Public Disclosure by ‘Small’ Traders

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Abstract

We model strategic trading by a rent-seeking insider, who exchanges without being spotted, and propose a comprehensive theory of market non-anonymity. Several novel results are established. They depend on asset value proprieties, beliefs, inter-temporal choices, and investors' characteristics. In equilibrium, under a regulation mandating public trade revelation, disclosures may shift prices. If they do, uninformed manipulations arise only in some instances. Specifically, insiders constrained on asset holdings earn more than they would without such a disclosure rule. Consequently, mandating disclosures is unnecessary, as informative trades will be revealed voluntarily. This result reveals a previously unexplored link to the literature on (uncertified/non-factual) announcements.

Keywords: Mandatory vs. voluntary public disclosure, securities regulation, insider trading, market manipulation.
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Public disclosure of inside statements always receives great attention in capital markets. Paradoxically, following the seminal work of Benabou and Laroque (1992), hereafter BL, on market manipulation and credibility, where insiders may produce false announcements and trade on the mispricing, there have been few attempts to develop conceptual models that study these strategic disclosures. Nowadays, the extent to which an inside statement conveys information is, more than ever, the object of a considerable debate. This is also true for statements that certify the undertaken trade, which the SEC and various European regimes, among others, require to be made public soon after the trade has been made. On this latter issue, three influential studies by Fishman and Hagerty (1995), John and Narayanan (1997), and Huddart et al. (2001), hereafter FH, JN, and HHL respectively, advance our understanding by focusing on big traders; but small-sized investors must also disclose trades publicly.[1]

This paper considers *small* traders—i.e., traders whose transactions cannot be spotted—who are subject to a so called capital constraint or risk limit,[2] and proposes a comprehensive theory of market non-anonymity. We examine public disclosure to interpret the effects of *mandatory* and *voluntary* reports about undertaken *trades,* and establish several novel results, including: (1) Disclosures do not always affect prices; (2) when they do, only in specific instances the investor, when uninformed, manipulates the market; and (3) for disclosure to be forthcoming, it does not have to be mandatory, as the investor will disclose informative trades voluntarily. The first two results depend on the asset value properties; on alternative (but correct) market beliefs associated with disclosure; on the weight assigned to present and future profits (that is, on the inter-temporal discount factor); and on the trader's characteristics, which translate into how likely he is to know about the real asset value today and to have inside information in the future. The third result not only tells us that regulators do not need to make laws against missed trade reporting and invigilate for it—rather, they need to identify who best should be allowed to report trades of a specific stock—it also represents the intermediate step to extend our study to the voluntary disclosure of *(uncertified/non-factual) announcements*, which can be spread, for example, through the media in concert with journalists (e.g., see Sobel (2000), p. 248) or by starting rumors, with predictions in line with the first two points above. These predictions do not rely on the assumption of a trader that (with positive probability) reports information 'honestly,' conversely imposed in previous models of inside announcements.

In order, let's first consider mandatory trade disclosure, with each trade compulsorily revealed after it is executed, and before the next order can be placed. A small trader could use public disclosure as a lever to move the asset price and enhance profits. Intuitively, while his orders do not affect prices, their disclosure could. However, if he is constrained on asset holdings, for any properties of the asset value, even public disclosure has no price impact—in other words,

[1] E.g., the Market Abuse Directive (EU Directive 2003/6/EC) lists traditionally small investors, such as managers, members of the supervisory board, employees/members of staff that could have private information, and their spouses, partners, and relatives. The (US) Securities Exchange Act refers to big traders—the 'principal stockholders'—but also to most firms' officers and directors on one side (SEC(2004), Section 16), and to relatively big traders on the other (SEC(2004), Section 13), the latter disclosing if the change in ownership amounts to at least 1% of the firm's stock. The Securities and Exchange Board of India Act refers to all the investors listed above, mandating those owning less than the 5% of the firm's stock to disclose when the change exceeds a very small quantity (e.g., Rs. 5 lakh in value), while setting a high threshold for bigger stockholders.

[2] This constraint makes the maximum number of shares that they may exchange today dependent on previous changes in their asset holdings. Consider an investor who currently holds no asset, and may trade up to a cap on total exposure equal to, say, 100 shares. If this trader starts by buying 30 units of the security, in another moment he may be buying again, up to a further 70 units, or sell, up to 130 units. This sort of position limit differs from that of an investor with unlimited trading capacity, assumed in HHL, or from that of a trader that can buy or sell up to an identical, finite quantity per trading-date, considered in FH and JN; it enriches, in a simple way, the strategy space by adding an inter-temporal dimension to how much the trader may exchange.

it is correctly believed to be *un*informative. To see what would happen otherwise, we consider a standard two-round trading model, and show that, if prices reacted *somehow* to disclosure (or its absence), when informed the trader would *in probability* deceive other market participants completely. Consequently, the market anticipates this behavior, ignoring disclosures, which makes our investor earn as much as under anonymity, where no signal is disclosed.

Indeed, only in some instances is a trader understood to possess private information just once, for contingent reasons. In general, because of his specific characteristics, he typically tends to be thought of as being in the position to acquire *new* private information again, at some (unknown, unless he is systematically informed with certainty) point in the future. To model this latter form of informational asymmetry, as in BL, we employ an infinite-horizon repeated framework.[3] Focusing on a two-round repeated structure, suppose for instance that, at any point in time, current disclosures are believed to be informative—specifically, the disclosure of a purchase is known to push the price just as far up as a sale disclosure pushes it down—unless (recent) past disclosures moved prices away from the real value.

As long as disclosures are known to affect current prices, at that repetition a trader that turns out to be informed may pick (or alternate between) one of the following two strategies. He may trade up to his maximum (which can but does not have to be common knowledge) to *lead* the price toward the right direction, earning as much as under anonymity, and subsequently profiting once again by reversing his position completely, in the same repetition, if the disclosure causes the price to overshoot the real value. Otherwise, he may *mislead* the market, trading in the opposite direction and reversing his position afterwards. The latter strategy—which in JN may be of equilibrium when the asset value distribution displays unequal mass below and above its mean—allows our trader to earn more than from leading in the current repetition, but only as much as under anonymity in the (next) future, when disclosures start to be ignored. Indeed, as in Allen and Gale's (1992) study, the market cannot determine if our investor is actually trading on information. Thus, when uninformed, he may manipulate, pretending to be informed—in jargon, *bluffing* (Harris (2002))—that is, randomly disclosing that he has bought or sold, which moves the price up or down respectively, then reversing his initial position. This strategy—first examined in FH, where the trader manipulates whenever uninformed—in expectation allows the investor to earn more than from not trading in the current repetition; but, if prices are pushed by chance in the wrong direction, future profits will be reduced. Hence, our trader may prefer to alternate between bluffing and *not trading*, or choose the latter.

The solution to this problem brings to the identification of three regions corresponding to different equilibria, in two of which disclosures are (at least partially) informative—the consequences being price shifts—and one where disclosures are not at all informative. Prices never shift when the weight granted to future profits is small, as if they did, the trader would systematically mislead the market. Conversely, provided he weighs future profits sufficiently, when (or as soon as) disclosures are believed to be informative, he prefers to lead the market whenever informed. Consequently prices react to disclosures. Specifically, the smaller the probability of acquiring information, the more he needs to weight future profits to opt for a non-manipulative strategy when uninformed; otherwise prices react only partially—in proportion to how often he is informed—rather than fully, as he manipulates whenever uninformed.[4] Put differently, there

[3] We make no reference to finite repetitions, as trivial. If our trader acquired private information repeatedly, with positive probability, only up to a certain moment in time—in other words, if he imagined that, at some future date, he was certainly not going to be informed any more—starting from the last repetition and solving backwards, the equilibrium in each repetition would coincide with that derived when no repetition occurs.

[4] The underlying structure is that of a new, important class of supergame—more precisely, of infinitely repeated games with discounting—whose result can be applied in areas of research other than public disclosure.

exists an equilibrium threshold in the likelihood that this trader is informed, which progressively increases as the weight given to future profits shifts from high to medium.[5] For each discount factor associated with this band of inter-temporal preferences, as the probability that he acquires information increases, uninformed manipulations occur less often, up to this threshold, above which he switches behavior, never trading when uninformed. Thus, a trader who is less likely to be informed (e.g., investors not directly involved in the firm's management) will manipulate, while one that is more likely to be informed (e.g., CEOs) will not.

Ever since Kyle (1985), an important strain of literature has focused on an insider that with positive probability leads prices towards the real value, undertaking reversals in case his strategic signal (e.g., the order flow, trade disclosure) causes the price to overshoot the privately known quotation. To model price overshooting, in principle every class of asset value properties is appropriate, other than that of a random variable with two possible realizations assumed in BL, FH, and JN, as these two priors would otherwise systematically bracket equilibrium prices. For tractability, however, this literature, which includes HHL, generally assumes normality. Instead, our predictions hold, whether or not the asset value distribution is continuous or (up to a certain degree) asymmetric, or its support unbounded. While overshooting is *not* due to the imprecision of the signal, the way the market interprets this signal plays a role. In fact, identical dynamics can be identified, whether the trader has to disclose trade *direction* or *size*, because a market response is to interpret any trade of the same direction identically. It follows that, when disclosures are believed to be informative, if the investor trades, he only exchanges up to his (un)observable maximum, which justifies the market reaction in question.

When (or as soon as) prices react to trade revelation, the investor expects to earn as much or more than he would without such a disclosure rule. Consequently, mandating disclosures is unnecessary, as informative trades will be advertised voluntarily. In detail, the trader decides to disclose not only when he knows that the resulting price will overshoot the privately known asset value, but also when it will *under*shoot this value (and thus no profitable reversal is possible). By doing so, he hides *this* information at no cost, so that the price following a disclosure turns out to shift the most (that is, as much as under mandatory disclosure), which ensures the highest occurrence of price overshooting, and the most profitable associated reversal. Clearly, an asset value distribution not preventing price overshooting is required to model voluntary disclosure of informative signals; otherwise, when informed, no small trader has an incentive to disclose.

Even when this investor cannot disclose certified trades, in principle he may still publicly produce uncertified announcements of any sort, provided he does not lie about relevant facts, which is forbidden under most regulations (e.g., SEC(2004), Section 10(b)). In this case, when (or as soon as) announcements are believed to be favorable/unfavorable, the equilibrium price following their disclosure shifts as it does when a certified purchase/sale turns out to be informative. This is why an investor that acquires new information repeatedly—whose equilibrium transactions coincide with those undertaken under the voluntary disclosure of certified trades—has all the incentives to produce these announcements after the initial purchase/sale. Specifically, his incentive to lead the market when informed, as well as his incentive not to manipulate when uninformed, turn out to be unaffected with respect to the case of a certified trade disclosure. Thus, three analogous regions of equilibria exist, in one of which manipulations arise. Indeed, a question exists in literature, whether requiring investors to publicly certify their trades prevents them from producing manipulative announcements (BL, p. 947). Our work suggests that, when mispricings are possible, this resolution makes traders indifferent about making announcements, but does not prevent equivalent trade-based manipulations.

[5] This band of inter-temporal preferences is the most relevant: Discount factors spanning from high to medium translate in interest rates ranging from nearly zero to values well above those in most world economies.

There are at least three ways to justify why the transactions of our insider cannot be spotted: First, with a large market compared to the position he can undertake—in other words, his maximum trading capacity is quantitatively negligible; second, with a market/trader of any size, and an indistinguishably large or low demand—indeed, in complex environments agents not processing all information turn to heuristic rules of thumb and weigh more salient information (Tversky and Kahneman (1974)); third, even a negligible trading pressure by a big insider can be justified, as BL do. They invoke the results in Kyle (1985, 1989) and Laffont and Maskin (1990), who show that in imperfectly competitive markets the trader can limit the leakage of information into prices. In this latter case, our predictions hold when the time between the first of a series of purchases/sales and its public disclosure is sufficient for the trader to buy/sell up to the cap on total exposure, splitting up the order into several smaller chunks. For large caps, this is possible only under those regulations that allow for a sufficient delay in reporting trades.[6] Conversely, this is always a possibility in case certified trades cannot be notified, whenever the insider produces announcements, the timing of whose disclosure is at the sender's discretion.

When a big investor, who is systematically informed (by assumption), has to disclose each trade *before* placing a new order, he reduces the dissemination of information dissimulating, that is adding a random component to his trades. This happens in HHL, where an investor with unlimited holdings earns substantially less compared to the case of no public disclosure, but one can conjecture that insiders with very large but *finite* total exposure caps dissimulate too. If so, our study suggests that, when disclosure is mandatory, it is the imposition of a *very tight* deadline to report trades that causes dissimulations. Ceteris paribus, when this trader—as well as one with a total exposure cap of any size—has enough time to place small orders, up to his maximum capacity, before reporting their execution, he opts for the latter alternative, which makes big traders earn more than with (now unnecessary) dissimulations, and allows for the possibility of a profitable reversal in case the price following these simultaneous disclosures overshoots the real value. The disappearance of this deceptive practice provides a rationale for allowing for long delays in reporting trades, or better, for making disclosures voluntary.

A regulatory concern relates to the tension between two elements implied by public disclosure. Advocates argue that higher transparency can increase price efficiency; opponents, that it will increase the set of manipulative behaviors. While mere speculations enable earlier information releases (Hart, 1977; Leland, 1992), the distortive effect of manipulations on prices is clearly undesirable. Though forbidden (e.g., see SEC, 2004, Section 9a2), manipulations are hard to prosecute, which is why an understanding of when and how to prevent them is imperative. This paper shows that disclosure by small traders cannot reduce price efficiency, only boost it or leave it unaffected. However, when manipulations arise, a regulator that aims to prevent them should refine market rules. In this case, our model tells us that such illegal conduct cannot be eliminated by suppressing the trade disclosure rule, unless the investor is also forbidden to produce announcements. On this front, this work examines whether two simple resolutions, the short-swing rule and public pre-trade non-anonymity, prevent manipulations without reducing price efficiency. Both resolutions have an independent interest; to the best of our knowledge, we are the first to model these issues.

The short-swing rule—which is contained in Section 16(b) of the SEC Security Exchange Act, but not prescribed in any EU Directive—constrains a class of investors already obliged to disclose their trades, namely, the firm's officers and directors, because it forces them to give up

[6] Rather than the US one, which in 2002 drastically reduced the possible delay, from one that depended on the trading-date—with insiders required to report within 10 days after the close of the calendar month during which the trade occurred—to a constant (but relative tight) one of 2 days, we are referring for example to Italy, Belgium, and France, with median delays of 5, 7, and 14 days respectively (Fidrmuc et al. (2011)).

profits from reversals if undertaken within 6 months from the first trade. For any properties of the asset value, this rule implies fully informative disclosures: On the one hand, *differently* from the case of an identical, finite quantity exchangeable per trading-date, it ensures that our trader does not manipulate when uninformed. On the other, it discourages this trader, when informed, from attempting deceptive strategies—conversely he leads, exchanging only in the beginning.[7]

To highlight the advantages and disadvantages of the US short-swing rule, which is imposed unconditionally, consider a trader who may acquire new inside information repeatedly. Even though in some instances this extra rule is ineffective—as deceptive strategies would have not been attempted anyhow—in others it prevents uninformed manipulation. However, there is an important drawback of SEC Section 16(b). In line with general concerns (Goldwasser (1999), p.48), a resolution discouraging manipulations can deter appropriate trading. In detail, provided the trader weights future profits heavily, the US short-swing rule is not only unnecessary but, when private information is sufficiently long-lived, also prevents the revelation of reversals (*or of their absence*), which would have shifted prices even closer to the fundamental value.

Pre-trade non-anonymity is a natural alternative to imposing trade disclosure. It consists of a public revelation of the forthcoming purchase or sale, together with the trader's identity, just before execution. A rule that forces (at least) the disclosure of the submitted order direction prevents the insider from trading in the market. This general result holds for any properties of the asset value and the noise traders' demand, and depends neither on the position limit to which the trader is subject, nor on whether he is small or large. Because the obligation to reveal orders before execution implies the lowest price efficiency level, this measure may be preferable only when the objective is to prevent an insider from profiting at the expense of other investors.

This paper continues as follows. Section I presents the assumptions. Section II studies the effects of a regulation that, following each purchase or sale, mandates public disclosure of trade direction. Section III investigates the foundation of mandatory and voluntary trade disclosure. At the end of this section, the analysis is extended to the case of a voluntary production of announcements. Section IV focuses on market beliefs. Section V extends our analysis in different directions, including that of trade size disclosure. Section VI evaluates the short-swing rule and public pre-trade non-anonymity. Section VII concludes.

I. Assumptions

Trading is modelled as a sequence of auctions, structured to give the flavor of a sequential equilibrium (Kreps and Wilson (1982)). As in Kyle (1985), a risky asset is exchanged for a riskless one among three kinds of traders. In a risk-neutral world, a *potential insider* (the *leader*, L) and noise traders submit orders to a market maker (M), that sets prices and clears the market.

The *ex-post* liquidation value of the asset, $\widetilde{v}$, is a random variable over $[-b,b]$, where $b>0$; $\widetilde{v}$ has zero mean; $F(\widetilde{v})$ is absolutely continuous with respect to the Lebesgue measure; and $f(\widetilde{v})$ is symmetric (in Section III, the absolute continuity and symmetry requirements are relaxed).

The timing is the following. Before $\widetilde{v}$ is exogenously revealed to the market at the end of the *period*, a sequence of *two rounds* (or auctions), $n \in \{1,2\}$, takes place. Round n consists of *three steps*. In Step 1, a public disclosure occurs; in Step 2, noise traders and L submit quantities (or orders); and in Step 3, the price is fixed and quantities are executed by M.

Two main states of the world are possible: $\widetilde{s} \in \{I, U\}$. In I the leader has information about $\widetilde{v}$, learning whether $\widetilde{v}>0$ or $\widetilde{v}<0$ in round $n=1$, and learning $\widetilde{v}=v$ in $n=2$. In U the leader does not know $\widetilde{v}$ at any round. State I occurs with probability q (for the case of a leader that, when

[7] As a result, if the short-swing rule were imposed when trade disclosure is not, this investor would have no incentive to voluntarily disclose his trades or produce announcements.

informed, already observes $\widetilde{v}=v$ in round $n=1$, see Section V). From now on, for brevity, we refer to a potential insider as an *insider* when, in a specific period, he actually possesses private information about $\widetilde{v}$; conversely, when he privately knows that $\widetilde{s}=U$, we say that he is *uninformed*.

The market maker's task is to set the clearing price in round n, p_n, efficiently; thus p_n is chosen to equal the asset expected value, conditional on the information available.

[See Fig. 1.]

At auction n the leader trades a quantity x_n, positive for a purchase, negative for a sale, and zero otherwise. The leader is constrained on asset holdings, in that he is restricted to *hold* $x_n \in [-x_L, x_L]$, where x_L, the cap on total exposure, is strictly positive and finite, and x_0 is normalized, without loss of generality, to 0.[8] Denote, with $\pi_n = x_n(v - p_n)$, the portion of L's profits attributable to the round $n \in \{1, 2\}$ trade, and assume that the *intra*-period discount factor equals 1. Noise traders' demand in n, the random variable $\widetilde{u}_n$, avoids the no-trade theorem problem (Milgrom and Stokey (1982)); $\widetilde{u}_n$ and $\widetilde{v}$ are independently distributed.

Define P and X, which are vectors of function, by $P=\langle P_1, P_2\rangle$ and $X=\langle X_1, X_2\rangle$, where P is M's pricing rule, and X is L's trading strategy. In detail, $p_n = P_n(\Omega_n)$, where Ω_n is M's information set at auction n; X_1: $\{U\} \cup (\{I\} \times \{\widetilde{v}>0, \widetilde{v}<0\}) \to [-x_L, x_L]$; $x_1 = X_1(\widetilde{v}=\text{sign}(v), \widetilde{s}=s)$; X_2: $\{U\} \cup (\{I\} \times [-b, b]) \to [-x_L - x_1, x_L - x_1]$; and $x_2 = X_2(\widetilde{v}=v, \widetilde{s}=s)$.

Definition 1 *An equilibrium is defined as: (i) A strategy by L that maximizes the overall sum of his discounted expected payoffs over time, given the price setting rule and the information L has when making each trade; (ii) a strategy by M that allows him to set each price equal to the asset expected value, given L's strategy and the information available (market efficiency condition); (iii) each player's belief about the other player's strategy is correct in equilibrium.*

As a distinctive assumption in this model, the orders that the potential insider submits have no inferable impact on the order-flow—in other words, public disclosure is the only information M conditions on.[9] To simplify the exposition, when this assumption holds, from now on we say (or imply, when not specified) that the leader is *small* (as opposed to *large*). Let's also assume that, as soon as $\widetilde{v}=v$ is exogenously revealed at the end of the period, the price immediately adjusts, and that the initial price, p_0, is normalized to $E[\widetilde{v}]=0$.[10]

Mandatory post-trade non-anonymity ($\mathcal{N}$) characterizes markets in which, at the very beginning of round n, the identity of agents placing orders in $n-1$ and whether they bought or sold are revealed (post-trade disclosure of submitted quantities and pre-trade non-anonymity are considered in Section V and VI.B respectively). Thus in $n=2$ the signal $\tau \in \{-1, 0, 1\}$ is released: $\tau=1$ implies that L bought in $n=1$; $\tau=-1$ implies a sale; $\tau=0$ implies no revelation in $n=2$ about the purchase or sale that L undertook in $n=1$. When disclosure is mandated, this setting coincides with inactivity in $n=1$. Because $\Omega_1=\{\varnothing\}$, $\Omega_2=\{\tau\}$, it follows that

[8] Other authors, before us, have assumed a symmetric upper- and lower-bound in the change of holdings (e.g., van Bommel (2003), Brunnermeier and Pedersen (2005)).

[9] Of the three ways, adduced in the introduction to this analysis, to justify a non-informative order-flow, $\widetilde{u}_n + x_n$, the first can be formalized with a distribution of $\widetilde{u}_n$, $g(\widetilde{u}_n)$, strictly positive for all $\widetilde{u}_n \in [-\infty, \infty]$, when $2x_L$ is quantitatively negligible. Under this structure, $E[x_n | \widetilde{u}_n + x_n] \approx E[x_n | \widetilde{u}_n]$. The second—i.e., that of an indistinguishably large or low demand—with a naïve market maker with diffuse priors about $\widetilde{u}_n$: If $g(\widetilde{u}_n)$ is unknown, then $E[x_n | \widetilde{u}_n + x_n]$ cannot be computed.

[10] We can think of p_0 being equal to $E[\widetilde{v}]$ as an implicit consequence of the market efficiency condition. This assumption does not play a role in the determination of any result in this work, in that no exchange takes place at the initial price. Nonetheless, it facilitates the exposition, allowing us to describe whether and how, within the same period, the prices set by M shift from this initial level.

P_1: $\{\varnothing\} \to [-b, b]$ and P_2: $\{-1, 0, 1\} \to [-b, b]$. Specifically, as long as trades get revealed after the order execution, price-driven markets—in which prices are set, then quantities placed and executed at this price—are equivalent to order-driven ones.[11] Anonymity ($\mathcal{A}$) characterizes markets in which no information is released.

II. Markets with post-trade mandatory disclosure

This section analyzes a regulation mandating disclosure of trade direction, first considering the benchmark case of a *non*-repeated sequence of two auctions, then a multi-period framework where this sequence is repeated up to infinite.

II.A. Single-period equilibrium with post-trade mandatory disclosure

Under $\mathcal{A}$, in equilibrium the market clears at the same price, p_n=0, at any auction. The equilibrium behavior of an insider aware of $\widetilde{v}$>0 (or $\widetilde{v}$<0) is such that $\sum_n x_n$ equals x_L (resp., $-x_L$)—in other words, such that he holds x_L (resp., $-x_L$) at the end of the period—while that of an uninformed leader is such that $\sum_n x_n \in [-x_L, x_L]$. This means that each type of leader can place any probability (also equal to 0 or 1) on all round n=1 trade quantities (x_1=0 included), no matter what information he observes. For instance, consider a trader that in n=1 systematically buys (or sells, or does not trade) *only* when he observes $\widetilde{v}$>0. Although in equilibrium M's beliefs about L's (pure or mixed) strategy are correct, absence of public *signals*—i.e., Ω_n=$\{\varnothing\}$—implies no price shift. At these prices, an uninformed leader is indifferent whether or not to trade at any round, as by purchasing or selling he earns 0 expected profits.

Under $\mathcal{N}$, in the standard two-round trading model, public trade disclosure by any small investor constrained on asset holdings is not informative. As under $\mathcal{A}$, an initial trade by L does not affect the short-run price, p_1—that is, because Ω_1=$\{\varnothing\}$, M sets p_1=p_0=0. Although its subsequent public disclosure might alter the long-run price, p_2, we show that in equilibrium M ignores any signal in the second round and sets p_2=0.

Proposition 1 *For mandatory trade disclosure, in the single period the 'unique beliefs' equilibrium is the following: M sets* p_n= 0*; type* $\widetilde{s}$= $I \wedge \widetilde{v}$>0 *and* $\widetilde{s}$= $I \wedge \widetilde{v}$<0 *trade in such a way that* $\sum_n x_n$= x_L *and* $\sum_n x_n$= $-x_L$ *respectively, providing they disclose the same signal* $\tau = \cdot$ *with equal probability (even* 0 *or* 1*); type* $\widetilde{s}$= U *trades in such a way that* $\sum_n x_n \in [-x_L, x_L]$.[12]

Proof. See Internet Appendix A. ■

It follows that, both under $\mathcal{A}$ and $\mathcal{N}$, the *per-period equilibrium payoff* of type $\widetilde{s}$=U equals 0, while that of the insider of type $\widetilde{v}$>0 (or $\widetilde{v}$<0) equals $x_L\xi$, where ξ=$E[\widetilde{v}|\widetilde{v}$>0].

To see why public disclosure of trades (as well as disclosure of no undertaken trade) is not informative, consider any candidate equilibrium pricing rule such that either the signal τ=−1 or τ=0 or τ=1 causes the price p_2 to shift from $p_{n\neq 2}$=0. For each of these pricing rules, derive L's optimal response, under the assumption that, when informed, L already observes $\widetilde{v}$=v in the first round. Holding this optimal trading strategy fixed, notice that the candidate pricing rule in question makes M reply to *all* types of insider belonging to either $[-b, 0)$ or $(0, b]$ with

[11] This degree of generality is due to a structure not allowing for information extraction from the order-flow.

