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NOTES

AN APPROACH TO THE ANALYSIS OF LAND UTILISATION*

How is a given amount of agricultural land allocated to different crops by a farmer? It may indeed differ as between food crops and cash crops, and also between small and large holdings. Because of the uncertainties of market and perhaps a historic propensity on the part of most farmers for growing and hoarding the foodgrains for their own consumption, the bulk of the acreage devoted to food crops may be determined largely by such factors as the level of rural population and their diet patterns; and there may be a wide range over which there would be no or little substitution between food and cash crops in such acreage in response to changes in relative profitability. Nonetheless, *in the margin* the food crop acreage may respond to relative profitability, the large size holdings being presumably more commercial in their operation. Thus the relative profitability may be taken as the mechanism of land allocation among various food crops as well as cash crops. In any case, this is going to be the basic postulate of our approach here, like all others in this respect.

A number of econometric models have been recently built to find answers to such a question in India, Pakistan, the Philippines, and the U.S.A. These studies nicely fall into two distinct categories.

Most of the empirical studies of acreage allocation relate the acreage (or production) directly to prices and such variables as yields and rainfall.[1] From the view point of logic, however, such methods are *indirect* in nature. The supply function is a theoretical concept derived from the hypothesis about profit maximization. In principle, production responds directly to relative profit opportunities and only indirectly to prices and yields. So unless the specific form of the function relating the acreage to prices and yields is carefully derived from a system of simultaneous equations of profit maximization, one is likely to commit the error of theoretically incorrect specification.[2]

It has been a common practice to test the sign of the coefficients of these models against the results we would expect on the basis of the economic theory of production. If this test is negative, it is necessary to go beyond the model to explain

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1. For example, Marc Nerlove: The Dynamics of Supply: Estimation of Farmers' Response to Price, The John Hopkins Press, Baltimore, 1958; Raj Krishna, "Farm Supply Response in India-Pakistan: A Case Study of the Punjab Region," *Economic Journal*, Vol. LXXII, No. 291, September, 1963, pp. 477-87; Walter P. Falcon, "Farmer Response to Price in a Subsistence Economy: The Case of West Pakistan," *The American Economic Review*, Vol. 54, No. 2, May, 1964, pp. 580-91; S. M. Hussain, "A Note on Farmer Response to Price in East Pakistan," *Pakistan Development Review*, Vol. IV, No. 1, Spring, 1964, pp. 93-106; M. Mangahas, A. E. Recto, and V. W. Ruttan, "Market Relationships for Rice and Corn in the Philippines," *Philippine Economic Journal*, Vol. V, No. 1, First Semester, 1966, pp. 1-27; and G. S. Acharya and Jati K. Sengupta, "Acreage Substitution between Jute and Rice," *Arthaniti*, Vol. IX, Nos. 1 and 2, January and July, 1966, pp. 28-46.

2. See footnote 15 below.

the discrepancy. Thus even when not part of the body of such econometric models the optimizing principle hovers over them like a ghost.[3]

On the other hand, we have the programming models of Day[4] and Henderson.[5] These are in the second category which follows the *direct* approach. In this approach the answer to the question posed above at the outset rests on two basic principles. First, the farmer compares the per acre expected net revenue (expected gross return minus per acre expected variable costs) of the several crops. He then allocates his given stock of land so as to maximize his total expected net revenue. But the allocation of his land to specific crop is constrained more generally than by the land alone. Henderson, for instance, specifies as a second principle that land allocation decisions for a given crop year are deviations from the pattern of the preceding year: "specifically, acreage plantings for each crop are constrained by maximum and minimum limits which indicate his desire for diversity and reluctance to depart from an established pattern."[6] Day has further generalized the Henderson model.

But the data requirements of the Henderson and Day models are rather prohibitive at the moment in the case of many a country. In any event, here we propose to build two models which are essentially *direct* in their approach; that is to say, they are based explicitly and directly on the principle of profit maximization like the Henderson and Day models; but unlike the latter they assume a Nerlovian adjustment process.[7] In this sense, our model can be placed somewhere in between the Nerlove-Raj Krishna-Falcon-type on the one hand, and the Henderson-Day-type on the other.

