Weighting Implied Volatilities from Soybean and Live Cattle Options*

by

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Abstract

This study addresses the problem of weighting implied volatilities to obtain a single volatility measure for estimating option prices. Four methods were examined. The smallest prediction error was found using an implied volatility weighted by the derivative of the option price with respect to its standard deviation.

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The volatility implied by options on future contracts is defined as that standard deviation which equates the theoretical Black formula to the observed option price. The measure is valuable as an ex ante predictor of futures price variance, and a substantial body of literature has found that this implied volatility (IV) is a more efficient predictor of option prices than historical volatility measures (Park and Sears; Jordan et.al.; Latane and Rendleman; Chiras and Manaster; Beckers; Hauser and Neff). Implied volatilities are of particular value to hedgers and commodity analysts in providing anticipatory futures price changes, but are also of value to researchers studying market efficiency (e.g. Trippi; Schmalansee and Trippi; Beckers; Park and Sears; Hauser and Neff; Chiras and Manaster; Latane and Rendleman; Day and Lewis).

The use of implied volatilities is, however, hampered by the fact that there are many options written on a single futures contract (referred to as a class) and sometimes substantial differences in their implied volatilities. For an analyst or hedger a single measure of the implied volatility for purposes of prediction and evaluation is ideal. Four approaches to finding this measure are evaluated in this paper. The first uses the IV of the option nearest to, or at the money (Beckers), the second uses a simple average of the implied volatilities (Trippi; Schmalansee and Trippi), the third weights the partial derivative of the option price with respect to the standard deviation (Latane and Rendleman) and the fourth weights the implied volatilities by the elasticity of the option price with respect to its standard deviation.

A problem which has yet to be resolved is which measure provides the best ex ante estimate of actual option premiums. The purpose of this paper, therefore, is to determine which of the above methods more closely approximates observed put option premia on soybeans and live cattle futures. The paper differentiates itself from previous research in that no study has compared the predicted and actual option premiums using all four methods on the same data set.

METHOD

This study first calculated the volatilities implied by put options prices on soybeans and live cattle futures. Soybean and live cattle options were used for two reasons. First, the underlying futures contracts are actively traded in high volume and second, soybeans and live cattle are representative of storable and non-storable commodities respectively. Thus, differences in commodity form are taken into consideration. For each option class a single valued implied volatility was determined according to the four methods outlined in the introduction. These single valued volatilities will be referred to, generically, as weighted implied volatilities (WIV). The WIV's are then substituted into the Black option pricing formula to estimate the option price (P). The methods employed are discussed in this section.

Put option prices, (P), on futures contracts are, according to Black's model (for a European put), a function of the current futures price (F), the striking price (S), the riskless interest rate (r), time to expiration (T), and the instantaneous standard deviation of the percentage changes in futures prices (v) and is notated as

(1) $P = (F, S, r, T, v).$

Specifics of the European option formula are discussed by Black, Merton, Asay, Wolf and others. (Barone-Adesi and Whaley provide a solution procedure for American options but find little difference in their premiums to European option formulas.) For any given option, F,S,r, and T are directly observable. However, v is not observed and has to be empirically estimated. Estimates can be obtained by taking the variance of the log change in futures prices but past research has found that these estimates do not perform well with the Black (or Black and Scholes) model. Accordingly researchers and commodity analysts calculate v directly from the option formula using an iterative numerical procedure. In this study implied volatilities were calculated using the Newton-Raphson method (Manaster and Koehler) and a polynomial approximation to the cumulative normal density function. IV's were generated for each option written on each futures contracts from 1986 through 1989.

To develop single valued WIV's for options in each class four methods were employed. The first applied equal weight to all of K options in each class. This is termed the average implied volatility (AVEIV) (Trippi; Schmalansee and Trippi),

$$(2) \quad AVEIV = \sum_{i=1}^{K} V_i \,/\, K$$

The second method uses the IV of the option nearest to, or at, the money (ATIV) (Beckers). This method assumes that most trading takes place with at-the-money options and therefore places zero weight on options which are in or out of the money.

The third method, (DERIV), uses a weighting scheme based on the derivative of the option pricing formula (equation 1) with respect to its standard deviation, P_v, (Latane and Rendleman),

$$(3) \quad DERIV = \left(\sum_{i=1}^{K} V_i^2 \; P_{vi}^2\right)^{1/2} \Big/ \left(\sum_{i=1}^{K} P_{vi}^2\right)^{1/2}$$

Latane and Rendleman argue that this method puts less weight on options within each class which are less sensitive to a precise specification of the market's underlying expectation of the standard deviation.

