#8637.

August 1990

The Welfare Significance and Non-significance of General Equilibrium Demand and Supply Curves

Walter N. Thurman

North Carolina State University, Dept. of economics and business

Raleigh, North Carolina

AAEA. 1990

Supply and demand - Mathematical models. 1990

#8637

1. Introduction

A recurring theme in the welfare economics literature is the measurement of multi-market effects of public policy in a single market. The way to accomplish this is to measure surplus changes in the one market using demand or supply curves that hold constant not prices in related markets, but supply and demand conditions. Two facts motivate interest in this technique. First is the ubiquity of multi-market policy effects. Second is the general difficulty in obtaining data from all affected markets.

The first exposition of this sort of analysis is found in Harberger (1964 and 1971). Later work by Just and Hueth (1979) and Just, Hueth, and Schmitz (1982, Appendix D) extended Harberger's analysis by modeling explicitly the market connections and by using duality theory to prove the welfare propositions. The 1982 work, in particular, provides a quite general framework for what here will be called general equilibrium welfare analysis. Just, Hueth, and Schmitz proved that, under competitive conditions, general equilibrium demand and supply curves can be used to measure in one market the sum of the surplus changes in all markets resulting from intervention in the one market.

Other important contributions are those by Anderson (1976) and Carlton (1979) who discussed the conditions under which the welfare effects of a tax on an input could be measured in an output market. Panzar and Willig (1978) pointed out the necessity of identical firms to their results.

In the current paper, I explore further the welfare interpretation of general equilibrium supply and demand curves. I come to two conclusions that are both practically important and unexpected given the analysis of Just, Hueth, and Schmitz and of Harberger. Specifically, the conclusions concern the definition and interpretation of general equilibrium demand and supply curves when there is more than one source of general equilibrium feedback.

First, when feedback from other markets shifts both supply and demand, neither general equilibrium curve, supply or demand, has welfare significance; but welfare triangles constructed from both of them do. Second, when feedback occurs only through demand channels, say, there is a uniquely defined general equilibrium demand curve. It is invariant (under some restrictions) to the policy being analyzed. But when there is feedback through both demand and supply the definition of the general equilibrium demand curve becomes policy dependent. Its definition changes depending on whether the policy is a price wedge, such as a tax or quota, or a quantity wedge, such as a government purchase and price support program. I establish these two points in a simple two-market framework where the welfare significance and non-significance of general equilibrium curves can most easily be seen.

2. Welfare Analysis with a General Equilibrium Demand Curve

Consider two goods that are substitutes in demand and are sold in competitive markets. A per-unit tax is placed on the first good, driving a wedge between demand and supply price. The increase in the price of the taxed good shifts out the demand for its substitute, thereby increasing the substitute's price and, in turn, shifting out the demand for the taxed good. The effects of the tax are shown in Figure 1 where $D_1(P_2^0)$ and $D_2(P_1^0)$ are the demands for Q_1 and Q_2 conditional on the initial equilibrium prices, P_1^0 and P_2^0. The per unit tax of τ causes a new equilibrium pair of demand prices, P_1^1 and

P_2^1, and a new supply price of $\tilde{P}_1^1$. The demands for both goods shift out from the demand price increases.

Now consider the welfare effects of the tax, both on consumers who suffer increases in the prices of both goods, and on producers who suffer a decrease in P_1 but benefit from an increase in P_2. The producers' loss in profits is their loss in producer surplus (see Just, Hueth, and Schmitz, ch. 4) which equals A+B (their loss in the Q_1 market) and -(C+D) (the negative of their gain in the Q_2 market). The consumers' losses are more complicated due to the fact that demand curves in both markets shift to the right. However, the size of these losses can be derived from the expenditure function of a representative consumer.

Let $m(P_1,P_2)$ be the minimum expenditure necessary to achieve utility level U at prices P_1 and P_2, where U is the initial level of utility. A money measure of the change in consumer welfare from the price increase is $\Delta CW = m(P_1^1,P_2^1)-m(P_1^o,P_2^o)$, the change in income necessary to compensate the consumer back to the original utility level after prices have risen. This is the compensating variation. Defining $m(\cdot)$ to be the expenditure function for the after-tax utility level would result in the equivalent variation.

The compensating variation can be related to Figure 1 by adding and subtracting $m(P_1^1,P_2^o)$ from the welfare measure:

$$\Delta CW = m(P_1^1,\ P_2^1) - m(P_1^o,\ P_2^o)$$

$$= m(P_1^1,P_2^o) - m(P_1^o,P_2^o) + m(P_1^1,P_2^1) - m(P_1^1,P_2^o)$$

$$= \int_{P_1^o}^{P_1^1} \frac{\partial m}{\partial P_1} (P_1,P_2^o)\, dP_1 + \int_{P_2^o}^{P_2^1} \frac{\partial m}{\partial P_2} (P_1^1,P_2)\, dP_2 \quad .$$

Or, using Shepherd's lemma:

$$(1) \qquad \Delta CW = \int_{P_1^o}^{P_1^1} D_1(P_1,P_2^o)\, dP_1 + \int_{P_2^o}^{P_2^1} D_2(P_1^1,P_2)\, dP_2 .$$

If the demands drawn in Figure 1 are the Hicksian demands, then the first integral of the last line of (1) is area E+F, the change in consumer surplus behind the initial demand curve for Q_1, $D_1(P_2^o)$. The second integral is area C+D+G, the change in consumer surplus behind the shifted demand for Q_2, $D_2(P_1^1)$.

