A DYNAMIC MODEL OF CAPITAL STRUCTURE FOR THE NONCORPORATE FIRM

by

Larry S. Karp
University of California at Berkeley

Robert A. Collins
University of Arkansas

#5335

A DYNAMIC MODEL OF CAPITAL STRUCTURE FOR THE NONCORPORATE FIRM

Abstract

A dynamic model of capital structure for the noncorporate farm is developed and analyzed. The model examines the effect on optimal capital structure of (1) bankruptcy risk, (2) the difference between the riskless rate and the expected return in agriculture, and (3) the difference between the off-farm wage and the implicit on-farm wage. If the difference between the wages is 0, a constant leverage is optimal under reasonable circumstances. The model predicts that older farmers require more leverage to induce them to remain in farming and that they tend to reduce their leverage as retirement approaches. The model is tested with cross sectional data.

A DYNAMIC MODEL OF CAPITAL STRUCTURE FOR THE NONCORPORATE FIRM

Introduction

Current financial difficulties in commercial agriculture highlight the importance of the capital structure of agricultural firms. This paper develops a theory to explain the noncorporate firm's choice of capital structure at a point in time and the evolution of capital structure over the lifetime of a proprietor.

The Modigliani-Miller theorem (Modigliani and Miller) provides conditions under which the value of a publicly traded firm is independent of its capital structure. Hellwig reexamines the theorem and concludes, "From a practical point of view, it seems reasonable to suppose that the Modigliani-Miller principle fails when there is a chance of bankruptcy" (p. 167). Myers reviews the various explanations that have been advanced to explain the capital structure of publicly traded firms. In answer to the question, "How do firms choose their capital structures?" he replies, "We don't know" (p. 575).

The theory of the capital structure of public firms is, at best, unresolved; the theory of the capital structure of noncorporate firms is practically unarticulated. The incompleteness of markets for noncorporate equity means that one cannot appeal to market arbitrage forces, which form the basis for models in the Modigliani-Miller tradition. The market value of proprietary equity is the liquidating value of the firm (the market value of assets minus liabilities). The value of proprietary equity is determined in the asset markets and does not necessarily reflect the management of the individual firm. Rather than attempting to maximize the market value of equity, the proprietor is likely to concentrate on the stream of income that can be withdrawn from the firm or on the expected liquidation value of the firm at retirement. The choice of capital structure affects these goals.

We assume that the farmer's only source of external financing is debt; given a level of equity, the farmer decides whether to remain in farming and, if so, how much debt to acquire. Since debt is, by an overwhelming margin, the most significant source of external finance for U. S. farms, the assumption is reasonable. Innes models the debt/external equity option for firms as an agency problem; his model provides conditions under which an all-debt contract results in equilibrium.

By assumption, bankruptcy occurs when debt equals assets. Bulow and Shoven demonstrate that there are a variety of circumstances where this assumption does not provide the optimal foreclosure rule. Their model describes the publicly held firm, but similar arguments hold for the proprietary firm. For the purposes of our model, it is important only that foreclosures occur under prescribed conditions; for simplicity, we take this to be where assets net of debt are less than or equal to zero.

Our chief concern is with the dynamics of capital structure. In order to keep the model tractable, we use a very simple description of the stochastics and assume that the farmer is risk neutral. This permits identification of the effects on capital structure of the following features: the underlying riskiness of the enterprise, constraints on the reinvestment of income, the level of equity, the time until retirement, and the opportunity cost of managing the enterprise.

The next section assumes that the opportunity cost of managing the enterprise is 0. This results in a very simple problem for which a closed-form solution can be obtained. The optimal leverage is independent of equity;

under plausible circumstances, the optimal leverage is a constant that balances risks and returns.

In the subsequent section the opportunity cost of managing the firm is assumed to be positive; we obtain an approximate solution for this problem. Young farmers who go bankrupt have greater possibilities of starting a second career than do old farmers, so the opportunity cost of managing the firm decreases over time. The optimal leverage depends on the level of equity and the time until retirement. This model provides two explanations for why young farmers would be expected to be more highly levered than old farmers: They tend to have less equity, and their opportunities outside farming are greater. In neither model is it optimal to plan to retire free of debt.

The model abstracts from small variations in income in order to concentrate on a single catastrophic event, bankruptcy. The probability that bankruptcy occurs over a unit of time depends on the leverage. For example, a firm with a debt/asset ratio of .8 will not survive a 25 percent loss in assets; the same loss with a debt/asset ratio of .7 is tolerable. If the firm does not go bankrupt, it earns a nonstochastic rate of return on assets. We model the evolution of the equity in continuous time using a jump process. This model is consistent with Bulow and Shoven's description of financial crises. They compare such a crisis with an "earthquake": ". . . The important assumption is that the expected future productivity of the firm's plant decreased discontinuously" (note 12, p. 442). This corresponds in our model to a discontinuous decrease in the value of assets due, for example, to a fall in land prices.

The following two sections elaborate the versions of the model with and without opportunity cost of managing the firm. Proofs are contained in a longer version of this paper available upon request. The subsequent section provides an empirical test of the hypotheses implied by the theory. A conclusion follows.

