected. The limitations stem from the underlying assumptions. Constant returns to scale is a convenient although not a necessary assumption. It is not possible, however, to relax the assumptions about competitive behavior, since these are required to eliminate the first-order derivatives. Furthermore, it is not possible in the quadratic framework to find a function other than the translog that will allow elimination of the derivatives using only data on prices and quantities; while this is restrictive in theory it is probably irrelevant in practice. The approach also relies on the sectors having production or cost functions that differ in their zero- and first-order parameters but not in their second-order parameters.

How serious are these limitations? Clearly no index number scheme will allow production structures to be completely different. Country-specific zero- and first-order parameters allow the intercept and slope of the production technology to vary, but not the rates of change of the slope. It also implies that the output compensated factor demand elasticities will be equivalent if prices and quantities are the same, or alternatively, the slopes of the factor demand functions are equal across countries.

The assumption of competitive markets is more complicated to evaluate because the issues are both conceptual and empirical. Conceptually, the presence of government policies are distorting the productivity measures if they are resulting in inefficiencies in production. If the producers are simply operating at a different location on the production possibility surface then there is no inefficiency in resource use, although there are likely to be social welfare implications. If the government programs constrain producers to be inside the frontier then

inefficiencies in resource use are introduced. A frontier production model would be one way to quantify these inefficiencies, and possibly explain productivity differences.

Two possible solutions to obtaining 'distortion-free prices' include calculating implicit and explicit subsidies per commodity and adjusting observed prices accordingly, or econometrically estimating the slope of the production technology and use the estimated marginal cost functions to obtain output prices. The latter, of course, hinges on having enough data to estimate the marginal cost or marginal value product functions. The first suggestion may be more reasonable given recent research on quantifying subsidies.

Another issue involves bilateral versus multilateral comparisons. The DF methodology discussed in the previous section is directly applicable to bilateral comparisons. By computing  $\theta_{ij}$  or  $\rho_{ij}$  for each pair of countries, one has information on the magnitude and rank for each region relative to the others. If one is interested in the specific magnitudes of the relative efficiency levels, then direct bilateral comparisons are needed. However, the bilateral comparisons are in theory not transitive; one cannot derive the bilateral comparison of countries  $i$  and  $k$  from information on the bilateral comparisons of these two countries with a third country  $j$ .

CCD compare productivity levels to a hypothetical country or sector and as such their approach is base-country invariant as long as the composition and/or number of countries is fixed. The multilateral comparisons, while eliminating the intransitivities of the comparisons, obscure the meaning of the magnitude of the productivity or efficiency differential.<sup>10</sup> If one is interested in summarizing a large number of possible comparisons, and the actual direct comparisons are of little interest then the CCD approach may be preferred. If the comparisons are motivated by a particular country's interests, or if the number of sectors is small, then the DF approach is in our view preferred since the benchmark is an actual country, and the efficiency differential is easy to interpret cardinally.

Both  $\theta_{ij}$  and  $\rho_{ij}$  are useful as components in understanding the larger notion of competitiveness. More specifically, a gap exists, both conceptually and empirically, between a country's competitiveness at the level of delivered goods and its costs of production, or cost efficiency, at the farm gate. This gap is important in agricultural comparisons because of the presence of government programs and distortions. The importance of the productivity measures as components of competitiveness also depends upon the time perspective. In the short run, exchange rates and trade policies are also major determinants of a country's competitiveness; in the long run, productivity and cost efficiency may become more dominant factors. Holding exchange rates, trade policies, transportation costs, etc., constant, higher efficiency means a country will be more competitive. Under these conditions,  $\theta_{ij}$  and  $\rho_{ij}$  are useful for assessing a country's overall efficiency level vis-a-vis its specific competitors.

Finally, the productivity measures and in particular, the econometric measures, may provide a preferred set of information for the competitiveness discussions, since they reflect the economic tradeoffs and the diversity in agricultural production. By contrast, costs of production measures are less reflective of real world situations and can be misinterpreted in the sense of lending "false precision" to the cost differences among sectors.

#### Data Development

The USDA is currently involved in efforts to create a consistent data base for the U.S., Canada, U.K., France, FRG, and Netherlands. The objective is to provide international data on the prices and quantities of inputs and outputs which will be useful for more than simply productivity analysis. While a detailed discussion of these efforts is beyond this paper, we briefly highlight some aspects of the ongoing efforts.

The accounting framework is modeled along the guidelines of the United Nations "System of National Accounts." This system is designed to give a systematic and comparable presentation of the activities of each agricultural sector. The agricultural accounts are based

on the concept of a "national farm" which represents a single agricultural holding producing the total output of agricultural products of a country's economy. Output is defined as the quantity sold plus stock changes and own-account fixed capital formation. Stocks consist of goods in progress (e.g., immature animals) and finished products from own production.

