THE "CONSTANT DIFFERENCE OF ELASTICITIES" (CDE) FUNCTIONAL FORM: A NEGLECTED ALTERNATIVE.

Contributed Paper presented at the

1989 Meeting of the American Agriculural Economics Association

Baton Rouge, Louisiana, July 30-August 2, 1989

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August, 1989

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#7617

ABSTRACT

The "Constant Difference of Elasticities" (CDE) Functional Form: A Neglected Alternative.

Unlike those flexible functional forms which are local approximations, the "Constant Difference of Elasticities" (CDE) function is globally well-behaved. Due to its theoretical properties and parsimony in the number of parameters, the CDE functional form offers many potential applications in production, consumption and trade analysis.

The "Constant Difference Elasticities" Functional Form: A Neglected Alternative

1.0 Introduction:

Since the 1960's, a large part of microeconomic research has focused on the development of empirical frameworks capable of describing producer or consumer behavior in general and unrestricted terms. Analytical tools known as flexible functional forms and viewed as second order Taylor approximations to an optimized indirect or direct objective function have been instrumental in improving the estimation and quantification of economic agent responses to market signals. Despite these qualities, however, flexible functional forms have been criticized on the grounds that: i) they are not parsimonious in the number of parameters (Pope), ii) they are only locally valid by satisfying curvature conditions implied by economic theory at a given approximation point (Simmon and Weiserbs, Caves and Christensen, White), and iii) they generate unstable elasticity estimates over the estimation sample range (Diewert and Wales 1987; Guilkey, Lovell and Sickles).

In order to tackle these problems, several solutions have been put forward in recent years. To begin, the violation of regularity conditions can be overcome by imposing curvature conditions at every sample data point and then estimating the parameters of the flexible functional form by constrained mathematical programming techniques (Ball; Kopp and Hazilla; Talpaz, Alexander and Shumway). An alternative solution is to find new functional forms which are globally well-behaved (Diewert and Wales 1987). Building on this latter idea, Diewert and Wales (1988) specified and estimated a semi-flexible functional form which is both globally valid and characterized by a reduced number of parameters.

The CRE or CDE[1] functions developed by Hanoch (1971, 1975, 1978) also alleviate the deficiencies associated with the flexible functional forms by being globally well-behaved and characterized by a reduced number of parameters. To arrive at such results, an implicitly additive (direct or indirect) structure is imposed to the analyzed firm's technology or consumer's preferences. Taking the example of single output technology with n inputs, a reduced number of parameters is obtained by representing all price effects of input demands by n free "substitution" coefficients instead of the n(n-1)/2 parameters required by most alternative flexible functional forms.

The objective of this paper is to provide detailed information regarding the CDE functional form - a form which has rarely been used in applied economics[2]. Specifically, the theoretical and practical strengths and weaknesses of this function are illustrated in the next two sections of the paper and examples of its potential use in empirical economics are given in the fourth section. The last section summarizes the main findings of this research and provides suggestions for further applications of the CDE function in the fields of applied economics.

2.0 Theoretical Foundations:

Duality theory allows for the representation of the two constrained optimum choices facing a firm: i) cost minimization subject to an output constraint or ii) output maximization for a minimum given cost. This is done by means of optimized indirect objective functions which, when differentiated with respect to input prices, yield theoretically consistent input demand relationships (see Appendix). Thus, the cost-minimization decision of a firm can be represented by a well-behaved cost function (Varian, p. 21). Compensated, constant-output input demand functions are then derived using Shepherd's lemma. Similarly, the constrained output maximization problem can be described by an indirect production function whose arguments are the prices of inputs normalized by total costs (Chambers, 1982). From this indirect production function, compensated constant-cost input demand functions are obtained using Roy's identity. The same mathematical result can be generated if the output maximization problem is represented by a reciprocal indirect production function which is positive, continuous, non-decreasing and quasi-concave in the vector of normalized input prices (Diewert, p. 126). Whether compensated input demands are specificied in terms of an output maximization or cost minimization framework does not really matter. Both modelling approaches are equivalent since they yield optimal input usage at a point where the technical rate of substitution among pairs of inputs is equal to the ratio of input prices. They only differ by the fact that the associated compensated input demand functions have different arguments. Parameterization of output and cost levels however shows both compensated input demand relationships to be equal (see (A.7) and (A.8) in Appendix).

