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# Is Anonymity the Missing Link Between Commercial and Industrial Revolution?\*

Stephen Broadberry<sup>†</sup>, Sayantan Ghosal and Eugenio Proto<sup>‡</sup>

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## Abstract

The Industrial Revolution is often characterized as the culmination of a process of commercialisation; however, the precise nature of such a link remains unclear. This paper models and analyses one such link: the impact of a higher degree of anonymity of market transactions on relative factor prices. Commercialisation raises wages as impersonal labour market transactions replace personalized customary relations. This leads, in equilibrium, to higher real wages to prevent shirking. To the extent that capital and labor are (imperfect) substitutes, the resulting shift in relative factor prices leads to the adoption of a more capital-intensive production technology which, in turn, results in a faster rate of technological progress via enhanced learning by doing. We provide evidence using European historical data that England was among the most urbanized and the highest wage countries at the onset of the industrial revolution.

**JEL classification:** N13, O14, O43

**Keywords:** *Commercialisation, Industrial Revolution, Anonymity, Efficiency Wages, Learning by Doing.*

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# 1 Introduction

A number of recent studies have pointed to the emergence of northwest Europe as a high wage economy during the early modern period, between the sixteenth and eighteenth centuries, with Britain overtaking the Netherlands to become the highest wage economy in Europe (van Zanden, 1999; Allen, 2001; Broadberry and Gupta, 2006), at the same time Britain was among the most urbanized countries in the world, in the first half of eighteenth century, at the onset of the industrial revolution (Malanima 2009).

Since one of the key features of the Industrial Revolution was the development of labour saving technology in Britain, it is natural to link the Industrial Revolution to these prior developments in factor prices and the global commercial environment in which they emerged (Broadberry and Gupta, 2009; Allen, 2009). Indeed, a long tradition in economic history links the transition to modern economic growth to the widespread commercialisation of Britain and other parts of northwest Europe between the late medieval period and the Industrial Revolution (Toynbee, 1890; Polanyi, 1944; Britnell and Campbell, 1995). However the precise nature of the links between the Commercial Revolution and the Industrial Revolution has remained unclear.

The definition of commercialization used in this paper means more than simply an increase in the proportion of output passing through the market (Britnell and Campbell, 1995: 1). Commercial development is characterized by a dramatic change in the informational structure among economic agents – as already emphasized in the theoretical literature by Banerjee and Newman (1998). In particular in this paper we are interested in the growing reliance on anonymous, impersonal relations, as against personalised customary relations, in labour markets following the commercial revolution. Our starting point is Greif (1994) who established an endogenous link between anonymous markets, trading relationships and prices taking as given the level of technology. The contribution of our paper is to establish the link between the degree of anonymity in labour markets and both capital accumulation and the evolution of technology, via the effect of relative prices.

In this paper we consider a model of endogenous growth via learning by doing with a labor market characterized by incomplete information, where agents' incentives to supply effort are endogenously determined by the prevailing degree of anonymity and wages. We distinguish between the probability that shirking is detected (as in a model of efficiency wages (Shapiro and Stiglitz's (1984))) and the probability that a shirking worker who

has been fired will be hired again in the future (along the lines of Greif (1994)). Our model thus combines elements of efficiency wages, learning-by-doing and endogenous growth (Hicks, 1932; Arrow, 1962; Romer, 1986) and the institutional enforcement of labour market discipline based on Greif (1994). At the same time, our model is able to deliver a simple closed form solution under the assumption of perfect factor substitutability. Nevertheless, we show the key results to be robust to the relaxation of this assumption with the more realistic assumption of imperfect substitutability.

Our analysis shows, first, that the growing commercialisation of the late medieval and early modern periods led to higher wages as the greater anonymity of market relations replaced customary relations in the allocation of labour. Note that this “efficiency wage” argument avoids the objection sometimes levelled at the literature on induced innovation that high wages do not reflect high labour costs because the labour is also highly productive. Efficiency wages imply higher wages in anonymous commercialized factor markets to induce the same effort as achieved in more personalized customary relationships backed up by close supervision. Second, we show that the resulting rise in the wage/cost of capital ratio led to the adoption of a more capital-intensive technology. Third, this led to a faster rate of technological progress through greater learning by doing on the capital intensive production technology.

Viewed from the perspective of Greif (1994), commercialisation can be interpreted as a switch in the “rules of the game” due to institutional or cultural changes, portrayed in Greif by the distinction between Genoese and Maghribi traders. However, it is worth noting that our model is consistent with a more continuous notion of commercialization that entails a more gradual process of unification of distinct markets, perhaps due to decreasing costs of trade, that dilutes the bonds of customary labour market relationships between employers and workers.

The paper proceeds as follows. In section 2, we present a literature review. Section 3 presents a theoretical model to establish the links between commercialisation and technological progress. We first present the model under the assumption that capital and labor are perfect substitutes; we then show that all results holds more generally when production factors are imperfect substitutes. Section 4 then provides an analytic narrative and a simple econometric exercise, using data relative to the historical transition to modern economic growth in northwest Europe, in which the Commercial Revolution of the early

modern period is linked to the Industrial Revolution via its effects on factor prices. Section 5 concludes. Proofs are contained in the Appendix.

## 2 Related literature

The approach taken here draws on ideas which have been used in the literature on the importance of high wages in stimulating the innovations of the Second Industrial Revolution in late nineteenth century America (Rothbarth, 1946; Habakkuk, 1962; David, 1975; Broadberry, 1997).<sup>1</sup> Until recently, there has been a reluctance to cast Britain in the role of a high wage producer at the time of the Industrial Revolution, since the vast literature on the standard of living debate emphasized the slowness of real wages to rise. However, recent work has emphasized international comparisons of the level of real wages and other factor prices, pointing clearly to Britain's unusual combination of factor prices (Allen, 2001; 2009; Broadberry and Gupta, 2006; 2009). This is important not only in explaining the adoption of modern technology, but also its non-adoption in other countries with different factor prices, a point emphasized in the theoretical literature by Zeira (1998) and in the historical literature by Broadberry and Gupta (2009), Allen, (2009) and Fremdling (2000).

The current paper suggests a link between the two main strands of the macro-development literature: the institutional approach (e.g. North, 1990; Greif, 2004; 2006; Acemoglu and Robinson, 2006) and unified growth theory (e.g. Galor and Weil, 1999; 2000; Galor and Moav, 2000; Hansen and Prescott 2002; Doepke, 2004; Galor, 2005; Cervellati and Sunde, 2005, Madsen, Ang and Banerjee, 2010). The institutional approach emphasizes the importance of trade and commercial development, which are supported by an appropriate institutional framework, while unified growth theory links technological progress to wages via an emphasis on human capital, with families choosing to have fewer but more highly educated children as technology becomes more complex. Our approach sees the two theoretical frameworks as complementary: establishing the right institutional framework to encourage the development of trade leads to higher wages, which then has the effects on technology traced out in unified growth models. Of course this does not preclude the operation of the Malthusian links between wages and population stressed in unified growth

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<sup>1</sup>See Acemoglu (2009) for a formal treatment of the link between labor scarcity and the rate of technological progress.

theory.

Furthermore, within the institutional approach, our paper provides a link between commercial development and economic growth that is complementary to the links suggested by Acemoglu et. al. (2005) and by Galor and Mountford (2006). Acemoglu et. al. focus on the impact of Atlantic trade on institutions, with growing trade strengthening the position of merchants in northwest Europe and enabling them to impose effective constraints on the executive, hence less extractive institutions. Galor and Mountford (2006) show that trade might have generated a demographic boom in non industrial countries, specialized in the production of unskilled-labor-intensive goods, hence fostering the process of divergence. Our approach focuses on an alternative link between trade and growth, with increasing commercialisation affecting factor prices, choice of technology and the rate of technological progress.

Our setting is also consistent with the idea that capital was an important engine of growth along the process of development (Galor and Moav 2004), since we emphasize the effect of high wages on the capital intensity of production and the subsequent rate of technological progress. Along similar lines Voigtländer and Voth (2010) emphasize the importance of the effect of growing capital inputs on TFP increases for the first phase of industrialization.<sup>2</sup>

### **3 The model**

In this section, we introduce our concept of anonymity and endogenous technological progress via learning by doing. We show that a higher degree of anonymity, arguably generated by the commercial revolution, made more difficult monitoring workers and led to an increase in wages and subsequently to more capital intensive production. This process eventually led to a technological increase in labour efficiency that we characterize as the industrial revolution.

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<sup>2</sup>And it is not inconsistent with the contributions investigating the importance of the Agrarian sector on the economic divergence like Galor, Moav and Vollrath (2004) and Vollrath (2011).

### 3.1 Workers

Time periods are indexed by  $t$ ,  $t = 0, 1, 2, \dots$ . There is a mass  $N$  of identical risk averse workers. There is a probability  $d$  that at each time  $t$ , the worker dies or permanently retires from working. Since the number of workers is fixed at each period, there are  $dN$  new workers in the economy so that the labor supply is always constant. Workers have an inter temporal discount factor which, for notational simplicity, we multiply by the probability of surviving next period,  $(1 - d)$ , and define the resulting product as  $\beta < 1$ .

At any period  $t$ , each worker can be either employed or unemployed and is endowed with a fixed amount of effort that can be costlessly provided. If she is unemployed she uses her effort in a backyard informal activity, which yields  $\mu A_t$ , where  $A_t$  is a technological parameter, linked to the general economic environment at any time  $t$ , which we will characterize later; if she is employed she earns a wage  $w_t$ .

