STATE-LEVEL NONPARAMETRIC TESTS OF PROFIT MAXIMIZATION

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Abstract

A nonparametric analysis of agricultural production behavior was conducted for each of the contiguous 48 states to test the joint hypothesis of profit maximization, convex technology, and nonregressive technical change. With minor to modest measurement error, the results are consistent with the joint hypothesis. They further document the importance of considering geographic variability in agriculture production behavior when modeling production relationships or government policies affecting production at the regional or national level.

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Few sectors of the economy come so close to fulfilling the assumptions of the competitive model as does agriculture. Most agricultural producers are clearly price takers. Whether they are profit maximizers, however, is not so clear. Some researchers have concluded that observed agricultural production behavior is largely consistent with the profit maximization hypothesis (Weaver; Shumway and Alexander). Some have concluded that producers are risk averse (Just; Lin, Dean, and Moore; Anderson, Dillon, and Hardaker), some that they are constrained profit maximizers (Lee and Chambers), and others that the evidence is ambiguous (Pope; Taylor). Despite the lack of consistent evidence, the assumption of profit maximizing behavior has been maintained frequently in econometric analysis of agricultural production (McKay, et al.; Lopez; Antle; Lee and Helmberger).

Two quite different approaches have been used to test the profit maximization hypothesis in agricultural production, the calculus (or parametric) approach and the algebraic (or nonparametric) approach. The calculus approach has been the most frequently applied. Differences in underlying assumptions of these two approaches create an important distinction relevant for empirical work. The calculus approach assumes that the entire production behavior (i.e., smooth technologies) is available for analysis, while the algebraic approach assumes only a finite number of observations is available (Varian, 1983).

With recent developments in the theory of duality and flexible functional forms, the behavioral hypothesis of producers can be tested parametrically. However, with the calculus approach a joint hypothesis is tested that always includes a particular functional form. When the estimated production or profit function fails to satisfy the hypothesis, it is almost impossible to determine whether it is the behavioral objective or the functional form that causes the failure.

The nonparametric approach on the other hand provides a complete test of the behavioral hypothesis in question without the necessity of appending auxiliary hypotheses concerning functional form. Since all existing data on production behavior consist of finite numbers of observations, the algebraic approach gives a more realistic conceptual setting for empirical analysis. In addition, nonparametric tests can be obtained at very low cost. The nonparametric technique is a potentially powerful heuristic tool for providing much insight into the potential usefulness of the data and for testing data consistency with a theoretic structure free of parametric specifications (Varian; Hanoch and Rothschild; Diewert and Parkan; Chavas and Cox; Fawson and Shumway). Yet, until very recently it has been seldom applied in agricultural production studies.

Two nonparametric procedures relevant for testing the profit maximization hypothesis are attributable to Varian (1984, 1985). The first test, developed in 1984, is a deterministic heuristic test for the joint hypothesis of profit maximization and a convex production technology. It was defined by Fawson and Shumway to incorporate monotonic nonregressive technical change and applied to time series data for the United States and ten farm production regions. The second test, developed by Varian in 1985, is a statistical test of the magnitude of measurement error required for consistency with the joint hypothesis. He derived a test statistic that permits this procedure to be interpreted in terms of the classical statistical framework of hypothesis testing.

The objective of this paper is to comprehensively apply both deterministic and stochastic nonparametric tests to determine whether agricultural production behavior in each of 48 states has been consistent with the joint hypothesis of profit maximization, convex technology, and monotonic nonregressive technical change.

Methodology

Nonparametric Tests

Suppose we have m possible goods, including both inputs and outputs. We represent a specific production plan by a netput vector $X=(X_1, \ldots, X_m)$ in R^m where X_i is output if $X_i > 0$ and input if $X_i < 0$. The set of all feasible production plans, Y, which is a subset of R^m, is called the production possibilities set. The set Y describes all patterns of inputs and outputs that are feasible. It is assumed that Y is a closed, convex, and negative monotonic set (Varian, 1984). The boundary points of a convex set reflect efficient production plans in the sense that there is no way to produce more output with the same inputs or to produce the same output with less inputs.

If producers seek to maximize economic profits and the technology remains constant over the sample, we can define profit at observation i (Π^i) simply as the inner product of the netput vector X^i and its associated price vector $P^i=(P_1, \ldots, P_m)^i$. Consistency with profit maximization requires

(1) $\Pi^i = P^iX^i \geqq P^iX^j$ for $\forall$ i,j=1,2, ... , n

where X^i is in Y. Varian (1984) refers to this condition as the weak axiom of profit maximization (WAPM). He shows that it is equivalent to the existence of a closed, convex, and negative monotonic production set that p-rationalizes the data, where "p" stands for profit.