The cost function approach has been favored by production economists to analyze substitution patterns among inputs, technological change and returns to scale. On the other hand, the indirect production function (or its reciprocal) approach is very appropriate in the specification of consumer demand functions since the level of utility (output) is not measurable. The use of the indirect reciprocal production function and the imposition of an implicitly additive structure form the basis behind the development of the CDE functional form.

2.1 <u>Indirect Implicit Additivity</u>:

The conventional strategy used to reduce the number of estimable parameters of a functional form is the imposition of separability restrictions on the structure of the underlying technology. Among the numerous concepts of separability developed by economists, a strongly separable or additive structure allows for the greatest reduction in the number of estimable parameters. In fact, if a direct (indirect) production function is additive in the levels of inputs (normalized input prices), the various substitution effects are proportional to the expansion (cost) effects (Frisch). The (nxn) matrix of Allen elasticities of substitution as defined for n inputs, can then be recovered from the knowledge of the input cost shares and the elasticities of inputs with respect to: i) output, in the case of a cost function approach; and, ii) costs in the case of the indirect production function approach (Deaton and Muelbauer). Although the use of this strategy to generate input demand is appropriate when data are scarce, it imposes strong restrictions on the structure of the underlying technology and the shape of the substitution effects.

Acknowledging these shortcomings, Hanoch suggested an additive structure be redefined in an implicit fashion. In so doing, it is possible to estimate a smaller number of estimable parameters, while at the same time computing Allen elasticities of substitution without imposing stringent conditions on the structure of the underlying technology. Following this line of thinking, a direct implicitly-additive structure can be imposed on the production function which can then be represented by a constant-ratio-elasticities-of-substitution (CRES) functional form. Similarly, due to the symmetry between the direct and indirect dual representations of a firm's technology, it is possible to define an indirect implicitly-additive reciprocal production function. In this latter case, it would mean that the isoquant surfaces are strongly separable or additive to the unit-cost prices.

A reciprocal indirect production function, K(.), is implicitly additive if it can be expressed in the following manner:

$$(1) \qquad K(y, v) = \sum_{i=1}^{n} K_i(y, v_i) \equiv 1$$

where y is the level of output produced ; v_i is the price of input i (designated by w_i) normalized by the minimum total cost (designated by C^*); and $K_i(.)$ are n functions of two variables and have properties similar to the general indirect reciprocal production function K(.).

Demand functions for inputs are obtained using the modified Roy's identity:

$$(2) \qquad x_i = \frac{K_i^i(y, v_i)}{\sum_{i=1}^{n} v_i \; K_i^i(y, v_i)} \quad \text{for } i = 1...n, \quad \text{where } K_i^i = \frac{\partial K_i(y, v_i)}{\partial v_i};$$

As formulated, the expression defining the demand for inputs is cumbersome and difficult to implement econometrically,. However, a simplification of the mathematical formulation is derived if x_i is divided by another input, say x_1, giving:

$$(3) \qquad \frac{x_i}{x_1} = \frac{K_i^i(y, v_i)}{K_1^1(y, v_1)} \qquad \text{for } i = 2....n.$$

The ratio, x_i/x_1, depends upon three variables, two of which are the "own" normalized prices of inputs i and 1. Consequently, the risk of multicollinearity relative to other flexible functional forms is minimized when such equations are estimated econometrically. In addition, despite the simplistic structure of (3), all the Allen elasticities of substitution, and indirectly the matrix of compensated price elasticities, can still be computed. To do so, the following formula is used (Hanoch 1975, p. 409]:

$$(4) \qquad \sigma_{ij} = \alpha_i(y, v_i) + \alpha_j(y, v_j) - \sum_{k=1}^{n} \alpha_k(y, v_k) s_k - \delta_{ij}\frac{\alpha_i}{s_i}$$

where s_k is the input cost share of input k and $\Sigma s_k = 1$;

$$\alpha_k = - \frac{v_k K_k^{kk}(v_k, y)}{K_k^k(v_k, y)};$$

K_k^{kk} and K_k^k are the second- and first- order partial derivatives, respectively, of the function, $K_k(.)$ with respect to v_k; and

$\delta_{ij} = 1$ if $i = j$ and 0 if $i \neq j$.