Aggregating across consumers and producers gives a loss of A+B+E+F in the first market and a net loss of G in the second. From this total loss of A+B+E+F+G should be subtracted tax revenue to arrive at the deadweight loss.

Now consider a measure of the welfare loss wholely calculated in the first market and based on D_1^*, the general equilibrium demand for Q_1. D_1^* connects points of equilibrium for consumers as the tax is imposed and both P_1 and P_2 rise. P_2 varies continuously along D_1^* so as to maintain equilibrium in the Q_2 market for each P_1; this is the general equilibrium relationship $P_2(P_1)$.[1]

---

[1]Equilibrium with the tax is described by the following two equations:

$$D_1(P_1,P_2) = S_1(P_1-\tau)$$

$$D_2(P_1,P_2) = S_2(P_2).$$

Taking τ as parametric, the equilibrium conditions define two equilibrium relations between the prices and the level of the tax:

$$P_1 = P_1(\tau) \text{ and } P_2 = P_2(\tau) .$$

The equilibrium relationship between P_1 and P_2 used in the definition of D_1^* is found by composition:

$$P_2(P_1) = P_2[\tau(P_1)] ,$$

where $\tau(P_1)$ is the inverse of the function $P_1(\tau)$. $P_2(P_1)$ can be shown to be strictly increasing if either the slope of S_2 is strictly positive or the own-price derivative of $D_2(P_1,P_2)$ is strictly negative.

It turns out that the change in surplus behind D_1^* captures not only the surplus change in the Q_1 market but the net surplus loss, consumer and producer, in the Q_2 market as well. The proof of this fact involves calculating the compensating variation by integrating along the equilibrium path from (P_1^o, P_2^o) to (P_1^1, P_2^1) rather than sequentially (first for P_1 and then for P_2) as was done in equation (1).

Using the fact that $P_2(P_1^o) = P_2^o$ and $P_2(P_1^1) = P_2^1$:

$$\Delta CW = m(P_1^1, P_2^1) - m(P_1^o, P_2^o) = m[P_1^1, P_2(P_1^1)] - m[P_1^o, P_2(P_1^o)] \ .$$

Applying the Fundamental Theorem of Calculus and integrating with respect to P_1 then implies:

$$\Delta CW = \int_{P_1^o}^{P_1^1} \frac{d}{dP_1} m[P_1, P_2(P_1)] \, dP_1$$

$$= \int_{P_1^o}^{P_1^1} \left\{ \frac{\partial}{\partial P_1} m[P_1, P_2(P_1)] + \frac{\partial}{\partial P_2} m[P_1, P_2(P_1)] \frac{dP_2}{dP_1} \right\} dP_1$$

$$= \int_{P_1^o}^{P_1^1} D_1[P_1, P_2(P_1)] \, dP_1 + \int_{P_1^o}^{P_1^1} D_2[P_1, P_2(P_1)] \frac{dP_2}{dP_1} \, dP_1 \ .$$

An important intermediate result is obtained by using the definition of D_1^* in the first integral and changing the variable of integration from P_1 to P_2 in the second:

$$(2) \qquad \Delta CW = \int_{P_1^o}^{P_1^1} D_1^*(P_1)\, dP_1 + \int_{P_2^o}^{P_2^1} D_2[P_1(P_2),P_2]\, dP_2 ,$$

where $P_1(P_2)$ is the inverse of the equilibrium relationship $P_2(P_1)$.

The first integral in (2) is the change in consumer surplus behind D_1^*, area E+F+H+I. The second integral is an integral of quantity along the equilibrium path in the second market: for any P_2, $P_1(P_2)$ gives the P_1 that equates demand with supply in the second market. Therefore, by definition of $P_1(P_2)$, $D_2[P_1(P_2),P_2] = S_2(P_2)$. Therefore, the second integral is:

$$\int_{P_2^o}^{P_2^1} D_2[P_1(P_2),P_2]\, dP_2 = \int_{P_2^o}^{P_2^1} S_2(P_2)\, dP_2 ,$$

which is the change in producer surplus in the second market. In terms of areas in Figure 1, (2) can then be written:

$$(3) \qquad \Delta CW = \Delta CS_1^* + \Delta PS_2,$$

where ΔCS_1^* is the general equilibrium change in consumer surplus measured in the taxed market and ΔPS_2 is the change in producer surplus in the second market. Finally, rearranging (3) gives the result that D_1^* incorporates the welfare of consumers of both goods and producers of Q_2:

$$(4) \qquad \Delta CS_1^* = \Delta CW - \Delta PS_2 .$$

The partial and general equilibrium analyses can be linked graphically first by re-writing (1) as:

$$(5) \qquad \Delta CW = \Delta CS_1(P_2^o) + \Delta CS_2(P_1^1) ,$$

where $\Delta CS_1(P_2^0)$ denotes the compensated change in consumer surplus in the first market, holding constant P_2 at P_2^0. $\Delta CS_2(P_1^1)$ is defined similarly. Next, equate the expressions for ΔCW in (3) and (5) to obtain:

$$(6) \qquad \begin{aligned} \Delta CS_1^* &= \Delta CS_1(P_2^0) + \Delta CS_2(P_1^1) - \Delta PS_2 \\ &= (E+F) \quad + (C+D+G) \quad - (C+D) \\ &= (E+F) \quad + G . \end{aligned}$$

The general equilibrium surplus change measures the sum of the change in consumer surplus in the Q_1 market conditional on P_2^0 (E+F) and the net loss in the Q_2 market (G).