Since effort cannot be observed, employed workers can either shirk or work (i.e. choose an effort level  $e \in \{0, 1\}$ ). An employed shirking worker uses her effort for the backyard activity earning  $\mu A_t$  in addition to the wage offered by the employer. She can be detected with probability  $1 - p$  and fired. In this case, a shirking worker who has been fired can look for a job in the next period by "hiding" among the pool of new workers  $dN$  and her probability of finding a new job is  $q\sigma$ , where (i)  $q$  (which is endogenous and will be determined later) is the probability, common to all individuals in the unemployment pool, of being hired and (ii)  $\sigma \in (0, 1)$  is a parameter, the probability of being detected by a new employer as having shirked in the past and accounts for the level of anonymity in the economy and is known by both workers and employers.

We interpret  $\sigma$  as the probability that the news that a worker has shirked in the past when employed reached a new employer. We assume that employers are committed, perhaps by customary rules, not to hire the workers if they can observe the signal, and this behavior is common knowledge among all workers.

The parameter  $\sigma$  measures the degree of anonymity in labour markets and we interpret  $\sigma$  as an index of commercialization of the economy, and consider a increase of  $\sigma$  as growing commercialization.<sup>3</sup>

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<sup>3</sup>Here, we assume that individuals who receive the signal about a shirking worker will not hire him. Grief shows that this strategy can be a sequential equilibrium when all employers shared information with each other about the performance of past employed workers do not hire a currently unemployed worker



A non-shirking worker will work in the firm until termination (which happens with probability  $d$  at each  $t$ ). We note that  $p + (1 - p)\sigma q$  is the probability that a shirking worker at time  $t$ , will still be alive at time  $t + 1$ . We define  $V_t^E(e)$  as the intertemporal utility of an employed worker that exercises effort  $e \in \{0, 1\}$  at time  $t$ .

We will now write down the conditions required to ensure that at the prevailing wages at time period  $t$ , choosing high effort  $e = 1$  is optimal for each employed worker. To this end, by the one-shot deviation principle (Blackwell (1965)), it is sufficient to show that no employed worker can gain by deviating and choosing low effort  $e = 0$  for one period at any  $t$ .

Fix a sequence of market wages  $\{w_t : t \geq 0\}$ .

The intertemporal utility for an employed non shirking worker is

$$V_t^E(1) = w_t + \beta V_{t+1}^E(1), \quad (1)$$

and we have the following expected discounted utilities for an employed worker who shirks once but does not shirk again in the future:

$$V_t^E(0) = w_t + \mu A_t + \beta((p + (1 - p)\sigma q)V_{t+1}^E(1) + (1 - (p + (1 - p)\sigma q))V_{t+1}^{US}), \quad (2)$$

where  $V_t^{US}$  is the intertemporal utility of an unemployed worker who has shirked at least once in the past but does not shirk again if employed in the future i.e.

$$V_t^{US} = \mu A_t + \beta(q\sigma V_{t+1}^E(1) + (1 - q\sigma)V_{t+1}^{US}). \quad (3)$$

Therefore, given the sequence of market wages, the no shirking constraint is met whenever:

$$V_t^E(1) \geq V_t^E(0). \quad (4)$$

We assume that at each  $t$ , each worker correctly anticipates future levels of  $V_t^E(e)$ ,  $e \in \{0, 1\}$

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who has shirked when employed in the past so that  $\sigma = 1$  (this scenario corresponds to the organization of Maghrebi traders described and analyzed as an equilibrium scenario by Grief (1994)). If no employer shares information with any other employer about the performance of past employed workers, then  $\sigma = 0$  (this scenario corresponds to the organization of Genoese traders described and analyzed as an equilibrium scenario by Grief (1994)). We take  $\sigma$  as exogenous and treat as a continuous variable although we note that we can adapt Grief (1994) to our setting to generate the extreme values,  $\sigma = 1$  and  $\sigma = 0$ , as equilibrium scenarios.

and  $V_t^{US}$ .

### 3.2 Production and firms

There is a fixed mass of one identical firms, indexed by  $i$ . We will assume that each firm has an increasing, strictly concave production function with Harrod-neutral (or labour augmenting) technological progress  $F(k, A_t l)$ .

To begin with, we assume perfect substitution between factors:

$$F(k, A_t l) = (\theta k + (1 - \theta)A_t l)^\alpha, \alpha < 1 \quad (5)$$

We will return to the general case (allowing for imperfect substitutability between the factors of production) in the next subsection.

We assume that  $A_t$  evolves over time according to

$$A_t = (1 - \rho)A_{t-1} + \rho a(K_{t-1}), 0 < \rho < 1. \quad (6)$$

The interpretation is that the prevailing technology in any period  $t$  is a weighted average of the technology prevailing in the preceding period and any new knowledge created in that period. We assume that the new knowledge created in the preceding period is an increasing function of the aggregate capital stock  $K_{t-1}$  at  $t - 1$ . This assumption can be interpreted as productivity growth through learning by doing (e.g. Arrow (1962) and Romer (1986)), specifically the stock of knowledge increases with the amount of capital used within each firm i.e.  $a(0) = 0$  and  $a'(K_{t-1}) > 0$ .

Firms borrow capital from an external capital market at an exogenously given interest rate  $r$ , the capital supply is perfectly elastic and, in equilibrium, make non-zero profits, given the assumption  $\alpha < 1$ , which implies decreasing return to scale on capital and labor factors. Therefore, profits of the firm can be interpreted as a return to a fixed factor of production namely entrepreneurship.

All firms are price-takers. At each  $t$ , each firm  $i$  takes the interest rate  $r$  and the technological parameter  $A_t$  as given. Although firms' choices at time  $t - 1$  influence the technology at time  $t$ , we make the standard assumption that the contribution of each firm is negligible and it is not internalized when the decision takes place: in effect, maximizing the sum of profits over time is equivalent to maximizing current period profits within each

time period. Therefore, at each  $t$ , each firm maximizes current period profits only i.e.

$$\max_{w_{i,t}, k_{i,t}, l_{i,t}} F(k, A_t l) - w_{i,t} l_{i,t} - r k_{i,t} \quad (7)$$

### 3.3 Market equilibria and steady state

We define a market equilibrium for a fixed  $\sigma$  as follows:

A market equilibrium is a sequence of  $(K_t^*, L_t^*, w_t^*, A_t^* : t \geq 1)$  such that at each  $t = 0, 1 \dots :$

1. Given  $r$  and  $A_t^*$ , for each firms  $w_{i,t} = w_t^*$ ,  $l_{i,t} = L_t^*$ ,  $k_{i,t} = K_t^*$  maximizes profits,
2. No employed worker shirks i.e.  $w_t^*$  satisfies the no shirking constraint(4),
3.  $A_t^* = A_{t-1}^* + a(K_{t-1}^*)$ .

At a steady state  $K_t = K_{t+1} = K^*$ ,  $L_t = L_{t+1} = L^*$  and  $A_t = A_{t+1} = A^*$  for all  $t$ . From (6), it follows that  $A^* = a(K^*)$ . Therefore, the steady state (long-run) values of the variables at a market equilibrium are denoted by  $(K^*, L^*, w^*, A^* = a(K^*))$ .

Next, we characterize the market equilibrium in our model first for the case of perfect substitutes and then for the general case with imperfect substitutes.

### 3.4 Market equilibrium with perfect substitutes

Consider, first, the profit maximizing problem for each firm (7) with

$$F(k, A_t l) = (\theta k + (1 - \theta) A_t l)^\alpha, \alpha < 1.$$

First, note that in equilibrium, for each firm, at each  $t$ , at an interior solution the first order conditions with respect to capital and labour  $k_{i,t}$  and  $l_{i,t}$  are:

$$(1 - \theta) A_t \alpha (\theta k_{i,t} + (1 - \theta) A_t l_{i,t})^{\alpha-1} = w_t^* \quad (8)$$

$$\theta \alpha (\theta k_{i,t} + (1 - \theta) A_t l_{i,t})^{\alpha-1} = r \quad (9)$$

we can rewrite (8) as:

$$k_{i,t} = \frac{-(1 - \theta) A_t l_{i,t} + \left(\frac{r}{\theta \alpha}\right)^{\frac{1}{1-\alpha}}}{\theta} \quad (10)$$

and substituting it in (9) we obtain simply

$$w_t^* = A_t \left( \frac{1-\theta}{\theta} \right) r \quad (11)$$

i.e. at each  $t$ , at an interior solution, the factor price ratio  $\frac{w_t}{r}$  is equal to the constant (factors of production are perfect substitutes) slope of the isoquant  $A_t \left( \frac{1-\theta}{\theta} \right)$ .

Let  $\omega = \left( \frac{1-\theta}{\theta} \right) r$ , so that  $w_t^* = \omega A_t$  or equivalently,  $\omega = \frac{w_t^*}{A_t}$ . We interpret  $\omega$  as wages measured in efficiency units of labour. In the case with perfect substitutes, as we shall see below,  $\omega$  will remain constant over time but as  $A_t$  will evolve over time, real wages  $w_t^*$  will change over time.