The fourth method, (ELIV), uses a weighting scheme based on the elasticity of the option pricing formula with respect to its standard deviation, (W) (Chiras and Manaster),

$$(4) \quad ELIV = \sum_{i=1}^{K} W_i V_i \Big/ \sum_{i=1}^{K} W_i ,$$

where W_i equals P_{vi} (V_i/P_i). Chiras and Manaster argue that rational investors are more concerned with the relative percentage change in the option price relative to its standard deviation, rather than the absolute change implied by Latane and Rendleman's method.

These four methods are not without criticism. The AVEIV is criticized because it puts equal weight on all options in a class without considering the relative importance of options which are in and out of the money. The use of ATIV can be criticized by the fact that the information content of options in or out of the money are lost. Day and Lewis criticize the DERIV and ELIV approaches on the basis that the former places more weight on IV's at-the-money while the latter places more weight on options out of the money.

Yet despite these criticisms examining the ex ante performance of the WIV's is still important. Clearly each method biases in one way or another the option price. This study detracts from the theoretical limitations assigned to each of the methods to follow a more pragmatic approach. That is, to determine which method provides the minimum prediction error.

Soybean and live cattle data used were compiled from the options and futures tapes of the Chicago Mercantile Exchange from 1986-1989. Interest rates were calculated as the annualized returns reflected in the nearby U.S. 90 day T-Bill futures. The IV's for each option were estimated to within a .0001 tolerance level of the actual put option price. WIV's were calculated for each class of options according to the four methods described above. These volatility measures were used to estimate same day premiums, and predict 1, 7, 21 and 60 day ahead premiums. The estimated forecast premiums are ex ante with respect to the volatilities only. The interest rates, futures prices and strike prices are all ex post measures.

To determine which of the four WIV's is the best ex ante predictor several measure are reported in this study including the mean prediction error (actual-predicted), the percentage error, and the standard deviation of the prediction error. These measures are reported for the total time period used, as well as control checks using the periods before and after the 1988 drought (May 1988).

It was anticipated that the production error was strongly related to the difference in the WIV relative to the actual IV, and from the literature cited, whether or not the option was in or out of the money.

To examine the ex ante predictive power of the various WIV's the following regression, with the intercept suppressed was run;

$$(5) \quad (P_i - \hat{P}_i) = B_1(IV_i - WIV) + B_2 DIN + B_3 DOUT + e,$$

where P_i is the actual premium, $\hat{P}_i$ is the premium estimated using the WIV's, DIN is a dummy variable with a value 1 if the option was in the money and 0 otherwise, DOUT is a dummy variable with value 1 if the option was out of the money and zero otherwise, and e is the residual error term. The purpose of (5) is two fold. First B_1 measures how much of the error in the estimate is due to the weighting method used and second, B_2 and B_3 measure the degree by which the option is in or out of the money biases the estimate.

RESULTS

This section presents the results of the study. Results are presented in two sections. First, summary statistics for the weighted implied volatilities and the errors in predicted values are presented and discussed. This is followed by a presentation and discussion of the regression results.

Summary Statistics

Table 1 summarizes the means and standard deviations (for the same day forecast) of the WIV's over the entire period of analysis and the pre and post drought periods. General results over the total period for live cattle indicate the WIV averages are very similar. The smallest WIV is attributed to ATIV with a value of .1441 on average. However the least variable WIV is AVEIV with a mean of .1540 and a standard deviation of .0235. The results for soybeans are however different. Like the live cattle options the lowest WIV is ATIV with an average WIV of .1634. ATIV also exhibits the lowest standard deviation which is .0312. There are substantial differences between soybeans and live cattle WIV's. Soybean volatility is consistently higher than live cattle volatility and the standard deviations were also higher. This relationship also holds for the pre and post drought control periods, with the sole exception being that the standard deviation of DERIV for the drought period is lower than the corresponding live cattle standard deviation.

The results in Table 1 provide information as to the relative WIV measures. Consistently ELIV gives the highest value, followed by AVEIV, DERIV and ATIV. It should also be noted that the mean WIV for AVEIV is also the average (unweighted) mean volatility measure (IV) of the total sample. Thus ELIV as a weighted volatility measure is consistently greater than the average IV, whereas ATIV and DERIV are less.

It is also worthy of note that the differences in the mean WIV's between the pre-drought and drought period are significantly different at the 1% level. It is surprising, however, that the mean volatilities were lower in the post drought period.