All the Allen elasticties of substitution depend on the substitution function, a_k, which in turn varies with the level of output and the "own" normalized prices of inputs v_k. Thus, the higher the function a_i, the higher σ_{ij} will be. A complementary relationship can be defined between inputs i and j, if the substitution functions are small, and if $a_i + a_j < \sum a_k s_k$.

2.2 CDE Functional Form:

To operationalize this notion of an indirect implicitly- additive production structure, the function K_i must be approximated by a CDE functional form given by:[3]

$$(5) \qquad K(y, v) = \sum_{i=1}^{n} B_i \, y^{e_i b_i} \left(\frac{W_i}{C^*}\right)^{b_i} \equiv 1$$

$$\text{with} \quad \log\left(y^{e_i b_i}\left(\frac{W_i}{C^*}\right)^{b_i}\right) \text{replacing} \left(y^{e_i b_i}\left(\frac{W_i}{C^*}\right)^{b_i}\right) \text{ for } b_i = 0.$$

B_i and e_i ei are called distribution and expansion parameters, respectively.

This function is globally valid if for $v_i \gg 0$, B_i and e_i are > 0, b_i is < 1, and either b_i is < 0 or $0 < b_i < 1$. Weaker conditions can be obtained if one b_i is > 1. Note that the above function must only be defined by 3n parameters. This represents a welcome alternative to other flexible functional forms such as the single output Translog cost function which needs (n+1)(n+2)/2 coefficients to be defined (Chambers, 1988).

By applying the modified Roy's identity and making use of expression (A.6) in the appendix, linearized CDE demand functions are derived and expressed in a logarithm ratio form:

$$(6) \quad \log\left(\frac{X_i}{X_1}\right) = A_i - g_i \log(y) - \alpha_i \log\left(\frac{W_i}{C*}\right) + \alpha_1 \log\left(\frac{W_1}{C*}\right) \text{ for } i = 2..n$$

$$\text{where } A_i = \log\left|\frac{B_i b_i}{B_1 b_1}\right|;\ g_i = e_i b_i - e_1 b_1, \text{ and}$$

α_k are substitution parameters equal to $1-b_k$. Note that validation conditions are also satisfied if $\alpha_k > 0$.

Allen partial elasticities of substitution are computed through the use of expression (4), giving:

$$(7) \quad \sigma_{ij} = \alpha_i + \alpha_j - \sum_{k=1}^{n} \alpha_k s_k - \delta_{ij} \frac{\alpha_i}{s_i}$$

Note that for $i \neq j \neq k$, $\sigma_{jk} - \sigma_{ik} = \alpha_i - \alpha_j$ = constant. This latter property of the CDE shows its exact nature as a functional form with constant two-input one-price elasticities of substitution (TOES). Hence, the CDE can be viewed as a generalization to n inputs of the CES cost function and some of its hybrid forms developed in the 1960's. Simple manipulation of parameters, α_i and e_i, in (5) or (6) reveals that the CDE function becomes a CES cost function if $g_i = 0$, or $e_i b_i = e_1 b_1$ and $\alpha_i = ..\alpha_1 = \alpha$ for all i = 1...n. Furthermore, if α is equal to one, the CDE is a Cobb-Douglas cost function. Other special and intermediate cases such as homogeneity and explicit additivity can be derived by imposing specific values to the expansion and substitution parameters (see Hanoch, 1975 for more details).

The CDE function also offers the advantage of taking into account a wide range of substitution relationships among factors of production. An analysis of expression (7) reveals that a high value associated with α_i would imply that input i is a net susbtitute with other inputs. On the other hand, small values of α_i and α_j would result into a complementary relationship between inputs i and j as long as $\alpha_i + \alpha_j < \Sigma \alpha_k s_k$.

Another interesting feature of the CDE function is that it is general enough in its formulation that it can also represent non-normal technologies which are characterized by the existence of inferior inputs. In fact, the output elasticities for inputs (λ_i) which are given by

$$(8) \quad \lambda_i = e_i + \sum_{k=1}^{n} e_k s_k \ [\alpha_i - \sum_{k=1}^{n} s_k \alpha_k] \quad - [e_i \alpha_i - \sum_{k=1}^{n} s_k e_k \alpha_k] .$$

are a function of the expansion and substitution parameters and the input cost shares. Then, depending on the values taken by these parameters, it is conceivable that negative values for the various λ_i's could arise.