With  $w_t^* = \omega A_t$ , we can decompose the value functions (1), (2) and (3) for each worker as follows:

$$V_t^E(1) = A_t v_t^E(1) \quad (12)$$

$$V_t^E(0) = A_t v_t^E(0) \quad (13)$$

$$V_t^{US} = A_t v_t^{US}. \quad (14)$$

where  $v_t^E(e)$  and  $v_t^{US}$  depend on  $\omega$ . Furthermore, we note that in equilibrium the no shirking constraint (4) must bind, therefore

$$v_t^E(1) = v_t^E(0). \quad (15)$$

It follows that solving recursively the system given by expressions (14), (13) and (12) for the steady state values of  $v_t^E(1)$ ,  $v_t^E(0)$  and  $v_t^{US}$  and using the equilibrium condition (15), we have

$$\omega = \frac{\mu(1 - \beta p(1 - q\sigma))}{\beta(1 - (p + (1 - p)q\sigma))}. \quad (16)$$

We note that for any given  $q$ ,  $\omega$  is increasing in  $\sigma$ - wages in efficiency units are higher the higher the level of anonymity. Now we can determine the equilibrium labor demand. In equilibrium nobody shirks, so the probability of finding a job for any non employed worker is  $q$ , and  $dL$  is number of new jobs in the economy, at the same time  $dN$  is the flow of new

employed workers, therefore the equation  $qdN = dL$  must hold. Let us then define

$$q = q(L) = \frac{L}{N}, \quad (17)$$

Using (16) and (17), we can then rewrite the no shirking constraint as

$$\omega(L, \sigma) = \frac{\mu(1 - \beta p(1 - q(L)\sigma))}{\beta(1 - (p + (1 - p)q(L)\sigma))}. \quad (18)$$

from which we note that, differently from the classical model with "efficiency wages", our equilibria are compatible with no unemployment  $N = L$ — i.e. it is possible that when  $q(L) = 1$ ,  $w < \infty$  for low values of  $\sigma$  i.e. when the degree of anonymity in the market isn't too high.

Moreover,

$$\lim_{L \rightarrow 0} \omega(L, \sigma) = \frac{\mu(1 - \beta p)}{\beta(1 - p)} \equiv \underline{\omega}.$$

independent of the value for  $\sigma$ .

Therefore, as long as

$$\frac{\mu(1 - \beta p)}{\beta(1 - p)} < \left( \frac{1 - \theta}{\theta} \right) r, \quad (19)$$

there is a positive level of steady state employment  $L$  for each value of  $\sigma$ . Further, as  $\mu(1 - \beta p(1 - q(L)\sigma))$  is increasing in both  $L$  and  $\sigma$  and  $\beta(1 - (p + (1 - p)q(L)\sigma))$  is decreasing in both  $L$  and  $\sigma$  so that the ratio  $\frac{\mu(1 - \beta p(1 - q(L)\sigma))}{\beta(1 - (p + (1 - p)q(L)\sigma))}$  is increasing in both  $L$  and  $\sigma$ , an increase in the value of  $\sigma$  must lower employment i.e. decrease  $L$ .

We depict the impact of a changing value of  $\sigma$  in Figure 1 below.

Figure 1 shows how (11) and (18) determine the equilibrium labor demand. The Non Shirking area is above the upward bending lines, AS and AS', determined by equation (18). For higher level of  $\sigma$  (i.e. higher market anonymity), the line rotates upward, going from AS to AS'. The straight line is the equilibrium wage, characterized by (11). At the equilibrium, the higher degree of anonymity  $\sigma'$  corresponds to a lower level of employment  $L'$ , as we can see in the figure comparing equilibrium  $E'$  with equilibrium  $E$ .

Intuitively, with an increase in anonymity, firms must offer higher wages for a given level of unemployment. However at a higher wage they would prefer to substitute all workers with capital: this is obviously not an equilibrium because with such a high level

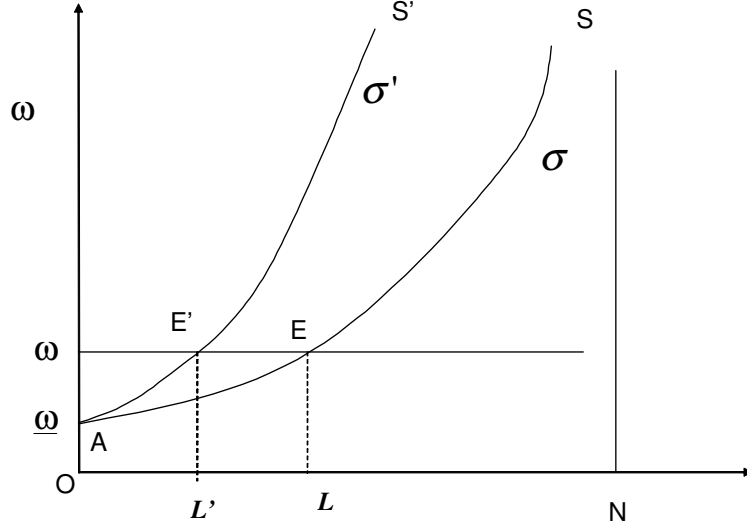


Figure 1: Labor market with perfect substitute production factors

of unemployment wages can go down so that firms can reemploy workers until the new level of unemployment is reached at a wage not higher than before. Therefore, with perfect substitutability, the only labor market effect of an increase in the degree of anonymity is a lowering of the level of employment. In the following section, where we relax the assumption of perfect substitutability, we show that the labor market will be also cleared by an increase in equilibrium wages. We will now analyze the effect of anonymity on capital allocation, which will have an impact on productivity in the long-run. Note also that although the wage in efficiency units  $\omega$  remains constant, real wages  $w_t$  will evolve over time as  $A_t$  changes over time.

Note from (18) that the equilibrium employment  $L$  is not dependent on  $A_t$  and  $K_t$ , so that we will write equilibrium employment as a function of the degree of anonymity alone i.e.  $L(\sigma)$ , with  $L'(\sigma) < 0$ . Using the demand for capital of a single firm, (10) and recalling that in equilibrium all firms are equal, so that  $K_t = k_{i,t}$  and  $L(\sigma) = l_{i,t}$  for all firms  $i$  at each  $t$ , aggregate capital at each time  $t$  is determined by the following equation:

$$K_t = -\frac{(1-\theta)}{\theta} A_t L(\sigma) + \frac{1}{\theta} \left( \frac{r}{\alpha\theta} \right)^{\frac{1}{1-\alpha}}. \quad (20)$$

Taken together with (6), (20) determines the evolution of capital and technology over time. The following proposition characterizes the dynamics for the case where capital and labour are perfect substitutes:

**Proposition 1** *Under condition (19), the relationship between anonymity, technology and capital in the long-run, when capital and labour are perfect substitutes, is given by the following:*

(i) *For each  $\sigma$ , there is a unique steady state with a positive capital stock  $K^* = K(\sigma)$ , a positive level of employment  $L^* = L(\sigma)$ , and technology  $A^* = a(K(\sigma)) > 0$ ;*

(ii) *The steady state capital stock  $K(\sigma)$ , technology  $A^* = a(K(\sigma))$  and real wages  $w^* = \omega a(K(\sigma))$  are all increasing in the degree of anonymity  $\sigma$ .*

(iii) *The steady state is locally a saddle and further, whenever  $a'(K^*) < \frac{\theta}{(1-\theta)L(\sigma)}$  the steady state is locally stable.*

**Proof:** See the appendix ■

The above proposition shows that there is a unique positive steady state value of the capital stock  $K^*$  corresponding to each value of  $\sigma$ . If the degree of anonymity increases to  $\sigma' > \sigma$ , what are the short-run and long-run effects? With  $k$  and  $l$  perfect substitutes, wages in efficiency units have to be constant and therefore, the labour market clears after a change in  $\sigma$  through a reduction in employment. In turn, a reduction in employment increases the demand for capital and therefore, results in a bigger capital stock, increased real wages and a higher level of technology in the long run. Further, as employment levels for a fixed  $\sigma$  do not depend on the current capital stock, if technology is not too sensitive to small change in capital stock, the steady state is locally stable.

In the following section, we relax the assumption of capital and labor being perfect substitutes and show that the main results will not change. The main difference is that, after an increase in  $\sigma$ , the labor market will perhaps more realistically clear through an increase of wages in efficiency unit,  $\omega$ .

### 3.5 Market equilibrium with imperfect substitutes

A key assumption that allowed explicit computation of the equations in the above analysis was that capital and labour were perfect substitutes in production. In this part of the

paper, we generalize the dynamics to the case where the factors of production are imperfect substitutes.

We will assume that the production function  $F(k, Al)$  is increasing in both its arguments, is strictly concave and smooth in  $k$  and  $l$ , that  $F''_{kl}(k, Al) < 0$  (i.e. the factors of production are (imperfect) substitutes), and  $\lim_{k \rightarrow 0} F_k = \lim_{l \rightarrow 0} F_l = \infty^4$ .

At each  $t$ , in equilibrium, the first order conditions characterizing profit maximizing input choices are:

$$F_k(k_t, A_t l_t) = r \quad (21)$$

$$A_t F_l(k_t, A_t l_t) = w_t^*. \quad (22)$$

Assume that at each  $t$ ,  $w_t^* = \omega_t A_t$  i.e. the market wage is linear in  $A_t$  so that, as before, we can decompose the value functions for each worker with  $V_t^E(1) = A_t v_t^E(1)$ ,  $V_t^E = A_t v_t^E(0)$  and  $V_t^U = A_t v_t^U$  where  $v_t^E(1)$ ,  $v_t^E(0)$  and  $v_t^U$  depend on  $\omega_t$ . The interpretation of  $\omega_t = \frac{w_t^*}{A_t}$  is the same as before (i.e. real wage expressed in efficiency units). Recalling that in equilibrium all firms are equal, so that  $K_t = k_{i,t}$  and  $L_t = l_{i,t}$  for all firms  $i$  at each  $t$ , we can determine the aggregate equations describing the steady state (where we have used (6)) as follows:

$$F_k(K^*, A(K^*)L^*) = r \quad (23)$$

$$F_l(K^*, A(K^*)L^*) = \omega_t. \quad (24)$$

with the no shirking constraint (4)

$$\omega^* = \omega(L^*, \sigma) = \frac{\mu(1 - \beta p(1 - q(L^*)\sigma))}{\beta(1 - (p + (1 - p)q(L^*)\sigma))}. \quad (25)$$

where  $q(L^*) = \frac{L^*}{N}$ .