Table 2 through 4 summarizes the differences between the actual put premiums and those estimated using the WIV. A negative element indicates that the actual put premium is overestimated. For both soybeans and live cattle the mean prediction errors are small for all of the forecasts considered. However, they are not statistically different from zero. As the forecast period increases so does the variance. This result is expected. For example the mean error for the soybean same-day prediction using AVEIV is -4.86 cents with a standard deviation of .1475 cents. The % error is only 3.17. For the 60 day forecast the mean prediction error is -3.15 cents (3.39%) but the standard deviation is more than doubled to 30.76 cents.

The results in Tables 2 through 5 indicate a general (but not wholly consistent) result that AVEIV and ELIV measures have a tendency to overestimate the actual premiums whereas ATIV and DERIV tend to underestimate the premiums. This generality holds for both the pre drought and drought periods.

There are also differences in the mean prediction errors of the pre drought and drought periods. The tendency in Tables 3 and 4 is for the mean prediction error, % error and standard deviation to be larger in the drought period. This result was expected, but a direct relationship between these differences and the WIV's cannot be deemed conclusive.

The results in Tables 2 through 4 provide an indication as to which volatility measure is best. In terms of the % prediction error the derivative weighted implied volatility appears to be best since in most cases the mean prediction error was less than 1%. Strengthening this conclusion is the fact that the % prediction error was mostly below 1% for both the pre drought and post drought period. In contrast the remaining WIV measures showed a substantial increase in the % error.

The results imply that the DERIV measure best captures the underlying behavioural characteristics of the put options. The explicit characteristic considered is that the option premia is positively related with its volatility. Thus, an anticipated increase in the volatility implies a marginal increase in the option price. Weighting the implied volatilities by this derivative appears to capture this effect. The practical implication of this result is that hedgers, researchers, and market analysts may fare well by using the derivative weighted implied volatility to estimate put premiums.

Regression Results

In the previous section results about the weighted implied volatilities and the prediction error associated with these volatilities were presented. The results indicated significant differences in the four WIV measures. It was implied that these differences accentuated the prediction errors. However, an alternative (and joint) hypothesis is that the prediction errors are also related to whether or not the actual option observation was in or out of the money. Given that most criticisms of the various measures are related to this point it is important that resulting degree of prediction error be measured. Equation (5), in its estimated form, is intended to examine the degree by which the WIV's, and whether or not the actual option was in or out of the money, affects the prediction error.

The results of regression (5) are presented in this section for soybeans and live cattle The regressions for live cattle are presented as equation 6 with t statistics in parenthesis.

6a)	AVDIF	=	5.1901 (IV-AVIV)	-	.0526 DIN	-	.0358 DOUT,	F = 10142
			(150.918)		(-51.95)		(-57.95)	
6b)	ATDIF	=	1.393 (IV-ATIV)	+	.0133 DIN	+	.0221 DOUT,	F = 4818
			(73.33)		(19.70)		(48.49)	
6c)	DERDIF	=	4.1800 (IV-DERIV)	-	.0213 DIN	-	.0193 DOUT,	F = 6320
			(136.30)		(13.88)		(-12.97)	
6d)	ELDIF	=	7.486 (IV-ELIV)	-	.0673 DIN	-	.0394 DOUT,	F = 18276
			(197.34)		(-53.99)		(-51.98)	

The dependent variables in (6) are the difference between the actual and estimated put option premiums for live cattle. The prefixes correspond to the type of WIV used. A positive coefficient implies that $d(P-\hat{P})$ is positive which implies that the predicted premium is less than the actual premium. Similarly a negative coefficient estimate implies that the predicted premium is greater than the actual premium. The variable (IV-WIV) is the difference between the actual implied volatility and the weighted implied volatility. For each of the WIV measures the estimated coefficient is positive. This implied that the predicted premiums overestimate the actual premiums when WIV is greater than IV and underestimates them when the WIV is less than IV. The results indicate that the error in prediction is explained in part by the difference between the actual IV's and the WIV's. The AVIV and ELIV measures tend to have the greatest effect on the dependent variable with coefficients of 5.19 and 7.49 respectively. That is, for example, a 1 cent difference in the volatility measures increases the prediction error by 5.19 cents. The ATDIF and DERDIF prediction errors are less affected by the IV differential (1.39 and 4.18 respectively) thus implying that obtaining WIV's from these methods provides the least prediction error.

The effect due to whether or not the option was in or out of the money are represented by the coefficients on DIN and DOUT. The results indicate that for AVIV, DERIV and ELIV the premium was overestimated and for ATIV it was underestimated when the option was both in and out of the money.