Because surplus areas behind general equilibrium curves capture multi-market effects and surplus areas behind partial equilibrium curves do not, the differences between the two surplus areas measure other-market effects. In Figure 1, the general equilibrium surplus change is E+F+H+I. Equation (6) reveals this area also to equal E+F+G. The equality of these two areas implies that H+I = G: the difference between the partial and general equilibrium surplus areas in market one is the net surplus change in market two.

The theoretical and practical importance of the analysis can be summarized as follows. Because D_1^* accounts for losses to consumers in both markets and to producers in the Q_2 market, the entire welfare analysis can be conducted in the Q_1 market with D_1^* and S_1. D_1^* and S_1 have welfare significance individually, and the welfare triangle constructed using them, B+F+I, is the total welfare loss net of tax revenue.

There are, of course, many types of policy interventions in markets, the tax example representing just one. The general equilibrium approach to welfare measurement is quite general, however, and applies to many other interventions.

For example, a production quota can be analyzed in essentially the same way as a tax. It, too, drives a wedge between supply price (marginal cost) and demand price. It gives rise to the same general equilibrium demand curve, D_1^*, and the same welfare analysis, although the transfer to the government in the tax case becomes a transfer to quota owners under a quota.[2]

Taxes and quotas both are examples of a type of intervention: one that drives a wedge between demand and supply prices. A second type of policy drives a wedge between quantities supplied and demanded. For example, consider the effects of a price support which also can be analyzed in Figure 1. Under the price support, the government buys enough Q_1 to support the price at $P_1^{\dagger}$. As in the case of the tax, the rise in P_1 induces an increase in P_2 through the shift in demand for the substitute good.

From the point of view of Q_1 producers and the government, the price support policy looks rather different from the tax. There are profit and government expenditure changes to calculate, all in the Q_1 market. But to consumers of Q_1 and Q_2 and to producers of Q_2, the effects of a price support at $P_1^{\dagger}$ are identical to the effects of a tax that raises P_1 to $P_1^{\dagger}$. The increase in P_1 gives rise to a shift in D_2 and, hence, an increase in P_2. The increase in P_2 feeds back into the Q_1 market and gives rise to D_1^*, the general equilibrium demand curve. D_1^* is the same under the quantity wedge caused by a price support as it is under the price wedge caused by a tax. Further, D_1^* retains its welfare significance. The consumer surplus change measured with

[2]Complicating issues that arise with a production quota are the competition for the right to produce and the distribution of quota among firms. These issues lie behind the definition of supply and demand curves and, hence, are not analyzable in Figure 1, where supply and demand curves are already defined. Ignoring these complications is equivalent to assuming away rent-seeking and assuming that the aggregate quota is distributed efficiently among firms, say, through the use of tradeable production permits.

respect to D_1^* aggregates the welfare changes to consumers of both goods and to producers of Q_2.

3. Welfare Analysis With a General Equilibrium Supply Curve

What has been established so far is that: (1) a general equilibrium demand curve has welfare significance in the case of demand substitution, and (2) the construction of the general equilibrium demand curve does not depend upon the policy being analyzed, at least if the policy is a price or quantity wedge. These results apply to a variety of situations in which there is demand substitution. When substitution occurs in supply rather than demand, a natural counterpart to D_1^* arises that also has welfare significance and whose definition is independent of policy. Figure 2 again analyzes the effects of a tax on Q_1, but now Q_1 and Q_2 are assumed to be substitutes in supply and independent in demand. The arguments are symmetrical to those of the previous section.

Imposing a tax of τ per unit drives a wedge between demand price, P_1, and supply price, $\tilde{P}_1$, in the taxed market. Due to the substitutability in supply, the reduction in $\tilde{P}_1$ shifts out the supply of the substitute good and lowers its price. The lower price of the second good, in turn, shifts out the supply of the taxed good. The final equilibrium is shown in Figure 2 with demand and supply prices of P_1^1 and $\tilde{P}_1^1$ in the first market and price P_2^1 in the second market.

The welfare effects of the tax fall on consumers, who face a higher price in the first market and a lower price in the second and on producers, who face lower prices in both markets. Again adopting the representative consumer's view, the consumer surplus loss is (A+B) - (C+D) which is the compensating variation if D_1 and D_2 are Hicksian demands for the utility level associated with the pre-tax prices. The effect on producer welfare is the effect on profits and again can be calculated as a sum of producer surplus changes.