The test presented above can be modified to take technical change into consideration. Following Fawson and Shumway, consider the joint hypothesis of profit maximization, convex technology, and monotonic nonregressive technical change. Modification of the test is accomplished by changing the index in (1) to i=1,2,...,n, and $j \leq i$. Implementation of the test is accomplished by constructing a binary matrix:

$$(2) \qquad B(i,j) = \begin{cases} 1 & \text{if } P^i X^i \geq P^i X^j, \text{ for } i=1,2,....,n \text{ and } j \leq i, \\ 0 & \text{otherwise.} \end{cases}$$

Only if the triangular matrix consists of entirely of ones are the data consistent with the joint hypothesis. For this reason, the deterministic test is "all or nothing". That is, the data must satisfy the hypothesis for all observations in order to not be rejected. Probabilities of Type I and Type II errors are not computed.

There are at least three possible reasons why the test could fail (Hanoch and Rothschild; Varian, 1985): (a) the data could consist of observations on the boundary of a well-behaved production possibilities set which are in some sense badly affected by choice errors, (b) producers do not always operate on the boundary, and/or (c) the observations are not perfect measurements. Focusing on the third possibility, it is obvious that measurement error affects all observations in all disciplines. For example, Morgenstern noted some years ago that national income data are often measured with a standard error in excess of 10 %. Consequently, if production data fail to satisfy WAPM by an amount smaller than the likely magnitude of the measurement error, then we might employ statistical procedures to decide whether to reject the joint hypothesis. Varian (1985) proposed a general nonparametric method for use with measurement error. The following test statistic can be interpreted in terms of the classical statistical framework of hypothesis testing.

The null hypothesis, H_0, is that the data (X^i, P^i) satisfy the joint hypothesis of profit maximization, convex technology, and monotonic nonregressive technical change. Assume that the true netput k quantity for observation i is related to the observed netput quantity in the following manner:

(3) $$Q_{ik} = X_{ik}(1 + \epsilon_{ik})$$

where Q_{ik} is the true netput quantity, X_{ik} is the observed netput quantity, and ϵ_{ik} is a random error term that is independently and identically distributed $N(0,\sigma^2)$. Since inputs are often measured in different units, equation (3) postulates a proportional rather than an additive error. Suppose that we could in some way observe the true data (Q_{ik}). Then we could calculate the test statistic:

(4) $$T = \Sigma_{i=1}^{n}\Sigma_{k=1}^{m} (Q_{ik}/X_{ik} - 1)^2 / \sigma^2 \quad \sim \chi^2 \text{ under } H_0.$$

Although Q_{ik} is not observed, it is nevertheless possible to calculate an observable lower bound on T. Consider the following quadratic programming problem:

(5) $$S = \min \Sigma_{i=1}^{n}\Sigma_{k=1}^{m} (Z_{ik}/X_{ik} - 1)^2 /\sigma^2$$

$$\text{subject to } \Sigma_{k=1}^{m}P_{ik}Z_{ik} \geq \Sigma_{k=1}^{m}P_{ik}Z_{jk}, \quad \forall\ i=1,2,\ldots,n \text{ and } j \leq i.$$

where Z_{ik} are the solutions to the quadratic programming problem. Under H_0, the true data (Q^i, P^i) satisfy the constraint. Hence the minimum of the sum of squares, S, must be no larger than the test statistic, T. This means that whenever S is greater than C_α, where C_α is the critical value for a given significance level, α, we reject H_0. Thus, we have at least the desired level of significance. In other words, Type I error will be no greater than α.

We can also derive a bound on the true unknown error variance in order to apply the above chi-square test. Let S be the value of our objective function, $S = R/\sigma^2$, where σ^2 is the true variance of the error term and R is the minimum of the sum of squared proportional residuals. The null hypothesis would be rejected if $S > C_\alpha$ or $\sigma^2 < R/C_\alpha$. Let $\bar{\sigma}^2 = R/C_\alpha$ be the critical value of σ^2 and $\bar{\sigma}$ be its square root. Then the critical value is a lower bound estimate of what the standard error of the data would have to be to not reject the null hypothesis at a particular significance level. This means that if one believed the data were measured with a standard error of less than the critical value, the null hypothesis must be rejected at the stated significance level. Whether or not we reject the null hypothesis depends on the magnitude of the critical value compared to our prior opinions concerning the likely magnitude of the unobserved measurement error.

Data

Annual data for each of 48 states for the period 1956 to 1982 were used in this study. Price data from 1939 were used in the specification of price expectations for Iowa and Texas. Price and quantity data for a nearly exhaustive array of outputs and variable inputs were taken from the state-level data set for U.S. agriculture compiled by Robert Evenson and his associates of Yale University. Major data sources included the USDA's Agricultural Statistics, Agricultural Prices, Field Crops Production Disposition and Value. Outputs were aggregated into a single index as were each of the specifications of expected output price. Variable inputs were aggregated into four categories (fertilizer, hired labor, machinery operation, and other inputs). The other input category included all remaining inputs except those specifically regarded as quasifixed, i.e., capital, land, and family labor. The Tornqvist aggregation procedure was used to compute all aggregate quantity categories.