The regression results for soybeans are presented as equation (7).

7a)	AVDIF =	2.3457 (IV-AVIV) (03.79)	-	.0792 DIN (-39.73)	-	.406 DOUT, (-40.56)	F =	4288
7b)	ATDIF =	.2917 (IV-ATIV) (26.10)	+	.0027 DIN (35.54)	-	.0289 DOUT, (-18.18)	F =	1050
7c)	DERDIF =	.5675 (IV-DERIV) (54.74)	-	.0243 DIN (2.55)	+	.0044 DOUT, (-31.32)	F =	474
7d)	ELDIF =	3.1626 (IV-ELIV) (115.59)	-	.1049 DIN (-45.17)	-	.0538 DOUT, (46.55)	F =	7046

The results for soybeans are similar to the live cattle results. Differences between IV and, AVIV and ELIV contribute most to the error in premium estimates while ATIV and DERIV contribute the least. AVIV and ELIV overestimate the premiums for options both in and out of the money. ATIV, tends to underestimate the premiums when options are in the money and overestimate them when they are out of the money, whereas DERIV tends to underestimate options out of the money and over estimates options in the money. The results imply that, at least for soybeans, the derivative weighted volatility is least sensitive to options which are in or out of the money. The evidence supports the use of DERIV as the primary method for evaluating put options on soybean futures (although using ATIV also provides a fairly good estimate).

CONCLUSIONS

The volatility of futures price movements is an important measure for hedgers, market analysts, and researchers. It provides an ex ante estimate of the market's perception of the futures price variance. It has been shown in past research that the implied volatility is a better ex ante predictor of variance than historical data when options are valued with the Black, or Black-Scholes, option pricing model. However, it has also been noted in past studies that options written on a single futures contract often have different valued implied volatilities. In response to this problem several researchers introduced various methods to weight the implied volatilities into a single valued, weighted implied volatility. Four of these methods were evaluated in this paper with the purpose of determining which method provides the best ex ante estimates of premiums on soybean and live cattle put options. The methods evaluated were, an average of the IV's; giving full weight to options nearest, or at, the money; weighting the IV's by the derivative of the put option price with respect to it's standard deviation; and finally weighting the IV's by the elasticity of the option price with respect to its standard deviation.

The results show that weighting the implied volatilities by the derivative of the option provided predictions which were, on average, better than the other measures. This conclusion was reached by examining sample means but was also supported by regression results which showed that prediction errors for options in and out-of-the-money were also relatively low. This conclusion provides a strong recommendation for volatility measurement to hedgers, analysts and researchers who routinely predict option prices using Black's option pricing model on commodity futures.

Table 1: Weighted Implied Volatilities for Soybeans and Live Cattle

	Observations	AVEIV	ATIV	DERIV	ELIV
		Total Period			
Soybeans	17488	.1800 (.0398)	.1634 (.0312)	.1670 (.0343)	.1857 (.0428)
Live Cattle	20199	.1540 (0235)	.1448 (.0244)	.1480 (.0272)	.1578 (.0254)
		Predrought			
Soybeans	10337	.1845 (.0397)	.1732 (.0304)	.1761 (.0365)	.1888 (.0417)
Live Cattle	13828	.1546 (.0240)	.1482 (.0242)	.1501 (.0243)	.1571 (.0255)
		Drought			
Soybeans	7151	.1735 (.0391)	.1492 (.0267)	.1539 (.0255)	.1814 (.0440)
Live Cattle	6371	.1528 (.0223)	.1375 (.0233)	.1434 (.0323)	.1596 (.0248)

Table 2: Mean Prediction Error by Weighted Implied Volatility for Soybean and Live Cattle Options