What is the impact of a change in the degree of anonymity  $\sigma$  on the steady state values of capital stock, employment and wages?

By examining the impact of a higher degree of anonymity on the steady state capital

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<sup>4</sup>An example of a production function that satisfies all these assumptions is  $F(k, Al) = (\theta k^\alpha + (1 - \theta)(Al)^\alpha)^{\frac{1}{\beta}}$ ,  $0 < \alpha < 1$ ,  $0 < \beta < 1$ .



labour ratio and wages, the following proposition generalizes Proposition 1 to the case where capital and labour are imperfect substitutes in production:

**Proposition 2** *Suppose that the production function is concave and satisfies the assumption that capital and labour are imperfect substitutes i.e.  $F''_{kl}(k, Al) < 0$ . The relationship between anonymity, technological and the equilibrium capital dynamics when capital and labour are imperfect substitutes is given by the following:*

(i) *For each  $\sigma > 0$ , there is a unique steady state with positive capital stock  $K^* = K(\sigma)$  and employment level  $L^* = L(\sigma)$ ;*

(ii) *The steady state stock capital labour ratio  $\frac{K^*}{L^*} = \frac{K(\sigma)}{L(\sigma)}$ , technology  $A^* = a(K(\sigma))$  and real wages  $w^* = \omega(L(\sigma))a(K(\sigma))$  are all increasing in the degree of anonymity  $\sigma$ ;*

(iii) *Assume that in the vicinity of the steady state, at each  $t$ , agents expect that future employment and wage levels will be the same as current employment levels. For each  $\sigma > 0$ , the steady state is locally a saddle. Further, there exists  $0 < \bar{\beta} < 1$  such that whenever  $\bar{\beta} < \beta < 1$  the steady state is locally stable.*

**Proof:** See the appendix. ■

The above proposition shows that there is a unique positive steady state value of the capital stock  $K^*$  corresponding to each value of  $\sigma$ . If the degree of anonymity increases to  $\sigma' > \sigma$ , what are the short-run and long-run effects?

Note that in the previous section, with  $k$  and  $l$  perfect substitutes, wages in efficiency units have to be constant and therefore, the labour market clears after a change in  $\sigma$  through a change in employment. Starting from the steady state capital stock and employment corresponding to  $\sigma$ , with imperfect substitutes in the model, a change in  $\sigma$  results in a change in (real) wages in the short-term i.e. in a change in  $\omega_t$  (as always  $A_t$  is fixed at  $t$  and will change from period  $t + 1$ ). In response to an increase in  $\omega_t$ , no firm will substitute labor with capital entirely given that the marginal productivity of capital will decrease, and the marginal productivity of labour will increase, as more capital is employed. Therefore, when capital and labour are imperfect substitutes in production, wages in efficiency units are no longer held constant and both wages in efficiency units and employment will adjust to clear the labor market.

In the long-run, an increase in the anonymity of the labor market results in a shift to a more capital intensive production and higher wages in efficiency units and via technological

progress (driven by learning by doing), the steady state capital and real wages associated with a higher level of anonymity (and hence, technology) is also an increasing function of  $\sigma$ .

In order to examine the local stability of the steady state, we assume that in the vicinity of the steady state, at each  $t$ , agents expect that future employment and wage levels will be the same as current employment levels. Under this assumption, when workers are patient enough, current wages in efficiency units aren't too sensitive to small changes in current labour market conditions. Therefore, small changes in current labour market conditions do not result in large changes in relative factor prices and therefore, capital stock and technology. Therefore, when workers are patient enough, the steady state is locally stable.

## **4 Historical evidence: the transition to modern economic growth in Northwest Europe, 1300-1850**

We now examine the transition to modern economic growth, combining historical evidence with the theoretical model presented in the previous section. We argue that the Commercial Revolution of the early modern period was an important staging post on the road to the Industrial Revolution because of the effects of growing commercialisation on factor prices. Increasing anonymity due to growing commercialisation led to an increase in the price of labour relative to the price of capital, which induced a substitution into a more capital intensive technology and an acceleration of technological progress through learning by doing. We argue further that the fact that commercialisation went further in Britain than in the rest of Europe helps to explain why the Industrial Revolution occurred first in Britain.

### **4.1 Growing commercialisation**

The growing commercialisation of the European economy can most easily be captured quantitatively in the share of the population living in urban areas, since towns were the centres of commerce. Table 1 provides data on the share of the population living in towns of at least 10,000 inhabitants. For Europe as a whole, the trend is unmistakably upwards from 1400. Looking at regional trends, however, urbanization shows a pattern of divergence within Europe. In the late medieval period, there were two main urban centres of commerce

in north Italy and in the Low Countries. While urbanization stalled in north Italy after 1500, there was a brief surge in Portugal and to a lesser extent Spain during the sixteenth century, following the opening up of the new trade routes to Asia and the New World. However, the most dramatic growth of urbanization in the early modern period occurred in the Netherlands in the sixteenth and seventeenth centuries and in England during the seventeenth and eighteenth centuries as those countries displaced the Iberian powers in long distance trade and commercialized their domestic economies to an unprecedented extent.

This growth of commercialisation had implications for the degree of anonymity in economic relations, in factor markets as well as in product markets and this, in turn, had implications for wages. When workers were employed in small-scale enterprise in rural locations where they formed part of a close-knit community, the problem of securing effort from workers could be solved through reliance on customary relations backed up by close supervision. As people moved to towns where they were unknown to their neighbors and potential employers, it became necessary for employers to find new ways to elicit effort. In the model above, this is captured by a change in  $\sigma$ , the degree of anonymity in the economy, which lies at the heart of the no-shirking constraint equations (18) and (25).

One approach to dealing with this increase in the degree of anonymity in market based relationships, which was widely adopted in large urban enterprises during the early stages of the Industrial Revolution, was payment by results or piece rates (Pollard, 1965: 189-191). Of course, piece rates had also been used in a rural setting during the early modern period as part of the putting out system, but their “discovery” in the context of urban industry in the eighteenth century was often greeted as “an innovation of major significance” (Pollard, 1965: 190). However, as Huberman (1996: 17-32) points out, attempts to manage the wage-effort bargain through piece rate payments in early nineteenth century Lancashire often met with little success unless accompanied by the payment of an efficiency wage above the spot-market rate. Rather than risk the prospect of losing a job with a wage above the spot market rate, a worker employed at the efficiency wage is deterred from shirking (Shapiro and Stiglitz, 1984).<sup>5</sup>

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<sup>5</sup>Although the Lancashire market for labour in cotton spinning in the early nineteenth century has often been portrayed as the archetypal spot market, Huberman (1996) cautions against this interpretation, arguing that it was more myth than reality. It is, moreover, a myth which is difficult to square with the central finding that has emerged from the new focus on comparative levels of real wages in Europe: that Britain was a high wage economy at the time of the Industrial Revolution (Allen, 2001; Broadberry and

## 4.2 Changing factor prices

Table 2 sets out the pattern of silver wages in Europe. The silver wage is the silver content of the money wage in the local currency, and is useful for comparing wages across countries on a silver standard. Note first that Northwestern Europe saw substantial silver wage growth in the century after the Black Death of the mid-fourteenth century and again during the early modern period after 1500, as well as during the Industrial Revolution period from the mid-eighteenth century, when Britain finally overtook the Netherlands decisively. Second, note that although southern Europe shared in the rise in the silver wage following the Black Death, from the mid fifteenth century the region was characterized more by fluctuations than by trend growth in the silver wage. Third, central and eastern Europe were also characterized more by fluctuations than by trend growth in the silver wage from the mid-fourteenth century. This is the pattern that would be expected from the conventional economic history of Europe, with the Mediterranean region playing the leading economic role during the first half of the millennium, but with northwest Europe forging ahead after 1500.

We have focused so far on wage differences within Europe, but a complete picture of the transition to modern economic growth also requires a consideration of wage differences between Europe and Asia. Broadberry and Gupta (2006) provide some evidence of this Great Divergence in the form of silver wage differences, shown here in Table 3. Silver wages in India and the Yangzi delta region of China were already lower than those in England by the beginning of the seventeenth century, and then fell further behind. Contrary to the revisionist claims of Pomeranz (2000), Parthasarathi (1998) and Frank (1998) that the richest parts of Asia remained at the same level of development as the richest parts of Europe until as late as 1800, they appear closer to the poorer parts of Europe.

We are interested in the incentives to adopt capital intensive technology. Hence we need also to examine the cost of capital, an important element of which is the rate of interest. Nominal interest rates for a number of countries are presented in Table 4. Note that since interest rates changed together across Europe, it is reasonable to assume them exogenous with respect to each single European economy, so that intra-European differences in the factor price ratio were driven by wage rate changes, as highlighted in our model. Table

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Gupta, 2006).

4 suggests a rate of interest in Europe around 10% in the late medieval period, falling to 5-6% in the aftermath of the Black Death, 1350-1400. There was a further reduction in European rates of interest during the first half of the eighteenth century, to around 3-4%. By this point, interest rates were substantially lower in Europe than in other parts of the world such as India, where rates remained at medieval levels. Growing commercialisation was thus accompanied by declining interest rates. The downward trend of interest rates in Europe, combined with the increase in wages, translates into an increase in the wage/cost of capital ratio, raising the incentives to substitute capital for labour in production. The greater increase of wage rates in northwest Europe meant that the incentive to adopt capital intensive production methods was also greater in that region.