I

Notations: A = total acreage;

A_i = acreage actually allocated to crop i $(i=1,2,\ldots,n)$; $A=\Sigma A_i$;

A^*_i = acreage *desired* to be allocated to crop i; $A=\Sigma A^*_i$;

z^* = *expected* total profit;

z^*_i = *expected* profit from crop i from the acreage A^*_i; $z^*=\Sigma z^*_i$;

p^*_i = *expected* price of crop i;

q^*_i = *expected* yield (per acre) of crop i in A^*_i;

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3. See Richard H. Day: Recursive Programming and Production Response, North-Holland Publishing Co., Amsterdam, 1963, pp. 3-4 and 100.
4. *ibid.*
5. J. M. Henderson, "The Utilisation of Agricultural Land: A Theoretical and Empirical Inquiry," *Review of Economics and Statistics*, August, 1959, pp. 242-59.
6. *ibid.*, p. 243.
7. Nerlove, *op. cit.*, pp. 59-62.

p_i = actual price of crop i ;

q_i = actual yield (per acre) of crop i in A_i;

C_i^* = *expected* cost of production of crop i in A_i^* ;

C_i = actual cost of production of crop i in A_i ;

γ_i, σ_i, λ_i, μ_i, k_i=parameters; their subscript i relates to the crop.

If no indicator of time is attached to a variable, then it is understood that the variable is related to the current period; A, for instance, stands for the total acreage in the current period t. In other periods, time would be indicated within brackets; $p_i(t-1)$, for instance, is the price of crop i in period $t-1$.

Model I: We postulate that the farmer allocates A to various crops so as to maximize the expected total profit z^*.

$$z^* = \Sigma z_i^* \quad .. \quad (1)$$

By definition, $z_i^* = (A_i^* \, p_i^* \, q_i^*) - C_i^*$ .. (2)

The process of expectation formation is difficult to establish empirically.[8] Let us assume that the arithmetic mean of the values in five preceding periods is the expected value of the variable in the current period.[9]

$$p_i^* = \frac{1}{5} \sum_{S=1}^{5} p(t-s) \quad .. \quad (3)$$

$$\text{and } q_i^* = \frac{1}{5} \sum_{S=1}^{5} q(t-s) \quad .. \quad (4)$$

The *desired* allocation of land is then determined as follows:

Maximize : $z^* = \Sigma \, (A_i^* \, p_i^* \, q_i^* - C_i^*)$ .. (5)

subject to the constraint that

$$\Sigma A_i^* = A \quad .. \quad (6)$$

8. For various functions of expectation formation, see John R. Hicks: Value and Capital, Second edition, Oxford University Press, London, 1961, p. 205; and also Marc Nerlove: Distributed Lags and Demand Analysis for Agricultural and Other Commodities, U.S.D.A., Washington, D.C., U.S.A., 1958, pp. 23-24.

9. *Cf.* "*Expected prices :* All eleven crops received some form of government price support in 1955.... The announced support prices for each unit are assumed to equal its expected prices..... *Expected yields:* Farmer's current yield expectations are generally based upon their past experience. In the present empirical analysis it is assumed that their expected yields equal the averages of their realised yields for the five preceding crop years... The five-year yields are widely used, and appear as reasonable as, if not more reasonable than, any alternative measure of expected yields." Henderson, *op. cit.*, pp. 249-50.

Here A is given. The first order conditions are:

$$p_i^* \, q_i^* - \frac{\partial C_i^*}{\partial A_i^*} = \lambda \quad i=1, 2,\ldots, n \qquad \ldots (7)$$

where λ is the Lagrangean multiplier. The second order conditions are assumed to be satisfied.

The cost function C_i^* has presumably a shape as shown in Figure 1.