Time Period	Number of Observations	AVEIV Mean	AVEIV %	ATWIV Mean	ATWIV %	DERIV Mean	DERIV %	ELIV Mean	ELIV %
				Soybeans					
0	17488	-.0486 (.1475)	3.17	0.284 (.0827)	1.85	.0059 (.1123)	.39	-.0824 (.1845)	5.37
1	17164	-.0483 (.1722	3.16	.0270 (.1252)	1.77	.0047 (.1521)	.31	-.0814 (.2003)	5.33
7	16010	-.0428 (.2363)	2.91	.0212 (.2148)	1.44	.0005 (.2301)	.03	-.0713 (.2527)	4.84
21	13430	-.0321 (.2982)	2.37	.0119 (.2891)	.88	-.0062 (.2993)	.46	-.0511 (.3043)	3.78
60	7301	-.0315 (.3076)	3.39	-.0132 (.3039)	1.42	-.0246 (.3130)	2.65	-.0377 (.3084)	4.07
				Live Cattle					
0	20199	-.0410 (.1107)	2.30	.0324 (.0562)	1.81	.0049 (.1272)	.274	-0.765 (.1574)	4.28
1	20030	-.0418 (.1331)	2.34	.0310 (.0893)	1.74	.0039 (.1461)	.22	-.0771 (.1717)	4.32
7	19004	-.0446 (.2035)	2.60	.0240 (.1745)	1.39	-.0015 (.2080)	.09	-.0791 (.2287)	4.61
21	16654	-.0389 (.2724)	2.47	.0198 (.2515)	1.26	-.0024 (.2698)	.15	-.0691 (.2888)	4.39
60	10645	.0171 (.3417)	1.52	.0460 (.3322)	4.10	.0335 (.3357)	2.99	.0034 (.3467)	.30

Table 3: Mean Prediction Error, Pre and Post Drought, Soybeans

Time Period	Number of Observations	AVDIF Mean	AVDIF %	ATDIF Mean	ATDIF %	DERDIF Mean	DERDIF %	ELDIF Mean	ELDIF %
				Pre Drought					
0	10337	-.0325 (.1223)	1.99	.0162 (.0809)	.99	.0014 (.1302)	.86	-.0552 (.1456)	3.38
1	10111	-.0326 (.1558)	2.01	.0147 (.1282)	.90	.0000 (.1729)	.00	-.0546 (.1686)	3.36
7	9302	-.0280 (.2282)	1.79	.0112 (.2201)	.71	-.0222 (.2456)	1.42	-.0458 (.2324)	2.92
21	7505	-.0169 (.2802)	1.17	.0073 (.2774)	.503	-.0037 (.2962)	.37	-.0267 (.2820)	1.84
60	3415	.0389 (.2788)	3.90	.0441 (.2803)	4.76	.0369 (.3002)	3.98	.0402 (.2739)	4.34
				Drought					
0	7151	-.0719 (.1751)	5.16	.0462 (.0821)	3.32	.0123 (.0791)	.88	-.1218 (.1218)	8.75
1	7053	-.0707 (.1909)	5.10	.0448 (.1185)	3.23	.0115 (.1156)	.83	-.1199 (.2332)	8.64
7	6708	-.0633 (.2456)	4.72	.0351 (.2063)	2.62	.0042 (.2065)	.31	-.1066 (.2746)	8.12
21	5925	-.0514 (.3185)	4.19	.0177 (.3031)	1.44	-.0095 (.3032)	.75	.0820 (.3279)	6.44
60	3886	-.0935 (.3182)	10.1	-.0635 (.3149)	6.84	-.0787 (.3141)	8.49	-.1061 (.3206)	11.44

Table 4: Mean Prediction Error, Pre and Post Drought, Live Cattle

Time Period	Number of Observations	AVDIF Mean	AVDIF %	ATDIF Mean	ATDIF %	DERDIF Mean	DERDIF %	ELDIF Mean	ELDIF %
				Pre Drought					
0	13828	-.0268 (.0797)	1.44	-.0219 (.0485)	1.17	-.0053 (.0469)	.28	-.0479 (.1192)	2.57
1	13697	-.0274 (.1137)	1.48	.0209 (.0908)	1.13	.0044 (.0926)	.237	-.0482 (.1409)	2.58
7	12911	-.0278 (.1910)	1.56	.0167 (.1745)	.93	.0012 (.1787)	.07	-.0474 (.2058)	2.65
21	11117	-.0153 (.2497)	.94	.0196 (.2374)	1.21	.0066 (.2424)	.40	-.0303 (.2561)	1.85
60	6816	.0611 (.3361)	5.23	.0746 (.3361)	6.38	.0681 (.3385)	5.83	.0565 (.3326)	4.83
				Drought					
0	6371	-.0718 (.1538)	4.45	.0551 (.0646)	3.42	.004 (.2157)	.25	-.1385 (.2051)	8.59
1	6331	-.0729 (.1632)	4.54	.0529 (.0818)	3.29	.0026 (.2212)	.16	-.1397 (.2112)	8.69
7	6093	-.0801 (.3077)	5.10	.0396 (.2778)	2.52	-.0072 (.3169)	.46	-.1463 (.3319)	9.32
60	3829	-.0611 (.3377)	4.45	-.0049 (.3189)	.47	-.0280 (.3215)	2.70	-.0910 (.3512)	8.77

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