Measurement errors are assumed to affect only the quantity data in this analysis. Possible errors in measuring actual prices are not considered. However, actual market prices are not necessarily the same as the expected prices that motivate production decisions. It is assumed here that observed input prices accurately reflect expectations of producers of these variables. Given the length of the production period for most agricultural commodities, however, it is clear that the relevant output prices are not known at the time most resources are committed to production. Therefore, one of the simplest specifications of expected output prices, the one-year lagged price, was chosen for purposes of the nonparametric tests.

Results

The empirical results of both nonparametric tests are reported in table

1 for each state. The states are grouped in the table by USDA farm production region. We first checked the observed data for deterministic consistency with the joint hypothesis. Violations of the WAPM inequalities under the joint hypothesis were observed for every state. The first column in table 1 reports the proportion of observations satisfying the WAPM inequalities under the joint hypothesis. It ranges from 23% for New Jersey to 94% for Arkansas. Since this test is "all or nothing", we conclude that the null hypothesis must be deterministically rejected for all 48 states. The WAPM inequalities are not satisfied for all observations in any state.

Results of the stochastic tests are reported in the remaining columns of this table. The quadratic program converged to the minimum solution for all but five states. Four of the five states that did not attain convergence are in the Northeast region. Sum of squared proportional residuals attributable to output and each input category in the quadratic programming solution are listed separately. The largest residuals in every converged solution were attributable to output quantities, and the next highest were due to the "other inputs" quantities. The reported critical values of the standard error of the data ranged from .009 to .079 for the 46 states reported. In 40 states, $\bar{\sigma}$ was less than .03. Since a proportional measurement error was postulated, the critical value means that one would reject the joint hypothesis at the 5% level of significance if it was believed that the quantity data were measured with a smaller percent standard error than $100*\bar{\sigma}$ (assuming that price data were measured without error). In other words, the critical value identifies the minimum measurement error of the data required for consistency with the joint hypothesis. These critical measurement errors are generally substantially smaller than anyone would be likely to attribute to these data. For all 46 states with a reported solution (including three that did not converge), measurement errors in the quantity data comparable to the magnitude Morgenstern argued were common in national income data would not have caused rejection of the joint hypothesis. Although a large percentage of observations in some states violated the WAPM inequalities, the violations could be fully explained by measurement error of magnitudes common in secondary data. Obviously, neither quantity nor price data are measured without error. Thus, had we examined measurement error in both series, the implied critical values would have been even lower.

A great variability in test results was found among states, both within and between farm production regions. The largest variability, both in the percent of observations satisfying the WAPM inequalities and in $\bar{\sigma}$, occurred in the Northeast and Mountain states regions. In the Northeast region, observations violating the WAPM ranged from 23 to 85%, and the largest and smallest values of $\bar{\sigma}$ among the 46 states were found there. In addition, most of the problems of nonconvergence of the quadratic programming solution were experienced in this region.

Summary and Conclusions

A nonparametric analysis of agricultural production behavior was conducted for each of 48 states under the joint hypothesis of profit maximization, convex technology, and nonregressive technical change. Both Varian's (1984) deterministic test as extended by Fawson and Shumway and Varian's (1985) stochastic test were applied to 27 years of annual production data in each state. In the stochastic test, it was assumed that measurement error occurred only in the quantity data.

Although great variability in test results were observed among states, some observations violated the joint hypothesis in each state using the deterministic test. Measurement errors of magnitudes common in secondary data yielded stochastic test results fully consistent with the joint hypothesis in each of 46 states. In 40 states, a standard error of the quantity data no greater than 3% was sufficient to not reject the joint hypothesis. Consequently, other than failure of the quadratic program to converge to an optimal solution in a few states, no evidence was found from these stochastic nonparametric tests to challenge the appropriateness of the profit maximization hypothesis frequently maintained in parametric studies of production agriculture. It is possible that reported parametric violations of this hypothesis have been due to auxiliary maintained hypotheses such as functional form.