Consider the supplies of Q_1 and Q_2 to come from many identical two-product firms. The industry profit function is the sum of the individual firms' profit functions and depends upon the prices of the two goods, P_1 and P_2. The prices of inputs are assumed constant and, therefore, are suppressed in the following discussion. The loss in profits resulting from the tax can be written as:

$$\Delta\Pi = \Pi(P_1^o,P_2^o) - \Pi(\tilde{P}_1^1,P_2^1)$$

$$= \Pi(P_1^o,P_2^o) - \Pi(\tilde{P}_1^1,P_2^o) + \Pi(\tilde{P}_1^1,P_2^o) - \Pi(\tilde{P}_1^1,P_2^1)$$

$$= \int_{\tilde{P}_1^1}^{P_1^o} \frac{\partial\Pi}{\partial\tilde{P}_1}(\tilde{P}_1,P_2^o)\,d\tilde{P}_1 + \int_{P_2^1}^{P_2^o} \frac{\partial\Pi}{\partial P_2}(\tilde{P}_1^1,P_2)\,dP_2 \quad .$$

Using Hotelling's lemma to equate competitive supply curves with price derivatives of the profit function implies that:

$$\Delta\Pi = \int_{\tilde{P}_1^1}^{P_1^o} S_1(\tilde{P}_1,P_2^o)\,d\tilde{P}_1 + \int_{P_2^1}^{P_2^o} S_2(\tilde{P}_1^1,P_2)\,dP_2 \; .$$

Or, defining $\Delta PS_1(P_2)$ as the change in producer surplus in the first market holding constant the second good's price at P_2 and defining $\Delta PS_2(P_1)$ symmetrically:

(7) $\quad \Delta\Pi = \Delta PS_1(P_2^o) + \Delta PS_2(\tilde{P}_1^1)$.

In terms of Figure 2, equation (7) reads: $\Delta\Pi = (E+F) + (C+D+G)$.

the total welfare effect gross of tax revenue. The total welfare effect can be derived wholely within the Q_1 market, however, by using the general equilibrium supply curve, S_1^*. Along S_1^*, at each $\tilde{P}_1$, P_2 takes on the value that clears the second market. The welfare significance of S_1^* can be seen by integrating along the general equilibrium price path from $[\tilde{P}_1^1, P_2(\tilde{P}_1^1)]$ to $[P_1^o, P_2(P_1^o)]$ where $P_2(\tilde{P}_1)$ is the equilibrium relationship between P_2 and $\tilde{P}_1$ induced by the dependence of both prices on τ. This approach to calculating the loss in profits gives:

$$\Delta\Pi = \Pi[P_1^o, P_2(P_1^o)] - \Pi[\tilde{P}_1^1, P_2(\tilde{P}_1^1)]$$

$$= \int_{\tilde{P}_1^1}^{P_1^o} \frac{d}{d\tilde{P}_1} \Pi\,[\tilde{P}_1, P_2(\tilde{P}_1)]\, d\tilde{P}_1$$

$$= \int_{\tilde{P}_1^1}^{P_1^o} \frac{\partial\Pi}{\partial\tilde{P}_1} [\tilde{P}_1, P_2(\tilde{P}_1)]\, d\tilde{P}_1 + \int_{\tilde{P}_1^1}^{P_1^o} \frac{\partial\Pi}{\partial P_2} [\tilde{P}_1, P_2(\tilde{P}_1)] \frac{d\tilde{P}_2}{d\tilde{P}_1}\, d\tilde{P}_1$$

$$= \int_{\tilde{P}_1^1}^{P_1^o} S_1[\tilde{P}_1, P_2(\tilde{P}_1)]\, d\tilde{P}_1 + \int_{\tilde{P}_1^1}^{P_1^o} S_2[\tilde{P}_1, P_2(\tilde{P}_1)] \frac{dP_2}{d\tilde{P}_1}\, d\tilde{P}_1 .$$

Noting that the first integrand is, by definition, the general equilibrium supply and changing variables of integration in the second integral yields:

$$\Delta\Pi = \int_{\tilde{P}_1^1}^{P_1^o} S_1^*(\tilde{P}_1)\, d\tilde{P}_1 + \int_{P_2^1}^{P_2^o} S_2[\tilde{P}_1(P_2), P_2]\, dP_2 .$$

Finally, by using the definition of $\tilde{P}_1(P_2)$ to substitute $D_2(P_2)$ for $S_2[\tilde{P}_1(P_2), P_2]$, we obtain:

$$(8) \qquad \Delta\Pi = \int_{\tilde{P}_1^1}^{P_1^o} S_1^*(\tilde{P}_1)\, d\tilde{P}_1 + \int_{P_2^1}^{P_2^o} D_2(P_2)\, dP_2 .$$

$$= \Delta PS_1^* + \Delta CS_2 ,$$

where ΔPS_1^* denotes the surplus change behind S_1^* and ΔCS_2 denotes the surplus change behind D_2.

Re-arranging the last line of (8) gives the welfare significance of S_1^*:

$$\Delta PS_1^* = \Delta\Pi - \Delta CS_2.$$

In Figure 2, the surplus area E+F+H+I represents the producer profit loss in both markets adjusted for the consumer surplus gain in the second market. A complete accounting for all the welfare effects is obtained by adding in the first market's consumer surplus loss of A+B. Net of tax revenue, the welfare loss is the triangle B+F+I.