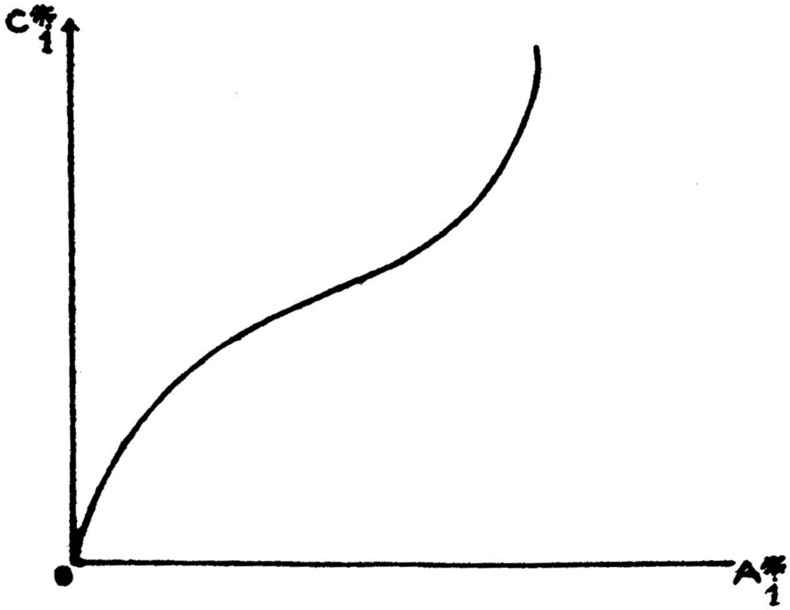

Figure 1

However, the relevant segment of the cost curve (the neighbourhood of the equilibrium point) can be approximated by a function such as:

$$C_i^* = k_i \, A_i^{*\,\sigma_i} \qquad \sigma_i > 1 \qquad \ldots (8)$$

Now from (7) and (8) we get:

$$p_i^* \, q_i^* - k_i \, \sigma_i \, A_i^{*\,\sigma_i - 1} = \lambda \qquad i = 1, 2, \ldots, n \qquad \ldots (9)$$

In Figure 2, A, B and C respectively are equilibrium points corresponding to:

