THEORETICAL PROBLEMS AND EMPIRICAL CONSIDERATIONS IN DYNAMIC DUAL MODELING

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The dynamic dual approach to characterizing production decision making over time maintains that some factors of production cannot instantaneously adjust to a long-run optimal level. Many of the actual reasons for sluggish adjustment are not directly observable. The adjustment cost function is introduced to guarantee the gradual accumulation of capital. This function is a "black box" that intends to summarize all of the forces that can lead to sluggish adjustment. As one may expect, when we go from the static to dynamic case the assumptions necessary to guarantee empirical tractability can be more difficult to accept, econometric estimation is more demanding, and theoretical restrictions are more cumbersome.

Naive price expectations assumptions are required. In the deterministic dynamic dual econometric models current relative prices are assumed to persist indefinitely when current period decisions are made. In the stochastic case, current relative prices are assumed to be known but these prices evolve stochastically. In the case of intertemporal cost minimization, the dynamic and variable factor demands are theoretically conditioned on the current and future production targets. Econometric implementation requires the assumption that current output level along with relative input prices will prevail indefinitely. The firm is assumed to revise its expectations and production plan as the base period changes and new prices and output are observed.

The use of the derivative property of the value function to obtain dynamic and variable factor demands and output supplies follows from the assumption that the value function exists and is at least twice differentiable. The value function is a solution to the dynamic programming (Hamilton-Jacobi) equation which is a nonlinear partial differential equation or partial differential - difference equation. Guaranteeing that the value function exists for all values of prices and initial capital stocks at each point in time can be a tough assumption to swallow.

Typical functional specifications focus on the quadratic specification of capital with other independent variables which results in a multivariate flexible accelerator and (along with aggregation restrictions) a normalizing equation that is linear in the parameters. While the third order derivatives are required to fully characterize the deterministic value function (and fourth order derivatives are needed in the Brownian motion driven process), one only sees specifications which can be viewed as second order expansions of the value function.

As one may expect, when truly dynamic linkages are present the dynamic and static restrictions on behavior can be quite different. To illustrate this consider the issue of nonjointness in production. In the static case, when relative prices are used, the nonjointness exists between outputs i and j if output i is independent of the price of output j which is equivalent to

$$\pi_{p_i p_j} = 0$$

where $\pi(\ )$ is the static profit function. In the dynamic case the discussion of jointness in production is confounded by a) the presence of inputs that can and cannot be allocated to the production of a specified output, b) the distinction between technological and economic (behavioral) sources of jointness, and c) the consideration of "short-run" and "long-run" nonjointness. It turns out that when homogeneity properties of the value function are maintained, the nonjointness restriction is

$$\partial Y_i/\partial p_j = - \dot{K}' V_{kp_ip_j} - (\partial \dot{K}/\partial p_j)' V_{kp_i}$$

$$rV_{p_ip_j} = \dot{Z} V_{zp_ip_j} + (\partial \dot{Z}/\partial p_j) V_{zp_i}$$

for $i \neq j$, where Y is output and p is its' price, K is a vector of allocatable capital inputs, Z is the nonallocatable capital input, V is the multiple-output value function and the subscripts indicate partial differentiation (see Stefanou for more details). In general, these dynamic restrictions are quite different from the static case. In a static setting Shumway et al. demonstrate that nonseparabilty of the restricted profit function can exist in the presence of complete input allocation and independent production functions if some inputs are constraining. It is demonstrated that this result holds true for the long-run as well. Restrictions that can be econometrically implemented are presented. The results indicate that one cannot, in general, apply the static theory results to the dynamic case (such as setting the cross price derivatives of the value function equal to zero and assuming the presence of separability in output prices).

Stochastic Dynamic Dual

The two cases of the stochastic evolution of the state considered here are continuous and discrete shocks. While we can integrate the two processes into the dynamic programming equation, there is no unified approach applicable to both cases. Consider the case of choosing an optimal investment plan in the presence of adjustment costs and the stochastic evolution of output price. The firm is constrained by the capital accumulation equation written in differential form as

$$(1) \quad dK = (I - \delta K)dt$$

where I is the rate of investment and δ is the constant rate of depreciation. The firm's instantaneous cash flow is $\pi(W, K, I) - ck$ where $\pi(\ )$ is the restricted, single-output short-run profit function and W is the variable factor wage normalized by the output price.