### 4.3 Factor prices and technology

Recent work by Broadberry and Gupta (2006; 2009) and by Allen (2009) emphasize the important role of factor prices in explaining the key technological choices of the Industrial Revolution period. Broadberry and Gupta (2009) analyze the shift of competitive advantage in cotton textiles between India and Britain. India was the world's major producer and exporter of cotton textiles during the early modern period, but was displaced from this position by Britain during the Industrial Revolution. Broadberry and Gupta (2009) point to the much higher wages in Britain than in India already in the late seventeenth century, when Indian cotton textiles were imported into Britain by the East India Company. This can be seen in the first column of Table 5. Combined with the smaller differences in the cost of raw cotton and the cost of capital, this presented British producers with a severe total factor input (TFI) price disadvantage. To get to a point where the free on board price was cheaper in Britain, required a shift to more capital intensive technology and a sustained period of technological progress to increase total factor productivity (TFP). For much of the eighteenth century, the fledgling British cotton industry required protection, although the point at which the shift in competitive advantage from India to Britain occurred varied by type of yarn or cloth (as a result of different input costs) and by market (as a result of transport costs).

Once the shift to capital-intensive technology had occurred, technological progress accelerated, as implied by equation (6) in the model. In Table 5, TFP growth shifted in Britain's favour at an annual rate of 0.3 per cent before 1770, rising to 1.5 per cent during

the period 1770-1820. This would be quite consistent with the 1.9 per cent per annum TFP growth rate estimated by Harley (1993: 200) for the British cotton industry between 1780 and 1860, together with slowly rising or stagnating productivity in India. This acceleration of TFP growth following the shift to capital intensive technology can be explained by the greater potential for learning on capital intensive technology.

#### 4.4 Real economic development

In Table 2, we examined the path of silver wages. However, an analysis of the transition to modern economic growth would not be complete without considering the path of real consumption wages and GDP per capita. The real consumption wage is obtained by dividing the silver wage with the silver price of basic consumption goods. Real consumption wages of European unskilled building labourers for the period 1300-1850 are shown in Table 6, taking London in the period 1500-49 as the numeraire. The first point to note is that real wages followed a similar pattern across the Black Death in the whole of Europe. Complete time series exist for comparatively few cities before 1500, but there is also scattered evidence for other cities. Taken together, the evidence supports the idea of a substantial rise in the real wage across the whole continent of Europe following the Black Death, which struck in the middle of the fourteenth century, wiping out between a third and a half of the population, when successive waves of the plague are cumulated (Herlihy, 1997). This episode of European economic history is thus broadly consistent with the Malthusian model, with a strong negative relationship between real wages and population (Postan, 1972: 27-40). In the first half of the fifteenth century, the real wage was quite uniform across the countries for which we have data, at about twice its pre-Black Death level. From the second half of the fifteenth century, however, Britain and Holland followed a very different path from the rest of Europe, maintaining real wages at the post-Black Death level and avoiding the collapse of real wages which occurred on the rest of the continent as population growth returned. Considering that in the same period Britain and Holland witnessed an increase in the level of urbanization, as noted above, we can argue that the anonymity is a candidate to explain this emergence of high wages in northwest Europe.

Table 7 presents the results of the latest research on the reconstruction of national income during the late medieval and early modern periods in a number of countries. The GDP per capita data show northwest Europe pulling ahead of the previously more de-

veloped Mediterranean Europe from the late sixteenth century. The national income data thus reinforce the conclusion from the silver wage and real wage data and from urbanization rates that Britain and Holland followed a different path from Italy and Spain. The Indian data confirm the conventional view that the Great Divergence was already underway during the early modern period, as Europe embarked upon a period of growing commercialisation which would ultimately end up with the Industrial Revolution and the transition to modern economic growth.

## 4.5 A simple econometric test

Our hypothesis is that commercialization led to high wages throughout Europe, in the short-run through the efficiency wage channel (increasing  $\omega_t$ ), and in the longer term because of the improved technology  $A_t$ . So far we have reviewed evidence showing that the urbanization— that we consider as a proxy for commercialization—, wage rate and GDP per capita data are related in a way which is at least consistent with the direction of causality proposed in the model. In what follows, we will estimate a simple econometric model linking real wages and urbanization.

There are at least two main problems in establishing causality from urbanization to wages: (i) a possible omitted variable bias since wages and urbanization can both be a by-product of underlying economic trends e.g. cycles or technological improvements, or other shocks; (ii) a possible problem of endogeneity., given that urbanization can be the effect rather than the cause of higher wages.

Given the constraints of available data, we will address (i) by controlling for country GDP and introducing the year fixed effect in the regressors, and (ii) by instrumenting urbanization at time  $t$ , with urbanization 50 years before.

Therefore, we estimate the following equation:

$$wage_{i,t} = c_i + \alpha * gdp_{i,t} + \beta * urbanization_{i,t} + year_t + \varepsilon_{i,t}; \quad (26)$$

where index  $i$  indicates the city, and  $t$  indexes time.

Data on *wages* are aggregated at city level and are taken from table 2, while data on *urbanization*, aggregated at country level, are from table 1. The variable *gdp* is the per capita gross domestic product displayed in table 7, as we said this provides a control for

the idiosyncratic economic cycles and technology, the years dummies  $year_t$  control for the effects generated by common shocks. Dummy variable  $c_i$  controls for city fixed effects.

Table 8 displays the results of the estimations. Column 1 shows the above described pattern between wages and urbanization, column 2 shows that this relation is still significant when we control for the aggregate shocks by introducing the year effect. In column 3 we introduce country GDP to control for some country level omitted variable. In column 4 we use a two stage least squares estimator by using  $urbanization_{t-1}$  (therefore 50 years before) as an instrument for  $urbanization_t$ . Finally in column 5 we introduce the wages at time  $t - 1$  in the 2SLS estimation to control for the effect of wages' serial correlations. In all specifications and consistently with our model, the coefficient of urbanization is positive and highly significant.

## 5 Concluding comments

We have argued that commercialisation played a pivotal role in the transition to modern economic growth. We see the growing commercialisation of the late medieval and early modern periods as leading to the acceleration of technological progress during the Industrial Revolution period via its effects on factor prices. The argument can be summarized as follows: (1) Commercialisation raised wages as a growing reliance on impersonal labour market relations in place of customary relations with a high degree of monitoring led to the adoption of efficiency wages. (2) The resulting rise in the wage/cost of capital ratio led to the adoption of a more capital-intensive production technology. (3) This led to a faster rate of technological progress through greater learning by doing on the capital intensive technology.

## References

- [1] Acemoglu, D. (2009) When Does Labor Scarcity Encourage Innovation? NBER Working Paper No. 14809
- [2] Acemoglu D. and J A. Robinson,(2006), Economic Origins of Dictatorship and Democracy. New York: Cambridge University. Press, 2006.



- [3] Acemoglu, D., Johnson, S. and J.A. Robinson, (2005), “The Rise of Europe: Atlantic Trade, Institutional Change and Economic Growth”, *American Economic Review*, 95: 546-579.
- [4] Allen, R.C. (2001), “The Great Divergence in European Wages and Prices from the Middle Ages to the First World War”, *Explorations in Economic History*, 38, 411-447.
- [5] Allen, R.C. (2009), *The British Industrial Revolution in Global Perspective*, Cambridge: Cambridge University Press.
- [6] Álvarez-Nogal, C. and Prados de la Escosura, L. (2009), “The Rise and Fall of Spain, 800-1850”, Universidad Carlos III, Madrid.
- [7] Arrow, K.J. (1962), “The Economic Implications of Learning by Doing”, *Review of Economic Studies*, 29, 155-173.
- [8] Banerjee A.V., and A.F. Newman (1998). “Information, the Dual Economy, and Development,” *Review of Economic Studies* 64, 631-653.
- [9] Blackwell, D. (1965), “Discounted Dynamic Programming,” *Annals of Mathematical Statistics* 36, 226–235.
- [10] Álvarez-Nogal, C. and Prados de la Escosura, L. (2009), “The Rise and Fall of Spain, 800-1850”, Universidad Carlos III, Madrid.
- [11] Britnell, R.H. and Campbell, B.M.S. (eds.), (1995), *A Commercialising Economy: England 1086 to c.1300*, Manchester: Manchester University Press.
- [12] Broadberry, S.N. (1997), *The Productivity Race: British Manufacturing in International Perspective, 1850-1990*, Cambridge: Cambridge University Press.
- [13] Broadberry, S.N., Campbell, B., Klein, A, Overton, M. and B. van Leeuwen, (2011), “British Economic Growth, 1270-1870”, London School of Economics.
- [14] Broadberry, S.N. and B. Gupta, (2006), “The Early Modern Great Divergence: Wages, Prices and Economic Development in Europe and Asia, 1500-1800”, *Economic History Review*, 59, 2-31.
- [15] Broadberry, S.N. and B. Gupta. (2009), “Lancashire, India and Shifting Competitive Advantage in Cotton Textiles, 1700-1850: The Neglected Role of Factor Prices”, *Economic History Review*, (forthcoming),