$$\lambda \gtreqless 0$$

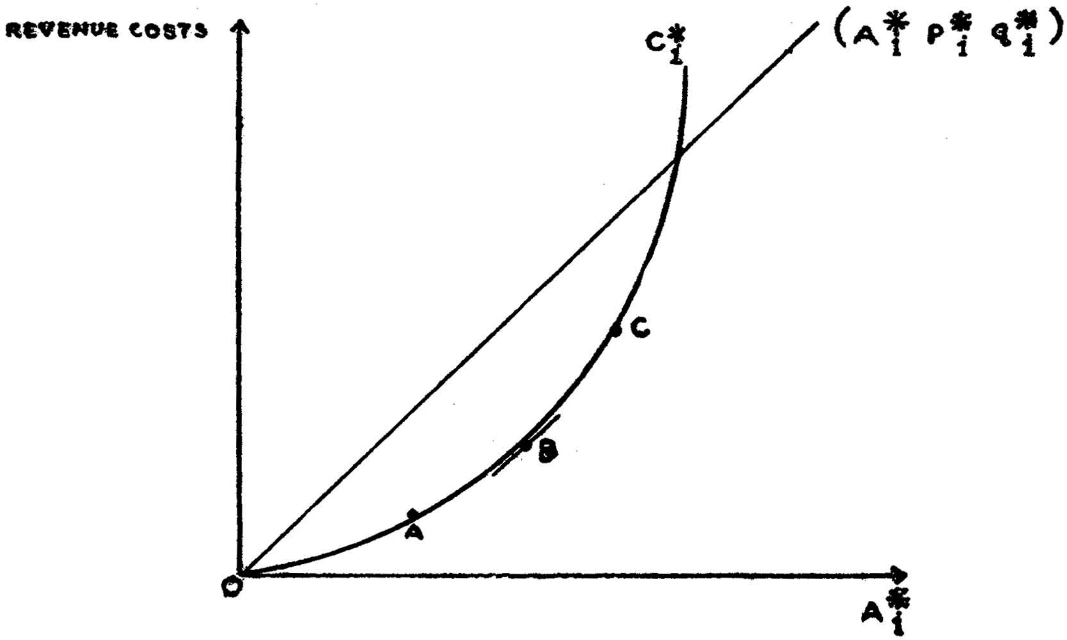

Figure 2

Only at B, marginal revenue equals marginal cost; at A it is greater than marginal cost whereas at C it is less (Figure 2).

Referring to (9), we get:

$$\log A^*_i = \frac{1}{1-\sigma_i} \log k_i\ \sigma_i + \frac{1}{\sigma_i - 1} \log (p^*_i\ q^*_i - \lambda) \qquad ..\ (10)$$

The Taylor's theorem gives us:[10]

$$\log (p^*_i\ q^*_i - \lambda) = \log p^*_i\ q^*_i - \frac{1}{p^*_i\ q^*_i} \lambda - \frac{1}{2(p^*_i\ q^*_i)^2} \lambda^2 + \qquad ..\ (11)$$

10. The Taylor's series expansion of log (x) in the neighbourhood of the point a : $\log x = \log a + \frac{1}{a}(x-a) - \frac{1}{2a^2}(x-a)^2 + \ldots$ is valid if $a > 0$ and $0 < x \leq 2a$. In (13), we have $x = p^*_i\ q^*_i - \lambda$ and $a = p^*_i\ q^*_i$. So $a > 0$. Note also that $p^*_i\ q^*_i > \lambda$; see (11). So our $x > 0$. Furthermore, *let* $p^*_i\ q^*_i - \lambda \leq 2p^*_i\ q^*_i$; that is, $x \leq 2a$. Therefore, the Taylor's theorem is applicable here.

Now (10) and (11) together yield (ignoring second and higher order terms of $\lambda/p^*_i q^*_i$) in (11) :

$$\log A^*_i = \frac{1}{1-\sigma_i} \log k_i \sigma_i + \frac{1}{\sigma_i - 1} \left[\log p^*_i q^*_i - \frac{1}{p^*_i q^*_i} \lambda - \ldots \right] \quad .. \quad (12)$$

But (12) cannot be estimated with statistical data, since A^*_i, the *desired* acreage, is not necessarily observable unless the *ex ante* and *ex post* acreage figures are assumed to be identical.

Here we introduce the Nerlovian-type adjustment process:[11]

$$\log A_i - \log A_i (t-1) = \gamma_i \left[\log A^*_i - \log A_i (t-1)\right]$$

$$0 \angle \gamma_i \underline{\angle} 1$$

$$\text{or } \log A^*_i = \frac{1}{\gamma_i} \log A_i - \frac{1-\gamma_i}{\gamma_i} \log A_i (t-1) \quad .. \quad (13)$$

In view of (12) and (13) we get:

$$\log A_i = a_0 + a_1 \log p^*_i q^*_i + a_2 \left(\frac{1}{p^*_i q^*_i} \right) + a_3 \log A_i (t-1) \quad .. \quad (14)$$

$$\text{where } a_0 = \frac{\gamma_i}{1-\sigma_i} \log k_i \sigma_i$$

$$a_1 = \frac{\gamma_i}{\sigma_i - 1}$$

$$a_2 = \frac{\lambda \gamma_i}{1-\sigma_i} \quad .. \quad (15)$$

$$\text{and } a_3 = (1-\gamma_i)$$

Note that we have already estimated p^*_i and q^*_i from (3) and (4) respectively. Now (15) can be estimated by the method of least squares. And once we know in a's of (14), the four parameters, namely, γ_i, σ_i, k_i and λ_i can be derived from (15).