Once again, the general equilibrium curve captures all the welfare effects from other markets. And in parallel to the analysis of quantity wedges in the previous section, the general equilibrium supply curve is invariant to the policy intervention, within the class of price or quantity wedges.[3]

The analysis of Figures 1 and 2 illustrates the quite general result found in Appendix D of Just, Hueth, and Schmitz. The analysis also suggests techniques of welfare measurement when there are feedback effects into the intervened-in market through both supply and demand channels. The following section pursues this suggestion, provides results not anticipated in the earlier literature, and suggests practical guidelines for the use of general equilibrium welfare measurement.

---

[3]It is interesting to note that for price and quantity wedges, both general equilibrium demand and supply curves are less elastic than their partial equilibrium counterparts. This holds true both for complements as well as substitutes in demand and supply.

4. The Welfare Significance of D^* and S^* with Two Channels of Equilibrium Feedback

Consider now the case in which a tax is imposed on good one when the two goods are substitutes both in demand and supply. For example, corn and soybeans are both alternative crops for many farmers and alternative inputs into animal feeds. In such circumstances, an increase in P_1 shifts out D_2 and shifts back S_2. Imposing a per unit tax of τ raises the demand price of the taxed good, P_1, and lowers the supply price, $\tilde{P}_1$. In the second market, these price changes shift out both S_2 and D_2 so that P_2 can increase or decrease. Figure 3 is drawn so that the demand substitution outweighs the supply substitution and P_2 increases.[4]

The after-tax equilibrium is shown as the pair of prices P_1^1 and $\tilde{P}_1^1$ in the Q_1 market and the single price P_2^1 in the Q_2 market. In the Q_1 market, D_1^* denotes points of consumer equilibrium for various levels of the tax, τ. S_1^* similarly represents points of producer equilibrium. In the Q_2 market the arrowed line is the equilibrium (Q_2,P_2) path as the tax rate is varied from 0 to τ. As before, consumers are assumed to be compensated to a constant utility level.

The natural extension of the previous analysis is to interpret the welfare significance of surplus areas behind S_1^* and D_1^*. However, in this instance, S_1^* and D_1^* do not have welfare significance individually, but the welfare triangles constructed from S_1^* and D_1^* do. To demonstrate this fact, we will again exploit the equilibrium dependence of P_2 and P_1 on τ which, if the relationships are

---

[4]In fact, with both supply and demand substitution, the signs of the changes in <u>both</u> P_1 and P_2 are ambiguous. This ambiguity does not affect the analysis but raises the possibility of a downward sloping general equilibrium supply curve or an upward sloping general equilibrium demand curve.

monotonic, implies an equilibrium functional relationship $P_2(P_1)$.[5] Again, for a given P_1 consistent with some Π, $P_2(P_1)$ is the price that equates supply and demand in the second market.

First, tally up the consumer welfare losses that result from the tax. In the case of Figure 3 consumers pay higher prices for Q_1 and Q_2, the compensating variation for which can be measured as follows:

$$\Delta CW = m(P_1^1, P_2^1) - m(P_1^o, P_2^o)$$

$$= m\,[P_1^1, P_2(P_1^1)] - m[P_1^o, P_2(P_1^o)]$$

$$= \int_{P_1^o}^{P_1^1} \frac{d}{dP_1} m[P_1, P_2(P_1)]\, dP_1$$

$$(9) \qquad = \int_{P_1^o}^{P_1^1} \frac{\partial}{\partial P_1} m[P_1, P_2(P_1)]\, dP_1 + \int_{P_1^o}^{P_1^1} \frac{\partial}{\partial P_2} m[P_1, P_2(P_1)] \frac{dP_2}{dP_1}\, dP_1$$

$$= \int_{P_1^o}^{P_1^1} D_1[P_1, P_2(P_1)]\, dP_1 + \int_{P_1^o}^{P_1^1} D_2[P_1, P_2(P_1)] \frac{dP_2}{dP_1}\, dP_1$$

$$= \int_{P_1^o}^{P_1^1} D_1^*(P_1)\, dP_1 + \int_{P_2^o}^{P_2^1} Q_2^*(P_2)\, dP_2 \,,$$

where $Q_2^*(P_2)$ is the equilibrium dependence of Q_2 on P_2 pictured along the path in Figure 3.

---

[5]If either P_1 or P_2 are non-monotonic in τ, the general results still hold, but the integrals in (9) must be broken up into intervals over which the relationships are monotonic. Nothing important is lost by ignoring this complication.

Noting that the first term on the right-hand side of the last line of (9) is the consumer surplus change behind D_1^*, (9) can be re-written as:

$$\Delta CS_1^* = \Delta CW - \int_{P_2^o}^{P_2^1} Q_2^*(P_2)\, dP_2 \ . \tag{10}$$

The non-significance of D_1^* under supply and demand substitution can now be seen. The integral on the right-hand side of (10) has no welfare significance and so D_1^* has no welfare significance.