- [16] Broadberry, S.N. and B. Gupta, (2011), "India and the great Divergence: An Anglo-Indian Comparison of GDP per capita, 1600-1871", London School of Economics and University of Warwick.
- [17] Buyst, E. (2009), "Estimates of Economic Growth in the Southern Low Countries/Belgium, c.1770-1846", Centre for Economic Studies, Katholieke Universiteit Leuven.
- [18] Cervellati, M. and U. Sunde, (2005), "Human capital formation, life expectancy, and the process of development". *American Economic Review*, 95(5), 1653-1672.
- [19] Clark, G. (1988), "The Cost of Capital and medieval Agricultural technique", *Explorations in Economic History*, 25, 265-294.
- [20] David, P.A. (1975), *Technical Choice, Innovation and Economic Growth*, Cambridge: Cambridge University Press.
- [21] Doepke, Matthias, (2004), "Accounting for Fertility Decline During the Transition to Growth", *Journal of Economic Growth*, 9(3):347-383.
- [22] Frank, A.G. (1998), *ReOrient: The Silver Age in Asia and the World Economy*, Berkeley: University of California Press.
- [23] Fremdling, R. (2000), "Transfer Patterns of British Technology to the Continent: The Case of the Iron Industry", *European Review of Economic History*, 4, 195-222.
- [24] Galor, O. (2005). "From stagnation to growth: Unified growth theory. In: P. Aghion and S. Durlauf (eds), *Handbook of economic growth*, Vol. 1A. Amsterdam: North-Holland.
- [25] Galor, O., Moav, O. (2000). "Ability biased technological transition, wage inequality and growth", *Quarterly Journal of Economics* 115, 469-498.
- [26] Galor, O., and Moav, O. (2004). "From physical to human capital accumulation: Inequality and the process of development". *Review of Economic Studies*, 71, 1001-1026.
- [27] Galor, O., Moav, O and D, Vollrath (2009) "Inequality in Landownership, the Emergence of Human-Capital Promoting Institutions, and the Great Divergence," *Review of Economic Studies*, 2009, 76, pp. 143-179

- [28] Galor, O. and A. Mountford (2006). "Trade and the Great Divergence: The Family connection", *American Economic Review* 96, 299-303.
- [29] Galor, Oded and David Weil, (1999), "From Malthusian Stagnation to Modern Growth", *American Economic Review Papers and Proceedings*, 89(2):150-154.
- [30] Galor, O., and Weil, D. (2000). "Population, technology and growth: From the malthusian regime to the demographic transition and beyond". *American Economic Review*, 90, 806-828.
- [31] Greif, A. (1994), "Cultural Beliefs and the Organization of Society: A Historical and Theoretical Reflection on Collectivist and Individualist Societies", *Journal of Political Economy*, 102, 912-950.
- [32] Greif, A. (2006) "Institutions and the Path to the Modern Economy, Lessons from Medieval Trade". New York, Cambridge University Press.
- [33] Habakkuk, H.J. (1962), *American and British Technology in the Nineteenth Century*, Cambridge: Cambridge University Press.
- [34] Hansen, G.D., and Prescott, E. (2002). "Malthus to Solow. *American Economic Review*", 92(4), 1205-1217.
- [35] Harley, C.K. (1993), "Reassessing the Industrial Revolution: A Macro View", in Mokyr, J. (ed.), *The British Industrial Revolution: An Economic Perspective*, Boulder, Colorado: Westview, 171-226.
- [36] Herlihy, D. (1997), *The Black Death and the Transformation of the West*, Cambridge, MA: Harvard University Press.
- [37] Hicks, J.R. (1932), *The Theory of Wages*, London: Macmillan.
- [38] Huberman, M. (1996), *Escape from the Market: Negotiating Work in Lancashire*, Cambridge: Cambridge University Press.
- [39] Maddison, A. (2003), *The World Economy: Historical Statistics*, Paris: Organisation for Economic Co-operation and Development.
- [40] Madsen, J. B. Ang J. B. and R. Banerjee (2010), "Four centuries of British economic growth: the roles of technology and population", *Journal of Economic Growth*, Volume 15, Number 4, Pages 263-290

- [41] Malanima, P. (2009), *Pre-Modern European Economy: One Thousand Years (10th - 19th Centuries)*, Leiden: Brill.
- [42] Malanima, P. (2011), “The Long Decline of a Leading Economy: GDP in Central and Northern Italy, 1300-1913”, *European Review of Economic History*, (forthcoming).
- [43] Moosvi, S. (2001), “The Indian Experience 1600-1900: A Quantitative Study”, in Panikkar, K.N., Byres, T.J. and Patnaik, U. (eds.), *The Making of History: Essays Presented to Irfan Habib*, New Delhi: Tulika, 328-358.
- [44] North, D.C. (1990), *Institutions, Institutional Change and Economic Performance*, Cambridge: Cambridge University Press
- [45] Pamuk, S. (2007), “The Black Death and the Origins of the ‘Great Divergence’ across Europe, 1300-1600”, *European Review of Economic History*, 11, 289-317.
- [46] Parthasarathi, P. (1998), “Rethinking Wages and Competitiveness in the Eighteenth Century: Britain and South India”, *Past and Present*, 158, 79-109.
- [47] Pfister, U. (2009), “German Economic Growth, 1500-1850”, *Historisches Seminar, Westfälische Wilhelms-Universität Münster*.
- [48] Polanyi, K. [1944] (1957), *The Great Transformation*, London: Bacon Hill.
- [49] Pollard, S. (1965), *The Genesis of Modern Management: A Study of the Industrial Revolution in Great Britain*, London: Arnold.
- [50] Pomeranz, K. (2000), *The Great Divergence: China, Europe, and the Making of the Modern World Economy*, Princeton, NJ: Princeton University Press.
- [51] Postan, M. (1972), *The Medieval Economy and Society: An Economic History of Britain in the Middle Ages*, London: Weidenfeld and Nicolson.
- [52] Romer, Paul M. (1986): “Increasing Returns and long-run Growth,” *Journal of Political Economy*, 94, 1002–37.
- [53] Rothbarth, E. (1946), “Causes of the Superior Efficiency of USA Industry as Compared with British Industry”, *Economic Journal*, 56, 383-390.
- [54] Shapiro, C. and Stiglitz, J.E. (1984), “Equilibrium Unemployment as a Worker Discipline Device”, *American Economic Review*, 74, 433-444.
- [55] Toynbee, A. (1890), *Lectures on the Industrial Revolution of the 18th Century in England*, (third edition), London: Longmans, Green.

- [56] van Zanden, J.L. (1999), "Wages and the Standard of Living in Europe, 1500-1800", *European Review of Economic History*, 3, 175-97.
- [57] van Zanden, J.L. and van Leeuwen, B. (2011), "The Origins of 'Modern Economic Growth'? Holland between 1347 and 1807", Utrecht University.
- [58] Voigtländer, N. and Voth, H.-J. (2010), "How the West 'Invented' Fertility Restriction", UPF Barcelona.
- [59] Vollrath, D, (2011) "The Agricultural Basis of Comparative Development," *Journal of Economic Growth*, 16(4), pp. 343-370
- [60] Zeira, J., 'Workers, machines, and economic growth', *Quarterly Journal of Economics*, 113 (1998), pp.1091-1117.

**TABLE 1: European urbanization rates (%)**

	1300	1400	1500	1600	1700	1750	1800	1870
Northwestern Europe								
Scandinavia	–	–	0.7	2.1	4.3	4.6	4.6	5.5
England (Wales)	4.0	2.5	2.3	6.0	13.2	16.4	22.1	43.0
Scotland	–	–	2.3	1.5	5.3	11.5	23.9	36.3
Ireland	0.8	2.1	–	1.0	5.1	5.1	7.3	14.2
Netherlands	–	–	17.1	29.5	32.5	29.6	28.6	29.1
Belgium	18.2	21.9	17.6	15.1	20.2	16.5	16.6	25.0
France	5.2	4.7	5.0	6.3	8.7	8.7	8.9	18.1
Southern Europe								
Italy CN	18	12.4	16.4	14.4	13	13.6	14.2	13.4
Italy SI	9.4	3.3	12.7	18.6	16.1	19.4	21	26.4
Spain	12.1	10.2	11.4	14.5	9.6	9.1	14.7	16.4
Portugal	3.6	4.1	4.8	11.4	9.5	7.5	7.8	10.9
Central-Eastern Europe								
Switzerland	3.0	2.0	2.8	2.7	3.3	4.6	3.7	8.2
Austria (Czech, Hung)	0.6	0.5	0.8	1.6	1.7	2.6	3.1	7.7
Germany	3.4	3.9	5.0	4.4	5.4	5.7	6.1	17.0
Poland	1.0	1.3	5.4	6.6	3.8	3.4	4.1	7.8
Balkans	5.2	4.6	7.7	13.3	14	12.3	9.8	10.6
Russia (European)	2.1	2.3	2.0	2.2	2.1	2.5	3.6	6.7
EUROPE	5.4	4.3	5.6	7.3	8.2	8.0	8.8	15.0

Source: Paolo Malanima (2009).

The urbanization rate is defined as the proportion of the population living in settlements of at least 10,000.

**TABLE 2: Daily silver wages of European unskilled building laborers**  
(grams of silver per day)

	1300	1350	1400	1450	1500	1550	1600	1650	1700	1750	1800
Northwestern Europe											
London	2.9	3.4	4.5	3.8	3.2	4.6	7.1	9.7	10.5	11.5	17.7
Amsterdam					3.1	4.7	7.2	8.5	8.9	9.2	9.2
Antwerp			3.5	3.1	3.0	5.9	7.6	7.1	6.9	6.9	7.7
Paris					2.8	5.5	6.6	6.9	5.1	5.2	9.9
Southern Europe											
Valencia			5.6	5.2	4.2	6.6	8.8	6.9	5.7	5.1	–
Madrid					–	6.3	8.0	–	5.1	5.3	8.0
Florence/Milan	2.2	4.5	3.8	3.5	2.9	3.8	4.7	4.1	3.2	2.9	3.1
Naples					3.3	3.5	5.3	4.8	4.8	3.8	3.8
Central-Eastern Europe											
Gdansk					2.1	2.1	3.8	4.3	3.8	3.7	4.8
Warsaw					–	2.5	3.2	2.7	1.9	3.4	4.9
Krakow			2.7	2.1	1.9	2.9	3.4	2.9	2.2	2.9	2.4
Vienna			4.0	3.2	2.7	2.6	4.4	3.5	3.2	3	2.1
Leipzig					–	1.9	3.5	3.9	3.7	3.1	4.4
Augsburg					2.1	3.1	4.0	4.7	4.2	4.3	–

Source: Broadberry and Gupta (2006: 7).