The last two properties attributed to the CDE show clearly that this functional form is "flexible" enough to accomodate both very specific and extreme technologies. This is not the case with the constant ratio of elasticity of substitution (CRES) function which can only take into account complementary and inferior inputs in very extreme situations.

Despite all its qualities, the adoption of the CDE functional form has a cost on the theoretical side. In fact, an indirect implicitly-additive structure is characterized by isoquant surfaces that are strongly separable with respect to unit-cost prices (Hanoch 1975). This implies a certain rigidity in the underlying technology and explains why it is preferable to use this kind of function for broad categories of inputs.

3.0 Empirical Implementation:

The system of CDE input demand functions has two major deficiencies from an estimation standpoint. First of all, only the substitution parameters, a_k, can be identified. The distribution and expansion parameters cannot be derived from (6) since only (n-1) equations are estimated. This means that the output elasticities for inputs (λ_i) cannot be computed. Several avenues can be adopted to get around this problem. One approach is to add an additional equation to the CDE system so that all coefficients can be determined. A second approach is to further specialize the CDE and its underlying production structure by imposing an additional constraint such as homogeneity or explicit additivity. Adopting an homogenous structure would make all expansion parameters, e_i, equal and thus identifiable. In this perspective, it is worth analyzing the case of a production sector characterized by constant returns to scale and perfect competition. Under such circumstances, marginal and average costs are the same and equal to output price. In addition, the expansion parameters, e_i, are all equal to one and the CDE input demand functions can be rewritten as follows:

$$(9) \quad \log\left(\frac{X_i}{X_1}\right) = A_i - \alpha_i \log\left(\frac{w_i}{p}\right) + \alpha_1 \log\left(\frac{w_1}{p}\right) \quad \text{for } i = 2..n$$

where w_i and p are the price of input i and output, respectively.

If the CDE functional form is explicitly additive, the product of parameters, $e_i b_i$ is equal to g for all i. As a result, the output variable is not an argument in the CDE input cost share ratio specification and the identification of the expansion parameters, e_i, is not an empirical issue anymore.

At first glance, the expression defining the CDE input demand function (equation (6)) may look simple and amenable to estimation. But this is not the case. In addition to a simultaneity problem (presence of total cost, C^*, as one right hand side endogenous variable), any system of CDE input demand equations can generate n different estimates for every parameter appearing in the behavioral equations, depending on which input serves as a common denominator in all factor demand functions. This difficulty of obtaining n different estimable CDE input demand specifications is partly overcome by expressing the input ratio in (6) in terms of input cost shares[4].

The end result of this operation is to develop a system of input cost share ratio equations which can be rewritten as

$$(10)\quad \log\left(\frac{S_i}{S_1}\right) = A_i + g_i \log(y) + b_i \log\left(\frac{W_i}{C^*}\right) - b_1 \log\left(\frac{W_1}{C^*}\right) \quad \text{for } i = 2..n$$

This new estimable version of the CDE input demand function is very attractive for the simple reason that expression (10) is similar in its formulation, but with a different specification, to the linearized <u>multinomial logit</u> model developed by Theil. This result implies that the CDE can be viewed as an exact and global validation of Theil's model in which the regularity conditions are satisfied everywhere in the input price space. In contrast, the other linearized versions of the multinomial logit model which have been applied so far to the estimation of input demand equations are local approximations with regularity conditions only fulfilled at a specific point (Considine and Mount).

Expressed in its shares form and appended with random terms, the system of CDE input demand functions is now a typical example of "seemingly unrelated" regression equations whose estimation by Full Information Maximum Likelihood (FIML) techniques is invariant relative to the dropped input cost share (Barten). As a result, n unique price or "substitution" parameters, b_i or α_i, can be determined regardless of the input cost share ratio specifications which are adopted. On the other hand, with this alternative representation of the CDE input demand functions, the identification of the expansion and distribution parameters (e_i and B_i) is still an unresolved issue.

4.0 <u>Empirical Applications of the CDE Functional Form</u>:

This section discusses two empirical applications of the CDE functional form which relate to the analysis of input demand functions. The first case illustrates the conditions under which CDE input demand functions can be estimated econometrically and with satisfactory results. The second application illustrates how a matrix of compensated price elasticities can be obtained with only a priori knowledge of its direct and diagonal elements. The suggested approach exploits some of the properties of the Allen elasticities of substitution associated with the CDE functional form.