[12] Two remarks are in order: (i) *Equilibrium beliefs uniqueness* refers to a unique component of equilibria, all of which are supported by the same set of beliefs and thus share the same pricing rule, even though these equilibria differ in L's trading strategy. (ii) The symbol $\wedge$ stands for *and*.

a price in the opposite partition of the support of $\widetilde{v}$.[13] In particular, this wrong price shift follows an identical first round order, x_1 (and thus an identical disclosure of trade direction). Consequently, the optimal trading strategy is unaffected when each of these types of insider only observes whether $\widetilde{v}<0$ or $\widetilde{v}>0$ in round $n=1$, which is why any of these candidate pricing rules still suffers from the same problem. Now recall that, since $f(\widetilde{v})$ is symmetric around 0, the probability of $\widetilde{v}$ being greater or smaller than p_0 is the same. It follows that any of these candidate pricing rules is (in expectation) wrong. In fact, at least half of the times, prices shift in the wrong partition of $\widetilde{v}$, regardless of whether in $n=1$ an insider knows $\widetilde{v}=v$ or $\widetilde{v}\gtrless 0$. In conclusion, no pricing rule such that $p_2\neq p_0$ can be an equilibrium one.

Part of the result is in line with the one in finitely repeated zero-sum games of incomplete information, in which it is impossible for the informed sender to mislead the uninformed receiver (Aumann and Maschler (1995)). Less intuitively, in the single period M does not make any use of the signal received, because L's preferences over actions are completely opposed to what can be roughly defined as M's preferences, which are to set prices efficiently. If prices somehow reacted to the trade disclosure (or its absence), the pricing rule would not be justified, and in this sense, M would be worse off and consequently would deviate. Otherwise, regardless of whether L actually possesses information, with probability greater than a half prices would move in the opposite direction with respect to $\widetilde{v}=v$, and in practice, M could do better by tossing a coin. This is mainly due to the position limit assumption (see Section III).

With respect to the equilibrium trading strategy depicted under $\mathcal{A}$, the one under $\mathcal{N}$ is constrained as follows. The probability that an insider of type $\widetilde{v}>0$ and one of type $\widetilde{v}<0$ place on round $n=1$ purchases is the same. Analogously, the probability that these types place on round $n=1$ sales is identical, as well as the probability placed on $x_1=0$. In this way, they hide their information completely and the pricing rule $p_n=0$ is justified (in fact, even when type $\widetilde{s}=U$ signals differently from what the informed types signal, M does not extract information from that). HHL shows that, when forced to disclose trades, a large insider dissimulates to reduce the revelation of his information. To do so, he plays a mixed strategy consisting of a first round trade that includes a random noise component. By contrast, in the present study the revelation of information following the first round trade is eliminated rather than reduced. To accomplish this, the insider can but does not have to employ mixed strategies, which is why dissimulation is not a driving force behind the present result. What matters is that *any* type of insider initially disregards his information and discloses (under probability) the same trade. By contradiction, suppose for example that the insider(s) of type $\widetilde{v}>0$ decided to signal $\tau=-1$ (or $\tau=0$, or $\tau=1$) *less*

[13] The result does not depend on the effective size of the cap on total exposure, x_L. To make some off-the-path manipulative attempts by a leader more explicit, consider the following candidate equilibrium pricing rules and the associated insider's best responses. Holding $p_1=0$ unchanged, first suppose that $P_2(\tau=1)$—i.e., the price in response to a disclosed purchase—is positive, $P_2(\tau=0)$ is non-negative, and $P_2(\tau=1)$ is negative (this is case $C3$ in the proof to Proposition 1). The round $n=1$ placed orders in response to these prices, as well as the disclosed trade directions, depend on the exact value that $P_2(\tau=-1)$, $P_2(\tau=0)$, and $P_2(\tau=1)$ assume. Specifically, not every type initially aware of $\widetilde{v}=v>0$ prefers to disclose a first round *sale*—which moves p_2 down, namely toward the wrong direction—unless both $P_2(\tau=1)$ and $\frac{P_2(\tau=0)}{2}$ are non-greater than $|P_2(\tau=-1)|$. Nonetheless, when this latter condition on prices is not satisfied, each type initially aware of $\widetilde{v}=v<0$ finds it optimal to *purchase* or *not to trade* in $n=1$ depending on whether $P_2(\tau=1)\geq\frac{P_2(\tau=0)}{2}$ or $0<P_2(\tau=1)\leq\frac{P_2(\tau=0)}{2}$ respectively, which causes p_2 to increase, namely to shift in the wrong direction. Second, suppose for instance that $P_2(\tau=-1)$ is positive and $P_2(\tau=0)$ and $P_2(\tau=1)$ are non-positive (this is case $C6$ in the proof to Proposition 1). When the leader initially observes $\widetilde{v}=v<0$, he finds it optimal to *sell* a tiny quantity in $n=1$—so that p_2 shifts up, namely in the wrong direction—and to continue selling up to his total exposure cap in $n=2$. In particular, this latter strategy highlights how trading in the so-called 'wrong direction'—i.e., buying and selling in n when $v<p_{n-1}$ and $v>p_{n-1}$ respectively—is not necessary to qualify a best reply as a manipulative attempt.

often than the insider(s) of type $\widetilde{v}<0$ do(es). For each of them, the optimal trading plan associated with this alternative signaling requirement implies a payoff that is equal to that achieved in equilibrium. However, this best reply is not an equilibrium response, because disclosure of a sale (resp., absence of disclosure; disclosure of a purchase) would shift p_2 *down*, a pattern which has been shown *not* to be compatible with that of an equilibrium pricing rule.

None of the equilibria in Proposition 1 is robust to a probability that M exogenously learns $\widetilde{v}=v$ at the end of the first rather than of the second auction. Even when this probability is small, an informed type is *not* indifferent any more about a first round trade or another. Instead, in response to $p_n=0$, an insider of type $\widetilde{v}<0$ prefers to sell in $n=1$, while one of type $\widetilde{v}>0$ prefers to buy. These replies cause M to deviate. Specifically, because type $\widetilde{s}=U$ now prefers inactivity, M sets $p_2(\tau=0)=0$ and $p_2(\tau=1)=-p_2(\tau=-1)=\xi$. At these new prices, however, a leader aware of $\widetilde{v}<0$ (or $\widetilde{v}>0$) buys (resp., sells) in $n=1$, which moves p_2 in the wrong direction, then reversing his initial position in the very likely event of an exogenous revelation of $\widetilde{v}=v$ to M only at the end of round $n=2$. In this case, it seems reasonable to improve our definition of equilibrium by adding a condition that makes M set prices efficiently, in the *weak* sense, *if* no pricing rule is justified otherwise. When this condition is added, since in equilibrium M turns out to be 'required' to ignore signals, and thus sets $p_n=0$, an insider of type $\widetilde{v}<0$ (or $\widetilde{v}>0$) initially sells (resp., buys). Thus, although the equilibrium trading strategy in Proposition 1 probably lacks of realism, the associated equilibrium prices and payoffs do not. Conversely, for any case studied in our work, other than that of a mere *mandatory* trade disclosure over a *finite* horizon, this extra equilibrium condition will not be necessary, because of the existence of equilibria that display robustness to a small probability of $\widetilde{v}=v$ being exogenously available to M in advance.

II.B. Informative post-trade mandatory disclosure

This subsection investigates whether *alternative* equilibria are possible, where disclosed trades become relevant. We will allow for an infinite repetition of the single period and refer to an equilibrium as a sequence of history-contingent replies that satisfy certain sequential conditions. When analyzing a problem with $t \in \mathbb{N}$ periods (where $\mathbb{N}$ includes 0), additional assumptions are needed. First, an *inter*-period discount factor, $\delta \in [0, 1)$, is assumed. In particular, δ and q are drawn by Nature at time $t=0$ (the only period in which L does not play), and do not vary over time. Second, the two active agents involved in the infinite repetition are the same market maker and leader. L's type changes over time: Immediately after the exogenous revelation of $\widetilde{v}=v$ to the whole market at the end of period t (but before period $t+1$ starts), $\widetilde{s}$ and $\widetilde{v}$ are drawn again by Nature. Both $\widetilde{s}$ and $\widetilde{v}$ are i.i.d. over periods. Third, for any repetition of the two auctions, p_0 and x_0 are normalized to 0.[14]

For an infinite repetition of the two auctions, consider the following M's strategy.

Definition 2 *Suppose M's strategy is to set* $p_1=0$ *and* $p_2=P_2^{\mathcal{N}}(\cdot)$ *in the first period, where* $P_2^{\mathcal{N}}$: $\tau=1 \rightarrow p_2=\mu$, $\tau=-1 \rightarrow p_2=-\mu$, $\tau=0 \rightarrow p_2=0$, *and* $\mu \geq 0$ *is the magnitude of the second round price shift. At the second round of the* t^{th} *period, if the outcome of all* $t-1$ *preceding periods has been* $\tau=1 \wedge v>0$ *or* $\tau=-1 \wedge v<0$ *or* $\tau=0$, *then play* $P_2^{\mathcal{N}}$; *otherwise, set* $p_2=0$.

The analysis is now restricted to what, for $\mu>0$, we call *trigger* strategy, which consists of a generic history-contingent pricing rule and a *punishment* scheme that makes M ignore subsequent disclosures if L *defects*—that is, when L causes the price to go in the wrong direction with respect to v. The punishment refers to the decrease in per-period expected profits suffered

[14]The amount of shares held at the end of period $t-1$ does not impact on period t space of actions. In fact, at the very end of period $t-1$, L can always rebalance his holdings, exchanging at the right price $\widetilde{v}=v$.

by L after defection. Specifically, Definition 2 implies that, as soon as M observes $vp_2<0$—i.e., a price manipulation occurs—at period j, from period $j+1$ onwards prices at any auction equal 0. Consequently, from period $j+1$, L's equilibrium trading strategy coincides with that undertaken under $\mathcal{N}$, when the two-round period is not repeated. Depending on δ, q, and $f(\widetilde{v})$, sub-classes of this trigger strategy are part of an equilibrium.

In particular, M can be thought of as representing the behavior of a semi-strong efficient market as a whole (BL), or as serving as an intermediary. Finally, as in Kyle (1985), M can be also interpreted as the reduced form of at least two competitive bidders per auction, where the winner—i.e., who posts the most attractive bid for L—clears the market at the winning price. In this case, to prevent multi-round collusion, Caldentey and Stacchetti (2010, p. 250) suggest imagining a large group of bidders, each of them bidding once and then quitting the market. Still, if prices were set by competitive bidders, a priori it is unclear whether a punishment strategy is implementable. Section IV explains why the notion of a unique market maker breaking even in expectation via the selection of any trigger strategy, and therefore even a *Grim* trigger—which applies a punishment consisting of M reverting to single period equilibrium behavior *forever* (see Friedman (1971))—is consistent with the idea of bidders setting prices competitively. Section IV also accounts for the multiplicity of equilibrium pricing rules.

II.B.1. Benchmark case ($q = 1$)

In general, a trader can alternate (with some probability, even 0 or 1) between trading *some* non-negative quantity in one direction and in the other. In this respect, providing at a certain period prices shift positively as stated in Definition 2, if an insider decides to incur the punishment, we say that he *misleads* M. If an insider decides to push the price in the right direction, he *leads* M. Define, with $\mathcal{M}(\mu)$ and $\mathcal{L}(\mu)$, how much L expects to earn per period from trading *optimally* while aiming to mislead and lead respectively. These two new strategies identified, let $\bar{\alpha}\in[0,1]$ be the probability with which he chooses the former rather than the latter.

Lemma 1 *Consider mandatory disclosure of trades. Given the pricing rule in Definition 2, by trading optimally a type $\widetilde{s}=I$ that decides to incur the punishment with probability $\bar{\alpha}$ earns $\bar{\alpha}\cdot\mathcal{M}(\mu)+(1-\bar{\alpha})\cdot\mathcal{L}(\mu)$ per period, where $\mathcal{M}(\mu)>\mathcal{L}(\mu)>x_L\xi$, $\forall\mu>0$, and:*

$$\mathcal{L}(\mu) = 2x_L\left\{\int_0^{\mu}(2\mu-\widetilde{v})f(\widetilde{v})d\widetilde{v}+\int_{\mu}^{b}\widetilde{v}f(\widetilde{v})d\widetilde{v}\right\}, \tag{1}$$

$$\mathcal{M}(\mu) = 2x_L\int_0^{b}(2\mu+\widetilde{v})f(\widetilde{v})d\widetilde{v}. \tag{2}$$

Proof. See Internet Appendix A. ■

At each period, if an insider of type $\widetilde{v}>0$ (or $\widetilde{v}<0$) decides to lead, it is optimal for him to trade $x_1=x_L$ (resp., $x_1=-x_L$), completely reversing this position afterwards by trading $x_2=-2x_L$ (resp., $x_2=2x_L$) in case $\widetilde{v}=v$ lies between p_2 and p_0, or not trading at all otherwise. As long as μ is strictly positive, since the insider has the chance to benefit from an additional price differential at the second round, the resulting per-period expected profits are greater than those after defection—in other words, if $\mu>0$, then $\mathcal{L}(\mu)>x_L\xi$. If this type decides to optimally mislead, he will initially sell (resp., buy) up to his cap on total exposure, always undertaking a complete reversal of the initial position afterwards. Only for $\mu=0$ we have that $\mathcal{M}(\mu)=\mathcal{L}(\mu)=x_L\xi$, case in which any strategy such that $\sum_n x_n$ equals x_L (resp., $-x_L$) is a best response.

Given M's trigger strategy, a leader informed with certainty chooses a level of $\bar{\alpha}$, $\bar{\alpha}^{*I}$, which maximizes his discounted expected profits over periods. In this case, whether to defect at a certain point in time only depends on how much the trader weighs future profits.

Proposition 2 *For mandatory disclosure of trades, an infinite repetition of the two-round trading period, and a leader acquiring new information every period (that is, when $q = 1$): (i) If $\delta \geq \delta_\nabla$, where $\delta_\nabla = \frac{\mathcal{M}(\mu=\xi)-\mathcal{L}(\mu=\xi)}{\mathcal{M}(\mu=\xi)-x_L\xi}$, an equilibrium exists in which disclosures affect prices. Specifically, M undertakes the strategy in Definition 2, setting $\mu = \xi$; L trades optimally in such a way that he never incurs the punishment. (ii) If $\delta < \delta_\nabla$, at each repetition the equilibrium coincides with that under $\mathcal{N}$, when no repetition of the period takes place.*

Proof. See Internet Appendix A. ■

Consider a situation in which $\mu > 0$. When the insider gives substantial weight to the profits from persistently leading the market optimally—an alternative to earning even more only once by misleading optimally, but then earning less forever—he opts for the former option with certainty. Thus, prices are not manipulated at the equilibrium, which is in pure strategies. In detail, disclosures being fully informative, an equilibrium price shift equal to ξ is justified. Conversely, when L does not weigh future profits enough, he would always mislead. However, M anticipates such misleading behavior, ignoring disclosures by setting $\mu = 0$. As a consequence, L effectively trades as he does in a single repetition of the two-round period.

Finally notice that, when $\delta = \delta_\nabla$, for any positive value of μ, insiders are indifferent towards leading and misleading optimally. In this case, depending on the probability with which each insider is believed to lead, infinite other equilibrium outcomes are possible, with price shifts that can assume any value between 0—when both insiders are believed to mislead with probability greater than or equal to $\frac{1}{2}$—to ξ included. Because δ_∇ is a point in the continuum, we refer only to the more informative equilibrium.

II.B.2. Generalized case ($\mathbf{q} \in (\mathbf{0}, \mathbf{1}]$): The manipulative-equilibrium threat

Consider a leader that is not informed with certainty. Whenever uninformed, this trader cannot undertake any insider activity. Still, provided that, at a certain moment in time, prices positively shift as hypothesized in Definition 2, with some probability the uninformed leader can *pretend to be informed*, that is, *bluff*, disclosing a purchase or a sale to move p_2 up or down respectively. When he does so, by trading *optimally* he expects to earn $\mathcal{P}(\mu)$ in that period, whether he opts for an initial purchase or a sale. Let $\bar{\beta} \in [0, 1]$ be the probability with which the uninformed leader decides to bluff as opposed to *not bluffing*, the latter strategy implying no trade undertaken in the first auction.

In case $\mu \neq 0$, an uninformed leader that decides to bluff finds it optimal to either buy or sell initially up to the cap on total exposure and completely reverse this position afterwards.

For an uninformed leader that decides to bluff, let $\bar{z} \in [0, 1]$ be the probability with which this type decides to do so by disclosing a purchase as opposed to disclosing a sale. Holding the price reaction in Definition 2 fixed, he is indifferent to the two options. In fact, because of the symmetry of the pricing rule, the associated per-period payoffs are identical. In addition, because of the symmetry of the punishment scheme and of $f(\widetilde{v})$, when $\mu \neq 0$, this choice does not even impact on the likelihood that type $\widetilde{s} = U$ accidentally causes the price to be wrong—an event that occurs with probability $\frac{\bar{\beta}}{2}$. However, for this symmetric pricing rule to be justified, beliefs in response to a purchase and a sale are restricted to assigning the same probability to type $\widetilde{s} = U$. For this reason, if L bluffs at the equilibrium, he chooses $\bar{z} = \frac{1}{2}$.

If type $\widetilde{s}$=U does not bluff, then x_1=0. No matter what his unobservable round n=2 trade is, this type expects to earn 0 per-period profits, that is less than $\mathcal{P}(\mu)$ whenever $\mu \neq 0$. Only for μ=0 we have that $\mathcal{P}(\mu)$=0, case in which any strategy such that $\sum_n x_n$=0 is a best response.

Lemma 2 *Consider mandatory disclosure of trades. Given the pricing rule in Definition 2, by trading optimally a type $\widetilde{s}$=U that decides to bluff with probability $\bar{\beta}$—i.e., to defect with probability $\frac{\bar{\beta}}{2}$—expects to earn $\bar{\beta}\cdot\mathcal{P}(\mu)$ in that period, where $\mathcal{P}(\mu)$=$2\mu x_L$>0, $\forall\mu$>0.*

Proof. See Internet Appendix A. ■

When q is not restricted to equal 1, another dimension is added to the problem presented in the previous subsection. At any period in which prices are expected to shift, L can randomize with probability $\bar{\alpha}$ (or $\bar{\beta}$) between misleading and leading (resp., bluffing and not bluffing) optimally when informed (resp., uninformed). In the subsequent period, this choice causes prices to shift again with probability $1-\bar{\alpha}$ (resp., $1-\frac{\bar{\beta}}{2}$). As long as $\mu \neq 0$, choosing $\bar{\alpha}\neq 0$ or $\bar{\beta}\neq 0$ implies a positive probability of incurring the punishment, taken into account when determining L's optimal strategy at the equilibrium, for every $\delta\in[0,1)$ and $q \in (0,1)$.

Consider a leader that is informed with probability q. The inter-temporal problem that he has to solve differs depending on whether or not in the current period—that is, period t=1—he possesses private information. Given M's trigger strategy, let $\bar{\alpha}^{*I}$ and $\bar{\beta}^{*I}$ (or $\bar{\alpha}^{*U}$ and $\bar{\beta}^{*U}$) be the levels of $\bar{\alpha}$ and $\bar{\beta}$ that maximize $E\left[\Pi^I\right]$ (resp., $E\left[\Pi^U\right]$), that is the discounted sum of profits that L expects to earn over time when in period t=1 he is (resp., is not) informed.

The next lemma defines L's best response.

Lemma 3 *Consider mandatory trade disclosure, an infinite repetition of periods, and a leader that acquires new information every period with probability $q \in (0,1)$. Given the pricing rule in Definition 2, identify the pairs $\bar{\alpha}^{*I},\bar{\beta}^{*I} = \arg\max_{\bar{\alpha},\bar{\beta}} E\left[\Pi^I\right]$ and $\bar{\alpha}^{*U},\bar{\beta}^{*U} = \arg\max_{\bar{\alpha},\bar{\beta}} E\left[\Pi^U\right]$, where*

$$E\left[\Pi^I\right] = \bar{\alpha}\cdot\mathcal{M}(\mu) + \left(1-\bar{\alpha}\right)\cdot\mathcal{L}(\mu) + \bar{\alpha}\frac{\delta}{1-\delta}\cdot q x_L \xi + \left(1-\bar{\alpha}\right)\delta\cdot\mathcal{S}(q,\delta,\mu,\bar{\alpha},\bar{\beta}), \quad (3)$$

$$E\left[\Pi^U\right] = \bar{\beta}\cdot\mathcal{P}(\mu) + \left(1-\frac{\bar{\beta}}{2}\right)\delta\cdot\mathcal{S}(q,\delta,\mu,\bar{\alpha},\bar{\beta}) + \frac{\bar{\beta}}{2}\frac{\delta}{1-\delta}\cdot q x_L \xi, \quad (4)$$

and

$$\mathcal{S} = \frac{q[\bar{\alpha}\cdot\mathcal{M}(\mu) + (1-\bar{\alpha})\cdot\mathcal{L}(\mu)] + (1-q)\bar{\beta}\cdot\mathcal{P}(\mu) + \frac{\delta}{1-\delta}[q\bar{\alpha} + \frac{(1-q)\bar{\beta}}{2}]q x_L \xi}{1-\delta\,\frac{2(1-q\bar{\alpha})-\bar{\beta}(1-q)}{2}}. \quad (5)$$

In the current period, the best response of a leader of type $\widetilde{s}$= I (or $\widetilde{s}$= U) is $\bar{\alpha}^{*I}$ (resp., $\bar{\beta}^{*U}$) when $\mu \neq 0$, and equals to the one in the single repetition of the period otherwise.

Derivation of $\mathcal{S}$ in Lemma 3. See Appendix. ■

The function $\mathcal{S}$ embeds the following elements. The leader does not know whether he will be informed at each future date but knows that at any date he will have learned whether he possesses new private information before signaling. In the decision process, L accounts for the probability of acquiring new information, how much he weighs future profits, and the consequences of each signal on the direction of present and future price shifts.[15]

The next lemma defines the level of μ at which the pricing rule in Definition 2 is efficient.

Lemma 4 *Consider mandatory trade disclosure, an infinite repetition of periods, and a leader that in every period acquires new information with probability q and trades optimally given the pricing rule in Definition 2. The market efficiency condition holds for $\mu = \mathbf{1}(\bar{\alpha}^{*I} < \frac{1}{2})[1-(1-q)\bar{\beta}^{*U}](1-2\bar{\alpha}^{*I})\xi$, where $\mathbf{1}(\cdot)$ is the indicator function.*

Before defection, beliefs formed in response to disclosed trades (or absence of disclosure) account directly for the current and indirectly for the planned choices by a leader aware about prices being restricted to shift as prescribed in Definition 2. Disclosures are informative—that is, μ is positive—*only if* a trader that is currently informed leads with probability greater than $\frac{1}{2}$. In this case, provided L does not bluff when currently uninformed, a level of μ equal to $1 - 2\bar{\alpha}^{*I}$ ensures efficient pricing. This level has to be reduced—i.e., multiplied by $1 - (1-q)\bar{\beta}^{*U}$—in case L bluffs with positive probability when uninformed.

Below we propose the closed-form solution to the general problem in markets with mandatory post-trade disclosure. More general conditions for this result to hold are presented in Corollary 3. In the next section the result is extended, and commentary provided.

Proposition 3 *For mandatory disclosure of trades and an infinite repetition of the two-round period, three regions over the space in $\delta \in [0,1)$ and $q \in (0,1]$ can be identified. They correspond to different equilibria in which M undertakes the strategy in Definition 2. In detail, (1) if $\delta \geq \Delta(q, \mu = \xi)$, in every period M sets $\mu = \xi$, and L plays $\bar{\alpha}^{*I} = \bar{\beta}^{*U} = 0$; (2) if $\nabla(q, \mu = q\xi) \leq \delta < \Delta(q, \mu = \xi)$, M sets $\mu = q\xi$, and L plays $\bar{\alpha}^{*I} = 0, \bar{\beta}^{*U} = 1$ up to the j^{th} repetition, where j is the first period after which M observes $vp_2 < 0$; and (3) if $\delta < \nabla(q, \mu = q\xi)$, at each repetition the equilibrium coincides with that under $\mathcal{N}$, when no repetition of the period takes place. Specifically, $\Delta(q,\mu) = \frac{\mathcal{P}(\mu)}{\mathcal{P}(\mu) + \frac{q}{2}[\mathcal{L}(\mu) - x_L \xi]}$ and $\nabla(q,\mu) = \frac{\mathcal{M}(\mu) - \mathcal{L}(\mu)}{\frac{1+q}{2}\mathcal{M}(\mu) - \frac{1-q}{2}\mathcal{L}(\mu) + (1-q)\mathcal{P}(\mu) - q x_L \xi}$.*

For any distribution of $\tilde{v}$ satisfying the initial conditions, these three regions always exist.

Proof. See Internet Appendix A. ■

When L repeatedly acquires information with probability $q \in (0,1]$, the equilibrium is derived as follows. Holding $\mu > 0$ fixed, notice that: (1) For $\delta \geq \Delta(q, \mu > 0)$, the pairs $\bar{\alpha}^{*I} = 0, \bar{\beta}^{*I} = 0$ and $\bar{\alpha}^{*U} = 0, \bar{\beta}^{*U} = 0$ maximize the functions $E\left[\Pi^I\right]$ and $E\left[\Pi^U\right]$ respectively. Thus, L's best response consists of leading when informed and not trading otherwise. For $\bar{\alpha}^{*I} = \bar{\beta}^{*U} = 0$, a level of μ equal to ξ guarantees price efficiency. Holding $\mu = \xi$ fixed, L does not deviate from the original strategy. Consequently, when $\delta \geq \Delta(q, \mu = \xi)$, in equilibrium disclosures are fully informative and no manipulation arises. (2) For $\nabla(q, \mu > 0) \leq \delta \leq \Delta(q, \mu > 0)$, the pairs $\bar{\alpha}^{*I} = 0, \bar{\beta}^{*I} = 1$ and $\bar{\alpha}^{*U} = 0, \bar{\beta}^{*U} = 1$ maximize $E\left[\Pi^I\right]$ and $E\left[\Pi^U\right]$ respectively. Hence, L's best reply is to lead when informed and bluff when uninformed. For $\bar{\alpha}^{*I} = 0, \bar{\beta}^{*U} = 1$, a level of μ equal to $q\xi$ guarantees price efficiency. At this level of μ, no deviation by L from the initial strategy occurs. It follows that:

[15] For a leader that is currently informed (or uninformed), his best response today, $\bar{\alpha}^{*I}$ (resp., $\bar{\beta}^{*U}$), coincides with his best planned response when informed (resp., uninformed) tomorrow. The assumption of an insider learning only about $\tilde{v} > 0$ or $\tilde{v} < 0$ (rather than $\tilde{v} = v$) in round $n = 1$ simplifies the analysis. Otherwise, the multi-period problem of a leader that is currently informed—but not that of one that is currently uninformed—is affected (see Section V.B.2 for details).