Derived from the database underlying Allen (2001: 429).

**TABLE 3: Silver wages of unskilled labourers  
(grams of silver per day)**

**A. Silver wages in England and India**

Date	England	India	India/England
1550-99	3.4	0.7	0.21
1600-49	4.1	1.1	0.27
1650-99	5.6	1.4	0.25
1700-49	7.0	1.5	0.21
1750-99	8.3	1.2	0.14
1800-49	14.6	1.8	0.12

**B. Silver wages in England and China**

Date	England	China	China/England
1550-1649	3.8	1.5	0.39
1750-1849	11.5	1.7	0.15

Source : Broadberry and Gupta (2006)

**TABLE 4: Interest rates (% per annum)**

	England	Flanders	France	Italy	Germany	India
1201-1250	10.3		10.8	8.6		
1251-1300	10.2	10	11.1	10.6	10.8	
1301-1350	11.2			12.9	10.1	
1351-1400	4.5			8.1	9.7	
1401-1450				9.6	8.5	
1451-1500	4.0	6.4	9.2	7.6	6.5	
1501-1550	4.6		8.2		5.3	
1551-1600	6.0	4.3	8.3			
1601-1650	6.0	3.9	6.6			
1651-1700	5.3	4.4				8
1701-1750	4.3	3.8	4.2			10
1751-1800	4.0	2.7	4.8	4.7		12

Source: Clark (1988: 273-274); Moosvi (2001: 337-9, 342, 351-2).



**TABLE 5: Comparative GB/India costs and prices (India =100)****A. Cost**

	Wage W/W*	Raw Cotton P C/C*	Cost of Capital R/R*	TFI Price
c.1680	400	182	137	206
c.1770	460	320	113	270
c.1790	663	480	106	357
c.1820	517	127	61	150

**B. Prices and TFP**

	TFI price	FOB Price P/P*	TFP A/A*
c.1680	206	200	103
c.1770	270	200	135
c.1790	357	147	243
c.1820	150	53	283

Source: Broadberry and Gupta (2009).

**TABLE 6: Daily real consumption wages of European unskilled building labourers (London 1500-49 = 100)**

	1300	1350	1400	1450	1500	1550	1600	1650	1700	1750	1800
Northwestern Europe											
London	57	75	107	113	100	85	80	96	110	99	98
Amsterdam					97	74	92	98	107	98	79
Antwerp			101	109	98	88	93	88	92	88	82
Paris					62	60	59	60	56	51	65
Southern Europe											
Valencia			108	103	79	63	62	53	51	41	
Madrid						56	51		58	42	
Florence/Milan	44	87	107	77	62	53	57	51	47	35	26
Naples					73	54	69		88	50	33
Central-Eastern Europe											
Gdansk					78	50	69	72	73	61	40
Warsaw						75	66	72	45	64	82
Krakow			92	73	67	74	65	67	58	63	40
Vienna			115	101	88	60	61	63	61	50	27
Leipzig						34	35	57	53	44	53
Augsburg					62	50	39	63	55	50	

Source: Broadberry and Gupta (2006: 7);

derived from the database underlying Allen (2001: 429).

**TABLE 7: GDP per capita levels (in 1990 international dollars)**

	1250	1300	1350	1400	1450	1500	1550	1600	1650	1700	1750	1800	1850
Northwestern Europe													
England	737	730	767	1095	1172	1164	1138	1144	1215	1649	1688	2085	3006
Holland			876	1195	1373	1454	1432	2662	2691	2105	2355	2408	1886
Belgium						929	1089	1073	1203	1264	1357	1497	
France						727		841		986		1230	
Southern Europe													
Spain	1249	1249	1388	1145	1160	1160	1294	1219	1175	1145	1190	1249	1487
Italy		1482	1376	1601	1668	1403	1337	1244	1271	1350	1403	1244	1350
Central-Eastern Europe													
Germany						1332		894	1130	1068	1162	1140	1428
Poland						462		516		566		636	
Austria						707		837		993		1218	
India								684	648	630	587	576	560

Source: Netherlands: van Zanden and van Leeuwen (2011); France, Austria, Poland: Maddison (2005), England: Broadberry et al. (2011).

Italy: Malanima (2011); Belgium: Buyst (2009), Blomme and van der Wee (1994); Germany: Pfister (2009);

Spain: Álvarez-Nogal and Prados de la Escosura (2009); India: Broadberry and Gupta (2011).

Table 8: **Effect of Urbanization on Wages:** Independent Variable is wage in the main european cities. In columns 4 and 5, urbanization is instrumented by using urbanization 50 years before. (std errors in brackets)

	OLS (1) 1300-1800 b/se	OLS (2) 1300-1800 b/se	OLS (3) 1300-1800 b/se	2SLS (4) 1300-1800 b/se	2SLS (5) 1300-1800 b/se
Urbanization	0.459*** (0.061)	0.360*** (0.056)	0.318*** (0.073)	0.649*** (0.158)	0.658*** (0.195)
GDP			0.002* (0.001)	0.005*** (0.002)	0.004 (0.004)
Wage[t-1]					0.226 (1.065)
Year effects	No	Yes	Yes	Yes	Yes
City effects	Yes	Yes	Yes	Yes	Yes
N	74	74	66	20	20

# A Appendix

## A.1 Proof of Proposition 1:

(i) It follows from (6) and (20) that the steady state level of capital stock  $K^*$  must be the solution to the equation

$$K = f(K) \quad (27)$$

where  $f(K) = -\frac{(1-\theta)}{\theta}a(K)L(\sigma) + \frac{1}{\theta}\left(\frac{r}{\alpha\theta}\right)^{\frac{1}{1-\alpha}}$ . The LHS of (27) is increasing in  $K$  while  $f'(K) = -\frac{(1-\theta)}{\theta}a'(K)L(\sigma) < 0$  as  $a'(K) > 0$ . Moreover,  $\lim_{K \rightarrow 0} f(K) = \frac{1}{\theta}\left(\frac{r}{\alpha\theta}\right)^{\frac{1}{1-\alpha}} > 0$ . Therefore, there exists a unique positive steady state capital stock  $K^*$  and technology  $a(K^*)$ .

(ii) The steady state capital stock satisfies the equation

$$K^* = -\frac{(1-\theta)}{\theta}a(K^*)L(\sigma) + \frac{1}{\theta}\left(\frac{r}{\alpha\theta}\right)^{\frac{1}{1-\alpha}}$$

By taking the total derivative of the preceding equation with respect to  $K^*$  and  $\sigma$  we obtain that

$$\frac{dK^*}{d\sigma} = -\frac{\frac{(1-\theta)}{\theta}a(K^*)L'(\sigma)}{1 + \frac{(1-\theta)}{\theta}a'(K^*)L(\sigma)} > 0$$

as  $L'(\sigma) < 0$  (from (18)) and  $a'(K^*) > 0$  (by assumption).

(iii) Examining the local stability of the steady state requires us to linearize the equations (6) and (20) at the steady state to obtain

$$\begin{bmatrix} 1 & -\frac{(1-\theta)}{\theta}L(\sigma) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} dK_t \\ dA_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \rho a'(K^*) & 1 - \rho \end{bmatrix} \begin{bmatrix} dK_{t-1} \\ dA_{t-1} \end{bmatrix}$$

so that

$$\begin{bmatrix} dK_t \\ dA_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{(1-\theta)}{\theta}L(\sigma) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ \rho a'(K^*) & 1 - \rho \end{bmatrix} \begin{bmatrix} dK_{t-1} \\ dA_{t-1} \end{bmatrix}$$

which after further simplification yields

$$\begin{bmatrix} dK_t \\ dA_t \end{bmatrix} = \begin{bmatrix} \frac{(1-\theta)}{\theta}L(\sigma)\rho a'(K^*) & (1-\rho)\frac{(1-\theta)}{\theta}L(\sigma) \\ \rho a'(K)0 & 1 - \rho \end{bmatrix} \begin{bmatrix} dK_{t-1} \\ dA_{t-1} \end{bmatrix}.$$

By computation, it is easily checked that the determinant of the matrix on the RHS of the preceding equation is zero so that if  $\lambda_1$  and  $\lambda_2$  denote the two eigenvalues of the said matrix,

$$\begin{aligned}\lambda_1 \lambda_2 &= 0, \\ \lambda_1 + \lambda_2 &= \frac{(1-\theta)}{\theta} L(\sigma) \rho a'(K^*) + (1-\rho)\end{aligned}$$

so that the steady state is always a saddle and is a sink if  $\frac{(1-\theta)}{\theta} L(\sigma) \rho a'(K^*) + (1-\rho) < 1$  or equivalently,  $a'(K^*) < \frac{\theta}{(1-\theta)L(\sigma)}$ . ■

## A.2 Proof of Proposition 2:

(i) We first show that there is a unique positive steady state capital stock  $K^*$  and that employment level  $L^*$  exists. Note that the steady state is a solution to the equations:

$$\begin{aligned}F_k(K, a(K) L) &= r, \\ F_l(K, A(K) L) &= \omega(L, \sigma).\end{aligned}$$

Consider the equation  $F_k(K, a(K) L) = r$ . Under the assumption that  $F_{kk} < 0$ ,  $F_{ll} < 0$ ,  $F_{kl} < 0$ ,  $a'(K) > 0$  from the equation  $F_k(K, a(K) L) = r$  there exists an implicit function  $g_1(L) = K$  with

$$g_1'(L) = -\frac{a(K)F_{kl}}{F_{kk} + F_{kl}a'(K)} < 0$$

while from the equation  $F_l(K, a(K) L) = \omega(L, \sigma)$  there exists an implicit function  $g_2(L) = K$  with

$$g_2'(L) = -\frac{a(K)F_{ll} + \omega_l(L, \sigma)}{F_{kk} + F_{kl}a'(K)} < 0.$$

Steady state employment  $L^*$  is the solution to  $g_3(L) = g_2(L) - g_1(L) = 0$ . As  $\lim_{k \rightarrow 0} F_k = \lim_{l \rightarrow 0} F_l = \infty$ ,  $\lim_{L \rightarrow 0} g_3(L) = \infty$  while  $\lim_{L \rightarrow \infty} g_3(L) = 0$  so that there exists  $L^* = L(\sigma) > 0$  such that  $g_3(L^*) = g_2(L^*) - g_1(L^*) = 0$ . Finally, note that  $K^* = K(\sigma) = g_2(L^*) = g_1(L^*) > 0$ .