A similar result holds for S_1^*. Proceeding as in the previous section, the loss in profits due to the tax is:

$$\begin{aligned}
\Delta\Pi &= \Pi(P_1^o, P_2^o) - \Pi(\tilde{P}_1^1, P_2^1) \\
&= \Pi[P_1^o, P_2(P_1^o)] - \Pi[\tilde{P}_1^1, P_2(\tilde{P}_1^1)] \\
&= \int_{\tilde{P}_1^1}^{P_1^o} \frac{d}{d\tilde{P}_1}\, \Pi[\tilde{P}_1, P_2(\tilde{P}_1)]\, d\tilde{P}_1 \\
&= \int_{\tilde{P}_1^1}^{P_1^o} \frac{\partial}{\partial \tilde{P}_1}\, \Pi[\tilde{P}_1, P_2(\tilde{P}_1)]\, d\tilde{P}_1 + \int_{\tilde{P}_1^1}^{P_1^o} \frac{\partial}{\partial P_2}\, \Pi[\tilde{P}_1, P_2(\tilde{P}_1)] \frac{dP_2}{d\tilde{P}_1}\, d\tilde{P}_1 \\
&= \int_{\tilde{P}_1^1}^{P_1^o} S_1[\tilde{P}_1, P_2(\tilde{P}_1)]\, d\tilde{P}_1 + \int_{\tilde{P}_1^1}^{P_1^o} S_2[\tilde{P}_1, P_2(\tilde{P}_1)] \frac{dP_2}{d\tilde{P}_1}\, d\tilde{P}_1 \\
&= \int_{\tilde{P}_1^1}^{P_1^o} \overset{*}{S}_1(\tilde{P}_1)\, d\tilde{P}_1 + \int_{P_2^1}^{P_2^o} S_2[\tilde{P}_1(P_2), P_2]\, dP_2
\end{aligned}$$

$$= \Delta PS_1^* + \int_{P_2^1}^{P_2^o} Q_2^*(P_2)\, dP_2 ,$$

where $P_2(\tilde{P}_1)$ is the equilibrium functional relationship between P_2 and $\tilde{P}_1$ induced by the dependence of each on τ and $\tilde{P}_1(P_2)$ is its inverse. $Q_2^*(P_2)$ is, again, the equilibrium path in the second market as the tax is raised from zero to τ and P_2 increases from P_2^o to P_2^1. Therefore, it follows that:

$$\Delta PS_1^* = \Delta\Pi + \int_{P_2^o}^{P_2^1} Q_2(\overset{*}{P}_2)\, dP_2 . \tag{11}$$

Just as (10) demonstrates that the surplus area behind D_1^* is not meaningful, (11) demonstrates that the area behind S_1^* is not meaningful. S_1^* has no welfare significance because the RHS integral has no welfare significance. It is evident from (10) and (11), however, that the single-market welfare analysis is salvaged by combining the two general equilibrium measures:

$$\Delta CS_1^* + \Delta PS_1^* = \Delta CW + \Delta\Pi . \tag{12}$$

Subtracting tax revenue from (12) gives the familiar triangle, measured with respect to D_1^* and S_1^*, as the net welfare cost of the tax across the related markets. In the general context of supply and demand substitution, this is the result claimed by Harberger and proved by Just, Hueth, and Schmitz.

The fact that D_1^* has no welfare significance sheds light on the special-case results of the previous sections. In the current case, as before, the surplus area behind D_1^* gives the total welfare loss to consumers in both markets minus an equilibrium integral of quantity in the second market (equation 10). In the case where only demand shifts in the second market, the equilibrium path moves along the stationary supply curve. The integral thus provides the change in producer surplus. When both S_2 and D_2 shift in response to the tax, the quantity integral has no particular meaning. The same argument applied to (11) reveals why the change in general equilibrium producer surplus measures the consumer surplus change in the related market when there is only supply substitution.

A further complicating issue when there is both supply and demand feedback is that the definitions of S^* and D^* become policy-dependent. For example, Figure 4 displays an analysis of the same Q_1 and Q_2 markets when a price support is introduced. The effect of the price support is to shift out D_2 and shift back S_2 (Q_1 and Q_2 are substitutes both in demand and supply) thereby unambiguously increasing P_2. The increase in P_2 shifts out D_1 and shifts back S_1. The resulting equilibrium curves in the first market are S_1^* and D_1^*. It is evident that S_1^* is different in Figures 3 and 4 because S_1^* is flatter than the partial equilibrium curves in Figure 3 but steeper in Figure 4. In Figure 4 as in Figure 3, D_1^* and S_1^* can be used together for a correct multi-market analysis, but have no meaning individually.

5. Conclusion

The non-significance result has important practical implications. With only one channel of feedback, say through demand substitution, one can legitimately conduct a partial analysis by estimating and analyzing D_1^* (see, for example, Thurman and Wohlgenant, and Rucker and Thurman.) The presence of a second feedback channel invalidates such a partial analysis; only a joint analysis of consumer and producer surplus change is correct.

The policy dependence of D^* and S^* is practically important in assessing empirical estimates of general equilibrium supply or demand curves. For example, knowing that the general equilibrium supply is less elastic than the partial equilibrium supply in the case of a quantity wedge (price support) and more elastic for a price wedge (tax) tells the analyst whether or not an empirically-estimated general equilibrium supply function is consistent with the assumed market structure.

Finally, it should be noted that D_1^* and S_1^* regain their individual welfare significance when there are two feedback channels, but with the demand substitution and supply substitution occuring in different markets. That is, if Q_1 is a substitute in demand with Q_2, but a substitute in supply with Q_3, then D_1^* can be used to measure welfare effects on consumers in the Q_1 and Q_2 markets and on producers in the Q_2 market. S_1^* can be used to measure welfare effects on producers in the Q_1 and Q_3 markets and on consumers in the Q_3 market. Legitimate partial analyses based on either S_1^* or D_1^* are then possible.