<u>Continuous Case</u>

The real wage is assumed to evolve in a stochastic manner and is expressed as an Ito equation

$$(2) \quad dW = g(W)dt + \sigma(W)dB$$

where B(t) is Brownian motion with $E[dB] = 0$ and $E[(dB)^2] = dt$. The intertemporal objective is

$$(3) \quad J(w, c, k) = \max_{I} E_t[\int_t^{\infty} e^{-rs}[\pi(W, K, I) - ck]ds]$$

subject to (1) and (2) with $K(t) = k$, $W(t) = w$ and E_t indicates the expectation starting at time t. The stochastic dynamic programming equation is

$$(4)\ rJ(w, c, k) = \max_{I} \pi(w, k, I) - ck + (I - \delta k)J_k + g(w)J_w + .5\sigma(w)^2 J_{ww}$$

which is a partial differential equation. The first order condition is $-\pi_I = J_k$. The optimal I depends on J_k which depends on (w, c, k). Thus, even though the price evolution functions of g(w) and σ(w) do not directly influence the first order conditions the fact that g(w) and σ(w) do influence optimal investment via J_k. Differentiating the optimized dynamic programming equation leads to variable and dynamic factor demands and output supply

$$\dot{K}^* = J_{kc}^{-1}[rJ_c + k - g(w)J_{wc} - .5\sigma(w)^2 J_{wwc}]$$

$$x^* = (g_w(w) - r)J_w + \dot{K}^* J_{kw} + [g(w) + \sigma_w(w)\sigma(w)]J_{ww} + .5\sigma(w)^2 J_{www}$$

$$Y^* = rJ + ck - \dot{K}^* J_k - g(w)J_w - .5\sigma(w)^2 J_{ww}$$

While a functional specification to the second order can capture the stochastic effect of the price evolution process, a third order expansion is preferred to account for the across equation restrictions. As presented here the Ito process assumes no serial correlation. To conserve degrees of freedom g(w) and σ(w) can enter the $\dot{K}^*$, x^* and Y^* equations as fixed by estimating a discretized version of (1) first and transforming it to be stationary.

Discrete Case

Consider the situation of a highly regulated market where the producer assumes the output price will either remain unchanged or increase a discrete known amount τ. If the nominal wage is assumed to be fixed and known, the real wage is assumed to evolve according to

$$(5) \quad dW = b(W)dt + \theta(W)d\beta$$

where

$$Pr(d\beta = \tau \mid \beta(t) = w\} = \mu(w)dt + o(dt)$$

$$Pr(d\beta = 0 \mid \beta(t) = w\} = 1 - \mu(w)dt + o(dt).$$

Using the intertemporal objective in (3) subject to (1) and (5) with $K(t) = k$ and $W(t) = w$, we can show the stochastic dynamic programming equation is

$$(6) \ rJ(w, c, k) = \max_{I} \pi(w, k, I) - ck + (I - \delta k)J_k + b(w)J_w + [J(w + b(w)dt + \theta(w)\tau, c, k) - J(w, c, k)]\mu(w)$$

which is a partial differential - first-order difference equation. Differentiating the optimized dynamic programming equation leads to variable and dynamic factor demands and output supply

$$\dot{K}^* = J_{kc}^{-1}[(r + \mu(w))J_c + k - b(w)J_{wc} - J_c(w + b(w)dt + \theta(w)\tau, \bullet)]$$

$$x^* = (b_w(w) - \mu(w) - r)J_w + \dot{K}^* J_{kw} + gb(w)J_{ww} + J_w(w + b(w)dt + \theta(w)\tau, \bullet)\mu(w)$$

$$Y^* = (r - \mu(w))J + wx^* + ck - \dot{K}^* J_k + b(w)J_w + J(w + b(w)dt + \theta(w)\tau, \bullet)\mu(w)$$

In the discrete case a second order specification is sufficient to capture the impact of the stochastic evolution of price. As in the continuous stochastic drift case, one may resort to estimating the parameters of (5) initially and then inserting them as fixed coefficients in the estimation of $\dot{K}^*$, x^* and Y^* in order to preserve degrees of freedom.

Mixed Continuous - Discrete Case

The amplitude of the discrete jump, θ, may randomly evolve over time according to

$$(7) \quad d\theta = h(\theta)dt + \alpha(\theta)dB_\theta$$

where $E\{dB_\theta\} = 0$ and $E\{(dB_\theta)^2\} = dt$. This case presents a mixed Brownian motion - jump process. Using the intertemporal objective in (3) subject to (1), (5) and (7) with $K(t) = k$, $W(t) = w$ and $\theta(t) = \tau$, the stochastic dynamic programming equation is

$$rJ = \max_I \pi(w, k, I) - ck + \dot{K}J_k + b(w)J_w + h(\theta)J_\theta + .5\alpha(\theta)^2 J_{\theta\theta} + [J(w + b(w)dt + \theta(w)\tau, c, k) - J(w, c, k)]\mu(w)$$

which is a second-order partial differential - first-order difference equation. The optimized dynamic programming equation can be differentiated to generate variable and dynamic factor demand and output supply equations as above.

Conclusions

The dual approach to dynamic modeling has tremendous potential in applied work. Using the dynamic programming equation as the starting point, concepts in static production theory such as returns to scale, productivity growth and substitution effects can be formally defined for the firm not at a steady-state position. The difficulty in econometric estimation of dynamic dual equations depends on the naivety of the behavioral assumptions concerning expectations one is willing to accept. While these assumptions may often be condemned as unrealistic, the challenge is to move further into relaxing models of dynamic optimizing behavior and not to retrench to the relatively better known domain of static behavioral modeling.

References

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