(ii) We examine how the steady state values of the key endogenously determined variables change due to changes in  $\sigma$ . After substituting for wages using the no shirking constraint (4) and noting that at the steady state  $A(K^*) = a(K^*)$ , the total derivative of

(23) and (24) at the steady state is given by the expression

$$\begin{bmatrix} F_{kk}^* + F_{kl}^* a'(K^*) & F_{kl}^* a(K^*) \\ F_{kl}^* + F_{ll}^* a'(K^*) & F_{ll}^* a(K^*) - \omega_l^* \end{bmatrix} \begin{bmatrix} dK^* \\ dL^* \end{bmatrix} = \begin{bmatrix} 0 \\ \omega_\sigma^* \end{bmatrix} d\sigma$$

where  $\omega_\sigma^* = \omega_\sigma(L^*, \sigma)$ . The determinant,  $D'$ , of the preceding matrix can be written as

$$D' = -(F_{kk}^* + F_{kl}^* a'(K^*)) \omega_l^* + a(K^*) (F_{kk}^* F_{ll}^* - (F_{kl}^*)^2) > 0$$

as  $F(\cdot)$  is strictly concave,  $F_{kl}^* < 0$ ,  $a'(K^*) > 0$  and  $\omega_l^* > 0$ . Therefore,

$$\begin{bmatrix} dK^* \\ dL^* \end{bmatrix} = \frac{1}{D'} \begin{bmatrix} F_{ll}^* a(K^*) - \omega_l^* & -F_{kl}^* a(K^*) \\ -F_{kl}^* + F_{ll}^* a'(K^*) & F_{kk}^* + F_{kl}^* a'(K^*) \end{bmatrix} \begin{bmatrix} 0 \\ \omega_\sigma^* \end{bmatrix} d\sigma$$

so that

$$\begin{bmatrix} dK^* \\ dL^* \end{bmatrix} = \frac{1}{D'} \begin{bmatrix} -F_{kl}^* a(K^*) \omega_\sigma^* \\ (F_{kk}^* + F_{kl}^* a'(K^*)) \omega_\sigma^* \end{bmatrix} d\sigma$$

and

$$\begin{aligned} \frac{dK^*}{d\sigma} &= -\frac{F_{kl}^* a(K^*) \omega_\sigma^*}{D'} > 0 \\ \frac{dL^*}{d\sigma} &= \frac{(F_{kk}^* + F_{kl}^* a'(K^*)) \omega_\sigma^*}{D'} < 0 \end{aligned}$$

as, from (25),  $\omega_\sigma^* > 0$ .

(iii) In order to characterize the local stability of the steady state, we need to characterize how real wages  $w_t$  change in the vicinity of the steady state as equilibrium employment  $L_t^*$  changes.

Consider the scenario described by (1), (2), (3) and the no shirking constraint (4). As the no shirking constraint (4) holds as an equality in equilibrium, setting  $v_t^E(1) = v_t^E(0)$  and equating the RHS of (1) and (2) we obtain that at each  $t$ ,

$$v_{t+1}^E(0) = \frac{\mu}{\beta(1 - (p + (1-p)q_t\sigma))} + v_{t+1}^{US}.$$

Substituting for  $v_{t+1}^E(0)$  in (2) we obtain that at each  $t$

$$v_t^{US} = \frac{\mu(1-p(1-q_t\sigma))}{(1-(p+(1-p)q_t\sigma))} + \beta v_{t+1}^{US}$$

which yields that at each  $t$ ,

$$v_t^{US} = \sum_{t' \geq t} \beta^{t'-t} \left( \frac{\mu(1-p(1-q_t\sigma))}{(1-(p+(1-p)q_t\sigma))} \right)$$

and therefore,

$$v_t^E(0) = \frac{\mu}{\beta(1-(p+(1-p)q_{t-1}\sigma))} + \sum_{t' \geq t} \beta^{t'-t} \left( \frac{\mu(1-p(1-q_t\sigma))}{(1-(p+(1-p)q_t\sigma))} \right).$$

Further, by computation, from (1) we obtain that at each  $t$ ,

$$v_t^E(1) = \sum_{t' \geq t} \beta^{t'-t} \omega_t.$$

Equating  $v_t^E(0)$  and  $v_t^E(1)$  in the vicinity of the steady state and equating the two as required by the no shirking constraint (4) we obtain that at each  $t$ ,

$$\sum_{t' \geq t} \beta^{t'-t} \omega_t = \frac{\mu}{\beta(1-(p+(1-p)q_{t-1}\sigma))} + \sum_{t' \geq t} \beta^{t'-t} \left( \frac{\mu(1-p(1-q_{t'}\sigma))}{(1-(p+(1-p)q_{t'}\sigma))} \right).$$

At this point, we will assume that in the vicinity of the steady state, at each  $t$ , agents expect that future employment and wage levels will be the same as current employment levels so that  $L_{t'}^e = L_t$  and  $\omega_{t'}^e = \omega_t$  for all  $t' \geq t$  where the superscript  $e$  denotes the expected level of future employment and future wages. In other words, in order to characterize the dynamics in the vicinity of the steady state, we do not assume that agents have rational expectations.

Under this assumption, it follows that we can write at each  $t$ ,

$$\omega_t = \frac{\mu(1-\beta)}{\beta(1-(p+(1-p)q_{t-1}\sigma))} + \left( \frac{\mu(1-p(1-q_t\sigma))}{(1-(p+(1-p)q_t\sigma))} \right).$$

so that at each  $t$  in the vicinity of the steady state  $d\omega_t = g_t^* dL_{t-1} + f_t^* dL_t$  where  $f^* = \frac{\mu(1-p(1-q(\frac{L^*}{N})\sigma))}{(1-(p+(1-p)q(\frac{L^*}{N})\sigma))}$  and  $g^* = \frac{\mu(1-\beta)}{\beta(1-(p+(1-p)q(\frac{L^*}{N})\sigma))}$ .



Taking the above computation into account and noting that at the steady state  $A(K^*) = a(K^*)$ , examining the local stability of the steady state requires us to linearize the equations (6), (23) and (24) at the steady state to obtain

$$\begin{bmatrix} F_{kk}^* & a(K^*)F_{kl}^* & 0 \\ F_{kl}^* & a(K^*)F_{ll}^* - f_l^* & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dK_t \\ dL_t \\ dA_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & g_l^* & 0 \\ \rho a'(K^*) & 0 & 1 - \rho \end{bmatrix} \begin{bmatrix} dK_{t-1} \\ dL_{t-1} \\ dA_{t-1} \end{bmatrix}$$

where  $F_{ij}^* = F_{ij}(K^*, a(K^*)L^*)$ ,  $i, j = k, l$  and  $\omega_l^* = \omega_l(L^*, \sigma)$ .

The matrix on the LHS of the preceding equation is invertible. Its determinant,  $D$ , is

$$D = -F_{kk}^* \omega_l^* + a(K^*) [F_{kk}^* F_{ll}^* - (F_{kl}^*)^2] > 0$$

by strict concavity of the production function,  $F_{kk}^* < 0$  and  $[F_{kk}^* F_{ll}^* - (F_{kl}^*)^2] > 0$  and as  $\omega_l^* < 0$ .

It follows that

$$\begin{bmatrix} dK_t \\ dL_t \\ dA_t \end{bmatrix} = \frac{1}{D} \begin{bmatrix} a(K^*)F_{ll}^* - \omega_l^* & -a(K^*)F_{kl}^* & 0 \\ -F_{kl}^* & F_{kk}^* & 0 \\ 0 & 0 & D \end{bmatrix} \begin{bmatrix} dK_{t-1} \\ dL_{t-1} \\ dA_{t-1} \end{bmatrix}$$

so that

$$\begin{bmatrix} dK_t \\ dL_t \\ dA_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & F_{kk}^* g_l^* & 0 \\ \rho a'(K^*) & 0 & 1 - \rho \end{bmatrix} \begin{bmatrix} dK_{t-1} \\ dL_{t-1} \\ dA_{t-1} \end{bmatrix}$$

By computation, the eigenvalues of the matrix on the RHS of the preceding equation must satisfy the equation  $\lambda_1 (\lambda_2 - F_{kk}^* g_l^*) (\lambda_3 - (1 - \rho)) = 0$  so that it immediately follows that two eigenvalues always have a modulus strictly less than one so that the steady state is locally a saddle and whenever  $|F_{kk}^* g_l^*| = \left| \frac{F_{kk}^* \mu q'(\frac{L^*}{N})(1-\beta)}{N\beta(1-(p+(1-p)q\sigma))} \right| < 1$  a sink. Clearly, if  $\beta$  is close enough to one,  $\left| \frac{F_{kk}^* \mu q'(\frac{L^*}{N})(1-\beta)}{N\beta(1-(p+(1-p)q\sigma))} \right| < 1$ . ■