4.1 Econometric Estimation of Demand for Feed Ingredients in the European Community:

As the CDE functional form permits the estimation of input demand functions with a reduced number of parameters, it is ideally suited to model production sectors characterized by a large number of inputs and limited data availability. In such circumstances, the rapid exhaustion of degrees of freedom and potential multicollinearity problems among explanatory variables prevent the application of flexible functional forms such as the translog or the normalized quadratic. An example is the feed concentrate market in the European Community (EC). Here, high support prices for cereals have induced many EC farmers to substitute grains for other cheaper feeds, thus leading to a demand for a large variety of feed ingredients . This section describes a successful application of the CDE functional form to estimate aggregate demand for feed ingredients in Denmark, United Kingdom and Ireland[5].

In this econometric exercise, it has been assumed that feed/livestock production processes in the above-mentioned EC countries can be represented by an aggregate single output CDE technology which possesses the following characteristics: i) six ingredients are used to feed livestock, including three grains (wheat, corn and other coarse grains), one cereal substitute (brans) and two high protein feeds (soymeal and other high protein feeds), ii) the ouput variable is a quantity aggregator of cattle and hog inventories and "feather" products expressed in a common livestock unit, and finally, iii) a time trend is incorporated into the CDE function in order to capture the effect of technological change in livestock feeding. Taking into account all these factors allows us to represent this feed technology by the following modified CDE function:

$$(11) \quad K(LIV_t, w_{it}, C_t^*, T) = \sum_{i=1}^{n} B_i \left(LIV_t\right)^{e_i b_i} \left(\frac{w_{it}}{C_t^*}\right)^{b_i} EXP^{(f_i T)} \equiv 1 \text{ for } i = 1 \ldots 6$$

where LIV_t designates total livestock production in period t, w_{it} is the price of feed ingredient i, C_t^* is the total feed cost, T is a time trend which takes values 0 in 1960, 1 in 1961 and so on, EXP denotes the exponential function, and the subscript i takes the follwing values: 1 for wheat, 2 for corn, 3 for other coarse grains, 4 for brans, 5 for other high protein feeds and 6 for soymeal.

The use of expression (10) now yields a system of linearized input cost share ratios for five of the feed ingredients represented by

$$(12) \quad \log\left(\frac{s_{it}}{s_{6t}}\right) = A_i + g_i \log(LIV_{it}) + b_i \log\left(\frac{w_{it}}{C_t^*}\right) - b_6 \log\left(\frac{w_{6t}}{C_t^*}\right) + k_i T$$

$$\text{where } A_i = \log\left|\frac{B_i b_i}{B_6 b_6}\right|, \quad g_i = e_i b_i - e_6 b_6, \quad k_i = f_i - f_6.$$

As shown by (12), each system equation is a function of two feed ingredient prices (w_{it} and w_{6t}) normalized by total feed cost (C_t^*), livestock output (LIV_t) and the time trend (T). In addition, the soymeal input cost share (variable s_{6t}) is the common denominator to all other feed ingredients and a common cross-equation restriction is imposed on the normalized price of soymeal. This constrained system of five linearized input cost share ratio equations has been estimated by FIML over the period 1963-1984 (see Table 1)[6]. In this process, the simultaneity arising from the presence of total feed cost among the right hand side variables has been overcome by replacing this latter variable by its predicted values obtained by regressing the

the observed total feed cost against a set of instrumental variables[7].

From an econometric standpoint, the performance of the CDE input demand system is quite satisfactory: except for the demand expression for wheat, all of them have high R^2; although most of the DW statistics lie in the inconclusive region, a likelihood ratio test rejects the null hypothesis of first order auto-correlation among the residuals; finally, 70% of the estimated coefficients are statistically different from zero at a 5% significance level.

All estimated price parameters (b_i) but one are smaller than one. The coefficient associated with the normalized price of brans is slightly over one, this indicating that the CDE functional form estimated for the feed concentrate market in Denmark, United Kingdom and Ireland is only locally valid.

With a direct price elasticity of -1.04, the feed demand for corn exhibits the most responsive reaction to its own prices among all the feed ingredients. The remaining feed inputs are price inelastic with direct elasticities ranging from -0.5610 for soymeal to -0.0036 for brans (see table 2).