(2.a) When q=1 and $\nabla(q{=}1, \mu{=}\xi) \leq \delta \leq \Delta(q{=}1, \mu{=}\xi)$, since no manipulation occurs, disclosures are again fully informative. (2.b) When q<1 and $\nabla(q{<}1, \mu{=}q\xi) \leq \delta \leq \Delta(q{<}1, \mu{=}\xi)$, disclosures are partially informative until a manipulative attempt causes prices to shift in the wrong direction, an event that occurs by the end of the k^{th} period with probability $1-(\frac{1+q}{2})^k$.[16] (3) For $\delta \leq \nabla(q, \mu{>}0)$, the arguments maximizing the two functions do not always coincide. This has no implications for L's strategic behavior because $\bar{\alpha}^{*I}{=}\bar{\alpha}^{*U}{=}1$. Put differently, if prices shifted, L would always mislead the market as soon as he is informed. In equilibrium, M ignores disclosures and L trades as he does in the single period.

Notice that, over the segment $\delta{=}\Delta(q{<}1, \mu{>}0) \wedge q{<}1$ (or $\delta{=}\nabla(q, \mu{>}0)$), any pair $\bar{\alpha}^{*I}{=}0$, $\bar{\beta}^{*U} \in [0,1]$ (resp., $\bar{\alpha}^{*I} \in [0,1]$,$\bar{\beta}^{*U}{=}1$) is also a leader's best reply. In this case, infinite equilibria are possible, where the price shift varies from $\mu{=}q\xi$ to $\mu{=}\xi$ (resp., from $\mu{=}0$ to $\mu{=}q\xi$). In line with the argument presented below Proposition 2, we refer only to the most informative one.

The three regions identified in Proposition 3 always exist. In fact, the functions $\nabla(q, \mu)$ and $\Delta(q, \mu)$ are continuous and $\nabla(q{=}1, \mu{=}\xi){<}\Delta(q{=}1, \mu{=}\xi)$. In particular, $\nabla\,(q{=}1, \mu{=}\xi){=}\delta_{\nabla}$ (as shown in the benchmark case), $\lim_{q\to 0} \Delta\,(q, \mu{=}\xi) \to 1$, and $\frac{\partial(\Delta(q,\mu=\xi))}{\partial q}{<}0$. Figure 2 contains an example with $\widetilde{v}\sim U[-1,1]$ to provide a graphical idea of the closed-form solution to the issue.

[See Fig. 2.]

For any q<1, whenever δ assumes values just below $\Delta(q{<}1, \mu{=}\xi)$, the potential insider continues leading when informed, but starts bluffing when uninformed. This is due to the fact that, for any pair δ and $q \in (0,1)$ and a positive μ, the *overall* incentive that an informed leader has to mislead (rather than lead) optimally today is smaller than the one that the same leader has to bluff (rather than not to bluff) optimally today when uninformed. On the one hand, per period the *extra*-payoff from misleading optimally, $[\mathcal{M}(\mu){-}\mathcal{L}(\mu)]$, is smaller than that from bluffing optimally, $[\mathcal{P}(\mu)-0]$.[17] On the other, while a misleading strategy implies a punishment with certainty, a bluffing strategy implies a defection only with probability $\frac{1}{2}$. Hence, starting from any pair δ and $q \in (0,1)$ associated with a non-manipulative outcome, by gradually decreasing δ, at some point a switch in the equilibrium occurs, to one where L has no incentive to mislead, but has incentive to bluff.

III. Foundation of mandatory/voluntary disclosure

First we focus on mandatory and voluntary trade disclosure, then extend the study to the voluntary disclosure of (uncertified/non-factual) announcements.

III.A. Voluntary vs. mandatory trade disclosure

To study the foundation of mandatory and voluntary *trade* disclosure, and highlight the role of the position limit to which L is subject, together with the role of the asset value properties, we start with a comparison with FH. The corollaries refer to a leader constrained on asset holdings.

In FH, for a disclosure to be forthcoming, it must be mandatory, the reason being that disclosures reduce the informed trader's profits. Given the single period made of $n \in \{1,2\}$ rounds, where p_0=0, suppose that a negligible leader, informed with probability q, can trade a

[16] For k=1, the probability of a defection equals $\epsilon = \frac{1-q}{2}$. For k=2, it equals $\epsilon{+}\epsilon(1{-}\epsilon)$, that is the probability of defection today plus that of a defection in period t=2, provided a punishment has not yet occurred. By the end of period $t{=}k$ a defection occurs with probability $\epsilon+\epsilon(1-\epsilon)+..+\epsilon(1-\epsilon)^{k-1}{=}\epsilon\frac{1-(1-\epsilon)^k}{1-(1-\epsilon)}{=}1-(\frac{1+q}{2})^k$.

[17] In fact, $\mathcal{M}(\mu){-}\mathcal{L}(\mu){<}\mathcal{P}(\mu) \therefore 2x_L[\int_0^\mu 2\widetilde{v}f(\widetilde{v})d\widetilde{v}{+}\int_\mu^b 2\mu f(\widetilde{v})d\widetilde{v}]{<}2\mu x_L \therefore \int_0^\mu(\mu-\widetilde{v})f(\widetilde{v})d\widetilde{v}{>}0$, for all μ>0.

(divisible) unit x_L per round, and that $\widetilde{v}\in\{-b,b\}$ has equally likely priors.[18] Under mandatory disclosure, when L initially sells (or buys), at the equilibrium p_1=0 and p_2=$-bq$ (resp., p_2=bq). At these prices, an insider aware of $\widetilde{v}$<0 (or $\widetilde{v}$>0) sells (resp., purchases) x_L twice, which is a trading strategy that, however, is less profitable than under $\mathcal{A}$. Conversely, type $\widetilde{s}$=U randomizes with equal probability between trading x_1=x_L, x_2=$-x_L$ and x_1=$-x_L$, x_2=x_L, earning a per-period payoff equal to $x_L bq$>0. Because the informed trader's loss from disclosure equals in magnitude the uninformed trader's gain, L's *ex-ante* payoff is higher with disclosure if $q<\frac{1}{2}$.

Conversely, in our model, under mandatory trade disclosure, the per-period payoff of any type of leader is equal or greater than under $\mathcal{A}$. Specifically, provided disclosures affect prices, the expected profits of an informed leader are always higher. Thus, if L were to choose in which market to exchange, $\mathcal{N}$ or $\mathcal{A}$, he would always at least weakly prefer the former.

Corollary 1 *When disclosed trades affect equilibrium prices, the leader prefers a system mandating disclosure to $\mathcal{A}$, and is indifferent otherwise.*

Now, let's consider a market in which L can voluntarily decide whether or not to disclose an undertaken purchase or sale. Since in this market the signal τ=0 is more opaque than when disclosures are mandatory, the conditions for an equilibrium with informative trades to exist are clearly harder to satisfy. Nonetheless, within the infinitely repeated structure, equilibria exist where the leader voluntarily discloses trades that shift prices.

Corollary 2 *For voluntary trade disclosure, in the single period a unique beliefs equilibrium exists, where type $\widetilde{s}$=$I\wedge\widetilde{v}$>0 and $\widetilde{s}$=$I\wedge\widetilde{v}$<0 disclose the same signal τ with equal probability, trading in such a way that $\sum_n x_n$=x_L and $\sum_n x_n$=$-x_L$ respectively; type $\widetilde{s}$=U attaches any probability to any signal, trading in such a way that $\sum_n x_n\in[-x_L,x_L]$; and p_n=0. When the period is infinitely repeated, alternative equilibria exist, where M undertakes the strategy in Definition 2. Specifically, if $\delta\geq\Delta(q,\mu=\xi)$, type $\widetilde{s}$=$I\wedge\widetilde{v}$>0 (or $\widetilde{s}$=$I\wedge\widetilde{v}$<0, or $\widetilde{s}$=U) signals τ=1 (resp., -1; 0), while M sets μ=ξ. If $\nabla(q,\mu=q\xi)\leq\delta<\Delta(q,\mu=\xi)$, up to the j^{th} repetition, any type $\widetilde{s}$=I signals and trades as before; with equal probability, type $\widetilde{s}$=U signals as type $\widetilde{s}$=$I\wedge\widetilde{v}$>0 and $\widetilde{s}$=$I\wedge\widetilde{v}$<0 do, trading x_1=x_L,x_2=$-2x_L$ and x_1=$-x_L$,x_2=$2x_L$ respectively; and M sets μ=$q\xi$; from period j+1 on, the equilibrium coincides with that in the single repetition of the period.*

Proof. See Internet Appendix A. ■

As far as the single period is concerned, no equilibrium exists such that prices at the second round shift following the voluntary disclosure of an undertaken transaction (or its absence). To see it, consider any of these off-the-path pricing rules and derive the optimal response from a leader that observes $\widetilde{v}$=v already in round n=1 and can—but does not have to—disclose trades. Given this leader's best response, M turns out to reply to at least half of the types of insiders—those below or those above 0—with prices that shift in the wrong direction. For the same reasons adduced for the case of a mandatory disclosure, the pricing rule in question is not justified, no matter whether an insider observes $\widetilde{v}\gtrless 0$ or $\widetilde{v}$=v in the first round. In equilibrium, unlike mandatory disclosure, the probability that an insider of type $\widetilde{v}$>0 and one of type $\widetilde{v}$<0 place on a round n=1 purchase (or sale; or absence of disclosure) does not necessarily have to be the same. Nonetheless, the probability that these types signal τ=1 (or -1, or 0) is identical—and can take any value from 0 to 1 (included)—so that the

[18]When $\widetilde{v}\in\{-b,b\}$, assuming that in round n=1 the insider learns only whether $\widetilde{v}\gtrless 0$ rather than $\widetilde{v}$=v does not make a difference, but makes a direct comparison between FH and our model possible.

information revelation is eliminated and the pricing rule p_n=0 is justified. Indeed, holding this latter pricing rule fixed, by trading as prescribed in equilibrium but signaling differently, each type of insider earns identical profits. However, they do not opt for any of these alternative strategies, since this would cause M to deviate and set an off-the-path pricing rule. Finally, consider those equilibria where *no disclosure* ever occurs and x_1 equals x_L or $-x_L$ or 0 when $\widetilde{s}=I\wedge\widetilde{v}>0$ or $\widetilde{s}=I\wedge\widetilde{v}<0$ or $\widetilde{s}=U$ respectively. These equilibria are robust to a small probability that M exogenously learns $\widetilde{v}=v$ at the end of the first rather than of the second round.

Within the infinitely repeated structure, when L weighs future profits sufficiently, alternative equilibria exist, where until defection (if any) prices and (voluntary) disclosures—as a function of the state of the world—are identical to those set in Proposition 3. The reason for this is that the '*relevant payoff structure*'[19] coincides with that analyzed when disclosures are mandatory. Suppose that, at a specific period, $\mu>0$, and consider an insider who is aware, for instance, of $\widetilde{v}>0$ (the case in which he is aware of $\widetilde{v}<0$ is symmetric). If this trader does not aim to incur the punishment, he can choose between two options, disclosing a purchase (which requires him to submit an initial buy order) or not disclosing any trade (which does not prevent him from placing either a buy or a sell order). Clearly, the former option is better, provided the insider buys up the maximum in the first round and subsequently reverses the initial position if $v<p_2$. By doing so, he expects to earn $\mathcal{L}(\mu>0)$ in that period. Conversely, the only way this insider has to incur the punishment is to sell initially and disclose the undertaken sale. In particular, by trading optimally—selling as much as possible in round n=1 and buying back up to the total exposure cap in n=2—he expects to earn $\mathcal{M}(\mu>0)$. Finally, an uninformed leader can pretend to be informed, disclosing either an undertaken purchase or sale. In either case, by trading optimally, he expects to earn $\mathcal{P}(\mu>0)$ in that period. Alternatively, type $\widetilde{s}=U$ can avoid disclosure, which assures him that he will not incur the punishment at the end of the period. In this case, no matter what the quantity traded in each of the two rounds is, he expects to earn 0 profits. For this reason, while for $\nabla(q,\mu=q\xi)\leq\delta<\Delta(q,\mu=\xi)$ the pre-defection equilibrium trading outcome (as a function of the states of the world) coincides with that in Proposition 3, for $\delta\geq\Delta(q,\mu=\xi)$, type $\widetilde{s}=U$ can trade differently.

From a regulatory perspective, the model suggests that it is not essential to impose public disclosure, as long as an investor with a cap on total exposure can voluntarily communicate trades. This result relies on general asset value properties, generalized even further below. Conversely, the length of punishment plays no role: The trader discloses voluntarily simply because he always earns at least as much as he does under $\mathcal{A}$, both when informed and uninformed. Voluntary dissemination of information results from the investor's will to communicate trades, which reveals a link to the literature on uncertified/non-factual messages.

The next corollary highlights which asset value properties drive the results obtained so far, when L is constrained on asset holdings. To explain the corollary, we consider a situation where disclosures are *mandatory* and present, in sequence, two examples that refer to a symmetric distribution of $\widetilde{v}$, centered around 0 (an event which, for the time being, is assumed not to be possible). The first example helps our understanding of the second, in which specific conditions on $f(\widetilde{v})$ for an informed type to send meaningful signals are identified.

The distinguishing feature of the first example is that, whenever L turns out to be informed about $\widetilde{v}>0$ (or $\widetilde{v}<0$), he is *forced* to exchange $x_1=x_L$ (resp., $x_1=-x_L$). Whether the two-round period is repeated or not, in equilibrium type $\widetilde{s}=U$ (who has *not* been constrained in the direction of the initial trade) randomizes with equal probability between trading $x_1=x_L$, $x_2=-2x_L$

[19]The term *relevant* refers to the per-period payoff that the leader achieves—in case the market conditions on signals—from *optimally* misleading, leading, bluffing, and not bluffing, and to the indirect implications that the pursuit of one specific payoff or another has on the probability of a punishment occurring.

and $x_1{=}-x_L$, $x_2{=}2x_L$. Because in this example the disclosure by an informed type is indirectly assumed to be informative, the equilibrium price p_2 following a purchase (resp., sale) shifts to $q\xi$ (resp., $-q\xi$), a value that allows type $\widetilde{s}{=}U$ to achieve a positive payoff—rather than 0, which is how much this type gets under $\mathcal{A}$—from a reversal. In other words, the first round equilibrium orders by any type of leader and equilibrium prices coincide with those in FH. Nonetheless, and different from FH, mandatory disclosure allows an informed leader to earn either more than or as much as what he earns when disclosures are concealed, depending on the asset value properties. To see this, define, with $r{>}0$, the realization of $\widetilde{v}$ that is closest to 0 from the right. When $f(\widetilde{v})$ is such that $q\xi \le r$, rather than undertaking an unprofitable reversal, the insider prefers not to trade in $n{=}2$, which is why his per-period payoff equals that achieved under $\mathcal{A}$. Conversely, when $r{<}q\xi$, any insider aware of $|v|{<}|q\xi|$ reverses the initial position, earning more than under $\mathcal{A}$.

The second example refers to a leader who is *not forced* to undertake any particular action in any first round. When $\delta \ge \Delta(q{,}\mu{=}\xi)$, at a specific period, if he turns out to be informed (or uninformed), he expects to earn more than (resp., as much as) under $\mathcal{A}$, provided that disclosures are believed to be informative *and* at the same time $f(\widetilde{v})$ is such that $r{<}\xi$. This latter condition ensures that L has an incentive to lead, in that those types of insider aware of $-\xi{<}v{<}0$ (resp., $0{<}v{<}\xi$) increase their profits by reversing the initial position in $n{=}2$, exchanging at a price $P_2(\tau{=}-1){=}-\xi$ (or $P_2(\tau{=}1){=}\xi$). Specifically, any symmetric distribution of $\widetilde{v}$ is such that the latter types find the reversal profitable, unless $\widetilde{v}{\in} \{-b, b\}$, in which case the reversal does not generate any additional revenue and thus there is no incentive to lead. When $\nabla(q, \mu{=}q\xi) \le \delta{<}\Delta(q, \mu{=}\xi)$, at a specific period, any type of leader expects to earn more than under $\mathcal{A}$, provided disclosures are believed to be informative *and* $f(\widetilde{v})$ is such that $r{<}q\xi$, a condition that allows any insider to increase his profits by reversing his initial leading position, exchanging at a price $P_2(\tau{=}-1){=}-q\xi$ (or $P_2(\tau{=}1){=}q\xi$), whenever he learns about $-q\xi{<}v{<}0$ (resp., $0{<}v{<}q\xi$). However, in this case the existence of two possible realizations of $\widetilde{v}$ above (or below) 0 does not guarantee that the condition $r{<}q\xi$ is satisfied. The intuition proposed in this second example is generalized here.

Corollary 3 *Relax the assumptions of a symmetric $f(\widetilde{v})$ and a $F(\widetilde{v})$ being absolutely continuous w.r.t. the Lebesgue measure, and consider $\widetilde{v}{\in}\mathcal{V}{\subset}\Re$ such that $E[\widetilde{v}]$ is normalized to 0 and:*

R1 : $\Pr(\widetilde{v}{<}0){=}\Pr(\widetilde{v}{>}0){=}\frac{1}{2}$; *R2 :* $\Pr(-\gamma{<}\widetilde{v}{<}0){=}\Pr(0{<}\widetilde{v}{<}\gamma){\neq} 0$;

R3 : $E[\widetilde{v} \mid \underline{b}{\le} \widetilde{v} \le -\gamma]{=} -E[\widetilde{v} \mid \gamma \le \widetilde{v} {\le} \bar{b}]$; *R4 :* $E[\widetilde{v} \mid -\gamma{<}\widetilde{v}{<}0]{=}-E[\widetilde{v} \mid 0{<}\widetilde{v}{<}\gamma]$;

where $\underline{b}{=}\min v \in\mathcal{V}$, $\bar{b}{=}\max v \in\mathcal{V}$, and γ equals ξ (or $q\xi$) if $\delta \ge \Delta(q{,}\mu{=}\xi)$ (resp., $\nabla(q{,}\mu{=}q\xi) \le \delta{<}\Delta(q{,}\mu{=}\xi)$). Under restrictions from R1 to R4, all the preceding results still hold. In particular, those in the single period only require R1 to be satisfied.

Notice that $|\underline{b}|$ does not have to equal $\bar{b}$. More generally, as is clear from *R3* and *R4*, even for the results in the infinitely repeated framework, a symmetric $f(\widetilde{v})$ is no longer required. *R1* has two implications. On the one hand, it ensures an equal probability mass above and below $E[\widetilde{v}]$, a restriction that is sufficient to guarantee that the results in the single period hold. For instance, the proof to Proposition 1 relies neither on the support of $\widetilde{v}$ being continuous, nor on the number of types of insider above and below p_0 being equal, nor on the specific distance between each type of insider and 0, nor on whether a realization of $\widetilde{v}$ above (or below) 0 is more likely than another realization lying on the same side of the support. On the other hand, *R1* implicitly tells us that $\widetilde{v}{=}0$ is either a zero-probability event or simply not possible, depending on whether or not the support of $\widetilde{v}$ is continuous around the initial price. This ensures that, whenever informed, a leader is clearly aware whether the fundamental value is above or below 0. In this way, in the infinitely repeated framework, no ambiguity arises about whether a signal pushed the market price in the wrong direction or not. For the results in Proposition 3 and Corollaries 1 and 2 to hold, *R2* is necessary to ensure that the investor has an incentive to lead. In fact, when this restriction holds, he can earn more than under $\mathcal{A}$ whenever he learns about $-\gamma<v<0$ and $0<v<\gamma$ by reversing the initial position in the second auction, exchanging at a price equal to $P_2(\tau{=}{-1}){=}{-\gamma}$ and $P_2(\tau{=}1){=}\gamma$ respectively. *R2* implicitly requires the existence of at least four distinguishable realizations of $\widetilde{v}$, two greater than 0, and two smaller. Specifically, for $\nabla(q,\mu{=}q\xi) \leq \delta{<}\Delta(q,\mu{=}\xi)$ (or $\delta \geq \Delta(q,\mu{=}\xi)$), at least one realization of $\widetilde{v}$ has to lie somewhere over both $(-q\xi, 0)$ and $(0, q\xi)$ (resp., $(-\xi, 0)$ and $(0, \xi)$). When *R2* is satisfied, it follows that at least one realization of $\widetilde{v}$ is lying somewhere over both $[\underline{b}, -\xi]$ and $[\xi, \overline{b}]$. On the contrary, the results in the single period holds even when only two realizations, one greater and one smaller than p_0, are possible. Finally, *R2–R4* ensure that an insider aware of $\widetilde{v}{<}0$ and one aware of $\widetilde{v}{>}0$ achieve the same payoff from leading (or misleading).

III.B. Voluntary production of (un)favorable announcements

In this subsection we consider the disclosure of messages, voluntarily selected and sent at no cost, when the market is able to interpret any sort of signal in (up to) three distinctive ways, whatever meaning is assigned to each different class of messages—that is, no matter what the beliefs following a message belonging to one specific class or another are.

Different from the voluntary (but truthful) disclosure of trades—in which case the following exogenously fixed mapping exists: '*L buys in* $n{=}1$'$\rightarrow$ $\tau{=}1$; '*L sells in* $n{=}1$'$\rightarrow$ $\tau{=}{-1}$—a priori uncertified/non-factual messages are not associated with any transaction undertaken. Hence, their disclosure is equivalent to the disclosure of non-necessarily truthful trades.

Consider a leader that, at the beginning of the t^{th}-period second action, sends a message $\phi_{i,m}{\in}\ \Phi_m \subset \Phi$, $m \in \{-1, 0, 1\}$, where $\phi_{i,m}$ is a priori not correlated with any unobservable trade, Φ is the universe of non-costly (verbal or non-verbal) messages, $\Phi_m \cap \Phi_{\neg m}{=}\varnothing$ and $\Phi_m \neq \varnothing$. In particular, inactivity by a leader that decides not to send any message is a signal per se. The corollary below defines equilibria when uncertified/non-factual messages are sent. When the single period is not repeated, signals are never informative. This is because, given a pricing rule with prices that react somehow to a specific signal or another, and L's associated best response, the pricing rule in question turns out to be wrong in expectation. Conversely, within an infinitely repeated framework, signals can become informative, as long as a clear punishment scheme is defined. Here, suppose that M's trigger strategy is to set $p_1{=}0$, $p_2(\phi_{i,1}){=}{-p_2(\phi_{i,-1})}{=}\mu'{\geq}\ 0$, and $p_2(\phi_{i,0}){=}0$ in the first period. Suppose also that, at any subsequent period, if the outcome of all the preceding periods has been either $\phi_{i,1}{\wedge}v{>}0$ or $\phi_{i,-1}{\wedge}v{<}0$ or $\phi_{i,0}$, M continues playing as he did before, and sets $p_n{=}0$ otherwise.

Corollary 4 *Consider a market where uncertified/non-factual messages are publicly sent. Under R1, in the single period a unique beliefs equilibrium exists, where type* $\widetilde{s}{=}I{\wedge}\widetilde{v}{>}0$ *and* $\widetilde{s}{=}I{\wedge}\widetilde{v}{<}0$ *disclose the same signal* $\phi_{i,m}$ *with equal probability, trading in such a way that* $\sum_n x_n {=}x_L$ *and* $\sum_n x_n{=}{-x_L}$ *respectively; type* $\widetilde{s}{=}U$ *attaches any probability to any signal, trading in such a way that* $\sum_n x_n{\in}\ [-x_L, x_L]$*; and* $p_n{=}0$*. When the period is infinitely repeated, under R1–R4, alternative equilibria exist. Specifically, if* $\delta \geq \Delta(q,\mu{=}\xi)$*, type* $\widetilde{s}{=}I{\wedge}\widetilde{v}{>}0$ *(or* $\widetilde{s}{=}I{\wedge}\widetilde{v}{<}0$*, or* $\widetilde{s}{=}U$*) signals* $\phi_{i,1}$ *(resp.,* $\phi_{i,-1}$*;* $\phi_{i,0}$*) and trades optimally in such a way that* $x_1{=}x_L$ *(*$-x_L$*,* 0*), while M sets* $\mu'{=}\xi$*. If* $\nabla(q,\mu{=}q\xi) \leq \delta{<}\Delta(q,\mu{=}\xi)$*, up to the* j^{th} *repetition, any type* $\widetilde{s}{=}I$ *signals and trades as before; with equal probability, type* $\widetilde{s}{=}U$ *signals as type* $\widetilde{s}{=}I{\wedge}\widetilde{v}{>}0$ *and* $\widetilde{s}{=}I{\wedge}\widetilde{v}{<}0$ *do, trading* $x_1{=}x_L$*,*$x_2{=}{-2x_L}$ *and* $x_1{=}{-x_L}$*,*$x_2{=}2x_L$ *respectively; and M sets* $\mu'{=}q\xi$*; from period*

$j+1$ *on, the equilibrium coincides with that in the single repetition of the period.*

Proof. See Internet Appendix A. ■

Predictably, in the single period, prices do not react to messages. Recall that the market is not only unable to condition on trades that are disclosed voluntarily. It also cannot extract meaningful information when trades are mandated, in which case no discretion other than that on the trade to be made is left to the investor. Thus, when none of the messages is tied to a specific transaction, the general result cannot be other than confirmed. In particular, all the equilibria where, in the first auction, an insider aware of $\widetilde{v}>0$ (or $\widetilde{v}<0$) purchases (resp., sells) x_L and an uninformed leader does not trade display robustness to a small probability that M exogenously learns $\widetilde{v}=v$ at the end of the first rather than of the second round.

Within the infinitely repeated structure, as long as signals are believed to be informative, a leader that weighs future profits enough finds it optimal to send messages that push the market price in the right direction whenever informed, at the same time trading in a way that maximizes his profits. In fact, a justifiable price shift $\mu'>0$ allows an informed trader to earn more than under anonymity any time the fundamental value turns out to lie between the equilibrium price p_2 and the starting price p_0. Only when the signal $\phi_{i,1}$ (or $\phi_{i,-1}$) in expectation conveys information concerning an increase (resp., decrease) of the asset fundamental value, we can call this message *favorable* (resp., *unfavorable*). Clearly, the notion of consistent rather than truthful behavior (or signal) should be adopted.

With reference to van Bommel's (2003) study, which is often cited when referring to a trader that spreads rumors (e.g., Kyle and Viswanathan (2008)), the structure proposed herein is more general, and allows for several innovative existence results. Indeed, the two models in van Bommel (2003) are more a characterization of a pure strategy equilibrium rather than a proof of existence and for different reasons they are not quite right. The present work contributes to the literature by reconducting them to a unique problem and establishing a firmer foundation for the issue of information-based manipulations (see Internet Appendix B).