Gross capital formation is divided into two categories: (1) gross fixed capital formation; and (2) changes in stocks. (1) includes non-residential structures, other construction, vehicles and other machinery and equipment. Estimates of capital stock are constructed as weighted sums of all past investment where the weights are the asset's efficiency as of a given age. The efficiency function is approximated by a rectangular hyperbola which incorporates several types of physical depreciation (e.g., straight line, geometric, etc.) as special cases.

Since the efficiency function is applied to a broad type of assets, a distribution of service lives is assumed. This is done by constructing a "cohort" efficiency function which is a weighted average of efficiency functions calculated using various asset ages. The weights are determined by a discard density function. The capital stock estimates reflect a concave physical depreciation pattern with a truncated normal distribution of retirement ages.

Own-account capital formation is not recorded in gross capital expenditure. However, the number of animals added to the fixed capital stock can be derived. These animals are valued at their opportunity costs if prices for replacement animals are not reported. Data have been compiled on the effects of age on productivity, culling rates which allow construction of cumulative survival functions, and genetic trends in certain animal traits such as milk yield, weaning weight, fleece weight, and litter size. The genetic trend measures embodied technical change. This information allows us to construct measures of relative efficiency which are, in turn, used to aggregate past investment.

The stocks of goods held in storage exclude commodities held by intervention authorities such as marketing boards or government held stocks. Animals included as fixed capital are also excluded. The stock estimates are for the beginning of the reference period.

The stock of land is an implicit quantity index of land in farms. It is assumed that farmland within a state (region) is homogeneous in quality, hence aggregation is at the state (regional) level. The measure of labor input reflects the changing composition of the farm work force. Data on hours worked and wage rates are cross-classified by occupational and demographic characteristics. Self-employed farmers are imputed the mean wage of hired workers with the same occupational and demographic characteristics.

### Conclusions

In understanding international comparisons and how they relate to a country's competitiveness, both rates of growth and differences in productivity levels are important. Econometric and index number methods are applicable for making international comparisons. The econometric approach is based on estimation of the parameters of the production technology. The economic approach to index numbers generates formulas that are easily computed using price and quantity data and, most importantly, the indexes "mesh" with economic production theory. The index number approach reviewed in this paper is based on the assumption of optimizing behavior on the part of producers; constant returns to scale is often assumed (although this is not necessary).

This paper has focused on the economic approach to index number measures of productivity because we believe these measures are capable of being implemented in the near future and do not require extensive time-series on outputs and inputs. While the approach provides a relatively easy way of ordering countries by their productivity levels, it should not be used to replace econometric or other methods of estimation in providing a detailed understanding of relative productivity levels.

## Footnotes

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1. Antle and Capalbo review topics in modern production theory that relate to the productivity literature.
2. Capalbo and Vo provide a review of intertemporal productivity rates for the U.S.; Nadiri and Kravis have produced surveys of international comparisons of productivity; Hoque et al. have synthesized the empirical research on international agricultural productivity. Other research includes the FAO report and papers by Behrens and DeHaen, Hayami and Ruttan, Yamada and Ruttan, and Nguyen.
3. The Laspeyres indexing procedure implies a Leontief production function; the geometric index implies a Cobb-Douglas production technology; and the Tornqvist index implies a homogenous translog production function.
4. An alternative accounting method is the axiomatic approach to index numbers (see Diewert, 1986).
5. Denny and Fuss (1983c) provide for a generalization of the quadratic lemma to the case of discrete variables.
6. If nonquadratic approximations are used, then the replacement for (1) will involve terms which include second order derivatives of the production function. These second order derivatives are related to the curvature of the production function and

consequently to the price elasticities of factor demand. Unless one knows the elasticities it will not be possible to use accounting methods to evaluate (1).

$$7. \rho_{ij} = \frac{1}{2} \left( \left. \frac{\partial g}{\partial d_i} \right|_{i=i} + \left. \frac{\partial g}{\partial d_i} \right|_{i=j} \right) (d_i - d_j)$$

8. The  $\theta_{ij}$  and  $\rho_{ij}$  can also be defined for the multiple output case using the further assumption of perfect competition in the output markets (see DF 1980).

9. The country  $i$  Malmquist productivity index can be defined as

$$(a) m^i (y^i, y^i, x^i, x^i) \equiv d^i (y^i, x^i) / d^i (y^i, x^i)$$

where  $y^i$  and  $x^i$  are country  $i$ 's observed output and input vectors, and  $d^i$  is country  $i$ 's output distance or deflation function. Since  $d^i (y^i, x^i) \equiv 1$ , the right hand side of (a) is interpreted as the maximum inflation factor for the country  $j$  output vector such that the resulting inflated output vector,  $m^i y^i$ , and  $x^i$  are on the production surface of country  $i$ . If country  $i$  has a lower productivity than country  $j$ , from the perspective of country  $i$ 's production structure, then  $m^i < 1$  and vice versa.

10. As DF indicate, intransitivities arise because of "cross-overs": at certain data points country  $j$  may be more efficient than country  $i$  and the reverse at other data points.

The existence of intransitivities can be of interest in and of themselves.

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