A Simple Model with Bankruptcy Risk

The farmer's equity at a point in time is $E(t)$ and his debt/asset ratio is δ. The probability of bankruptcy occurring over an interval of time dt is proportional to the increasing, convex function $\gamma(\delta)$.

The rate of return on equity net of borrowing costs is the concave function $I(\delta)$ that first increases and then decreases. If the rate of return on assets exceeds the cost of borrowing, an increase in financial leverage increases the rate of return on equity since more assets work for each dollar of equity; this is the leverage multiplier effect. The expected net rate of return on assets declines, however, because of the higher borrowing costs associated with the firm's increased probability of bankruptcy. As long as the multiplier effect exceeds the increased borrowing costs, $I(\ )$ increases; at some value of δ less than 1, $I(\ )$ reaches a maximum and thereafter decreases.

The proportion of income withdrawn from the firm is w. With these definitions, the stochastic differential for equity is

$$dE = (1 - w)\ I[\delta(t)]\ E(t)\ dt + Ed\pi \tag{1}$$

where

$$Pr(d\pi = -1) = \gamma[\delta(t)]\ dt + o(dt)$$

and

$$Pr(d\pi = 0) = 1 - \gamma[\delta(t)]\ dt + o(dt)$$

where $o(dt)$ denotes terms of order dt. The deterministic portion of dE is $(1 - w)\ IEdt$, which is the retained portion of earnings over an interval dt;

when $d\pi = 0$, equity increases at the expected rate; $d\pi = -1$ means that the firm is bankrupt. Equation (1) is a jump process.

The manager is assumed to maximize the expectation of the present value of withdrawals plus the liquidation value at retirement. At time t, given equity E(t), his problem is

$$(2) \qquad J(E, t) = \max_{\substack{\delta\varepsilon[0,1] \\ w\varepsilon[\underline{w},\infty]}} \mathcal{E}_t \left[\int_t^T e^{-\rho s} wIEds + e^{-\rho T} E(T) \right]$$

subject to (1). The value function J( ) gives the farmer's expectation of the present value of the firm. Provided that this is greater than the liquidation value, $e^{-\rho t}E$, the farmer wishes to remain in business.

The optimization problem in (2) can be reinterpreted as one of maximizing the expected value of a retirement portfolio consisting of equity in the firm and riskless bonds. With this interpretation, a withdrawal of wIEdt implies an investment of the same amount at the riskless rate, ρ.

The withdrawal rate, w, is unbounded above, so that at any point the farmer can liquidate the firm and receive equity E(t). The lower bound on w is $\underline{w}$. In many cases it is reasonable to assume that $\underline{w} = 0$, i.e., the farmer is capable of retaining all earnings of equity but has no source of outside funds other than debt. A positive value of $\underline{w}$ is appropriate if the proprietor is required to consume more as his equity increases, or if he is committed to a particular balance in the retirement fund consisting of riskless bonds and equity in the firm. A negative value of $\underline{w}$ implies that the proprietor is able to obtain funds at the riskless rate (e.g., by drawing on his retirement fund). The additional equity he can obtain in this manner is proportional to his earnings, I( )E. We assume that $\underline{w}$ is given, although in a more general model it might be regarded as a control variable.

We summarize the important assumptions implicit in (1) and (2):

(i) The farmer is risk neutral.

(ii) There are two states of the world: either bankruptcy occurs or the farmer earns a nonstochastic return.

(iii) Bankruptcy occurs whenever $E(t) \leq 0$.

(iv) The functions I( ) and γ( ) do not depend on time.

For the remainder of this section, we also assume

(v) The value of the program when there is no equity is 0: $J(0, t) = 0$.

Assumption (v) is not innocuous. It implies that the opportunity cost of the farmer's labor equals the (implicit) wage that he receives as a proprietor. More typically, the wage under alternative employment may be greater than the implicit wage. The effect of relaxing assumption (v) is considered in the next section. The assumption is useful because it clarifies the effect of the functions I( ) and γ( ) and the parameter $\underline{w}$ on the choice of δ.

We define the quantity $\hat{\delta}$ as the leverage that maximizes $I(\delta) - \gamma(\delta)$. At $\hat{\delta}$, the marginal risk and the marginal expected return of increasing the leverage are equal. The quantity $I(\hat{\delta}) - \gamma(\hat{\delta})$ gives the maximized risk-adjusted expected rate of return to equity. The optimal leverage equals $\hat{\delta}$ at retirement, provided that the firm is still in operation. The optimal leverage and withdrawal policy is described in the following proposition.

PROPOSITION 1.

(i) If $\rho = I(\hat{\delta}) - \gamma(\hat{\delta})$, it is optimal to set $\delta = \hat{\delta}$; withdrawal policy is irrelevant.

(ii) If $\rho > I(\hat{\delta}) - \gamma(\hat{\delta})$, it is optimal to liquidate immediately.