IV. Robustness (Part I): Market beliefs

Within the infinitely repeated structure, an unlimited number of alternative trigger strategies can be part of an equilibrium. For the same pair δ and q, on the one hand, the way prices shift following the same disclosures can differ; on the other, equilibria exist where, at some point following a defection, prices can start shifting again. Internet Appendix C proposes a guided tour through the wide universe of multiple equilibria, listing five minimal restrictions on beliefs such that, *if* any price shift at period t occurs in equilibrium, the way this price reacts in response to a specific signal or another, disclosed at period t, is unique—we term this result '*price-shift uniqueness*'—and equal to $q\xi$ or ξ in magnitude, depending on whether $\nabla(q,\mu{=}q\xi) \leq \delta<\Delta(q,\mu{=}\xi)$ or $\delta \geq \Delta(q,\mu{=}\xi)$ respectively.

Below we underline how the equilibrium prices that the unique market maker sets when breaking even by selecting a trigger strategy—no matter whether supported by a specific punishment scheme or another—coincide with those set by competitive bidders, and that this result directly follows from the third equilibrium condition, the one on beliefs.

IV.A. Competition and punishment equivalent bidding outcomes

Even the winning price resulting from competition among bidders can turn out to be in some sense the punishment equivalent to L's intrinsic misbehavior against past bidders. To see it, rather than a unique M, consider a set of at least two *competitive* bidders per auction, *bidding once and then quitting*. In this context, the following needs to be spelled out. First, in defining the equilibrium, *a strategy by each bidder that maximizes his expected payoff* is required, in alternative to the market efficiency condition. Second, bidders are assumed to be risk-neutral and to have the same initial beliefs conditional on past history. Third, because each bidder trades only once, in the context of an infinite repetition of the period, no discount factor δ is considered when computing his realized payoff. Nonetheless, each bidder cares about past and future history, and about δ, which affects L's signaling strategy over time.

Clearly, at a specific (per-period second) round, the only *initial* beliefs that always turn out to be confirmed in equilibrium, independently of future beliefs, are those about any history of disclosures that are not informative at that round.

Still, any equilibrium achievable with a single market maker that breaks even in expectation can also be achieved when competitive bidders come to play. For instance, consider a pair δ and q which is such that an equilibrium Grim trigger supports pre-defection informative disclosures. When competitive bidders are taken into account, an equilibrium exists, where they set prices identical to those offered by a unique M selecting the Grim trigger in question. This equilibrium exists because of the *awareness* all players share about post-defection prices being set by bidders who disregard disclosures, which justifies pre-defection beliefs and equilibrium prices. More in general, given the definition of equilibrium employed herein, it is the awareness of what future bidders may or may not believe—and therefore about any implied punishment equivalent bidding strategy by those bidders competing over future prices—that supports equilibrium responses by current bidders, when the latter believe that the history of disclosure currently observed is somehow informative.

V. Robustness (Part II): Private information arrival and trade size disclosure

This section discusses alternative versions of our model, with a potential insider constrained on asset holdings. The following assumptions are relaxed: (1) A public disclosure about the direction of trade, but not its size; (2) a quality improvement (from each first to second round) in the private information possessed by an informed leader.

We show that equilibria exist, the outcomes of which are in line to those derived so far.

By twisting the first assumption, our structure is sufficient to account for the full range of consequences that the following four regulations—which are alternatives to the mandatory or voluntary disclosure of trade direction—imply: Mandatory trade size disclosure; voluntary disclosure of trade size when trade direction cannot be revealed separately; voluntary trade size disclosure when revelation of trade direction is mandatory; voluntarily disclosure of *either* trade direction *or* trade size *or* nothing.

The second assumption is relaxed by analyzing a leader that, when informed, observes $\widetilde{v}=v$ from the first auction. Even in this case, the model is such that an equilibrium characterization can be made, both when examining a market in which the disclosure of trades is regulated (in one of the six ways listed above) and when studying uncertified/non-factual announcements.

Specifically, an analysis that focuses on two auctions per period is enough to understand the implications of a framework that, depending on the case, allows the trader to choose between a number of signals that is either equal, greater, or smaller than the number of possible realizations of $\widetilde{s}$ and $\widetilde{v}$ observed by the leader in the first auction. When the single repetition of the period is taken into account, this result is presented under the more general assumption of a non-specified but finite number of auctions.

V.A. Single repetition of the period

This subsection considers a period made of *any* finite sequence of auctions, $n \in \{1, .., N\}$, where the leader's trading strategy, $X=\langle X_1, .., X_N\rangle$, is such that $X_{n>1}$: $\{U\} \cup (\{I\} \times [-b, b])$

$\to[-x_L-\sum_{i=1}^{n-1}x_i, x_L-\sum_{i=1}^{n-1}x_i]$ and $x_{n>1}=X_{n>1}(\widetilde{v}=v, \widetilde{s}=s)$, to highlight that, for *any* non-degenerate random variable $\widetilde{v}\in\mathcal{V}$, and no matter whether in the first round an insider learns only $\widetilde{v}\gtrless 0$ or $\widetilde{v}=v$, a unique beliefs equilibrium *exists*, where M ignores disclosures, setting $p_{n\in\{1,..,N\}}$, the price at each auction, equal to 0.

For what concerns the revelation of certified trades, this result holds for any combination of provision for order direction and order size disclosure considered in this work. At the equilibrium, in each of the first $N-1$ rounds, *any* insider aware of $\widetilde{v}>0$ (or $\widetilde{v}<0$) trades in such a way that $\sum_n x_n=x_L$ (resp., $\sum_n x_n=-x_L$), provided at round $n\in\{1,..,N-1\}$ he sends a signal—observable with a round of delay—which is (under probability) identical to the one that any other type of insider would send *at the same round*. Conversely, any sequence of signals can be part of the uninformed leader's equilibrium strategy, which is such that $\sum_n x_n\in[-x_L, x_L]$. Provided at the same round all types of insider send the same signal with equal probability (even 0 or 1), this result holds even when L can only produce uncertified/non-factual messages (a priori uncorrelated with the undertaken trade) at *any* step of *any* round, that is, even when these messages becomes publicly observable in $n=1$.

To see why these equilibria exist, suppose that M believes that signals are not informative. As a consequence, at each auction he will ignore them and set the price $p_{n\in\{1,..,N\}}$ equal to $E[\widetilde{v}]$, which we normalize to 0. Holding this pricing rule fixed, note that, at any round but the last one, each type of leader is indifferent about exchanging one quantity or another (even 0), provided he trades optimally in round N. The reason being that, for each of these types—but not, of course, among types—the per-period payoff associated to any of these alternative sequences of transactions is identical. In particular, each of these trading plans is (part of) a best reply, in that it is not possible to earn more otherwise. It follows that, when all types of insider signal identically, the pricing rule is justified.

V.B. Infinite repetition of the period

We examine an infinitely repeated two-round period. To ease exposition, we refer below to a real asset value, $\widetilde{v}$, whose properties are those defined in Section I and, for what concerns any regulation about public trade disclosure, to *symmetric* Grim triggers with the following three main characteristics. (1) At *each* second round before defection, (1.a) the function P_2 is identical and such that the revelation about a purchase (or about a specific purchased quantity) causes a positive price shift that equals in magnitude the negative shift following the revelation about a sale (resp., about an identical quantity, when sold); (1.b) when the regulation mandates (or allows for) trade size revelation, P_2 is non-decreasing in the disclosed quantity x_1; (1.c) absence of any disclosure causes the price not to shift; (2) L is thought of as defecting when, at the end of a certain period, it happens that $p_2v<0$; and (3) as soon as a defection is observed, M punishes by reverting to single period equilibrium behavior forever. When appropriate, the implications of alternative Grim punishment schemes will be analyzed. Specifically, since we are dealing with Grim triggers, we only refer to L's strategy and M's pricing rule before defection (if any).

V.B.1. Trade size disclosure when the insider learns information gradually

Consider an insider that in the first round observes $\widetilde{v}\gtrless 0$, and learns $\widetilde{v}=v$ only in the second.

When mandatory/voluntary trade size disclosure is taken into account, the following four regulations can be identified. For each of them, at least one equilibrium with *informative* disclosures exists, whose outcome in terms of traded quantities (as a function of the states of the world) and prices (as a function of traded quantities) is identical to that proposed in Proposition 3, where a regulation that imposes disclosure of trade direction but conceals trade size was examined. Further details about the equilibria in question are presented below.

First, let's consider mandatory trade size disclosure (or voluntary disclosure of trade size when trade direction cannot be revealed separately, in which case the signal x_1=0 implies absence of disclosure), and focus on pre-punishment pricing rules such that, at the second round of each period, $P_2(x_1) = -P_2(-x_1) \geq 0$. For $\nabla(q, \mu{=}q\xi) \leq \delta < \Delta(q, \mu{=}\xi)$ (or $\delta \geq \Delta(q, \mu{=}\xi)$), an equilibrium exists where $P_2(x_1{=}x_L)$ equals γ, which we defined in Corollary 3. Specifically, for an argument in line with that produced when studying the voluntary disclosure of trade direction (see Section III), even when the regulation allows for the sole voluntary disclosure of trade size, in equilibrium L reveals undertaken purchases and sales.

Second, let's consider a regulation that allows for a voluntary trade size disclosure when revelation of trade direction is mandatory (or a regulation that allows the voluntary disclosure of either trade direction or trade size or nothing)—the consequences being that the signal $\{\tau{=}0, x_1{=}0\}$ implies no effective exchange (resp., no revelation about any trade undertaken) in n=1, and $\{\tau \neq 0, x_1{=}0\}$ implies no trade size revelation—and focus on a per-period pre-punishment pricing rule P_2^ς: $\{-1, 0, 1\} \cup [-x_L; x_L] \to [-b, b]$, which maps the pair $\tau{=}\cdot, x_1{=}0$ in the same way as the function $P_2^\mathcal{N}$ does with $\tau{=}\cdot$, and which is such that $P_2^\varsigma(\tau{=}\cdot, x_1 \geq 0) = -P_2^\varsigma(\tau{=}\cdot, -x_1) \geq 0$. For $\nabla(q, \mu{=}q\xi) \leq \delta < \Delta(q, \mu{=}\xi)$ (or $\delta \geq \Delta(q, \mu{=}\xi)$), an equilibrium exists, where $P_2^\varsigma(\tau{=}1, x_1{=}x_L)$ and $P_2^\varsigma(\tau{=}1, x_1{=}0)$ both equal γ. Before defection, each type of insider is indifferent whether or not to disclose trade size (resp., between the mere disclosure of trade direction and the revelation of trade size, two alternatives that are both preferred to absence of disclosure). In equilibrium, a leader that observes $\widetilde{v}{>}0$ (or $\widetilde{v}{<}0$) reveals the purchased (resp., sold) quantity with probability $\overline{\varsigma}_t \in [0, 1]$ (resp., $\underline{\varsigma}_t \in [0, 1]$), while with probability $1{-}\overline{\varsigma}_t$ (resp., $1{-}\underline{\varsigma}_t$) he only discloses information about trade direction. Specifically, for $\nabla(q, \mu{=}q\xi) \leq \delta < \Delta(q, \mu{=}\xi)$, type $\widetilde{s}{=}U$ pretends to be informed, disclosing how much he initially purchased or sold—as opposed to revealing only the direction of the trade—with probability $\overline{\varsigma}_t$ and $\underline{\varsigma}_t$ respectively. Notice also that there exist pre-defection pricing rules P_2^ς in response to which *no* type of insider is indifferent between disclosing trade size and trade direction: For $\nabla(q, \mu{=}q\xi) \leq \delta < \Delta(q, \mu{=}\xi)$ (or $\delta \geq \Delta(q, \mu{=}\xi)$), when $P_2^\varsigma(\tau{=}1, x_1{=}0)$ equals γ and $P_2^\varsigma(\tau{=}1, x_1{=}x_L)$ is smaller than γ, in equilibrium all types of L (resp., of insider) only disclose trade direction; conversely, when $P_2^\varsigma(\tau{=}1, x_1{=}0) < P_2^\varsigma(\tau{=}1, x_1{=}x_L) = \gamma$, they disclose trade size.

Finally notice that, because of the number of possible realizations of $\widetilde{s}$ and $\widetilde{v}$ observed by L in each first round, which is the same as in the previous sections, *no* sophistication of the notion of defection triggering the Grim punishment—that is, the second restriction (out of three) that characterizes the trigger strategy defined at the beginning of Section V.B—can in any way lead to a further increase of the information embedded into prices.

V.B.2. The case of an informed type immediately aware of $\widetilde{\mathbf{v}}$=v

Let's consider a potential insider that, when informed, already learns $\widetilde{v}{=}v$ in the first round. Below we explain that, when drawing our attention to any of the alternative signaling channels studied so far, three regions over the space in $\delta \in [0, 1)$ and $q \in (0, 1]$, characterized by high, intermediate, and low values of δ, can be identified—call them upper, intermediate, and lower region respectively. For each pair δ and q lying over the upper (or intermediate; or lower) region, an equilibrium with fully (resp., partially; non-) informative disclosures exists, where the pricing rule and the leader's strategy coincide with those employed when $\delta \geq \Delta(q, \mu{=}\xi)$ (resp., $\nabla(q, \mu{=}q\xi) \leq \delta < \Delta(q, \mu{=}\xi)$; $\delta < \nabla(q, \mu{=}q\xi)$) by the same market maker and a leader that, when informed, only observes $\widetilde{v} \gtrless 0$ in round n=1.

To simplify the exposition, we focus on a regulation that mandates revelation of trade direction and prevents revelation of its size, and consider the trigger strategy in Definition 2. Indeed, for what concerns voluntary disclosure of the sole trade direction (or the disclosure of uncertified/non-factual messages, or any of the four alternative regulations dealing with trade size disclosure listed in Section V.B.1), the reasoning is analogous. The motive for these similarities relates to the equivalence of the relevant payoff structures.

As for the case examined in Section II, which differs from the one in question—namely, the mandatory disclosure of the sole trade direction—in the rate of arrival of private information, from period t=2 forward the leader only has expectations about the profits from leading or misleading when $\tilde{s}$=I, and from bluffing or not when $\tilde{s}$=U. Because of the symmetric space of actions, trigger strategy, and $f(\tilde{v})$, the incentive to mislead (as opposed to leading) does not even depend on $\tilde{v}$ being higher or lower than 0. Conversely, unlike the case studied in Section II, here in period t=1 the incentive to mislead depends on $\tilde{v}$=v. In detail, with respect to a situation where an insider only observes whether $\tilde{v} \gtrless 0$ in round n=1, the multi-period problem of a leader that is currently informed is affected as follows. For every inter-temporal strategy characterized by a current-period realization $\tilde{s}$=$I \wedge \tilde{v}$=v, a further control variable is introduced, to distinguish the insider's choice in period t=1 from his planned choice when informed in any future period. Consequently, the equation in Lemma 4 changes, so that μ reflects the expectation of all informed types' period t=1 leading behavior.

As an intermediate step in the identification of the three regions, we show that, in order to understand the role of the informed types in the determination of the informative equilibrium outcome, it is sufficient to draw attention to those aware of $|v| \geq |\mu|$ rather than those that know $v \in (-\mu, \mu)$. To see why this is the case, define, with $\mathcal{X}(\mu, v)$, the extra-payoff that an insider earns in the current period from optimally misleading rather than leading. In particular, while $\mathcal{X}(\mu{>}0, |v|{<}|\mu|){=}|2x_L v|$ depends on the specific value of $v \in (-\mu, \mu)$ that he observes, $\mathcal{X}(\mu{>}0, |v| \geq |\mu|){=}2x_L\mu$ does not. Two remarks are in order. First, given the trigger strategy in Definition 2, a characteristic that all the equilibria with informative disclosures share is that each insider aware of $|v| \geq |\mu|{>}0$ leads. This is due to the combined effect of the following two elements. On the one hand, as we said, those that observe $v \geq \mu$ (or $v \leq -\mu$) all have the same incentive to mislead today, which is why their equilibrium behavior is identical.[20] On the other, if the latter misled, a trigger strategy with $\mu{>}0$ would not be justified, in that in expectation the price shift would be too large. Second, every insider aware of $|v| \geq |\mu|$ is more tempted to mislead today than any type aware of $v \in (-\mu, \mu)$, in that $\mathcal{X}(\mu{>}0, |v|{<}|\mu|){<}\mathcal{X}(\mu{>}0, |v| \geq |\mu|)$. This means that, if δ and q are such that all types $\tilde{s}$=$I \wedge |v| \geq |\mu|$ lead—which as we have explained is always the case when the equilibrium is informative—every type $\tilde{s}$=$I \wedge v \in (-\mu, \mu)$ leads too, the latter having a smaller incentive to mislead.

Clearly, for very high values of δ and any $q \in (0, 1]$, no manipulation arises and disclosures are fully informative, so that μ equals ξ. In fact, since L weighs future profits heavily, he prefers to lead when informed and not to bluff otherwise.[21] Now, starting from any pair $\delta \simeq 1$ and $q \in (0, 1)$ and gradually shifting the parameter δ down, at some point a first switch in the equilibrium occurs, to one with uninformed manipulations that cause μ to equal $q\xi$. Specifically, in line with Proposition 3, this first switch *always* takes place before a further decrease of δ causes the equilibrium to switch again, to one where no disclosure is informative. The driving force for this result is that, for any pair δ and $q \in (0, 1)$ and a positive μ, the *overall* incentive that type $\tilde{s}$=U has to bluff (rather than not to bluff) optimally today is greater than the *overall* incentive that a leader aware of $\tilde{v}$=v has from misleading (rather than leading) optimally today. To see

[20]Given $\mu{>}0$, if δ and q are such that L is indifferent about misleading and leading (or about bluffing and non-bluffing), for an argument in line with the one presented below Proposition 2 and 3, here we refer only to the reply implying the most informative equilibrium, namely to the latter behavior.

[21]This relates to the fact that, as long as μ is positive, by leading optimally, an investor aware of $|v|{<}|\mu|$ or $|v| \geq |\mu|$ earns respectively more than or as much as what he gets, when μ=0, from trading optimally.

it, let's consider those insiders aware of $|v| \geq |\mu|$, who have the highest incentive to mislead. Because $[\mathcal{P}(\mu) - 0]{=}\mathcal{X}(\mu{>}0, |v| \geq |\mu|)$, the per-period extra-payoff that type $\widetilde{s}{=}U$ achieves when bluffing (rather than not bluffing) equals the one that type $\widetilde{s}{=}I \wedge |v| \geq |\mu|$ achieves from misleading (rather than leading). Nonetheless, the different inter-temporal consequences that these two choices imply are such that, for an insider that knows $|v| \geq |\mu|$—and thus for any type of insider—choosing to mislead today is overall less appealing than it is for type $\widetilde{s}{=}U$ to choose to bluff today. It follows that, over the space in $\delta \in [0, 1)$ and $q \in (0, 1]$, immediately below the upper region, there is an intermediate region, where the weight granted by L to future profits is not high enough to prevent him from manipulating today when uninformed, but is still too high for a misleading behavior to be a best reply. Two final remarks follow.

First, given any of the alternative signaling channels considered above, the model tells us that, by increasing the number of non-strategically equivalent states of the world—that is, by allowing a leader constrained on asset holdings either to observe $\widetilde{v}{=}v$ even in the first auction or to be uninformed with positive probability less than 1 (or both)— in equilibrium manipulative attempts occur *only if* (but not *if*) the trader repeatedly acquires private information with probability $q{<}1$ and at the same time the state $\widetilde{s}{=}U$ is drawn.

Second, for a leader that, when informed, learns $\widetilde{v}{=}v$ from the beginning of the period, consider again public trade disclosure (a similar argument can be drawn for what pertains to uncertified/non-factual announcements). In terms of equilibrium outcome, given the symmetric trigger strategy defined at the beginning of Section V.B, the level of information embedded in prices does not increase when a structural switch in the signaling channel is examined, from one where only three signals (i.e., $\tau{=}{-}1$, $\tau{=}0$, and $\tau{=}1$) to one where infinite alternative signals (i.e., the exact quantity traded) can be publicly observed.[22] However, when the latter channel is taken into account, provided the notion of defection triggering the Grim punishment is refined, for some pairs δ and q up to infinite other informative equilibria can be identified, where the level of information reflected in prices is higher. Nonetheless, none of these equilibria is a perfect separating one, where each type signals differently.[23]

VI. Further regulatory issues

In this section, we begin by studying the US short-swing rule. To assess its implications for market quality, attention is drawn to price-level efficiency on one side, and manipulative behaviors on the other. In fact, regulators generally perceive an increase in the former as a possible target; however, consensus exists on the latter harming market integrity. In this respect, no synthetic index of market quality or price-level stability is generally accepted. Next, we explore the implications of a regulation mandating public pre-trade non-anonymity.

[22] Even when the number of possible alternative signals is the highest, namely when considering a regulation which allows to voluntarily disclose trade size and mandates revelation of trade direction (or to voluntarily disclose either trade size or trade direction or nothing), whether or not a pre-defection pricing rule is such that $P_2^c(\tau{=}1, x_1{=}0){=}P_2^c(\tau{=}1, x_1{=}x_L)$ only impacts on whether, in equilibrium, L decides to disclose trade size too (resp., trade direction or trade size), as explained when characterizing the case in which the insider, at each first auction, only observes $\widetilde{v} \gtrless 0$ (see Section V.B.1).

[23] To sustain the perfect separating equilibrium, the trigger strategy should be such that, when a type of leader turns out to signal anything other than what only he is meant to send, a punishment follows. However, at this candidate equilibrium, no type has any incentive to avoid the punishment. The main reason for this relates to the fact that it impossible for any type, at each second round before defection, to benefit from a reversal of the initial position, in that the market efficiency condition requires the price following a specific signal to equal the type of leader who sent this signal. Not only is the per-period payoff following a perfect revelation of L's type never greater than what the same type achieves, in equilibrium, under $\mathcal{A}$, but it is also smaller than what this type gets from defecting optimally, which is why this trigger strategy is not justified.

VI.A. (Dis)advantages of the US short-swing rule

Very big stockholders, firms' officers and principals on one side, and traders listed in Section 13 on the other, they are all forced by the SEC to disclose undertaken trades publicly. However, only officers and principals are subject to a further restriction, Section 16(b). We investigate whether this extra rule is strictly necessary or beneficial.

When the short-swing rule is imposed, type $\widetilde{s}=U$ earns *negative expected* profits from a reversal, which is always a dominated strategy. Nonetheless, the introduction of this restriction *does not automatically guarantee* that manipulations do not occur any more.

To see it, consider a two-round trading model, and a fundamental value $\widetilde{v} \in \{-b, b\}$.

When L can trade *up to an identical, finite quantity per round*, under mandatory trade disclosure, a unique equilibrium exists where, with respect to the case in which the short-swing rule is not set (considered in Section III.A), the behavior of the informed types and the pricing rule coincide. An insider aware of b (or $-b$) purchases (resp., sells) x_L twice, and the price following the disclosure of a purchase (resp., sale) equals qb (resp., $-qb$). At this equilibrium, for any $q \neq 1$, an uninformed leader *manipulates*, initially randomizing with equal probability between a purchase and sale, but placing *no further order* in the second round. In fact, Section 16(b) does not discourage this type from trading in n=1, in which case he *expects* to earn 0 profits. By doing so, with respect to a situation in which he is inactive, type $\widetilde{s}=U$ causes round n=2 prices to shift less, and therefore any informed type to earn more.

Under the assumption of an asset value $\widetilde{v}$ with two equally likely priors, let's now focus on a leader *with constrained asset holdings*, who is subject to the US short-swing rule. Among the different equilibria that arise, there exists a class of them in which a leader that observes $\widetilde{v}=-b$ (or observes $\widetilde{v}=b$, or is uninformed) trades $x_1=-x_L$ (resp., trades $x_1=x_L$; places any probability, also equal to 0 or 1, on all round n=1 trade quantities, x_1=0 included) and *never* trades afterwards, without being affected, in terms of payoffs, by the consequences that a disclosed sale or purchase have on prices. Differently from the case in which Section 16(b) is *not* set and disclosures are believed *not* to be informative, by undertaking a round n=1 sale (or purchase), type $\widetilde{s}=U$ moves prices, a result which is clearly not quite credible. In fact, in contrast with a situation where L can trade only up to an identical, finite quantity per round, here the imposition of the short-swing rule causes type $\widetilde{s}=U$ to be indifferent whether or not to place a first round order, as no other type benefits from this manipulative attempt. To account for this credibility matter, we invoke the following equilibrium refinement.

Definition 3 *When a type of leader is indifferent whether or not to place orders at any round, this type opts for no order submission, unless this choice causes another type to earn less.*

When this criterion is invoked, 'useless' manipulations by a leader with constrained asset holdings disappear, in that *all* the equilibria but those where type $\widetilde{s}=U$ is inactive are eliminated. In fact, it is easy to show that no equilibrium exists where this type earns a round n positive payoff (left to the reader). The equilibria surviving this refinement are such that the price following the revelation of a sale or that of a purchase equals $-b$ and b respectively, and equals 0 otherwise. At these equilibria, with probability $\psi \in [0, 1]$ a leader aware of $\widetilde{v}=-b$ (or $\widetilde{v}=b$) trades $x_1=-x_L$ (resp., $x_1=x_L$) and, recalling that reversals are dominated, x_2=0, while with probability $1 - \psi$ he trades x_1=0, $x_2=-x_L$(resp., $x_2=x_L$). In particular, the equilibrium where ψ equals 1 is the *only one* displaying robustness to a small probability that M exogenously learns $\widetilde{v}=v$ at the end of the first rather than of the second auction.

Finally, let's consider again a leader, with a cap on total exposure, who is subject to Section 16(b), generalizing the analysis to the case of a non-degenerate random variable $\widetilde{v}\in\mathcal{V}$, and a period made of N rounds. In this case, an equilibrium *exists* where *any* price following the revelation of a *first* sale or of a *first* purchase equals $E[\widetilde{v}|\widetilde{v}<0]$ and $E[\widetilde{v}|\widetilde{v}>0]$ respectively, and equals 0 otherwise; while an uninformed leader never trades, one aware of $\widetilde{v}< E[\widetilde{v}]$ (or $\widetilde{v}> E[\widetilde{v}]$) sells (resp., buys) x_L in n=1, and does not trade afterwards. This equilibrium is robust to a small probability that the market exogenously learns $\widetilde{v}=v$ at the end of the first round.