Table 1. Results of Nonparametric Tests

State	Observations Satisfying WAPM under Joint Hypothesis	Sum of Squared Proportional Residuals						Critical Value of Standard Error ($\bar{\sigma}$)[a]
		Output	Fertilizer	Hired Labor	Machinery Operation	Other Inputs	Total(R)	
	(%)							
Northeast								
NY[b]	36.5	.03936	.00007	.00053	.00051	.00619	.04666	.01691
NJ	23.3	.78596	.00911	.06980	.02453	.12411	1.01351	.07883
PA	81.0	.01233	.00002	.00009	.00015	.00156	.01415	.00931
DE	85.4	.03047	.00012	.00008	.00021	.00603	.03691	.01504
MD	81.2	.02392	.00010	.00016	.00025	.00278	.02721	.01292
ME[b]	57.9	.08968	.00046	.00179	.00158	.02731	.12082	.02722
NH[b]	39.7							
RI[b]	46.3							
VT	51.1	.03677	.00005	.00026	.00042	.00721	.04471	.01656
CT	24.6	.26072	.00107	.03202	.00458	.05280	.35119	.04640
MA	27.2	.17779	.00018	.01073	.00260	.02537	.21667	.03645
Lake States								
MN	74.1	.03519	.00015	.00003	.00033	.00182	.03752	.01517
WI	52.6	.03242	.00006	.00007	.00033	.00306	.03594	.01484
MI	63.2	.02158	.00010	.00013	.00045	.00145	.02371	.01206
Corn Belt								
OH	78.3	.03597	.00020	.00008	.00046	.00252	.03923	.01551
IN	81.5	.05739	.00044	.00006	.00042	.00326	.06157	.01943
IL	76.5	.07097	.00046	.00006	.00063	.00326	.07538	.02150
IA	77.8	.02909	.00030	.00002	.00019	.00211	.03171	.01394
MO	61.6	.05817	.00029	.00008	.00074	.00518	.06446	.01988
Northern Plains								
ND	75.7	.17875	.00055	.00037	.00689	.00700	.19356	.03445
SD	59.3	.14393	.00027	.00009	.00232	.01328	.15989	.03131
NE	81.7	.03065	.00009	.00003	.00030	.00278	.03385	.01441
KS	72.5	.05394	.00015	.00005	.00069	.00358	.05841	.01892
Appalachia								
VA	72.2	.02936	.00015	.00027	.00050	.00231	.03259	.01414
WV	25.9	.07641	.00019	.00077	.00315	.01115	.09167	.02371
KY	77.2	.02862	.00011	.00010	.00043	.00148	.03074	.01373
NC	78.8	.03689	.00016	.00021	.00040	.00171	.03937	.01554
TN	68.3	.02983	.00015	.00014	.00048	.00326	.03386	.01441
Southeast								
SC	73.3	.06176	.00075	.00079	.00129	.00516	.06975	.02068
GA	83.6	.02923	.00026	.00014	.00032	.00511	.03506	.01466
FL	86.8	.10071	.00103	.00412	.00072	.00447	.11105	.02609
AL	85.4	.02056	.00012	.00007	.00022	.00309	.02406	.01215

Table 1 (continued)

State	Observations Satisfying WAPM under Joint Hypothesis	Sum of Squared Proportional Residuals						Critical Value of Standard Error ($\bar{\sigma}$)[a]
		Output	Ferti- lizer	Hired Labor	Machinery Operation	Other Inputs	Total(R)	
	(%)							
Delta States								
MS	61.6	.05817	.00029	.00008	.00074	.00518	.06446	.01988
AR	93.9	.02584	.00006	.00020	.00034	.00288	.02932	.01341
LA	83.6	.05312	.00027	.00049	.00094	.00497	.05979	.01915
Southern Plains								
OK	73.3	.05781	.00007	.00019	.00106	.00465	.06378	.01977
TX	82.0	.04228	.00010	.00027	.00055	.00693	.05013	.01753
Mountain States								
MT	68.8	.11406	.00038	.00099	.00334	.01162	.13039	.02827
ID	85.2	.01328	.00004	.00011	.00020	.00116	.01479	.00952
WY	32.0	.10387	.00017	.00286	.00250	.01620	.12560	.02775
CO	85.7	.03059	.00009	.00032	.00054	.00303	.03457	.01456
NM	54.5	.04901	.00004	.00075	.00074	.01038	.06092	.01933
AZ	81.7	.02417	.00005	.00052	.00017	.00454	.02945	.01344
UT[b]	28.3	.18573	.08683	.14528	.26913	.04701	.73398	.06708
NV	78.0	.05838	.00002	.00179	.00137	.00583	.06739	.02033
Pacific States								
WA	87.3	.04023	.00009	.00073	.00056	.00317	.04478	.01657
OR	74.3	.06439	.00013	.00118	.00105	.00440	.07115	.02089
CA	86.0	.04556	.00003	.00117	.00016	.00283	.04975	.01746

[a] The critical value of the standard error of the data ($\bar{\sigma}$) was calculated at the 95% significance level of a chi-square distribution with 135 degrees of freedom.

[b] The quadratic programming solution did not converge.

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