REFERENCES

Anderson, James E., "The Social Cost of Input Distortions: A Comment and a Generalization" American Economic Review (1976) 66:235-238.

Carlton, Dennis W., "Valuing Market Benefits and Costs in Related Output and Input Markets" American Economic Review (1979) 69:688-696.

Harberger, Arnold C., "The Measurement of Waste," American Economic Review (1964) 54:58-76.

Harberger, Arnold C., "Three Basic Postulates of Applied Welfare Economics: An Interpretive Essay," Journal of Economic Literature (1971) (:785-97.

Just, Richard E. and Darrell L. Hueth, "Multimarket Welfare Measurement," American Economic Review (1979) 69:947-54.

Just, Richard E., Darrell L. Hueth, and Andrew Schmitz Applied Welfare Economics and Public Policy (1982) Prentice-Hall, Englewood Cliffs, N.J.

Panzar, John C. and Robert D. Willig, "On the Comparative Statics of a Competitive Industry with Inframarginal Firms," American Economic Review (1978) 68:474-478.

Rucker, Randal R. and Walter N. Thurman, "The Economic Effects of Supply Controls: The Simple Analytics of the U.S. Peanut Program," forthcoming in the Journal of Law and Economics (October 1990).

Thurman, Walter N. and Michael K. Wohlgenant, "Consistent Estimation of General Equilibrium Welfare Effects," American Journal of Agricultural Economics (1989) 71: 1041-1045.