There are two interesting properties of this model. First, since we can compute the parameters k_i and σ_i from the regression equation (14) and the definitional equation (15), it is possible to get an empirical estimate of the cost function, *without directly using cost data anywhere in estimating this model.*

11. Nerlove: The Dynamics of Supply, *op. cit.*, p. 62.

The cost

$$C_i = k_i A_i^{\sigma_i} \qquad .. \quad (16)$$

being now known, we can deduct it from the revenue ($A_i \, p_i \, q_i$) and calculate the actual *profit* (or shall we call it rent) earned in crop i in every period.

Second, note that λ is the "marginal utility" of land. If the estimation of (15) shows that λ is negative, then one could conclude that even though the farmer is allocating the given amount of land rationally *between* crops (assuming that the econometric model satisfies the necessary statistical tests), the *total* amount of land under cultivation in the country is uneconomic for overall profit maximization; the equilibrium point is at C in Figure 2. And so on.

However, this model has a limitation. If λ is assumed to be constant, then A^*_i depends mainly upon its own productivity, and so forth; it would not respond to changes taking place in respect of other crops. As such even though (14) may turn out well as a satisfactory *econometric* model to describe what had happened during a certain period, analytically it is deficient[12] for crop acreage forecasting in future.

II

Model II: A different version of the above model can be constructed to take care of the above mentioned "Marshallian problem."

For any particular crop i, and any other alternative crop j, we get from (9):

$$p^*_i \, q^*_i - k_i \, \sigma_i \, A_i^{*\,\sigma_i - 1} = p^*_j \, q^*_j - k_j \, \sigma_j \, A_j^{*\,\sigma_j - 1} \qquad .. \quad (17)$$
$$i, j = 1, 2, \ldots, n$$

$$\text{or } \log A^*_i = \frac{1}{1-\sigma_i} \log k_i \, \sigma_i + \log \left[p^*_i \, q^*_i - p^*_j \, q^*_j + k_j \, \sigma_j \, A_j^{*\,\sigma_j - 1} \right] \qquad .. \quad (18)$$
$$i, j = 1, 2, \ldots, n$$

With any given i, (18) holds for each and every j. So we propose to construct an overall weighted index of all p^*_j ($j=1, 2, \ldots, n$; $i \neq j$), and a similar index of all q^*_j ($j=1, 2, \ldots, n$; $i \neq j$) as follows:

$$p^* = \sum_{j \neq i} p^*_j \left\{ A_j(t-1) \, q_j(t-1) \right\} \Big/ \sum_{j \neq i} A_j(t-1) \, q_j(t-1) \qquad .. \quad (19)$$

$$\text{and } q^* = \sum_{j \neq i} q^*_j \, A_j(t-1) \Big/ \sum_{j \neq i} A_j(t-1) \qquad .. \quad (20)$$

12. This resembles the problems associated with the Marshallian assumption of constant marginal utility of income. See P. A. Samuelson: Foundations of Economic Analysis, Harvard University Press, Cambridge, Massachusetts, U.S.A., 1947, pp. 189-95.

Likewise, the concept of the weighted index of marginal costs of alternative crop acreages could be put in the form:

$$\mu^* = \sum_{j \neq i} \left[\frac{A_j\ (t-1)}{A\ (t-1)} \right] \sigma_j\, k_j\, A_j^{*\ \sigma_j - 1} \qquad .. \quad (21)$$

Then (18) can be rewritten as:

$$\log A_i^* = \frac{1}{1-\sigma_i} \log k_i\, \sigma_i + \log\, [p_i^*\, q_i^* - p^*\, q^* + \mu^*] \qquad .. \quad (22)$$

Again, in view of the adjustment process (13) for crop i, (22) yields:

$$\log A_i = b_0 + b_1 \log\, [(p_i^*\, q_i^* - p^*\, q^*) + \mu^*] + b_2 \log A_i\ (t-1) \qquad .. \quad (23)$$

$$\left.\begin{aligned} \text{where } b_0 &= \frac{\gamma_i}{1-\sigma_i} \log k_i\, \sigma_i \\ b_1 &= \frac{\gamma_i}{\sigma_i - 1} \\ \text{and} \quad b_2 &= (1 - \gamma_i) \end{aligned}\right\} \qquad .. \quad (24)$$