To highlight advantages and disadvantages implied by the imposition of the US short-swing rule on a leader constrained on asset holdings, let's refer to this latter equilibrium. In case N=2, with respect to the equilibrium in Proposition 3, while for $\delta<\nabla(q,\mu=q\xi)$ the introduction of Section 16(b) makes disclosures informative, for $\nabla(q,\mu=q\xi)\leq\delta<\Delta(q,\mu=\xi)$ it *also* eliminates uninformed manipulations that would otherwise have occurred; conversely, for $\Delta(q,\mu=\xi)\leq\delta$, this additional rule neither reduces manipulations—which would have not arisen in any case—nor improves price efficiency. The negative effect of Section 16(b) is that, following a first disclosure, which we explained to be fully informative, since this rule prevents reversals, in some instances it compromises any further revelation of information that the disclosure of an undertaken reversal (or its absence) would have conveyed otherwise. This happens when private information is sufficiently long-lived—that is, at each period, a sequence of *at least three* rounds takes place. In this case, when the short-swing rule *is not added,* equilibria arise, where a leader *repeatedly* acquiring new information over time never manipulates and price efficiency is *higher,* providing δ is sufficiently high. Specifically, an equilibrium exists where, by trading in round n=1 and not trading in n=2 (because the cap on total exposure has been reached already), even absence of disclosure at the beginning of n=3 moves prices at that round (see Internet Appendix D for a characterization of this equilibrium). This outcome suggests some reflections about the *unconditional* introduction of the short-swing rule, which in some instances is not successful.

The predictions presented in this subsection are robust, in two further respects. Under Section 16(b), the results are unaffected if L, when informed, already learns $\widetilde{v}=v$ rather than $\widetilde{v}\gtrless E[\widetilde{v}]$ in n=1. Traded quantities and price responses (as a function of the state of the world) do not change in equilibrium, when the regulation mandates *trade size* disclosure.

VI.B. Public pre-trade non-anonymity

Public pre-trade disclosure characterizes markets in which, while placing orders, each investor is mandated to reveal his identity, together with information concerning (at least) the direction of the submitted quantity.

First we analyze the case of a mandatory disclosure of order direction, when no order size can be disclosed (under mandatory order size disclosure, or when order direction is mandatory and order size is voluntary, the derivation of the equilibrium is similar, and left to the reader). Then we refine beliefs according to Definition 3. When this criterion is invoked, as long as at least the direction of orders is compulsorily revealed to the public, prices do not shift because the potential insider prefers to stay out of the market.

A distinguishing feature of all the following results is that their derivation does not depend on the maximum quantity that L can trade per round. In the analysis, we refer to an investor that, with probability q, observes $\widetilde{v}=v$ from the very first of a finite number of auction. Even though, for simplicity, this trader is assumed to be small, in the end we will explain why this assumption can be relaxed without affecting the equilibrium outcome, which does not depend on how informative the order-flow is.

A solution is provided for any non-degenerate random variable $\widetilde{v}\in\mathcal{V}$, whose support lower- and upper-bound are denoted with $\underline{b}\in\Re$ and $\overline{b}\in\Re$ respectively. We will show that, unless $\overline{b}=-\underline{b}=\infty$, *alternative trading strategies* can be part of an equilibrium. Moreover, when the probability that $\widetilde{v}$ equals $\underline{b}$ (or $\overline{b}$) is positive—which is the case for discrete and (several) mixed distributions—*alternative pricing rules* can be justified. Nonetheless, we will see that, by invoking the refinement in Definition 3, the equilibrium surviving the criterion will be unique.

Consider a regulation such that, as soon as an order is submitted—that is, before the price is set—the leader has to disclose whether he is undertaking a purchase or a sale. In detail, at the very beginning of round $n \in \{1, .., N\}$, the signal $\S_n \in \{-1, 0, 1\}$ is released, where $\S_n$=1 (or $\S_n$=−1; or $\S_n$=0) implies that L is submitting a buy (resp., a sell; no) order in n.[24] In this context, it follows that the pricing rule, P=$\langle P_1, .., P_n \rangle$, is such that the function P_n: $\{-1, 0, 1\}^n \rightarrow [\underline{b}, \overline{b}]$ depends on all the orders placed by L until that auction n (included).

To derive the equilibrium, a key step consists of focusing on the last auction, N. First notice that, by not trading, L earns 0 profits, no matter where $P_N(\S_1, .., \S_N{=}0)$ lies. Second, suppose that L is signaling $\S_N{=}-1$ (the argument is symmetric when the leader signals $\S_N{=}1$). Because any type of leader aware (at least in expectation, if $\widetilde{s}{=}U$) of $\widetilde{v}{=}v{>}P_N(\S_1, .., \S_N{=}-1)$ prefers not to trade rather than to sell in N, *only* a type aware (at least in expectation) of $\widetilde{v}{=}v \leq P_N(\S_1, .., \S_N{=}-1)$ can be the one that sends this signal. In particular, if this latter type earns a *positive* round N payoff, then the pricing rule is wrong. This is because $P_N(\S_1, .., \S_N{=}-1)$ turns out to be *strictly* greater than the expected asset value conditional on the information available, unless *every* type aware (at least in expectation) of $\widetilde{v}{=}v{<}P_N(\S_1, .., \S_N{=}-1)$ earns even more from purchasing in N, in which case—for an analogous argument—the price $P_N(\S_1, .., \S_N{=}1)$ turns out to be *strictly* smaller than what it should be. It follows that a pricing rule is justified if it is such that every type of leader aware (at least in expectation) that $\widetilde{v}$ is different from $P_N(\S_1, .., \S_N{=}-1)$ and $P_N(\S_1, .., \S_N{=}1)$ strictly prefers to signal $\S_N{=}0$. Specifically: (i) As long as a '*perfect revelation*' of the investor's type at any previous auction has *not yet* occurred, $P_N(\S_1, .., \S_N{=}-1){=}\underline{b}$ (or $P_N(\S_1, .., \S_N{=}1){=}\overline{b}$) is the sole price response that causes every type of leader but that aware of $\widetilde{v}{=}\underline{b}$ (resp., $\widetilde{v}{=}\overline{b}$) not to sell (resp., not to purchase) in N. Given this price response, an investor that observes $\widetilde{v}{=}\underline{b}$ (resp., $\widetilde{v}{=}\overline{b}$) weakly prefers to disclose $\S_N{=}-1$ (resp., $\S_N{=}1$), earning as much as he achieves when he does not trade in N (an action that is always feasible), namely 0. (ii) If L's type has already been perfectly identified in a specific auction $n{<}N$, the leader earns a round N payoff equal to 0. In fact, no matter whether he submits a buy, a sell, or no order in N—an action that depends on the position limit to which L is subject, if any—the price p_N will not shift from the correct price already set in n.

In conclusion, although L's action in round N depends on past events—namely, on his action and M's pricing rule at any previous auction—in equilibrium the payoff that L achieves from selecting one round N best response or another is independent of past history, in that he always earns a round N payoff equal to 0. Thus, while deriving L's inter-round equilibrium actions, round N can be treated separately from the first $N-1$ auction, because L's inter-temporal choice up to round N (excluded) is not affected by his decision in this latter round. Now, consider only the first $N-1$ auctions. Focusing on the new 'last round'—that is, round $N-1$—the same conclusions reached when analyzing round N can be drawn. Following this logical process, we note that L's inter-termporal choice at each round is not affected by his decision in any future round. The payoff he achieves from selecting a best round n response or another equals 0, no

[24] In order driven markets, at each round, it does not matter whether the signal is sent before or after the associated order submission, provided the price is set after the signal is sent. Order submission and signal disclosure are assumed to occur in separate steps, to emphasize the distinction between how much L submits on one side, and how much information concerning a submission—namely, order direction or order size—is disclosed via a public announcement on the other. The analysis is unaffected when we study price-driven markets, provided the round n disclosure about a forthcoming round n trade is made before the price is formed. Otherwise, no departure from post-trade disclosure would occur.

matter what equilibrium action L and M play at any other past or future auction.

In equilibrium, (i) *until* the round in which a perfect revelation of the type occurs (included), a potential insider aware (at least in expectation, if $\tilde{s}=U$) of $\underline{b}<v<\bar{b}$ does not submit orders at any round. Indeed, for supports of $\tilde{v}$ *bounded* on the left (or right), a leader that observes $\tilde{v}=\underline{b}$ (resp., $\tilde{v}=\bar{b}$) randomizes with any probability—even 0 or 1, and not necessarily equal within rounds—between selling (resp., purchasing) any quantity and not trading. Conversely, (ii) from the round following a perfect revelation of the trader *onwards*, any type of leader places any probability on each feasible action, given his position limit.

For what concerns equilibrium prices, *before* a *first* order is placed, they equal $E[\tilde{v}]$ at any round, *unless* either $\tilde{v}=\underline{b}$ or $\tilde{v}=\bar{b}$ has positive mass, in which cases, depending on beliefs, an initial lack of submissions *may* shift prices and, in some instances, lead to a perfect revelation of the type (see Internet Appendix E). In case an initial series of missed submissions does not perfectly reveal L's type, a perfect revelation occurs as soon as L submits a *first* order, which shifts prices to $\underline{b}$ or $\bar{b}$ depending on whether this submission is a sell or a buy order respectively.

Whether the cap on total exposure (or the quantity that the potential insider is allowed to submit per round), x_L, is negligible or not, and in the latter case, whether x_L is finite or equal to ∞, does not play a role in the determination of these equilibria. In other words, the associated outcomes do not depend on the leader being a small or a large investor. In fact, focusing on the derivation of the results above, it is clear that, even when only the order direction has to be mandatorily disclosed, the price at round n does not depend on the past and present order-flow, $\{x_1+\tilde{u}_1, .., x_n+\tilde{u}_n\}$, because $\{\S_1, .., \S_n\}$ turns out to be a sufficient statistic for $\{\S_1, .., \S_n, x_1+\tilde{u}_1, .., x_n+\tilde{u}_n\}$ with respect to $\tilde{v}$. Thus, not only the support of $\tilde{u}_n$ can be bounded. Any specification about the properties of the noise traders' demand is acceptable.

When the criterion in Definition 3 is invoked, asset value properties no longer play a role. A unique equilibrium survives this refinement. At this equilibrium, L never submits orders and $P_n(\S_i=0, \forall i \in \{0, .., n\})=E[\tilde{v}]$. In fact, denoting with $\lambda \in \{1, .., N\}$ the first round in which L places an order, the equilibrium price responses $P_{n\in\{\lambda,..,N\}}(\S_i=0, \S_\lambda=-1, \forall i<\lambda)=\underline{b}$ and $P_{n\in\{\lambda,..,N\}}(\S_i=0, \S_\lambda=1, \forall i<\lambda)=\bar{b}$ represent an implicit threat that makes any type of leader *at least weakly prefer* inactivity to any other strategy. By deciding not to trade at any auction, neither type $\underline{b}$ nor type $\bar{b}$ causes any other type to experience a payoff reduction. Therefore, given our restriction on beliefs, *every* type of leader now *prefers* not to trade at all.

To sum up, refining beliefs in the way we suggested, a clear result is derived. A regulation mandating at least pre-trade disclosure of order directions keeps the potential insider away from the market. This result is independent of (i) the asset value statistical properties, (ii) the size of L, (iii) the position limit to which L is subject, and (iv) the noise traders' demand.

VII. Conclusion

The present article studies public disclosure of inside statements by 'small' investors, who exchange without being spotted, and develops a comprehensive theory of market non-anonymity that brings several novel results of concern to investors and regulators.

First, we examine the effects of a regulation mandating investors to publicly certify trades undertaken. The analysis reduces regulators' concerns about this form of disclosure. In fact, only in specific instances will a trader with constrained asset holdings manipulate when uninformed. Asset value properties, market beliefs, inter-temporal choices, and investors' characteristics play a role. The divergence with which different regulations list the investors and the conditions (on allowed delay and on minimal exchanged quantity) to report trades confirms how a consensus on who best should disclose has not yet been reached. On this front, the solution to the problem of a trader who is in the position repeatedly to acquire new inside information indicates that, if prices react to current disclosures, those traders who are less likely to be informed (e.g., investors not directly involved in the firm's management) tend to undertake uninformed manipulations; conversely, those who are more likely to be informed (e.g., CEOs) tend *not* to manipulate when *un*aware about elements that will affect the fundamental value. Actually, the SEC obliges also principal stockholders to disclose their trades. In this respect, our study highlights that, by allowing for a sufficient delay in reporting trades, even these big investors—instead of dissimulating, when informed, to reduce the leakage of inside information—will behave similarly to small-sized traders, breaking down each pre-decided order into several small chunks.

The second but most important result of this article is that mandating trade revelation is unnecessary. In fact, under mandatory disclosure, our trader turns out to achieve a higher payoff compared to the case of no public disclosure. Therefore, by changing the regulation and making trade reporting not compulsory, any time the price is known to react to current disclosures, the investor turns out to have all the incentives to trade as before, voluntarily revealing to the public any transaction undertaken immediately after having exchanged up to his (privately known) maximum. Not only does this result indicate that there is no need to enforce trade reporting with punitive laws or invigilation, nor to study which delay to allow in publicizing trades. It also reveals a link to the strain of literature on (uncertified or non-factual) announcements in capital markets, upon which we improve by getting over the assumption of a truthful or honest insider.[25] Rather, truthfulness or honesty are entirely derived at the equilibrium. As for the revelation of certified trades, we show that informative disclosures occur voluntarily, except when the fundamental value is constrained to two possible realizations, in which case meaningful voluntary disclosures cannot be modeled. In particular, when the market interprets a non-factual message as favorable/unfavorable, even in this case prices react as they do following the disclosure of a certified purchase/sale, namely the kind of transaction that the investor actually undertakes in secret before disclosing that non-factual message. Hence, in those instances where investors manipulate, requiring them to certify their trades does not prevent the price from moving accidentally in the opposite direction with respect to the real asset value. In fact, "actions *do not* speak louder than words". Still, because of its fast operating time, certifying trades electronically may guarantee a higher chance that the signal reaches the public before inside information reaches its end time. Consequently, electronically certified trades may allow for higher levels of price efficiency over time, together with a higher incident of possible price overshooting, which ultimately represents the goal for whose achievement the insider discloses voluntarily.

Finally, the imposition of two alternative rules is modeled. The US short-swing rule ensures that any otherwise appealing deceptive aim is not pursued. However, its unconditional adoption has drawbacks. Public pre-trade non-anonymity keeps insiders away from the market, yet this measure implies the lowest price efficiency level.

To conclude, the smallness assumption in terms of price impact makes our model fairly tractable, and allows to generalize the analysis in different dimensions (e.g., that of the fundamental value distribution), with predictions that are robust in many respects. In particular, the results pertaining to the revelation of certified transactions hold for several combinations of provision for order direction and order size disclosure. By questioning which combination of factors drives each of our results, this article also helps us to understand better the determinants for a

[25] With the exception of the uncertified revelation of trades—whose truthfulness is often enforced (at least on paper) by vigilance, preventing any lying about relevant facts—for what concerns the production of non-factual messages, truthfulness (even when the message makes some reference to inside information) and honesty are generally hard to verify and interpret respectively, and thus not enforceable (see also BL, p. 947). Hence a priori it is difficult to reconcile this moral conduct with that of profit-maximizing traders.

number of important predictions in literature, from which ours differ. Because of its simplicity, the present analytical framework represents an ideal benchmark to which future research can refer to measure and refine our knowledge or challenge the policy implications derived herein.

Appendix

Derivation of $\mathcal{S}$ in Lemma 3. For an infinite horizon, with M's strategy held fixed, by defecting at period t=1, L's expected profits from t=2 on (discounted to t=1) equal $\frac{\delta}{1-\delta}qx_L\xi$; by not defecting in t=1, they equal $\delta\mathcal{S}$, where $\mathcal{S}$ also depends on q, δ, μ. To underline it, we write $\mathcal{S}(q,\delta,\mu,\bar{\alpha},\bar{\beta})$. In particular, $\mathcal{S}=\sum_{i=0}^{\infty}\delta^i W_{i+1}$, where:

$$W_1=q[\bar{\alpha}\cdot\mathcal{M}(\mu)+(1-\bar{\alpha})\cdot\mathcal{L}(\mu)]+(1-q)\bar{\beta}\cdot\mathcal{P}(\mu), \tag{6}$$

$$W_{j+1}=\bar{\alpha}q^2x_L\xi+q(1-\bar{\alpha})W_j+(1-q)(1-\bar{\beta})W_j+\frac{(1-q)\bar{\beta}}{2}W_j+\frac{(1-q)\bar{\beta}}{2}qx_L\xi, \forall j>1, \tag{7}$$

which can be written as: $W_{j+1}=\gamma+\varphi W_j$, $\forall j>1$, where $\gamma=[q\bar{\alpha}+\frac{(1-q)\bar{\beta}}{2}]qx_L\xi$, $\varphi=[\frac{2(1-q\bar{\alpha})-\bar{\beta}(1-q)}{2}]$. This is a first order linear difference equation. Thus: $W_{j+1}=\gamma[\sum_{i=0}^{j-1}\varphi^i]+\varphi^j W_1=\gamma\frac{1-\varphi^j}{1-\varphi}+\varphi^j W_1$. It follows that:

$$\mathcal{S}=\sum_{i=0}^{\infty}\delta^i\Big[\varphi^i W_1+\gamma\frac{1-\varphi^i}{1-\varphi}\Big]=\frac{W_1+\frac{\delta}{1-\delta}\gamma}{1-\delta\varphi}. \tag{8}$$

The series converges if $|\delta\varphi|<1$, which is always verified, because $0\le\delta<1$ and $0\le\varphi\le 1$. In fact: (i) $\varphi\le 1 \therefore -2q\bar{\alpha}-\bar{\beta}(1-q)\le 0$, and (ii) $0\le\varphi \therefore 0\le 2(1-q\bar{\alpha})-\bar{\beta}(1-q) \therefore q(2\bar{\alpha}-\bar{\beta}) \le 2-\bar{\beta}$, which holds whenever $\bar{\alpha}\in[0,1]\wedge\bar{\beta}\in[0,1]\wedge q\in(0,1]$. It is also easy to check that $\frac{\delta}{1-\delta}qx_L\xi<\delta\cdot\mathcal{S}$, $\forall\delta>0\wedge\mu>0$. ■

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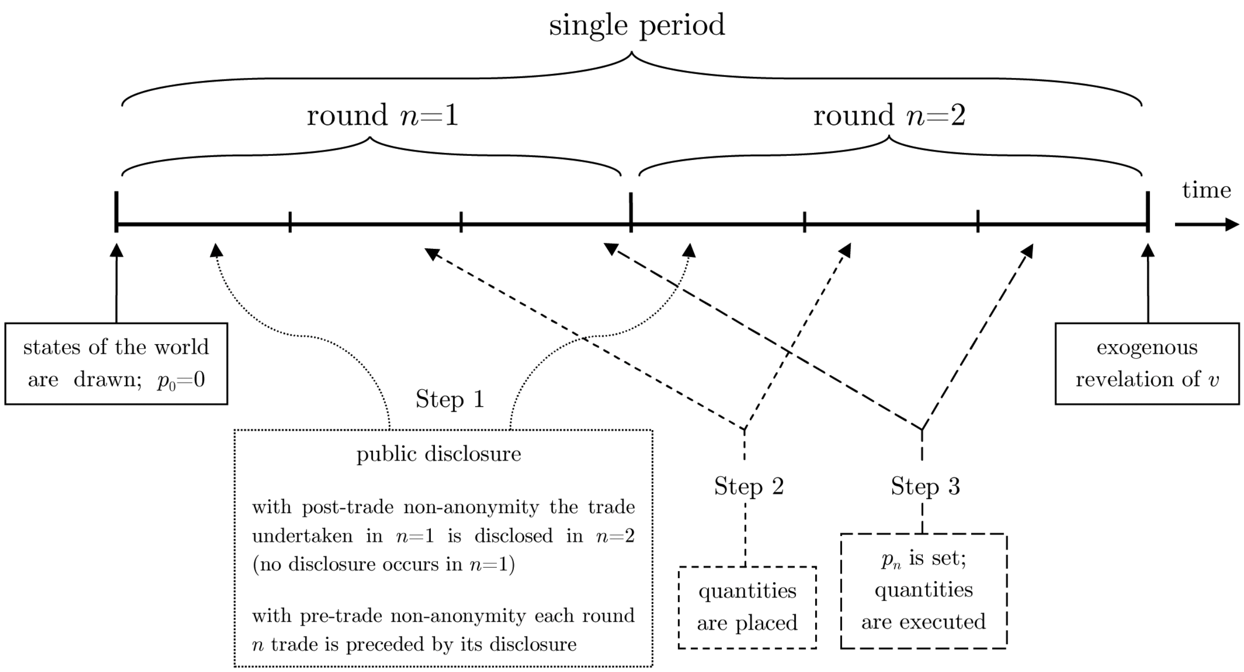

Fig. 1. Timing of events in the single period.

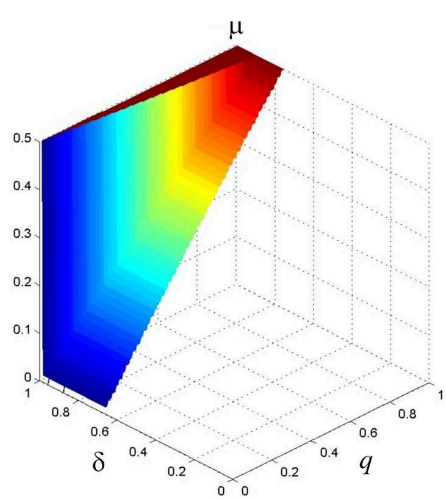

Fig. 2. Behavior of μ (equilibrium price shift following trade disclosure) for each pair δ and q (inter-period discount factor and probability that L is informed over time respectively) in the case of a fundamental value $\widetilde{v} \sim U[-1,1]$. Notes: The white area coincides with μ=0 (M never conditions on disclosed trades).

Not for Publication: Web-based Technical Appendix to "Public Disclosure by 'Small' Traders"

Internet Appendix A

Proof of Proposition 1. For any possible pricing rule such that $p_1{=}0$ and that $\exists\tau : P_2(\tau) \neq 0$, we prove the following. (I) Assuming an insider that observes $\widetilde{v}{=}v$ even in $n{=}1$, derive each type of insider's optimal strategy, $X(\widetilde{s}{=}I \wedge \widetilde{v}{=}v)$. Holding X fixed and inverting it to make the information possessed by L explicit, we show that, when M is replying to at least half of the types of insider—those belonging either to $[-b,0)$ or $(0,b]$—contradictions arise, in that he sets either $P_2(\tau{=}\cdot\,,X){=}E[\widetilde{v}\mid\widetilde{v}{=}v{<}0]{>}0$ in response to the disclosure by *each* leader aware of $\widetilde{v}{=}v{<}0$, or $P_2(\tau{=}\cdot\,,X){=}E[\widetilde{v}\mid\widetilde{v}{=}v{>}0]{<}0$ in response to the disclosure by *each* leader aware of $\widetilde{v}{=}v{>}0$. (II) When in $n{=}1$ the insider only observes whether $\widetilde{v}{<}0$ or $\widetilde{v}{>}0$, the price that M sets in round $n{=}2$ in response to at least one of the two types of insider turns out to lie over $(0,b]$ (or $[-b,0)$) when L observes $\widetilde{v}{<}0$ (resp., $\widetilde{v}{>}0$).

(I) Eight cases (from $C1$ to $C8$) representing all the possible combinations of M's strategy profiles can be identified.

$C1$: $P_2(\tau{=}1) \geq 0 \wedge P_2(\tau{=}0) \geq 0 \wedge P_2(\tau{=}{-1}) \geq 0$. Given this strategy profile, the following sub-cases can be identified. (i) When $P_2(\tau){=}0$, $\forall\tau$, no contradiction of the sort described above arises. (ii) When at least one, but not every, signal $\tau{=}i$ causes $P_2(\tau{=}i)$ to equal 0, the best response by an insider aware of $\widetilde{v}{=}v{<}0$, $X(\widetilde{s}{=}I\wedge\widetilde{v}{=}v{<}0)$, is such that $\tau \neq i$. To see it, it is sufficient to notice that, in case $P_2(\tau{=}1)$ (or $P_2(\tau{=}0)$; or $P_2(\tau{=}{-1})$): (ii.a) equals 0, an insider aware of $\widetilde{v}{=}v{<}0$ that decides to signal $\tau{=}1$ (resp., $\tau{=}0$; $\tau{=}{-1}$) cannot do any better than trading in such a way that $x_1{+}x_2{=}{-x_L}$, earning $x_L v$; (ii.b) differs from 0, the strategy $\langle x_1{=}x_L; x_2{=}{-2x_L}\rangle$ (resp., $\langle x_1{=}0; x_2{=}{-x_L}\rangle$; $\langle x_1{\lesssim}\, 0; x_2{\lesssim} -x_L\rangle$) allows each type $\widetilde{s}{=}I\wedge\widetilde{v}{=}v{<}0$ to earn more than $x_L v$.[26] Holding X fixed, we have that $P_2(\tau \neq i, X){=}E[\widetilde{v}\mid\widetilde{v}{=}v{<}0]{>}0$, which is a contradiction. Finally, (iii) when $P_2(\tau){>}0$, $\forall\tau$, any response X by each type $\widetilde{s}{=}I\wedge\widetilde{v}{=}v{<}0$ is such that $P_2(\tau{=}\cdot\,,X){=}E[\widetilde{v}\mid\widetilde{v}{=}v{<}0]{>}0$.

$C2$: $P_2(\tau{=}1) \leq 0 \wedge P_2(\tau{=}0) \leq 0 \wedge P_2(\tau{=}{-1}) \leq 0$. This case is symmetric to $C1$.

$C3$: $P_2(\tau{=}1){>}0 \wedge P_2(\tau{=}0) \geq 0 \wedge P_2(\tau{=}{-1}) < 0$. Given this pricing rule, the strategy $X{=}\langle x_1{=}{-x_L}; x_2{=}2x_L\rangle$ strictly dominates any other, provided the insider observes $\widetilde{v}{=}v{>}\max\{0;\zeta;\epsilon\}$, where $\zeta{=}P_2(\tau{=}{-1}){+}P_2(\tau{=}1)$ and $\epsilon{=}P_2(\tau{=}{-1}){+}\frac{P_2(\tau=0)}{2}$.[27] It follows that, as long as $\max\{\zeta;\epsilon\} \leq 0$, each type aware of $\widetilde{v}{=}v{>}0$ prefers X. Holding $X_1(\widetilde{s}{=}I \wedge \{\zeta;\epsilon\} \leq 0{<}v) {=}{-x_L}$ fixed, we have that $0{>}P_2(\tau{=}{-1},X){=}E[\widetilde{v}\mid\widetilde{v}{=}v{>}0]$, $\forall v{>}0$, which is a contradiction.