An examination of average cross-price elasticities indicates a wide range of both expected and unexpected results concerning the substitution relationships between feed ingredients. Among the expected results, the pairwise substitution relationships among all cereals and between brans and wheat, and brans and corn is noteworthy. Similarly, the two protein rich ingredients are weak net substitutes. Another expected finding is the small but positive relationship between soymeal and and energy-rich ingredients. This latter result can be justified on the grounds that high support prices for cereals in the EC induces farmers to purchase cheaper feed ingredients such as soymeal which, although rich in protein, competes with grains as a source of energy.

Among the unexpected cross-effects is the fact that bran and other coarse grains are net complements. This result, which seems to have strengthened over time, is quite surprising since both ingredients are significant sources of energy and should substitute for one another. A plausible explanation for this peculiar phenomenon is the fact that these ingredients are used for different purposes in feeding livestock in the EC member countries under study. The other unexpected conclusion is that all the cross-price elasticities are small, with the exception of corn.

4.2 Computation of Synthetic Compensated Price Elasticities:

From expression (6), it can be seen that CDE-generated Allen elasticities of substitution and indirectly, compensated price elasticities are a function of all the input cost shares, s_k , and the parameters, α_k, for k = 1..n. As a result, if s_k and the direct Allen elasticities of substitution, σ_{kk}, are known a priori, all coefficients, α_k, are determined by resolving a system of n linear equations with n unknowns, represented by the following expression:

$$(13) \quad \sigma_{kk} = 2\alpha_k - \sum_{k=1}^{n} \alpha_k s_k - \frac{\alpha_k}{s_k}$$

Once the α_k's have been calculated, they are fed into (7) to determine the remaining off-diagonal elements of the matrix of Allen elasticities of substitution and indirectly, the compensated cross-price elasticities. In this procedure, an iterative search consisting of changing at the margin the pre-established values of the direct Allen elasticities would be needed as long as the implied substitution parameters, α_k, violate the validation conditions of the associated CDE function.

The above technique of computing synthetic price elasticities can also facilitate the econometric estimation of CDE input demand functions when few data points are available and a large number of inputs are under consideration. In such circumstances, the known coefficients (b_i or α_i) are incorporated into the estimable CDE input demand specification. Then, the remaining parameters (intercepts and coefficients associated with the other exogenous variables) which are not linked by any cross-equation restriction at all can be estimated by restricted two-stage least squares techniques applied to each equation.

To illustrate the advantage of the above modeling approach, it has been applied to the Italian feed concentrate market which was represented by a conceptual CDE model similar to the one developed for the aggregate Danish, British and Irish feed sector. This time, the total number of data points for the six feed ingredients under study was limited (ten observations from 1976 to 1984), precluding a full estimation of the associated CDE input demand functions.

The assumed values for the direct Allen elasticities of substitution have been selected on the basis of previous work undertaken on the EC feed concentrate markets (Hillberg; McKenzie, Paarlberg and Huerta; Surry and Moschini) and the importance of each feed ingredient in feeding livestock in Italy (See Table 3). Based on these considerations, the estimated cross-price elasticities conform to a-priori expectations. The only unexpected finding is that soymeal and other high protein feeds are net complements. However, we should not attach too much importance to this result because of the very small value of the associated cross-price elasticities.

5.0 Concluding Remarks:

This paper has brought to the fore the principal theoretical and empirical features of the CDE function when applied to production analysis. This functional form, which has been largely neglected by the economics profession, possesses a certain number of attractive properties that are worth considering. It is a globally well-behaved and can accomodate extreme and specific technologies . What makes this model specification very appealing is the fact that a reduced number of parameters are needed to estimate input demand relationships, thus avoiding potential multicollinearity problems among explanatory variables. However, the gains made on the empirical side are offset by the imposition of an implicitly additive structure on the underlying technology.

Despite this weakness, the CDE is very helpful in estimating input demand relationships in situations characterized by a large number of inputs and a limited data sample. An econometric application to the EC feed concentrate market supports this point and show clearly that reliable elasticities estimates can be generated with this functional form. Another feature of this function is that it can generate a complete matrix of theoretically consistent price elasticities using only estimated values of the diagonal elements as a starting point (i.e. own-price elasticities or direct Allen elasticities of substitution).