Still (23) cannot be estimated with statistical data, unless μ^* in every period is known.[13] We shall therefore assume that μ^* as defined in (21) and used in (23) is constant.[14] It means that the acreage allocated to a given crop i can be diverted at the margin to such an "alternative composite crop" that the marginal cost (with reference to acreage) in the latter is constant. This assumption may not be "justifiable" in strict theory; but it may be expedient in the econometric analysis.[15]

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13. Note that p_i^*, q_i^*, p^* and q^* can be computed from (3), (4), (19) and (20) respectively.

14. The term log $[(p_i^*\, q_i^* - p^*\, q^*) + \mu^*]$ cannot be expanded around $(p_i^*\, q_i^* - p^*\, q^*)$ by Taylor's series. For the theorem is not necessarily applicable since $(p_i^*\, q_i^* - p^*\, q^*)$ may be negative.

15. Note how prices and yields of *other* crops appear in (23). Compare with this, the estimating equation of Raj Krishna :

$$\left[\text{Actual acreage of crop i}\right]_t = a_0 + a_1 \left[\begin{array}{l}\text{Prices of crop i}\\ \text{deflated by an}\\ \text{index of prices}\\ \text{of the alterna-}\\ \text{tive crops}\end{array}\right]_{t-1} + a_2 \left[\begin{array}{l}\text{Yield of crop i}\\ \text{deflated by the}\\ \text{yields of other}\\ \text{crops}\end{array}\right]_{t-1}$$

$$+ a_3 \left[\text{Total acreage}\right]_{t-1} + a_4 \left[\begin{array}{l}\text{Actual acreage}\\ \text{of crop i}\end{array}\right]_{t-1} + a_5 \left[\text{Rainfall}\right]_{t-1};$$

(Contd.)

Now, μ^* being constant by assumption, (23) can be estimated by an *iterative* method. We know that μ^*, the "marginal cost" of the "alternative composite crop" acreage, is positive; and that it also cannot be very far off from the corresponding "marginal revenue," *i.e.*, $p^* q^*$. Symbolically, then

$$0 \angle \mu^* \overset{>}{\underset{\angle}{=}} p^* q^* \qquad .. \quad (25)$$

There is a theorem to the effect that the maximum likelihood estimates of the parameters in (23) as well as of μ^* are produced by that value of μ^* (together with other coefficients) which maximizes the multiple correlation coefficient of (23).[16] So we can iterate and regress for alternative values of μ^* in (23), and accept that set of estimates of the parameters for which the multiple correlation coefficient is the highest.

Again the parameters of the original equations are just identifiable in the reduced form in the sense that given the b's of (23), we can calculate γ_i, σ_i and k_i from (24). So we can get empirical cost functions for acreages of individual crops, and calculate the profits (rents) for every year.

III

To sum up, starting from the principle of profit maximization we have developed two dynamic models of land utilisation. Both the versions can be estimated with statistical data. We also observe that although all the previous econometric models in this respect do assume, implicitly or explicitly, the hypothesis of profit maximization, the estimating equations in practically all of them do not seem to have the logical foundation of economic theory. It is also interesting to note that our models do not require any additional data in comparison with the others.

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or that of Falcon :

$$\left[\begin{array}{c}\text{Per cent change in cotton acreage}\\ t-1 \text{ to } t\end{array}\right] = a_0 + a_1 \left[\frac{\text{Price of cotton}_{\;t-1}}{\text{Weighted price of rice, bajra, jowar, corn and sugarcane}_{\;t-1}}\right]$$

We have ignored the error terms.

16. Nerlove : The Dynamics of Supply, *op. cit.*, pp. 189-92; and Arnold C. Harberger (Ed.): The Demand for Durable Goods, University of Chicago Press, Chicago, U.S.A., 1960, pp. 95-96.

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