[26] The symbols $\gtrsim$ and $\lesssim$ stand for *just greater than* and *just smaller than* respectively.

[27] To see it, consider an insider that observes $\widetilde{v}{=}v{>}0$. First notice that any alternative strategy such that $x_1{<}0$ makes this type earn smaller profits. Second, while for $v \geq P_2(\tau{=}1)$ we have that $\langle x_1{=}{-x_L}; x_2{=}2x_L\rangle \succ \langle x_1{>}0, x_2{=}\cdot\rangle$, on the contrary, for $0{<}v{<}P_2(\tau{=}1)$ we have that the strategy $\langle x_1{=}{-x_L}; x_2{=}2x_L\rangle$ strictly dominates $\langle x_1{=}x_L; x_2{=}{-2x_L}\rangle$ (which strictly dominates any alternative strategy such that $x_1{>}0$) only *if* $\{-x_L v{+}2x_L[v - P_2(\tau{=}{-1})]\}{>}\{x_L v - 2x_L[v - P_2(\tau{=}1)]\} \therefore v{>}\zeta$. Third, while for $v \geq P_2(\tau{=}0)$ we have that $\langle x_1{=}{-x_L}; x_2{=}2x_L\rangle \succ \langle x_1{=}0, x_2{=}\cdot\rangle$, on the contrary, when $0{<}v{<}P_2(\tau{=}0)$ we have that the strategy $\langle x_1{=}{-x_L}; x_2{=}2x_L\rangle$ strictly dominates $\langle x_1{=}0; x_2{=}{-x_L}\rangle$ (which strictly dominates any alternative strategy such that $x_1{=}0$) only *if* $\{-x_L v{+}2x_L[v - P_2(\tau{=}{-1})]\}{>}\{-x_L[v - P_2(\tau{=}0)]\} \therefore v{>}\epsilon$.

Now we show that, when M is replying to any type of insider aware of $\widetilde{v}=v<0$ with the pricing rule in $C3$, contradictions arise, provided $\max\{\zeta;\epsilon\}>0$.

Before we proceed, the following intermediate results need to be established. From an insider perspective: (i) When $\widetilde{v}=v<0$, the strategy $\langle x_1=x_L; x_2=-2x_L\rangle$ (or $\langle x_1=0; x_2=-x_L\rangle$) strictly dominates any other strategy such that $x_1>0$ (resp., $x_1=0$). (ii) When $v \leq P_2(\tau=-1)<0$, both $\langle x_1=x_L; x_2=-2x_L\rangle$ and $\langle x_1=0; x_2=-x_L\rangle$ also strictly dominate any strategy such that $x_1<0$. (iii) When $P_2(\tau=-1)<v<0$, the strategy $\langle x_1=x_L; x_2=-2x_L\rangle$ (or $\langle x_1=0; x_2=-x_L\rangle$) strictly dominates $\langle x_1=-x_L; x_2=2x_L\rangle$ (which dominates any alternative strategy such that $x_1<0$) only if $x_L v - 2x_L[v - P_2(\tau=1)]$ (resp., $-x_L[v - P_2(\tau=0)]$) is strictly greater than $-x_L v+2x_L[v - P_2(\tau=-1)]$, that is only if $v<\zeta$ (resp., $v<\epsilon$). (iv) When $\widetilde{v}=v<0$, if $P_2(\tau=1)>\frac{P_2(\tau=0)}{2}$ (or $P_2(\tau=1)=\frac{P_2(\tau=0)}{2}$; or $P_2(\tau=1)<\frac{P_2(\tau=0)}{2}$), the profits that an insider earns from playing $\langle x_1=x_L; x_2=-2x_L\rangle$ are greater than (resp., equal to; smaller than) those from playing $\langle x_1=0; x_2=-x_L\rangle$.

As a consequence of the results at point i, ii, iii, and iv, when $\max\{\zeta;\epsilon\}>0$, the following conclusions can be drawn. (a) Suppose that $P_2(\tau=1)>\frac{P_2(\tau=0)}{2}$. (a.i) If $\zeta>0$, no matter which value ϵ assumes, then each type $\widetilde{s}=I\wedge\widetilde{v}=v<0$ strictly prefers $X=\langle x_1=x_L; x_2=-2x_L\rangle$ to any other strategy. Holding X fixed, it follows that $0<P_2(\tau=1, X)=E[\widetilde{v} \mid \widetilde{v}=v<0]$, $\forall v<0$, which is a contradiction. (a.ii) The remaining sub-case, namely the one of $\zeta \leq 0<\epsilon$, is not of interest. In fact, making the condition on ζ and ϵ explicit, it follows that it refer to a situation where $P_2(\tau=-1)+P_2(\tau=1) \leq 0<P_2(\tau=-1)+\frac{P_2(\tau=0)}{2} \therefore P_2(\tau=1)<\frac{P_2(\tau=0)}{2}$, which is not a possibility, being the case in question—i.e., point a—the one of $P_2(\tau=1)>\frac{P_2(\tau=0)}{2}$. (b) Suppose that $P_2(\tau=1)=\frac{P_2(\tau=0)}{2}$ (case in which $P_2(\tau=0)>0$ for sure). This condition on prices implies that $\zeta=\epsilon$. Thus, the only relevant sub-case to be studied is the one of $\zeta=\epsilon>0$. In this instance, each insider aware of $\widetilde{v}=v<0$ replies by randomizing between $\langle x_1=x_L; x_2=-2x_L\rangle$ and $\langle x_1=0; x_2=-x_L\rangle$. Holding the trading strategy by each of these types of insider fixed, regardless of the probability with which he initially buys or does not trade (even 0 or 1), the price in response to his disclosure turns out to lie above 0, which is a contradiction. (c) Suppose that $P_2(\tau=1)<\frac{P_2(\tau=0)}{2}$ (case in which $P_2(\tau=0)>0$ for sure). (c.i) If $\epsilon>0$, no matter which value ζ assumes, then each type $\widetilde{s}=I\wedge\widetilde{v}=v<0$ strictly prefers $X=\langle x_1=0; x_2=-x_L\rangle$ to any other strategy. Holding X fixed, it follows that $0<P_2(\tau=0, X)=E[\widetilde{v} \mid \widetilde{v}=v<0]$, $\forall v<0$, which is a contradiction. (c.ii) The remaining sub-case, namely the one of $\epsilon \leq 0<\zeta$, is not of interest. In fact, making the condition on ϵ and ζ explicit, it follows that $P_2(\tau=-1)+\frac{P_2(\tau=0)}{2}\leq 0<P_2(\tau=-1)+P_2(\tau=1) \therefore \frac{P_2(\tau=0)}{2}<P_2(\tau=1)$, which is not a possibility, being the case in question—i.e., point c—the one of $P_2(\tau=1)<\frac{P_2(\tau=0)}{2}$.

$C4$: $P_2(\tau=1) > 0 \wedge P_2(\tau=0) \leq 0 \wedge P_2(\tau=-1)<0$. This case is symmetric to $C3$.

$C5$: $P_2(\tau=-1) \geq 0 \wedge P_2(\tau=0) \geq 0 \wedge P_2(\tau=1) < 0$. If $\widetilde{v}=v>0$, it can be shown that buying a negligible quantity in $n=1$ and buying again up to the maximum capacity in $n=2$, that is $\langle x_1 \gtrsim 0; x_2 \lesssim x_L\rangle$, dominates any other strategy. However, holding $X_1(\widetilde{v}=v>0)=x_1 \gtrsim 0$ fixed, it follows that $0>P_2(\tau=1, X)=E[\widetilde{v} \mid \widetilde{v}=v>0]$, a contradiction.

$C6$: $P_2(\tau=-1) > 0 \wedge P_2(\tau=0) \leq 0 \wedge P_2(\tau=1) \leq 0$. This case is symmetric to $C5$.

$C7$: $P_2(\tau=0) > 0 \wedge P_2(\tau=1) \leq 0 \wedge P_2(\tau=-1) \leq 0$. The following sub-cases can be identified. (i) For $P_2(\tau \neq 0)=0$, we end up in case $C1$. (ii) For $P_2(\tau \neq 0) \neq 0$, each insider aware of $v>0$, for example, strictly prefers $\langle x_1=x_L; x_2=0\rangle$ to $\langle x_1=0; x_2=\cdot\rangle$, which means that he signals in a way that pushes p_2 below 0. (iii) For $P_2(\tau=1)=0 \wedge P_2(\tau=-1)<0$ (or $P_2(\tau=1)<0 \wedge P_2(\tau=-1)=0$), each insider aware of $v>0$ strictly prefers $\langle x_1=-x_L; x_2=2x_L\rangle$ (resp., $\langle x_1 \gtrsim 0; x_2 \lesssim x_L\rangle$) to any other strategy. The price response to the behavior by each of these types in such that $0>P_2(\tau{=}-1,X){=}E[\widetilde{v} \mid \widetilde{v}{=}v{>}0]$ (resp., $0>P_2(\tau{=}1,X){=}E[\widetilde{v} \mid \widetilde{v}{=}v{>}0]$). $C8$: $\quad P_2(\tau{=}0)<0 \wedge P_2(\tau{=}1)\geq 0 \wedge P_2(\tau{=}-1)\geq 0$. This case is symmetric to $C7$.

(II) Notice that, in response to a pricing rule, *if all* types of leader already aware, in round $n{=}1$, of $\widetilde{v}{=}v{>}0$ (or $\widetilde{v}{=}v{<}0$) prefer to submit a specific order $x_1\in[-b,b]$—alternatively, *if* they are indifferent about placing a specific round $n{=}1$ order or another—*then* a leader that in $n{=}1$ only observes $\widetilde{v}{>}0$ (resp., $\widetilde{v}{<}0$) displays an identical preference over actions.
Because in part I we showed that, in response to a pricing rule such that $p_2{=}0$ and that $\exists\tau : P_2(\tau)\neq 0$, each type of insider aware of either $\widetilde{v}{=}v{<}0$ or $\widetilde{v}{=}v{>}0$ places an identical first round order, x_1, which causes contradictions to arise, it follows that, when in round $n{=}1$ the insider is only aware of whether $\widetilde{v}{<}0$ or $\widetilde{v}{>}0$, the best reply X by *either* type $\widetilde{s}{=}I\wedge\widetilde{v}{<}0$ *or* type $\widetilde{s}{=}I\wedge\widetilde{v}{>}0$ is such that $P_2(\tau{=}\cdot,X){=}E[\widetilde{v} \mid \widetilde{v}{<}0]{>}0$ or $P_2(\tau{=}\cdot,X){=}E[\widetilde{v} \mid \widetilde{v}{>}0]{<}0$ respectively. ■

Proof of Lemma 1. Consider the case of $\mu{>}0$. To model L's behavior, let's introduce an auxiliary random variable, $\widetilde{\alpha}$, which (without loss of generality) has the following propriety: $\widetilde{\alpha}\sim U[0,1]$. For a leader that decides to mislead rather than lead with probability $\bar{\alpha}$: (i) If $\widetilde{\alpha}\geq\bar{\alpha}$, then: (i.a) When $0{<}\widetilde{v}{=}v{<}\mu$ (or $0{>}\widetilde{v}{=}v{>}-\mu$), case that happens with probability $2[\mathrm{F}(\mu)-\mathrm{F}(0)]$, buying (resp., selling) a quantity x_L in $n{=}1$ and reversing this position in $n{=}2$ by selling (resp., buying) x_L and continuing selling (resp., buying) an extra quantity x_L is the optimal strategy if L decides to trade in two rounds. Besides, trading in two rounds dominates trading only in one. (i.b) When $\widetilde{v}{=}v\geq\mu$ (or $\widetilde{v}{=}v\leq-\mu$), case that happens with probability $2[\mathrm{F}(b)-\mathrm{F}(\mu)]$, buying (resp., selling) up to x_L in $n{=}1$ or in $n{=}2$ and then waiting up to public revelation of $\widetilde{v}{=}v$ dominates buying (resp., selling) a positive quantity in both rounds. In $n{=}1$, L still does not know $\widetilde{v}{=}v$; thus buying (resp., selling) up to x_L in $n{=}1$ dominates doing it in $n{=}2$ because, if L traded only in $n{=}2$, with probability $\Pr(0{<}v{<}\mu)$ he would miss the opportunity to profit by subsequently reversing his position, in the manner explained above. (ii) If $\widetilde{\alpha}{<}\bar{\alpha}$, L's optimal strategy is to trade $x_1{=}-x_L$ (or $x_1{=}x_L$) when $\widetilde{v}{>}0$ (resp., $\widetilde{v}{<}0$) and reverse his position up to the limit capacity in $n{=}2$. ■

Proof of Proposition 2. First we find $\bar{\alpha}^{*I}$ that maximizes L's discounted expected profits over periods. In details, $\bar{\alpha}^{*I}{=}\arg\max_{\bar{\alpha}} E[\Pi]$, where $E[\Pi]{=}\mathcal{T}{+}\varkappa\{\mathcal{T}{+}\varkappa\{\mathcal{T}{+}\varkappa\{...\}\}\}{=}\frac{\mathcal{T}}{1-\varkappa}$, $\mathcal{T}{=}\bar{\alpha}\cdot\mathcal{M}(\mu){+}(1{-}\bar{\alpha})\cdot\mathcal{L}(\mu){+}\bar{\alpha}\frac{\delta x_L\xi}{1-\delta}$, and $\varkappa{=}\delta(1{-}\bar{\alpha})$. Notice that $\delta \gtreqless \delta_\nabla \rightarrow \frac{\partial E[\Pi]}{\partial\bar{\alpha}} \lesseqgtr 0$. Thus, (i) If $\delta\geq\delta_\nabla$, L's best response is to set $\bar{\alpha}^{*I}{=}0$. Holding L's optimal strategy fixed, consider M's initial pricing rule. For $\mu{=}\xi$, we have an equilibrium. Since $0{<}\delta_\nabla{<}1$, some economies such that $\delta\geq\delta_\nabla$ always exist. (ii) If $\delta\leq\delta_\nabla$, L's best response is to set $\bar{\alpha}^{*I}{=}1$. Holding L's optimal strategy fixed, for $\mu\neq 0$ contradictions arise. Providing L replies as he does in equilibrium when no repetition of the single-period occurs, then $\mu{=}0$ is justified. ■

Proof of Lemma 2. Consider the case of $\mu{>}0$. To model the behavior of type $\widetilde{s}{=}U$, without loss of generality two new auxiliary random variables, $\widetilde{\beta}\sim U[0,1]$ and $\widetilde{z}\sim U[0,1]$, are introduced. For a leader that decides to bluff rather than not to bluff with probability $\bar{\beta}$: (i) If $\widetilde{\beta}\geq\bar{\beta}$, case in which L does not trade in $n{=}1$, any probability (also equal to 0 or 1) placed on all round $n{=}2$ trade quantities ($x_2{=}0$ included) implies an ax ante per-period profits equal to 0. (ii) If $\widetilde{\beta}{<}\bar{\beta}$ and $\widetilde{z}\geq\bar{z}$ (or $\widetilde{z}{<}\bar{z}$), buying (resp., selling) a quantity x_L in $n{=}1$ and selling (resp., buying) a quantity $2x_L$ in $n{=}2$ is the optimal strategy, which makes L earn under expectation $\mathcal{P}(\mu){=}\int_{\widetilde{v}}\{\,x_L(-1)(-\mu)+x_L[\widetilde{v}{-}(-\mu)]\,\}\,f(\widetilde{v})d\widetilde{v}{=}2\mu x_L{>}0$ per period. ■

Proof of Proposition 3. For $\mu{>}0$, we prove only that (a) if $\delta{>}\Delta(q,\mu)$, then $\bar{\alpha}^{*I}{=}\bar{\beta}^{*U}{=}0$; (b) if $\delta{=}\Delta(q,\mu)$, then either $\bar{\alpha}^{*I}{=}\bar{\beta}^{*U}{=}0$ or $\bar{\alpha}^{*I}{=}0,\bar{\beta}^{*U}{=}1$; (c) if $\nabla(q,\mu){<}\delta{<}\Delta(q,\mu)$, then $\bar{\alpha}^{*I}{=}0$, $\bar{\beta}^{*U}{=}1$; (d) if $\delta{=}\nabla(q,\mu)$, then either $\bar{\alpha}^{*I}{=}0,\bar{\beta}^{*U}{=}1$ or $\bar{\alpha}^{*I}{=}1$; and (e) if $\delta{<}\nabla(q,\mu)$, then $\bar{\alpha}^{*I}{=}1$.

To find the maximum of $E\left[\Pi^I\right]$ and $E\left[\Pi^U\right]$, consider $\frac{\partial E\left[\Pi^I\right]}{\partial\bar{\alpha}}$, $\frac{\partial E\left[\Pi^I\right]}{\partial\bar{\beta}}$, $\frac{\partial E\left[\Pi^U\right]}{\partial\bar{\alpha}}$ and $\frac{\partial E\left[\Pi^U\right]}{\partial\bar{\beta}}$. All the denominators (function of $\bar{\alpha}$ and $\bar{\beta}$) are squared. Each numerator is not function of the variable we are deriving for. Thus the maximum is on one of the support's boundaries of $\bar{\alpha}$ and $\bar{\beta}$.

Let's fix $\mu{>}0$ and study the corner solutions when L in $t{=}1$ is of type (i) $\widetilde{s}{=}I$ or (ii) $\widetilde{s}{=}U$.

(i) Considering the function $E\left[\Pi^I\right]$, what follows can be derived: (i.a) $\delta \gtreqless \Delta(q,\mu)$ $\rightarrow E[\Pi^I \mid \bar{\alpha}{=}\bar{\beta}{=}0] \gtreqless E[\Pi^I \mid \bar{\alpha}{=}0,\bar{\beta}{=}1]$, (i.b) $\delta \gtreqless \frac{\mathcal{M}(\mu)\,-\,\mathcal{L}(\mu)}{\mathcal{M}(\mu)\,-\,\mathcal{L}(\mu)\,+q[\mathcal{L}(\mu)\,-\,x_L\xi]} \rightarrow E[\Pi^I \mid \bar{\alpha}{=}\bar{\beta}{=}0] \gtreqless$ $E[\Pi^I \mid \bar{\alpha}{=}1]$, and (i.c) $\delta \gtreqless \nabla(q,\mu) \rightarrow E[\Pi^I \mid \bar{\alpha}{=}0,\bar{\beta}{=}1] \gtreqless E[\Pi^I \mid \bar{\alpha}{=}1]$.
First, focusing on cases (i.a) and (i.c), we have that $\Delta(q,\mu){>}\nabla(q,\mu) \therefore 2\mathcal{P}(\mu){>}\ \mathcal{M}(\mu){-}\mathcal{L}(\mu)$, which can be shown to be always verified. Thus, the sufficient condition for the pair $\bar{\alpha}{=}0,\bar{\beta}{=}0$ to guarantee the highest expected profits is $\delta{>}\Delta(q,\mu)$. For $\delta{=}\Delta(q,\mu)$, we have that $E[\Pi^I \mid \bar{\alpha}{=}\bar{\beta}{=}0]{=}\ E[\Pi^I \mid \bar{\alpha}{=}0,\bar{\beta}{=}1]{>}E[\Pi^I \mid \bar{\alpha}{=}1]$. In particular, notice that $\frac{\partial(\Delta(q,\mu>0))}{\partial q}{=}$ $\frac{-2[\mathcal{L}(\mu>0)\,-\,x_L\xi]\mathcal{P}(\mu>0)}{[2\,\mathcal{P}(\mu>0)\,-qx_L\xi+q\,\mathcal{L}(\mu>0)]^2}{<}0$ and that $\lim_{q\to 0}\Delta(q,\mu{>}0){\rightarrow}\ 1$.
Second, focusing on cases (i.b) and (i.c), we have that $\nabla(q,\mu){<}\frac{\mathcal{M}(\mu)-\mathcal{L}(\mu)}{\mathcal{M}(\mu)-\mathcal{L}(\mu)+q[\mathcal{L}(\mu)-x_L\xi]}{\therefore}\ 2\mathcal{P}(\mu){>}$ $\mathcal{M}(\mu){-}\mathcal{L}(\mu)$, which is verified. Hence, the sufficient condition for the pair $\bar{\alpha}{=}1,\bar{\beta}{=}\cdot$ to guarantee the highest expected profits is $\delta{<}\nabla(q,\mu)$. For $\delta{=}\nabla(q,\mu)$, we have that $E[\Pi^I \mid \bar{\alpha}{=}1]{=}$ $E[\Pi^I \mid \bar{\alpha}{=}0,\bar{\beta}{=}1]{>}E[\Pi^I \mid \bar{\alpha}{=}0,\bar{\beta}{=}0]$.
The remaining pair, $\bar{\alpha}{=}0,\bar{\beta}{=}1$, ensures the highest expected profits when $\nabla(q,\mu){<}\delta{<}\Delta(q,\mu)$.

(ii) Given the function $E\left[\Pi^U\right]$, the pair $\bar{\alpha}{=}0,\bar{\beta}{=}0$ guarantees the highest expected profits when $E[\Pi^U \mid \bar{\alpha}{=}\bar{\beta}{=}0]$ is simultaneously greater than $E[\Pi^U \mid \bar{\alpha}{=}0,\bar{\beta}{=}1]$, $E[\Pi^U \mid \bar{\alpha}{=}1,\bar{\beta}{=}0]$, and $E[\Pi^U \mid \bar{\alpha}{=}1,\bar{\beta}{=}1]$. It is possible to derive what follows: $\delta \gtreqless \Delta(q,\mu) \rightarrow E[\Pi^U \mid \bar{\alpha}{=}\bar{\beta}{=}0] \gtreqless$ $E[\Pi^U \mid \bar{\alpha}{=}0,\bar{\beta}{=}1]$, $\delta \gtreqless \frac{\mathcal{M}(\mu)-\mathcal{L}(\mu)}{\mathcal{M}(\mu)-\mathcal{L}(\mu)+q[\mathcal{L}(\mu)-x_L\xi]} \rightarrow E[\Pi^U \mid \bar{\alpha}{=}\bar{\beta}{=}0] \gtreqless E[\Pi^U \mid \bar{\alpha}{=}1,\bar{\beta}{=}0]$, and $\delta \gtreqless$ $\nabla(q,\mu) \rightarrow E[\Pi^U \mid \bar{\alpha}{=}0,\bar{\beta}{=}1] \gtreqless E[\Pi^U \mid \bar{\alpha}{=}1,\bar{\beta}{=}1]$. For $\delta{>}\Delta(q,\mu)$, it is easy to see that the pair $\bar{\alpha}{=}0,\bar{\beta}{=}0$ implies expected profits that are strictly greater than those associated to any other pair, while for $\delta{=}\Delta(q,\mu)$ we have that $E[\Pi^U \mid \bar{\alpha}{=}\bar{\beta}{=}0]{=}E[\Pi^U \mid \bar{\alpha}{=}0,\bar{\beta}{=}1]{>}E[\Pi^U \mid \bar{\alpha}{=}1]$. Proceeding as we did so far, it can be shown that, for $\delta{=}\nabla(q,\mu)$ (or $\delta{<}\nabla(q,\mu)$), there is at least a pair $\bar{\alpha}{=}1,\bar{\beta}{=}\cdot$ that generates an inter-temporal payoff equal to (resp., greater than) $E[\Pi^U \mid \bar{\alpha}{=}0,\bar{\beta}{=}1]$. ∎

Proof of Corollary 2. Here we consider only the single period. For any possible pricing rule such that $p_1{=}0$ and that $\exists\tau : P_2(\tau) \neq 0$, we prove the following. Assuming an insider that observes $\widetilde{v}{=}v$ even in $n{=}1$, derive each type of insider's best reply, consisting of a triple x_1,τ,x_2. Holding this strategy fixed, we show that M is setting either $p_2{>}0$ in response to the signal sent by *each* type $\widetilde{s}{=}I{\wedge}\widetilde{v}{=}v{<}0$, or $p_2{<}0$ in response to the signal sent by *each* type $\widetilde{s}{=}I{\wedge}\widetilde{v}{=}v{>}0$. To demonstrate the result, eight cases (from $C1$ to $C8$) representing all the possible combinations of M's strategy profiles are identified.
$C1$: $P_2(\tau{=}1) \geq 0 \wedge P_2(\tau{=}0) \geq 0 \wedge P_2(\tau{=}{-}1) \geq 0$. The analysis of this case is in line with that conduced under mandatory trade disclosure (see proof to Proposition 1, case $C1$).
$C2$: $P_2(\tau{=}1) \leq 0 \wedge P_2(\tau{=}0) \leq 0 \wedge P_2(\tau{=}{-}1) \leq 0$. This case is symmetric to that above.