Although the CDE function has been developed in the context of a single-output firm , it can be equally applied not only to other production problems but also to the analysis of consumer demand and international trade. Thus, in the context of a multi-output technology in which inputs are non-specific, compensated supply functions can be estimated if the revenue function of a firm is represented by a CDE. Similarly, compensated cross-price elasticities of supply can be derived synthetically in the same fashion as in the case of of the input cost minimization problem analyzed in the previous section. The approach of determining synthetic price elasticities can be extended to the computation of uncompensated price elasticities in the consumer and producer cases. The only additional requirements are that elasticities defining the expansion or income effects be known . With regard to trade analysis, the CDE function can be used to specify import demand or export supply relationships which are differentiated by place of origin or destimation, and thus can be viewed as a generalization of the Armington model.

A final area where the CDE function could be very useful concerns the modelling of producer and consumer sectors in computable general equilibrium (CGE) models. In fact, since this function is globally well-behaved and characterized by a reduced number of parameters, it would allow expansion of the general capabilities of these CGE models which have been specified on the basis of simple functions such as the Cobb-Douglas or the CES function . In this process, however, a certain number of problems such as calibration and algorithm resolution would have to be solved.

FOOTNOTES

1 The acronym CRE stands for "constant ratio of elasticities" and indicates the fact that this function generates elasticities of substitution whose ratios are constant. In the case of the CDE, the difference of elasticities of substitution is constant.

2 To the author's knowledge, only three econometric studies have used the CDE functional form and have been published in academic journals (Dar and Dasgupta, Hawkins, and Merrilees). Although the number of empirical applications is also limited, the CRES function is more widely known since Dixon and al. adopted in the ORANI model to represent the aggregate technology of theAustralian agricultural sector.

3 The exact mathematical form of the CDE function is obtained by solving the following differential equation:

$$\alpha_k = -v_k K^k_{kk}(v_k, y)/K^k_k(v_k, y).$$

The solution to this differential equation is

$$K_i(y,\ v_i) = B_i(y) v_i^{1-\alpha_i} \text{ for } \alpha_i \neq 1,$$

$$\text{and } K_i(y\ v_i) = B_i(y)\log(v_i) \text{ for } \alpha_i = 1 \text{ where } \alpha_i = 1 - b_i.$$

4 To derive this new estimable version of the CDE, mulitply and divide both sides of (6) by w_i/C^* and w_1/C^*, respectively.

5 The specification and estimation of the CDE demand for feed ingredients is part of a larger and disaggregated econometric model of the EC feed/livestock model capable of capturing all the substitution relationships among feed ingredients and of simulating some alternative programs to the existing EC cereal policy regime (Surry). To limit the size of this model, the EC feed concentrate market has been broken down into four homogenous regions including i)France, ii) Italy, iii)a group of countries composed of Netherlands, Belgium and Germany and iv) and a last group formed by the three above-mentioned countries who joined the EC in 1973.

[6] Lack of space prevented us from providing a detailed account on the definition and quality of the data used in this econometric exercise. For more information on data issues, see Surry (1988).

[7] In order to avoid multicollinearity problems, total feed cost has been regressed against five principal components extracted from a matrix of exogenous variables composed of prices of feed ingredients, livestock output and the time trend. These five principal components explain 95% of the overall variability of the matrix of exogenous variables.

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APPENDIX

Alternative and Symetric Representation of the Constrained Optimization Choices Facing a Firm

I. Cost Function:

$$\text{(A.1)} \qquad C(w, y) \equiv \min_x \{ w'x : f(x) \geq y \}$$

where w and x are price and quantity vectors of n inputs;

and y = f(x) is a production function.

Shepherd's Lemma:

$$\text{(A.2)} \qquad x_i \equiv \phi_i(w, y) = \frac{\partial C(w, y)}{\partial w_i} \quad \text{for } i = 1 \ldots n$$

II. Indirect Production Function:

$$\text{(A.3)} \qquad y = G^*(v) \equiv \max_x \{ f^*(x) : v'x \geq 1\}, x \geq O_n\} \text{ for } v \geq O_n$$

where $v = \frac{W}{C}$

Roy's Identity:

$$\text{(A.4)} \qquad x_i^* \equiv \frac{\frac{\partial G^*(v)}{\partial v_i}}{\sum_{i=1}^{n} v_i \frac{\partial G^*(v)}{\partial v_i}} = \psi_i(v)$$