Figure 1

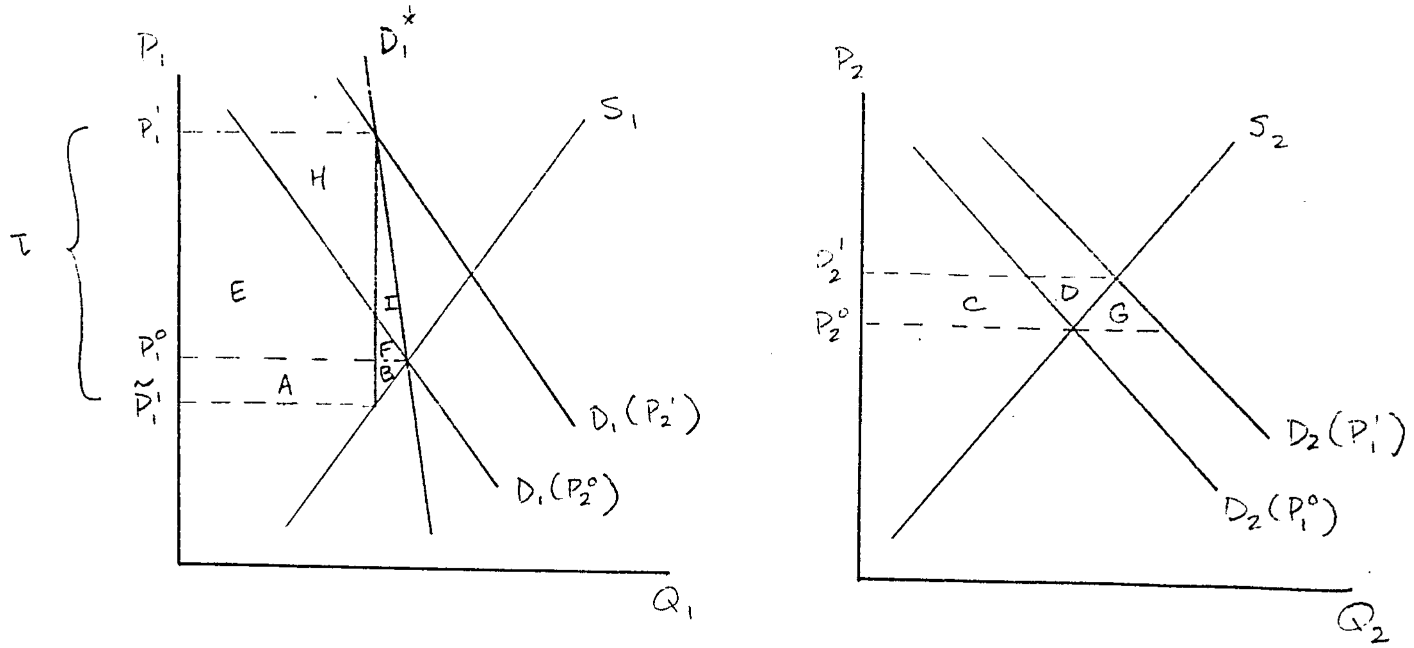

Figure 2

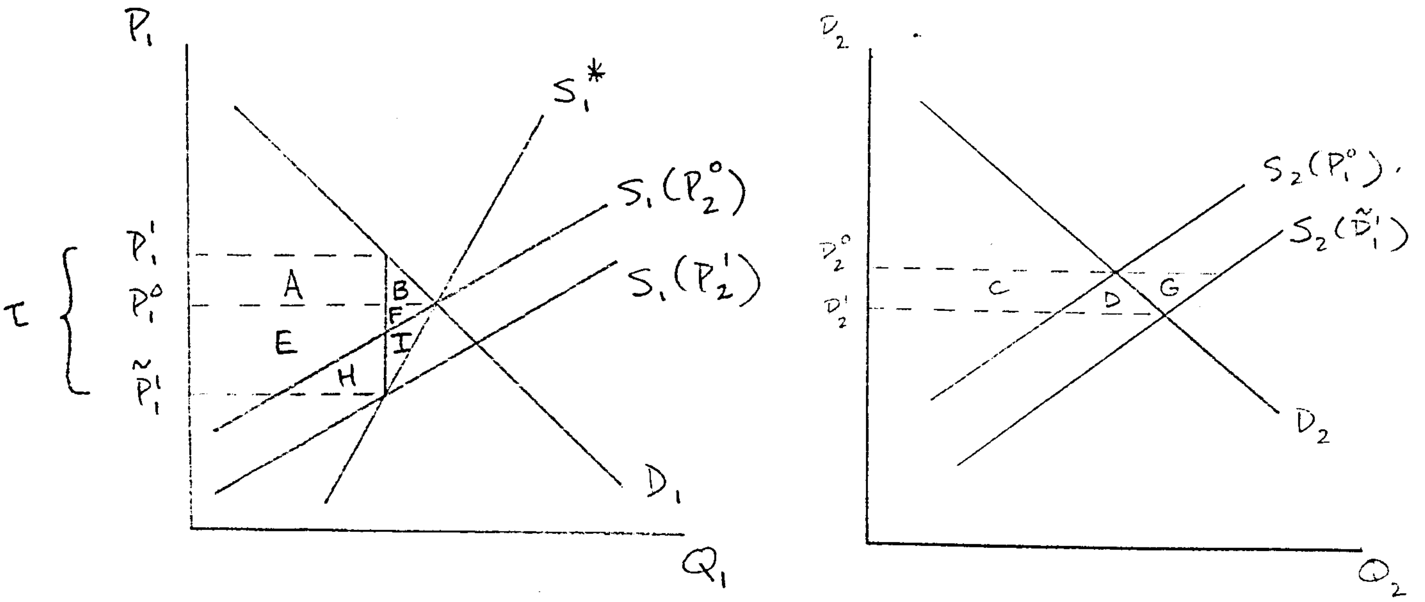

Figure 3

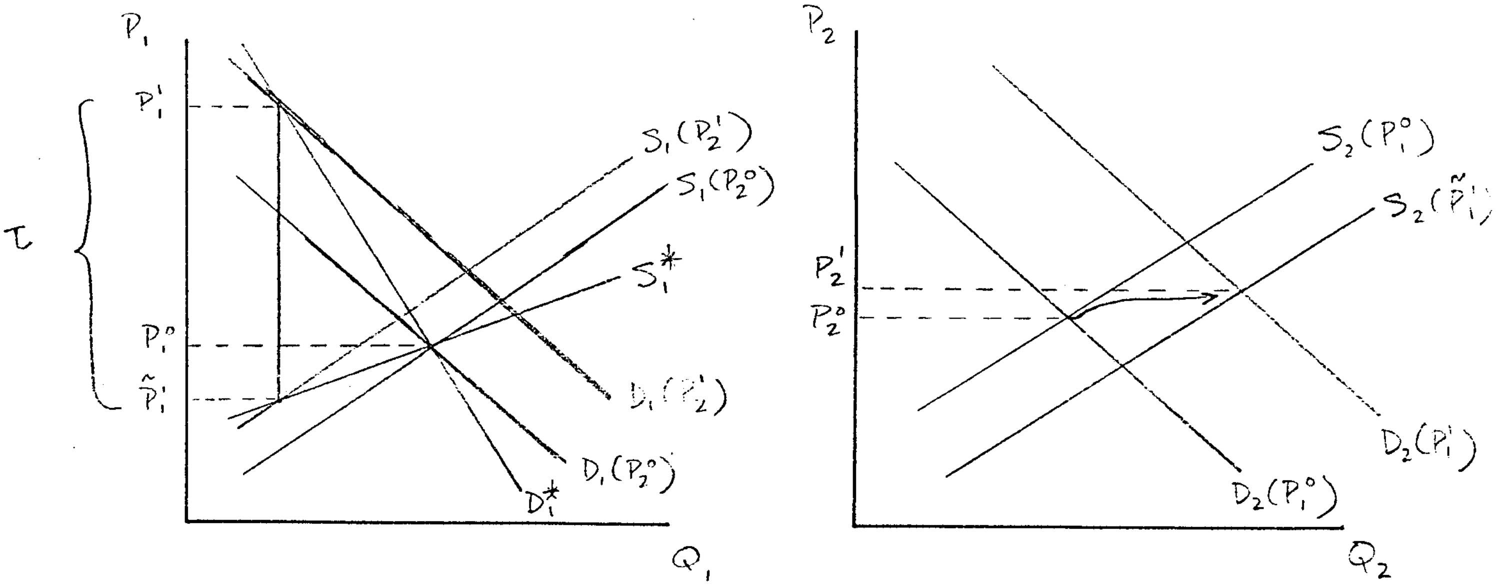

Figure 4