$C3$: $P_2(\tau{=}1){>}0 \wedge P_2(\tau{=}0) \geq 0 \wedge P_2(\tau{=}-1) < 0$. Given this pricing rule, from an insider perspective, disclosing $\tau{=}-1$ while trading $x_1{=}-x_L, x_2{=}2x_L$ strictly dominates any other strategy, provided he observes $\widetilde{v}{=}v{>}\max\{0;\zeta;\epsilon'\}$, where $\epsilon'{=}P_2(\tau{=}-1){+}P_2(\tau{=}0)$.[28] It follows that, as long as $\max\{\zeta;\epsilon'\} \leq 0$, each type of insider aware of $\widetilde{v}{=}v{>}0$ prefers to trade $x_1{=}-x_L$ and disclose the undertaken sale. Holding this strategy fixed, it turns out that, in response to each of these types, M is setting a price below 0, which is a contradiction.
Now we show that, when M is replying to any type of insider aware of $\widetilde{v}{=}v{<}0$ with the pricing rule in $C3$, contradictions arise, provided $\max\{\zeta;\epsilon'\}{>}0$.
Before we proceed, the following intermediate results need to be established. From the perspective of an insider aware of $\widetilde{v}{=}v{<}0$: (i) When he signals $\tau{=}1$ (or $\tau{=}0$), the profits from trading $x_1{=}x_L, x_2{=}-2x_L$ are greater than those from trading any other combination of quantities $x_1{>}0, x_2$ (resp., x_1, x_2). (ii) When $v \leq P_2(\tau{=}-1){<}0$, the profits from signaling $\tau{=}1$ (or $\tau{=}0$) while trading $x_1{=}x_L, x_2{=}-2x_L$ are greater than those from signaling $\tau{=}-1$ while trading any quantity $x_1{<}0, x_2$. (iii) When $P_2(\tau{=}-1){<}v{<}0$, the profits from signaling $\tau{=}1$ (or $\tau{=}0$) while trading $x_1{=}x_L, x_2{=}-2x_L$ are greater than those from signaling $\tau{=}-1$ while trading any quantity $x_1{<}0, x_2$ only if $x_Lv - 2x_L[v - P_2(\tau{=}1)]$ (resp., $x_Lv - 2x_L[v - P_2(\tau{=}0)]$) is strictly greater than $-x_Lv{+}2x_L[v - P_2(\tau{=}-1)]$, that is only if $v{<}\zeta$ (resp., $v{<}\epsilon'$). (iv) When he trades $x_1{=}x_L, x_2{=}-2x_L$, the profits from signaling $\tau{=}1$ rather than $\tau{=}0$ are greater (or equal; or smaller), provided $P_2(\tau{=}1){>}P_2(\tau{=}0)$ (resp., $P_2(\tau{=}1){=}P_2(\tau{=}0)$; $P_2(\tau{=}1){<}P_2(\tau{=}0)$).
As a consequence of the results at point i, ii, iii, and iv, when $\max\{\zeta;\epsilon'\}{>}0$, the following conclusions can be drawn. (a) Suppose that $P_2(\tau{=}1){>}P_2(\tau{=}0)$. (a.i) If $\zeta{>}0$, no matter which value ϵ' assumes, then each type $\widetilde{s}{=}I{\wedge}\widetilde{v}{=}v{<}0$ strictly prefers to trade $x_1{=}x_L, x_2{=}-2x_L$ and signal $\tau{=}1$ rather than to play any other strategy. Holding this strategy fixed, we have that, in response to each of these types, M is setting a price above 0, which is a contradiction. (a.ii) The remaining sub-case, namely the one of $\zeta \leq 0{<}\epsilon'$, is not of interest. In fact, making the condition on ζ and ϵ' explicit, it follows that $P_2(\tau{=}-1){+}P_2(\tau{=}1) \leq 0{<}P_2(\tau{=}-1){+}P_2(\tau{=}0) \therefore P_2(\tau{=}1){<}P_2(\tau{=}0)$, which is not a possibility, being the case in question—i.e., point a—the one of $P_2(\tau{=}1){>}P_2(\tau{=}0)$. (b) Suppose that $P_2(\tau{=}1){=}P_2(\tau{=}0)$ (case in which $P_2(\tau{=}0){>}0$ for sure). This condition on prices implies that $\zeta{=}\epsilon'$. Thus, the only relevant sub-case is the one of $\zeta{=}\epsilon'{>}0$. In this instance, each insider aware of $\widetilde{v}{=}v{<}0$ replies by randomizing between signaling $\tau{=}1$ and $\tau{=}0$ while trading $x_1{=}x_L, x_2{=}-2x_L$. Holding the strategy by each of these types of insider fixed, regardless of the probability with which he discloses $\tau{=}1$ or $\tau{=}0$ (even 0 or 1), the price in response to his disclosure turns out to lie above 0, which is a contradiction. (c) Suppose that $P_2(\tau{=}1){<}P_2(\tau{=}0)$ (case in which $P_2(\tau{=}0){>}0$ for sure). (c.i) If $\epsilon'{>}0$, no matter which value ζ assumes, then each type $\widetilde{s}{=}I{\wedge}\widetilde{v}{=}v{<}0$ strictly prefers to trade $x_1{=}x_L, x_2{=}-2x_L$ and signal $\tau{=}0$ rather than to play any other strategy. Holding this strategy fixed, it follows that, in response to each of these types, M is setting a price above 0, which is a contradiction. (c.ii) The remaining sub-case, namely the one of $\epsilon'{\leq}\ 0{<}\zeta$, is not of interest. In fact, making the condition on ϵ' and ζ explicit, it follows that $P_2(\tau{=}-1){+}P_2(\tau{=}0) \leq 0{<}P_2(\tau{=}-1){+}P_2(\tau{=}1) \therefore P_2(\tau{=}0){<}P_2(\tau{=}1)$, which is not a possibility, being the case in question—i.e., point c—the one of $P_2(\tau{=}1){<}P_2(\tau{=}0)$.

[28] To see it, consider an insider that observes $\widetilde{v}{=}v{>}0$. First notice that, if he signals $\tau{=}-1$, the profits from trading any alternative combination of quantities such that $x_1{<}0$ are smaller. Second, for $v \geq P_2(\tau{=}1)$ (or $v \geq P_2(\tau{=}0)$), the profits from signaling $\tau{=}1$ (resp., $\tau{=}0$) while trading any combination of quantities $x_1{>}0, x_2$ (resp., x_1, x_2) are smaller. Third, for $0{<}v{<}P_2(\tau{=}1)$ (or $0{<}v{<}P_2(\tau{=}0)$), it is easy to derive that the profits from signaling $\tau{=}1$ (resp., $\tau{=}0$) while trading any combination of quantities $x_1{>}0, x_2$ (resp., x_1, x_2) are smaller only *if* $\{-x_Lv{+}2x_L[v - P_2(\tau{=}-1)]\}$ is strictly greater than $\{x_Lv - 2x_L[v - P_2(\tau{=}1)]\}$ (resp., $\{x_Lv - 2x_L[v - P_2(\tau{=}0)]\}$), that is only *if* $v{>}\zeta$ (resp., $v{>}\epsilon'$).

$C4$: $P_2(\tau{=}1) > 0 \wedge P_2(\tau{=}0) \leq 0 \wedge P_2(\tau{=}{-1}){<}0$. This case is symmetric to that above.
$C5$: $P_2(\tau{=}{-1}) \geq 0 \wedge P_2(\tau{=}0) \geq 0 \wedge P_2(\tau{=}1) < 0$. Given this strategy profile and an insider informed about $\widetilde{v}{=}v{<}0$, notice that the profits from signaling $\tau{=}{-1}$ (or $\tau{=}0$) while trading $x_1 \lesssim 0, x_2 \lesssim -x_L$ (resp., $x_1{=}x_L, x_2{=}{-2x_L}$) are greater than those from signaling $\tau{=}1$ while trading $x_1{>}0, x_2$. In addition, if he signals $\tau{=}{-1}$ (or $\tau{=}0$), the profits that from trading $x_1 \lesssim 0, x_2 \lesssim -x_L$ (resp., $x_1{=}x_L, x_2{=}{-2x_L}$) are greater than those from trading any alternative combination of quantities $x_1{<}0, x_2$ (resp., x_1, x_2), unless $P_2(\tau{=}{-1}){=}0$ (resp., $P_2(\tau{=}0){=}0$), case in which he is indifferent between this strategy and any other such that $x_1{+}x_2{=}{-x_L}{\wedge}x_1{<}0$ (resp., $x_1{+}x_2{=}{-x_L}$).
Thus, when deriving the best response by an insider aware of $\widetilde{v}{=}v{<}0$, it is sufficient to check whether he prefers to signal $\tau{=}{-1}$ or $\tau{=}0$ while trading $x_1 \lesssim 0, x_2 \lesssim -x_L$ or $x_1{=}x_L, x_2{=}{-2x_L}$ respectively. Specifically: (a) If $\frac{P_2(\tau=-1)}{2}{>}P_2(\tau{=}0)$ (or $\frac{P_2(\tau=-1)}{2}{<}P_2(\tau{=}0)$), case in which $P_2(\tau{=}{-1}){>}0$ (resp., $P_2(\tau{=}0){>}0$) for sure, then each type $\widetilde{s}{=}I{\wedge}\widetilde{v}{=}v{<}0$ prefers the former (resp., the latter). Holding this strategy fixed, it turns out that, in response to each of these types, M is setting a price which lies above 0, which is a contradiction. (b) If $\frac{P_2(\tau=-1)}{2}{=}P_2(\tau{=}0){>}0$, each of these types is indifferent towards the two options. Holding his best response fixed, regardless of the probability with which he discloses $\tau{=}{-1}$ or $\tau{=}0$ (even 0 or 1), the price in response to his disclosure turns out to lie above 0, which is a contradiction. (c) If $\frac{P_2(\tau=-1)}{2}{=}P_2(\tau{=}0){=}0$, we end up in case $C2$.
$C6$: $P_2(\tau{=}{-1}) > 0 \wedge P_2(\tau{=}0) \leq 0 \wedge P_2(\tau{=}1) \leq 0$. This case is symmetric to that above.
$C7$: $P_2(\tau{=}0) > 0 \wedge P_2(\tau{=}1) \leq 0 \wedge P_2(\tau{=}{-1}) \leq 0$. Given this strategy profile, from an insider perspective, signaling $\tau{=}0$ while trading $x_1{=}x_L, x_2{=}{-2x_L}$ strictly dominates any other strategy, provided he observes $\widetilde{v}{=}v{<}\min\{0;\zeta';\epsilon'\}$, where $\zeta'{=}P_2(\tau{=}0){+}\frac{P_2(\tau=1)}{2}$.[29] It follows that, as long as $\min\{\zeta';\epsilon'\} \geq 0$, each type of insider aware of $\widetilde{v}{=}v{<}0$ prefers to trade $x_1{=}x_L$ and signal $\tau{=}0$. Holding this strategy fixed, it turns out that, in response to each of these types, M is setting a price above 0, which is a contradiction.
Now we show that, when M is replying to any type of insider aware of $\widetilde{v}{=}v{>}0$ with the pricing rule in $C7$, contradictions arise, provided $\min\{\zeta';\epsilon'\}{<}0$.
Before we proceed, the following intermediate results need to be established. From the perspective of an insider aware of $\widetilde{v}{=}v{>}0$: (i) When he signals $\tau{=}1$ (or $\tau{=}{-1}$), the profits from trading $x_1 \gtrsim 0, x_2 \lesssim x_L$ (resp., $x_1{=}{-x_L}, x_2{=}2x_L$) are greater than those from trading any alternative combination of quantities $x_1{>}0, x_2$ (resp., $x_1{<}0, x_2$). (ii) When $v \geq P_2(\tau{=}0){>}0$, the profits from signaling $\tau{=}1$ (or $\tau{=}{-1}$) while trading $x_1 \gtrsim 0, x_2 \lesssim x_L$ (resp., $x_1{=}{-x_L}, x_2{=}2x_L$) are greater than those from signaling $\tau{=}0$ while trading any quantity x_1, x_2. (iii) When $0{<}v{<}P_2(\tau{=}0)$, the profits from signaling $\tau{=}1$ (or $\tau{=}{-1}$) while trading $x_1 \gtrsim 0, x_2 \lesssim x_L$ (resp., $x_1{=}{-x_L}, x_2{=}2x_L$) are greater than those from signaling $\tau{=}0$ while trading any quantity x_1, x_2 only if $x_L[v - P_2(\tau{=}1)]$ (resp., $-x_L v{+}2x_L[v - P_2(\tau{=}{-1})]$) is strictly greater than $x_L v - 2x_L[v - P_2(\tau{=}0)]$, that is only if $v{>}\zeta'$ (resp., $v{>}\epsilon'$). (iv) If $P_2(\tau{=}{-1}){>}\frac{P_2(\tau=1)}{2}$ (or $P_2(\tau{=}{-1}){=}\frac{P_2(\tau=1)}{2}$; or $P_2(\tau{=}{-1}){<}\frac{P_2(\tau=1)}{2}$), the profits from signaling $\tau{=}1$ while trading $x_1 \gtrsim 0, x_2 \lesssim x_L$ are greater than (resp., equal to; smaller than) those from signaling $\tau{=}{-1}$

[29] To see it, consider an insider that observes $\widetilde{v}{=}v{<}0$. First notice that, if he signals $\tau{=}{-1}$, the profits from trading any alternative combination of quantities such that $x_1{<}0$ are smaller. Second, for $v \leq P_2(\tau{=}1)$ (or $v \leq P_2(\tau{=}{-1})$), the profits from signaling $\tau{=}1$ (resp., $\tau{=}{-1}$) while trading any combination of quantities $x_1{>}0, x_2$ (resp., $x_1{<}0, x_2$) are smaller. Third, for $P_2(\tau{=}1){<}v{<}0$ (or $P_2(\tau{=}{-1}){<}v{<}0$), it is easy to derive that the profits from signaling $\tau{=}1$ (resp., $\tau{=}{-1}$) while trading any combination of quantities $x_1{>}0, x_2$ (resp., $x_1{<}0, x_2$) are smaller only *if* $\{x_L v - 2x_L[v - P_2(\tau{=}0)]\}$ is strictly greater than $\{x_L[v - P_2(\tau{=}1)]\}$ (resp., $\{-x_L v{+}2x_L[v - P_2(\tau{=}{-1})]\}$), that is only *if* $v{<}\zeta'$ (resp., $v{<}\epsilon'$).

while trading $x_1{=}-x_L, x_2{=}2x_L$.
As a consequence of the results at point i, ii, iii, and iv, when $\min\{\zeta';\epsilon'\}{<}0$, the following conclusions can be drawn. (a) Suppose that $P_2(\tau{=}-1){>}\frac{P_2(\tau=1)}{2}$ (case in which $P_2(\tau{=}1){<}0$ for sure). (a.i) If $\zeta'{<}0$, no matter which value ϵ' assumes, then each type $\widetilde{s}{=}I\wedge\widetilde{v}{=}v{>}0$ strictly prefers to trade $x_1{\gtrsim}\,0, x_2{\lesssim}\,x_L$ and signal $\tau{=}1$ rather than to play any other strategy. Holding this strategy fixed, we have that, in response to each of these types, M is setting a price that lies below 0, which is a contradiction. (a.ii) The remaining sub-case, namely the one of $\epsilon'{<}0 \leq \zeta'$, is not of interest. In fact, making the condition on ζ' and ϵ' explicit, it follows that $P_2(\tau{=}-1){+}P_2(\tau{=}0){<}0 \leq P_2(\tau{=}0){+}\frac{P_2(\tau=1)}{2} \therefore P_2(\tau{=}-1){<}\frac{P_2(\tau=1)}{2}$, which is not a possibility, being the case in question—i.e., point a—the one of $P_2(\tau{=}-1){>}\frac{P_2(\tau=1)}{2}$. (b) If $P_2(\tau{=}-1){=}\frac{P_2(\tau=1)}{2}{=}0$, we end up in case $C1$. (c) Suppose that $P_2(\tau{=}-1){=}\frac{P_2(\tau=1)}{2}{<}0$. This condition on prices implies that $\zeta'{=}\epsilon'$. Thus, the only relevant sub-case is the one of $\zeta'{=}\epsilon'{<}0$. In this instance, each insider aware of $\widetilde{v}{=}v{>}0$ replies by randomizing between signaling $\tau{=}1$ and $\tau{=}-1$ while trading $x_1{\gtrsim}\,0, x_2{\lesssim}\,x_L$ or $x_1{=}-x_L, x_2{=}2x_L$ respectively. Holding the strategy by each of these types of insider fixed, regardless of the probability with which he discloses $\tau{=}1$ or $\tau{=}0$ (even 0 or 1), the price in response to his disclosure turns out to lie below 0, which is a contradiction. (d) Suppose that $P_2(\tau{=}-1){<}\frac{P_2(\tau=1)}{2}$ (case in which $P_2(\tau{=}-1){<}0$ for sure). (d.i) If $\epsilon'{<}0$, no matter which value ζ' assumes, then each type $\widetilde{s}{=}I\wedge\widetilde{v}{=}v{>}0$ strictly prefers to trade $x_1{=}-x_L, x_2{=}2x_L$ and signal $\tau{=}-1$ rather than to play any other strategy. Holding this strategy fixed, it follows that, in response to each of these types, M is setting a price that lies below 0, which is a contradiction. (d.ii) The remaining sub-case, namely the one of $\zeta'{<}0 \leq \epsilon'$, is not of interest. In fact, making the condition on ζ' and ϵ' explicit, we have that $P_2(\tau{=}0){+}\frac{P_2(\tau=1)}{2}{<}0 \leq P_2(\tau{=}-1){+}P_2(\tau{=}0) \therefore \frac{P_2(\tau=1)}{2}{<}P_2(\tau{=}-1)$, which is not a possibility, being the case in question—i.e., point c—the one of $P_2(\tau{=}-1){<}\frac{P_2(\tau=1)}{2}$.
$C8$: $P_2\,(\tau{=}0){<}0 \wedge P_2\,(\tau{=}1) \geq 0 \wedge P_2\,(\tau{=}-1) \geq 0$. This case is symmetric to that above. ■

Proof of Corollary 4. The analysis of the single period is in line with that in Corollary 2, and left to the reader. When considering the infinitely repeated structure, what follows needs to be proven.

When informed about $\widetilde{v}{>}0$, an insider that does not want to defect prefers to push the market price toward the right direction, signaling $\phi_{i,1}$ rather than $\phi_{i,0}$ (the case of L aware of $\widetilde{v}{<}0$ is similar). Specifically, when signaling $\phi_{i,1}$, the best thing he can do is to buy $x_1{=}x_L$ and then trade optimally, earning under expectation $\mathcal{L}(\mu)$. In fact, trading $\langle\, x_1{\simeq}\,0, x_2{=}\cdot\,\rangle$ or $\langle\, x_1{=}-x_L, x_2{=}\cdot\,\rangle$ are dominated. Signaling $\phi_{i,0}$ and trading $\langle\, x_1{=}\cdot, x_2{=}\cdot\,\rangle$ leads to a payoff which is smaller than $\mathcal{L}(\mu)$.

When informed about $\widetilde{v}{>}0$, an insider that wants to defect signals $\phi_{i,-1}$. In this case, he maximizes his profits by trading $\langle\, x_1{=}-x_L, x_2{=}2x_L\,\rangle$, earning under expectation $\mathcal{M}(\mu)$.

When the leader is uninformed, if he signals $\phi_{i,0}$, he avoids the punishment with certainty. In this case, no matter what the quantity traded in each of the two rounds is, he expects to earn 0 profits. Conversely, if he signals $\phi_{i,1}$ (or $\phi_{i,-1}$), he incurs the punishment with probability $\frac{1}{2}$. In this case, trading $\langle\, x_1{=}x_L, x_2{=}-2x_L\,\rangle$ (resp., $\langle\, x_1{=}-x_L, x_2{=}2x_L\,\rangle$) implies the highest expected profits, which equal $\mathcal{P}(\mu)$. ■

Internet Appendix B

On post-trade mandatory disclosure: Reconsidering van Bommel (2003). This appendix reconsiders van Bommel (2003), hereafter VB, which studies a Kyle's model with a risky asset exchanged among a leader with a negligible cup on total exposure, noise traders, M, and competitive followers. L sends rumors to followers, who reveal them to M through a change in asset demand. Two separate stage games (ending with the exogenous revelation of v) are presented. In the first, the existence of L, commonly known to be of type *"Honest"*, is assumed. He has to say *"buy"* if he observes $\widetilde{v}=v \geq 0$ (or *"sell"* if $\widetilde{v}=v<0$); when uninformed, L cannot spread any rumor. In the second model, L is known to be of type *"Bluffer"*, so when informed he has to play like an Honest, and when uninformed he has to say randomly either *"buy"* or *"sell"*.

Assuming $\widetilde{v}\sim U[-2,2]$, $\widetilde{\mathbf{u}}\sim \mathsf{N}(\mathbf{0},\mathbf{I})$, $cov(\widetilde{\mathbf{u}},\widetilde{v})=0$, VB defines the equilibrium price at round $n \in \{1,..,N\}$, p_n. As $n \to \infty$, it is argued that p_n asymptotically converges to a certain value.

Very recently, van Bommel (2008) tries to justify why in VB the leader does not trade in $n \in \{2,..,N-1\}$. The clarification does not consider any of the following matters, which seriously weaken the validity of the conjectures in VB; its content does not help in this sense.

The equilibrium price dynamic derived from assuming an Honest, and especially the one assuming a Bluffer, are not appropriate, mainly because $\widetilde{v}$ and the aggregate demand at auction n are treated as independent random variables, even though they are indirectly dependent ($\widetilde{v}$ affects L's rumor; this impacts on followers' demand, affecting the mean of the aggregate demand). Even considering the recent clarification by the author, the pricing rule is not justified.

A simpler approach saves the conclusion in VB. Rather than a stage game t made of infinite auctions, assume two auctions, and consider L spreading rumors directly to M. The (corrected) contribution is the following. When type Honest is imposed, if L says *"buy"* (or *"sell"*; or *".."*), then $p_2=1$ (resp., $p_2=-1$; $p_2=0$). With a Bluffer, if L says *"buy"* (or *"sell"*), then $p_2=q$ (resp., $p_2=-q$). The equilibria hold for a more general class of distributions than $\widetilde{\mathbf{u}}\sim \mathsf{N}(\mathbf{0},\mathbf{I})$. Followers do not play a role, so there is no need to assume about them any more.

To relax this peculiar notion of type assumption, VB allows an informed L to choose between two alternatives in $n=1$: the equilibrium trading and (imposed) signaling strategy, or *"cheat"* (i.e., spread a so called *"false"* rumor and trade in the opposite direction). It is argued that the rumor is not informative any more because, holding fixed M's best response to an insider forced to play according to his type, the insider cheats, reversing his position afterwards. However this only proves that, for this very specific pricing rule, a deviation by the insider occurs.[30]

Within an infinitely repeated framework, the sufficient condition for the sustainability of the so-called *"Honest equilibrium"* proposed in VB consists of an inter-period discount factor δ such that, when L is uninformed in $t=1$, the profits from being Honest forever are greater than those from being Bluffer in $t=1$, and Honest from $t=2$ on (this in case L does not incur the punishment in $t=1$). However, among other points, it is unclear why the sender should consider the opportunity of randomizing when uninformed at a certain date, but not when facing an identical situation in the future. Our methodology and results differ drastically. Specifically, for each pair $\delta\in(0,1)$ and $q \in (0,1)$, two extra levels of randomization—which become three, in Section V, when an informed leader learns $\widetilde{v}=v$ from the beginning of the period—are required, to assess the existence of informative equilibria.

While an ad hoc trigger strategy for the sustainability of the Honest equilibrium is imposed in VB, we consider a general Grim, showing that: (i) Another group of equilibria exists, similar to that presented by VB in the stage game with an imposed Bluffer type; (ii) for a general $f(\widetilde{v})$, irrespective of the value of $q \in (0,1)$, a level of δ exists, at which manipulations are *always* possible. Internet Appendix C studies other informative equilibria and manipulative behaviors.

[30] Consider for simplicity mandatory disclosure. There exist pricing rules such that: (i) L prefers not to disclose trades (this strategy is somehow equivalent to the no-rumor disclosure in VB). For instance, consider L observing $\widetilde{v}=v\simeq 0 \simeq P(\tau=1)\simeq P(\tau=-1)$ and $P(\tau=0)$ sufficiently far from v; (ii) no subsequent reversal of the initial position occurs.

Contrary to what is stated in VB (p.1502), not all $f(\widetilde{v})$ can be used. It is untrue that this kind of *"analysis uses a special case of the Crawford and Sobel (1982) signaling game"* (VB, p.1500): Cheap-talk games do not require private information to be exogenously revealed at any time.

Internet Appendix C

Price-shift uniqueness. This appendix lists minimal restrictions on beliefs that guarantee *price-shift uniqueness.* To start with, it is worth noticing that, any time signals are believed not to be informative in a specific period, L cannot do any better than trading as he does when that period is not repeated, a behavior that confirms M's initial beliefs. As a consequence, for *each* pair δ and q and a specific equilibrium pricing rule such that at period $t{=}1$ prices react to disclosures, infinite other equilibria exist, where prices start shifting according to the same rule from period $t{>}1$, as if history started from period t, while in the preceding $t-1$ periods prices do not react to news. Although no limit can be set to the initial number of periods in which disclosures are believed not to be informative, in the following analysis there is no loss in generality in assuming that, if prices shift, they start shifting from period $t{=}1$.

When selecting among triggers, it seems natural to think of the following minimal conditions.

Condition 1 *At period t, only $\tau{=}0$ (or $\phi_{i,0}$) is never interpreted as a defection.*

Condition 2 *At period t, $P_2(\tau{=}-1) \gtreqless 0 \Leftrightarrow P_2(\tau{=}1) \lesseqgtr 0$ (or $P_2(\phi_{i,\varpi}) \gtreqless 0 \Leftrightarrow P_2(\phi_{i,\varpi'}) \lesseqgtr 0$, where $\varpi \cdot \varpi' {<} 0$).*

Condition 1 requires the signal $\tau{=}0$ (or $\phi_{i,0}$), disclosed at period t, to be the only signal following which no punishment at period $t{+}1$ is applied, even if this signal causes the price at period t to move in the wrong direction with respect to v. Condition 2 states that, if $P_2(\tau{=}1)$ (or $P_2(\phi_{i,\varpi})$) shifts from 0, then $P_2(\tau{=}-1)$ (resp., $P_2(\phi_{i,\varpi'})$) should *somehow* shift too, but in the opposite direction, and vice versa.

Even when restricting our attention just to Grim triggers, if only the first or second condition is imposed, for a variety of pairs δ and q, equilibria exist where prices shift differently. This is shown in examples below. To simplify the argument, we focus on the case of mandatory trade disclosure and refer to the fundamental value properties defined in Section I.

First note that, when both conditions hold, the trigger in Definition 2 is not discarded.

The second condition alone is not enough to guarantee price-shift uniqueness. For instance, consider the following trigger strategy, which ensures that no punishment is applied when a sale is disclosed. The trigger differs from the one in Definition 2 in the function P_2': $\tau{=}1 \rightarrow p_2{=}\xi, \tau{=}0 \vee \tau{=}-1 \rightarrow p_2{=}-q\xi$,[31] and in the following sequential condition: *At the second auction of the t^{th} period, if the outcome of all $t-1$ preceding periods has been $\tau{=}1 \wedge v{>}0$ or $\tau{=}-1$, then play P_2'; otherwise, set* $p_2{=}0$. For sufficiently high δ and sufficiently small q, this alternative trigger strategy is part of an equilibrium in which no defection ever occurs. In detail, when uninformed, L trades $x_1{=}-x_L$, $x_2{=}2x_L$, expecting to earn positive profits; when L observes $\widetilde{v}{<}0$ (or $\widetilde{v}{>}0$), he trades $x_1{=}-x_L$ (resp., $x_1{=}x_L$), subsequently trading $x_2{=}2x_L$ if $-q\xi{<}v$ (resp., $x_2{=}-2x_L$ if $v{<}\xi$), or $x_2{=}0$ otherwise, expecting to earn more than under $\mathcal{A}$. This equilibrium depends on disclosed sales never being classified as defections, while it is irrelevant whether a disclosed inactivity is never considered to be a defection too. This is because L has

[31] The symbol $\vee$ stands for *or.*

no incentive to signal τ=0.[32] The first condition discards this alternative equilibrium.