III. Reciprocal Indirect Production Function:

$$\text{(A.5)} \qquad K^*(v) \equiv \frac{1}{G^*(v)} = \min_x \{ \frac{1}{f^*(x)} : v'x \leq 1 ; x \geq O_N\}$$

Modified Roy's Identity:

$$\text{(A.6)} \qquad x_i^* \equiv \frac{\frac{\partial K^*(v)}{\partial v_i}}{\sum_{i=1}^{n} v_i \frac{\partial K^*(v)}{\partial v_i}} = \psi_i(v)$$

IV. Relationships between Compensated and Constant Cost Input Demand Functions:

$$\text{(A.7)} \qquad x_i = \phi_i\left(w, G^*(v)\right) = \psi_i(v) = x_i^*$$

or

$$\text{(A.8)} \qquad x_i^* = \psi_i\left(\frac{w}{C(w,y)}\right) = \phi_i(w, y) = x_i$$

Table 1: Maximum Likelihood Estimates of the CDE Input Demand Coefficients for Denmark, United Kingdom and Ireland

Dependent Variable	Constant	Time	Livestock Output	Normalized Price of: Wheat (1)	Corn (2)	Other Coarse Grains (3)	Brans (4)	Other High Protein Feeds (5)	Soymeal (6)	R^2	DW
$\log(s_1/s_6)$	23.5526	-0.00644	-2.53125	0.09927					0.36027	0.3089	1.020
	(14.0494)	(-0.53352)	(-4.21351)	(0.17791)					(1.61572)		
$\log(s_2/s_6)$	-10.1105	-0.08122	0.65271		-0.21965				0.36027	0.7702	1.181
	(-24.4510)	(-4.26391)	(0.64658)		(-0.24260)				(1.61572)		
$\log(s_3/s_6)$	-4.0762	-0.00850	1.13922			0.67237			0.36027	0.9277	0.708
	(-3.2021)	(-12.801)	(3.10040)			(2.40351)			(1.61572)		
$\log(s_4/s_6)$	-2.4542	-0.06454	1.08194				1.02866		0.36027	0.9084	1.010
	(-1.5969)	(-10.455)	(2.82177)				(4.56958)		(1.61572)		
$\log(s_5/s_6)$	24.3442	-0.01656	-1.91317					0.85367	0.36027	0.9004	2.011
	(5.2023)	(-2.14297)	(-5.27907)					(3.77967)	(1.61572)		

Notes: Dependent variables are the ratios of input cost shares expressed in logarithms. With the exception of the time trend , all other explanatory variables are also expressed in logarithms. The normalized prices of feed ingredients are obtained by dividing each feed ingredient price by the total feed cost. Asymptotic t values are reported in parentheses under the corresponding coefficient estimates.

Table 2: Compensated Price Elasticities for Feed Ingredients
Denmark, United Kingdom and Ireland
(Average 1963-84)

	Wheat	Corn	Other Coarse Grains	Brans	Other High Prot. Feeds	Soymeal
Input Cost Shares	0.149	0.089	0.533	0.059	0.068	0.102
Quantities				Price		
Wheat	-0.706	0.146	0.395	0.022	0.038	0.105
Corn	0.242	-1.044	0.565	0.041	0.059	0.137
Other Coarse Grains	0.109	0.095	-0.238	-0.011	-0.002	0.047
Brans	0.056	0.063	-0.101	-0.004	-0.025	0.011
Other High Prot. Feeds	0.081	0.079	-0.007	-0.022	-0.160	0.029
Soymeal	0.155	0.123	0.256	0.007	0.020	-0.561

Table 3: Compensated Price Elasticities for Feed Ingredients
Italy
(Average 1976-84)

	Wheat	Corn	Other Coarse Grains	Brans	Other High Prot. Feeds	Soymeal
Input Cost Shares	0.035	0.492	0.147	0.123	0.028	0.175
Quantity			Price			
Wheat	-1.500	0.823	0.233	0.193	0.035	0.216
Corn	0.058	-0.300	0.098	0.054	0.009	0.081
Other Coarse Grains	0.055	0.329	-0.500	0.070	0.007	0.039
Brans	0.055	0.320	0.083	-0.500	0.006	0.0360
Other High Pro. Feeds	0.044	0.162	0.036	0.028	-0.250	-0.020
Soymeal	0.043	0.152	0.033	0.025	-0.003	-0.250