Likewise, the first condition alone is not enough to guarantee price-shift uniqueness. For example, consider a Grim trigger that satisfies the first condition, with a pre-defection pricing rule P_2'' such that $P_2''(\tau{=}{-1}){=}P_2''(\tau{=}0){=}0$ and $P_2''(\tau{=}1){=}\xi$. When L is informed about $\widetilde{v}{<}0$ (or uninformed), in the first round of each period he is indifferent about not trading and selling some quantity, provided he subsequently trades optimally in n=2. In fact, in either case he expects to earn $x_L\xi$ (resp., 0 profits)—that is, as much as under $\mathcal{A}$—without incurring punishment. It follows that, for sufficiently high values of δ, equilibria exist in which a type informed about $\widetilde{v}{<}0$ and an uninformed type hide their information completely, randomizing with identical probability (even 0 or 1) only between τ=0 and τ=−1. Indeed, the leader's objective is to earn more than under $\mathcal{A}$ whenever he turns out to know $\widetilde{v}{>}0$, in which case he expects to earn $\mathcal{L}(\mu{=}\xi)$ per period by disclosing a purchase and trading optimally. Since the second condition prevents $P_2''(\tau{=}{-1})$ from equalling 0 when $P_2''(\tau{=}1)$ differs from 0, this alternative equilibrium is eliminated.

Provided the first condition is satisfied, when changing the mapping P_2'' by gradually shifting the price response to the signal τ=−1 from 0 to *positive* values, for δ sufficiently high, informative equilibria can be identified immediately, in which L discloses inactivity today when he is aware of $\widetilde{v}{<}0$ or uninformed. In fact, in this case the signal τ=0 is the only one that allows him not to defect with certainty and earn as under $\mathcal{A}$ today, but more than under $\mathcal{A}$—that is, $\mathcal{L}(\mu{=}\xi)$—any time he is aware of $\widetilde{v}{>}0$ in the future. The joint effect of both these conditions discards this counterintuitive equilibrium too, since the price response to the signal τ=−1 is required to be *negative* when the price response to the signal τ=1 is positive.

Now, let's draw the attention just to Grim trigger strategies such that, before defection, the way period t prices react to period t disclosures is identical *among* periods. Under mandatory trade (or voluntary trade, or uncertified/non-factual message) disclosure, for the same pair δ and q, *more than one* pre-defection pricing rule can, in some instances, simultaneously satisfy the market efficiency condition *and* the two conditions above. However, as an *indirect* consequence of the next lemma, the associated *outcome* is identical, provided Condition 3 (presented below) holds *too*. This outcome coincides with that in Proposition 3 and Corollary 2 (both generalized in Corollary 3) for what concerns mandatory and voluntary trade disclosure respectively, and with that in Corollary 4 for what concerns uncertified/non-factual messages.

Lemma 5 *Consider trade (or uncertified/non-factual message) disclosure, an infinitely repeated structure, and beliefs that are restricted to be such that, at period t, Condition 1 and 2 hold. When $P_2(\tau{=}1){>}0$ (resp., $P_2(\phi_{i,m\neq 0}) \neq 0$) and $P_2(\tau{=}0)$ (resp., $P_2(\phi_{i,0})$) is 'sufficiently close' (but not necessarily equal) to 0, both types of insider prefer to lead, signaling $\tau \neq 0$ (resp., $\phi_{i,m\neq 0}$), rather than to signal τ=0 (resp., $\phi_{i,0}$). Prices that shift differently are never justified.*

Proof of Lemma 5. In the first part of this proof, part I, we consider mandatory and voluntary trade disclosure. In part II, we consider disclosure of uncertified/non-factual messages.

(I) First, we prove that a pre-defection pricing rule such that $P_2(\tau{=}{-1}) \leq 0 \leq P_2(\tau{=}1)$ and with $P_2(\tau{=}0)$ 'too far away' from 0 does not satisfy the market efficiency condition. Suppose that $P_2(\tau{=}0){<}0$ (the case of $P_2(\tau{=}0){>}0$ is symmetric). If type $\widetilde{s}{=}I{\wedge}\widetilde{v}{>}0$ decides to signal τ=1—that is, given the pricing rule in question, to lead—it is optimal for him to trade $x_1{=}x_L, x_2{=}{-2x_L}$ when $v{<}P_2(\tau{=}1)$, and $x_1{=}x_L, x_2{=}0$ when $v \geq P_2(\tau{=}1)$, earning under

[32] On the contrary, for high values of q, this alternative trigger is not justified. Rather than leading—i.e., signaling τ=1—an insider aware of $\widetilde{v}{>}0$ prefers to trade $x_1{=}{-x_L}$,$x_2{=}2x_L$—i.e., to signal τ=−1—in this way causing the price to shift in the wrong direction with certainty, without being punished for it.

expectation $2x_L\{\int_0^{P_2(\tau=1)}[2P_2(\tau=1)-\widetilde{v}]f(\widetilde{v})d\widetilde{v}+\int_{P_2(\tau=1)}^{b}\widetilde{v}f(\widetilde{v})d\widetilde{v}\}$. If he decides to signal τ=0—without being punished for that—under mandatory (or voluntary) trade disclosure, it is optimal for him to trade x_1=0, x_2=x_L (resp., x_1=$-x_L$, x_2=$2x_L$), earning under expectation $2x_L\int_0^b[\widetilde{v}-P_2(\tau=0)]f(\widetilde{v})d\widetilde{v}$ (resp., $2x_L\int_0^b[\widetilde{v}-2P_2(\tau=0)]f(\widetilde{v})d\widetilde{v}$). It follows that, if $P_2(\tau=0)$ is smaller than $4\int_0^{P_2(\tau=1)}[\widetilde{v}-P_2(\tau=1)]f(\widetilde{v})d\widetilde{v}$ (resp., $2\int_0^{P_2(\tau=1)}[\widetilde{v}-P_2(\tau=1)]f(\widetilde{v})d\widetilde{v}$), type $\widetilde{s}$=$I\wedge\widetilde{v}$>0 prefers to signal τ=0 rather than τ=1, causing the pricing rule not to be justified.

Second, to prove that a pre-defection pricing rule such that $P_2(\tau=1)\leq 0\leq P_2(\tau=-1)$ and $P_2(\tau=0)\neq 0$ is not justified, suppose that $P_2(\tau=0)<0$ (the case of $P_2(\tau=0)>0$ is symmetric). Type $\widetilde{s}$=$I\wedge\widetilde{v}$>0 prefers to signal τ=0 rather than leading, signaling τ=-1. It follows that his best response causes the price shift to be wrong with certainty.

Third, to prove that $P_2(\tau=1)\leq P_2(\tau=0)=0\leq P_2(\tau=-1)$ implies no departure from $\mathcal{A}$, we show that type $\widetilde{s}$=$I\wedge\widetilde{v}$>0 prefers to signal τ=0 rather than leading, signaling τ=-1 (for a symmetric argument, type $\widetilde{s}$=$I\wedge\widetilde{v}$<0 prefers to signal τ=0 rather than τ=1). In fact, if type $\widetilde{s}$=$I\wedge\widetilde{v}$>0 signals τ=0, under mandatory (or voluntary) trade disclosure, it is optimal for him to trade x_1=0,x_2=x_L (resp., x_1= $\cdot$,x_2=x_L-x_1), earning under expectation $x_L\xi$ per period. Conversely, if he decides to signal τ=-1, he can trades $x_1\lesssim 0$ and then trade optimally, buying or selling depending on the realization of $\widetilde{v}$=v, and expecting to earn less than $x_L\xi$; alternatively, if he trades x_1=$-x_L$, then he finds it optimal to trade x_2=$2x_L$ when $P_2(\tau=-1)<v$, and x_2=0 when $P_2(\tau=-1)\geq v$, expecting to earn $-x_L\xi+2x_L\int_{P_2(\tau=-1)}^{b}[\widetilde{v}-P_2(\tau=-1)]f(\widetilde{v})d\widetilde{v}$ per period, which is again less than $x_L\xi$.

(II) Let's now consider a pre-defection pricing rule such that $P_2(\phi_{i,\varpi'})\leq 0\leq P_2(\phi_{i,\varpi})$, where ϖ equals -1 (or 1) when ϖ' equals 1 (resp., -1), and with $P_2(\phi_{i,0})$ 'too far away' from 0. This pricing rule does not satisfy the market efficiency condition. Suppose that $P_2(\phi_{i,0})<0$ (the case of $P_2(\phi_{i,0})>0$ is symmetric). If type $\widetilde{s}$=$I\wedge\widetilde{v}$>0 decides to lead—that is, to send $\phi_{i,\varpi}$—it is easy to show that he finds it optimal to trade x_1=x_L, x_2=$-2x_L$ when $v<P_2(\phi_{i,\varpi})$, and x_1=x_L, x_2=0 when $v\geq P_2(\phi_{i,\varpi})$, in this way earning under expectation $2x_L\{\int_0^{P_2(\phi_{i,\varpi})}[2P_2(\phi_{i,\varpi})-\widetilde{v}]f(\widetilde{v})d\widetilde{v}+\int_{P_2(\phi_{i,\varpi})}^{b}\widetilde{v}f(\widetilde{v})d\widetilde{v}\}$. If he decides to send $\phi_{i,0}$—which is a signal that allows him not to be punished even though it pushes the price in the wrong direction—it is optimal for him to trade x_1=$-x_L$, x_2=$2x_L$, earning under expectation $2x_L\int_0^b[\widetilde{v}-2P_2(\phi_{i,0})]f(\widetilde{v})d\widetilde{v}$. Thus, if $P_2(\phi_{i,0})$ is smaller than $2\int_0^{P_2(\phi_{i,\varpi})}[\widetilde{v}-P_2(\phi_{i,\varpi})]f(\widetilde{v})d\widetilde{v}$, the pricing rule is not justified, because type $\widetilde{s}$=$I\wedge\widetilde{v}$>0 prefers to signal $\phi_{i,0}$ rather than $\phi_{i,\varpi}$. ■

To give an insight into this lemma, we refer to the case of mandatory/voluntary trade disclosure (for what concerns uncertified or non-factual messages, the intuition is slightly simpler than what is explained here and the related implications are in line with it). The two conditions above restrict the analysis to two classes of pre-defection pricing rules, $P_2(\tau=1)\leq 0\leq P_2(\tau=-1)$ and $P_2(\tau=-1)\leq 0\leq P_2(\tau=1)$, setting no condition on whether the missed disclosure of a purchase or a sale shifts prices. (i) When $P_2(\tau=1)\leq 0\leq P_2(\tau=-1)$, no equilibrium with informative disclosures arises. In fact, while for $P_2(\tau=0)\neq 0$ the market efficiency condition does not hold, for $P_2(\tau=0)=0$ an insider aware of $\widetilde{v}$>0 (or $\widetilde{v}$<0) prefers to signal τ=0 rather than leading—i.e., disclosing τ=-1 (resp., τ=1)—which causes no departure from $\mathcal{A}$ to occur. (ii) When $P_2(\tau=-1)\leq 0\leq P_2(\tau=1)$, (ii.a) if $P_2(\tau=0)$ is negative (or positive) and set 'too far away' from 0, the market efficiency condition does not hold. In fact, an insider aware about $\widetilde{v}$>0 (resp., $\widetilde{v}$<0) prefers to signal τ=0—that is, to pretend to be *un*informed, moving the price down (resp., up) without being punished for that—rather than leading. Instead, (ii.b) if $P_2(\tau=0)$ is 'sufficiently close' (or equal) to 0—in detail, for $P_2(\tau=0)$ such that $\int_0^{P_2(\tau_1=1)}[\widetilde{v}-P_2(\tau_1=1)]f(\widetilde{v})d\widetilde{v}\leq\frac{P_2(\tau_1=0)}{\varrho}\leq$

$\int_0^{-P_2(\tau_1=-1)}[-P_2(\tau_1=-1)-\widetilde{v}]f(\widetilde{v})d\widetilde{v}$, where ϱ equals 4 (or 2) when disclosures are mandatory (resp., voluntary)—both types of insider prefer to lead optimally rather than signaling $\tau=0$.

Condition 3 *If at any point in time the leader turns out to be indifferent, given his multi-period decision problem, between misleading (or bluffing) and leading (resp., not bluffing) optimally, he is believed to opt for the latter alternative with probability* 1.

When beliefs are restricted in such a way that Conditions 1 and 2 hold, *if* prices move, under *R1–R4*, the shift only follows a disclosed purchase or sale, turning out to be positive or negative respectively, but—because of the symmetry of $f(\widetilde{v})$, the space of actions, and the consequences that the misleading behavior of one or the other type of insider imply, and thanks to Condition 3—equal to $q\xi$ or ξ in magnitude, depending on whether $\nabla(q,\mu=q\xi)\leq\delta<\Delta(q,\mu=\xi)$ or $\delta\geq\Delta(q,\mu=\xi)$ respectively. Together, the market efficiency condition and Conditions 1 and 2 also imply that, if the signal $\tau=0$ is sent, it never shifts equilibrium prices. Indeed, equilibrium pricing rules exist, with prices that at a certain period respond, with a shift, to the signal $\tau=0$ disclosed at the same period. For instance, when $\delta\geq\delta_\nabla\wedge q=1$ or when $\nabla(q,\mu=q\xi)\leq\delta<\Delta(q,\mu=\xi)$ $\wedge q<1$, any Grim trigger with a pre-punishment pricing rule such that, at every period *before* defection, $P_2(\tau=1)=-P_2(\tau=-1)=q\xi$ and $0<|P_2(\tau=0)|\leq\varrho\int_0^{q\xi}(q\xi-\widetilde{v})f(\widetilde{v})d\widetilde{v}$ *also* satisfies the market efficiency condition and Conditions 1 to 3. Nonetheless, the associated equilibrium *outcome* coincides with that derived when prices shift according to the trigger in Definition 2. The reason being that, before defection (if any), no type of leader finds it optimal to disclose $\tau=0$. Thus, there is no loss in generality in assuming that, at each period, $P_2(\tau=0)=0$.

When the following two inter-temporal restrictions on beliefs *also* hold, for any pair δ,q, it is possible to identify a unique way in which prices at a certain period can shift in response to one disclosure or another, sent at that period. This result is presented below, in Proposition 4.

First notice that, even when the Grim punishment is taken into account and Conditions 1 to 3 hold, there exist equilibria such that, before defection, a (finite, well known) number of periods in which disclosures are believed to convey information concerning what L observes is *alternated* with a non-necessarily equal (but finite and well known) number of periods in which no disclosure is believed to be informative. The next condition restricts beliefs by eliminating this option. Otherwise, for the same pair δ,q, depending on how regularly, before defection, periods in which disclosures are believed to be informative are alternated with periods in which they are not, the incentive to mislead (or bluff) as an alternative to leading (resp., not to bluff) is affected, with clear consequences on the way pre-defection prices can react to disclosures.

Condition 4 *If at a certain period disclosures are believed to be informative, also at each subsequent period they are believed to be informative, in one way or another, until a defection occurs.*

Second, consider any pair δ and q such that a specific equilibrium pricing rule exists, where pre-defection price shifts are supported by a Grim punishment. For (almost[33]) all these pairs δ and q, an identical pre-defection pricing rule followed by a less severe punishment (that is, a *non-Grim* punishment) is also part of an equilibrium where, at some point after defection, prices start reacting to disclosures *again.* Condition 5 constrains beliefs formed in response to a disclosure—and prices set by a market maker holding those beliefs—as follows.

Condition 5 *Let beliefs be such that: (i) Before each defection, if prices shift, they shift as if, after defection, a Grim punishment occurs. (ii) After a specific defection, (at least in some*

[33] For an intuition concerning the weight of the adverb 'almost', see after Proposition 4.

periods) prices can shift, provided the implicit punishment following this defection represents a deterrent to support past prices, equivalent to the Grim punishment.

To see the implications of this condition, let's refer, for the sake of simplicity, to the result in Proposition 3. For $\delta \geq \nabla(q, \mu{=}q\xi)$, if prices start reacting again after defection, and in a way that *does not* represent a deterrent that is as strong as the Grim punishment, before a first defection the incentive to mislead (or bluff) as an alternative to leading (resp., not to bluff) can be affected.[34] Condition 5 eliminates this possibility.

Proposition 4 *Consider trade (or uncertified/non-factual message) disclosure, the infinitely repeated structure, and beliefs that are restricted in such a way that Conditions 1 to 5 hold. Under R1–R4, at any period disclosures can affect prices if and only if they are believed to be informative. At a specific period, if the equilibrium price* p_2 *increases (or decreases), for* $\delta \geq \Delta(q, \mu{=}\xi)$*, this shift equals* ξ *(resp.,* $-\xi$*) and follows the signal* $\tau{=}1$ *or* $\phi_{i,\varpi'}$ *(resp.,* $\tau{=}{-}1$ *or* $\phi_{i,\varpi}$*), sent by type* $\widetilde{s}{=}I \wedge \widetilde{v}{>}0$ *(resp.,* $\widetilde{s}{=}I \wedge \widetilde{v}{<}0$*); for* $\nabla(q, \mu{=}q\xi) \leq \delta{<}\Delta(q, \mu{=}\xi)$*, it equals* $q\xi$ *(resp.,* $-q\xi$*) and follows the signal* $\tau{=}1$ *or* $\phi_{i,\varpi'}$ *(resp.,* $\tau{=}{-}1$ *or* $\phi_{i,\varpi}$*), sent by types* $\widetilde{s}{=}I \wedge \widetilde{v}{>}0$ *(resp.,* $\widetilde{s}{=}I \wedge \widetilde{v}{<}0$*) or* $\widetilde{s}{=}U$*. For* $\delta{<}\nabla(q, \mu{=}q\xi)$*, no shift ever occurs.*

Given Conditions 1 to 5, for $\delta{=}\nabla(q, \mu{=}q\xi)$ and $\delta{=}\Delta(q, \mu{=}\xi)$, before a first defection (if any), equilibrium prices shift *only if*, after this defection, M believes that every disclosure is not informative—that is, if all post-defection prices equal 0. Conversely, for each pair δ, q such that $\nabla(q, \mu{=}q\xi){<}\delta{<}\Delta(q, \mu{=}\xi)$ or $\delta{>}\Delta(q, \mu{=}\xi)$, equilibria exist where, after defection, prices start reacting to disclosures again. In this case, not only the Grim punishment, but also other less severe punishments, represent equivalent threats that support (and therefore justify) pre-defection price shifts. In particular, for $\nabla(q, \mu{=}q\xi){<}\delta{<}\Delta(q, \mu{=}\xi) \wedge q{<}1$, an unlimited number of alternative post-defection equilibrium *outcomes* is possible. To see it, for each of these latter pairs δ and q, consider any equilibrium pricing rule such that, immediately after a first defection, M punishes by reverting to single period equilibrium behavior for a *minimum*, *finite*, number of periods which make the entire post-defection pricing rule in question sufficient to support all prices set before that defection. Clearly, infinite other equilibria exist where, following the same defection, M correctly believes that no disclosure is informative at all, for a finite number of periods greater than this minimum number.

Internet Appendix D

Informative disclosure of a reversal (or of its absence) when inside information is long-lived. This appendix considers a situation where trade disclosure is imposed when the short-swing rule is not, and characterizes an equilibrium where a leader that repeatedly acquires long-lived inside information and weights future profits sufficiently never manipulates and price efficiency is higher than under Section 16(b). For simplicity's sake, we refer to the case of $N{=}3$, where $P{=}\langle P_1, P_2, P_3\rangle$ is M's pricing rule (for $N{>}3$, the argument is similar). Specifically, in $n{=}3$, the signal $\tau' \in \{-1, 0, 1\}$ is released: $\tau'{=}1$ (or $\tau'{=}{-}1$; or $\tau'{=}0$) reveals that in $n{=}2$ the leader bought (resp., sold; did not trade); hence, because $\Omega_3{=}\{\tau, \tau'\}$, it follows that

[34] Consider the Grim trigger in Definition 2. When a weaker (or much weaker) punishment is threatened, for at least some (resp., all) pairs δ, q such that $\nabla(q, \mu{=}q\xi) \leq \delta{<}\Delta(q, \mu{=}\xi)$, equilibria where disclosures are never informative can arise. Similarly, for at least some pairs δ, q such that $\delta \geq \Delta(q, \mu{=}\xi)$—but never for pairs with an extremely high value of δ—equilibria with either manipulative or not informative (resp., equilibria with not informative) disclosures can arise.

P_3: $\{-1,0,1\}^2 \rightarrow [-b,b]$. Under the assumption that the statistical properties of $\widetilde{v}$ defined in Section I hold, consider a sequential condition, such that prices at period t react to disclosed trades, unless disclosure moved prices away from the fundamental value in any of the $t-1$ preceding periods. At the equilibrium, an uninformed leader never trades, while a leader aware of $\widetilde{v}{>}0$ behaves as follows (the strategy of one aware about $\widetilde{v}{<}0$ is symmetric): In the first auction, he buys x_L. Specifically, when $\widetilde{v}{=}v \in [0,\xi)$, he reverses his position up to the maximum capacity in round $n{=}2$; then, if $\widetilde{v}{=}v \in [\underline{\xi},\xi)$, where $\underline{\xi}{=}E[\widetilde{v}|0 \leq \widetilde{v} \leq \xi]$, this reversal is followed by a second reversal at the third auction—that is, $x_3{=}2x_L$; conversely, if $\widetilde{v}{=}v \in (0,\underline{\xi})$, then $x_3{=}0$. When $\widetilde{v}{=}v \in [\xi,b]$, he does not trade in the second action; then, if $\widetilde{v}{=}v \in [\xi,\overline{\xi})$, where $\overline{\xi}{=}E[\widetilde{v}|\xi \leq \widetilde{v} \leq b]$, he reverses his position up to the maximum capacity at the third auction; conversely, if $\widetilde{v}{=}v \in [\overline{\xi},b]$, then $x_3{=}0$. For what concerns equilibrium prices, following an initial purchase, at the second auction the price response $P_2(\tau{=}1)$ equals ξ, while at the third auction $P_3(^{\tau=1}_{\tau'=-1})$ and $P_3(^{\tau=1}_{\tau'=0})$ equal $\underline{\xi}$ and $\overline{\xi}$ respectively; symmetrically, following an initial sale, we have $P_2(\tau{=}-1){=}-\xi$, $P_3(^{\tau=-1}_{\tau'=1}){=}-\underline{\xi}$, and $P_3(^{\tau=-1}_{\tau'=0}){=}-\overline{\xi}$; finally, not only $P_1(\cdot)$ and $P_2(\tau{=}0)$, but also $P_3(\tau{=}0,\cdot)$, equal 0. Interestingly, by trading in round $n{=}1$ and not trading in $n{=}2$, absence of disclosure at the beginning of $n{=}3$ moves prices at that round. Absolute continuity of $F(\widetilde{v})$ and symmetry of $f(\widetilde{v})$ can be easily relaxed, and a more general set of restrictions that includes *R1–R4* identified.

Internet Appendix E

Pre-trade non-anonymity and the informational content of a missed submission. Here we analyze a regulation mandating public revelation of submitted orders, describing the effect that an initial lack of submissions by L has on prices. Two cases are in order.

(I) Consider the case in which both $\widetilde{v}{=}\underline{b}$ and $\widetilde{v}{=}\overline{b}$ have zero mass. At any round n taking place before a first order is *effectively* submitted, even when a missed order submission conveys relevant information about the fundamental value, the price response $P_n(\S_i{=}0, \forall i \in \{1,..,n\})$ equals $E[\widetilde{v}]$. To see it, denote, with $\Upsilon_{n,\underline{b}} {\in} [0,1]$ (resp., $\Upsilon_{n,\overline{b}} {\in} [0,1]$), the probability with which type $\widetilde{s}{=}I \wedge \widetilde{v}{=}\underline{b}$ (or $\widetilde{s}{=}I \wedge \widetilde{v}{=}\overline{b}$) is *correctly* believed to sell (resp., buy) at any of these rounds. Now, let's consider a situation where, for example, $\Upsilon_{1,\underline{b}}{=}0$ and $\Upsilon_{1,\overline{b}}{=}1$. In this case, the signal $\S_1{=}0$ implies that L is *not* aware of $\widetilde{v}{=}\overline{b}$ (otherwise, a buy order in round $n{=}1$ would have been placed with certainty). However, because the event $\widetilde{v}{=}\overline{b}$ is a zero-probability one, it follows that $P_1(\S_1{=}0){=}qE[\widetilde{v}|\widetilde{v}{\neq}\,\overline{b}]{+}(1-q)E[\widetilde{v}]{=}E[\widetilde{v}]$.

(II) Consider the case in which either $\widetilde{v}{=}\underline{b}$ or $\widetilde{v}{=}\overline{b}$ has positive mass. Before a first order is *effectively* placed, different price responses supported by alternative sets of beliefs are justified. To see it, let's focus on the case of beliefs formed in response to disclosures by a leader that employes pure strategies. Define, with $c \in \{1,..,N\}$ (or $d \in \{1,..,N\}$), the *first* rounds in which an insider aware of $\widetilde{v}{=}\underline{b}$ (resp., $\widetilde{v}{=}\overline{b}$) is *correctly* believed to submit a sell (resp., buy) order rather than no order. At each round $n{<}\min\{c,d\}$, since no type of leader trades, $P_{n<\min\{c,d\}}(\S_{n<\min\{c,d\}}{=}0)$ equals $E[\widetilde{v}]$. From round $n{=}\min\{c,d\}$ (included) onwards, until the auction in which a first order is placed (excluded), prices are set as follows. (i) If $c{<}d$, a missed order submission at round $n{=}c$ highlights that L does not observe $\widetilde{v}{=}\underline{b}$. Since he is either aware of $\underline{b}{<}v \leq \overline{b}$ or uninformed, the price at round $n \in \{c,..,d-1\}$, $P_{n\in\{c,..,d-1\}}(\S_{n\leq c}{=}0)$, equals $qE[\widetilde{v}|\underline{b}{<}v]{+}(1-q)E[\widetilde{v}]$. For a symmetric argument, (ii) if $d{<}c$, then $P_{n\in\{d,..,c-1\}}(\S_{n\leq d}{=}0)$ equals $qE[\widetilde{v}|v{<}\overline{b}]{+}(1-q)E[\widetilde{v}]$. Finally, (iii) if $c{=}d$, any missed disclosure at round $n{=}c$ causes the price from that auction (included) onwards, $P_{n\geq c=d}(\S_{n\leq c}{=}0)$, to equal $qE[\widetilde{v}|\underline{b}<v<\overline{b}]+(1-q)E[\widetilde{v}]$. In general, whenever the probability that $\widetilde{v}$ equals $\underline{b}$ (or $\overline{b}$) is positive, there exist infinite equilibria such that, following an initial series of missed submissions, a partial revelation of L's type occurs. However, given the same series of missed submission, a perfect revelation is possible only if $\widetilde{v}\in\{\underline{b},\overline{b}\}$.

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