(iii) If $\rho < I(\hat{\delta}) - \gamma(\hat{\delta})$, it is optimal to set $w = \underline{w}$. (a) For $\underline{w} > 0$, the optimal δ increases over time; (b) for $\underline{w} = 0$, it is optimal to maintain $\delta = \hat{\delta}$; and (c) for $\underline{w} < 0$, the optimal δ decreases over time.

This proposition has a very intuitive interpretation. If the discount rate, ρ, equals the risk adjusted rate, $I - \gamma$, where the latter is maximized, then the farmer is indifferent between liquidation and staying in business. If asset markets were perfect and management ability homogenous, competitive pressure would cause the liquidation value of the firm to equal its value as a going concern. If the discount rate is greater than the maximized value of the risk-adjusted expected return, the farmer does better by immediate liquidation.

In the case where the discount rate is less than the maximized risk-adjusted expected rate, the value of the firm as a going concern exceeds the liquidation value. The optimal leverage depends on the lower bound of the proportion of withdrawals. If $\underline{w} = 0$, the farmer chooses the leverage that maximizes the risk adjusted expected return on equity and holds it constant.

Bankruptcy Risk with Positive Opportunity Cost

This section replaces assumption (v) with

(v') $J(0, t) = e^{-\rho t} \frac{c}{\rho} (1 - e^{-\rho \tau})$, which uses the definition $\tau = T - t$, the time to retirement. This assumption can be interpreted as the statement that either the wage in the alternative employment exceeds the implicit wage by the amount c or that, in the alternative employment, the farmer accumulates a pension fund at the rate c. Replacing the constant c by a function $c(\tau)$ would complicate the solution without adding insight. We also make the following assumptions:

(vi) $I(\widehat{\delta}) > \rho + \gamma(\widehat{\delta}) \equiv \alpha$.

(vii) $\underline{w} = 0$.

We obtain an approximate solution to this problem which is valid for small c. This solution implies the following two results:

PROPOSITION 2. Older farmers require higher levels of equity to induce them to remain in farming.

PROPOSITION 3. (i) Farmers with high equity are less highly levered than farmers wth low equity. (ii) Given the same level of equity, old farmers are less highly levered than middle-aged farmers; the latter may be more highly levered than very young farmers. (iii) An individual farmer tends to decrease his leverage over time, conditional on not going bankrupt.

Empirical Test of Model

The principle implications of the model are stated in Propositions 2 and 3. Survey data from Arkansas farmers in 1986 (Collins) was used to test the model. The survey consisted of a stratified random sample of 2,500 farms selected from the nine crop and livestock reporting districts in Arkansas; the survey resulted in 989 usable survey forms. The average annual earnings of farm laborers in each county of Arkansas (Census of Agriculture) was used as a proxy for the individual farmer's opportuniy cost (the constant c in the previous section).

Since the data did not include time series, it was not possible to determine the extent to which the probability of future financial difficulty depends on current capital structure. That is, the function $\gamma(\delta)$ could not be estimated.

The model assumes constant returns to scale. Previous empirical tests of this hypothesis have been ambiguous. Using OLS, we regressed the rate of return on equity, defined as net cash flows divided by equity, against δ (debt/assets), δ^2, assets, and age. The results strongly support the hypotheses that the return to equity is increasing in leverage and is independent of scale. There is weak evidence that $I(\delta)$ is concave. The F statistic for the null hypothesis that all coefficients are insignificant exceeds 30 so that hypothesis is strongly rejected.

The data suggest that younger farmers have a higher expected rate of return on equity than older farmers; this may be due to different levels of education. This does not contradict the model which allows the function $I(\delta)$ to vary across individuals; it does, however, suggest an additional reason why older farmers may be less highly levered.

Proposition 2 states that older farmers require more equity to keep them from retiring than younger farmers. The minimum equity level, $\bar{E}$, is directly observable for those who choose to quit farming voluntarily. A sample of farmers who quit farming voluntarily was created by taking farmers who quit because of financial problems, better alternative occupation, or other non-health related reasons and who had positive equity. Farmers who indicated they were quitting because of health problems or retirement or those that had nonpositive equity were eliminated from the sample. The results are shown in table 1.

Table 1. Dependent Variable: Equity

Independent variable	$\hat{\beta}$	Standard error	t
Intercept	-176403	--	--
Age	7307.6	3910.69	1.87

$R^2 = 0.0907$

ADJ $R^2 = 0.0647$

Equity $= \beta_0 + \beta_1$ Age $+ \varepsilon$

$N = 36$

The coefficient on age has the expected sign and is significant at the 3.5 percent level for a one-tailed test. This provides a moderate level of support for Proposition 2.

Proposition 3 states that older farmers and farmers with higher levels of equity tend to be less highly levered. In addition, a higher opportunity cost implies higher optimal leverage. The observed leverage was regressed against age, equity, and opportunity cost. All coeffcients were highly significant and had the expected sign. Since equity is an explanatory variable and also appears in the denominator of the independent variable, there is the potential for spurious correlation. We, therefore, interpreted Proposition 3 in terms of debt rather than debt/assets. The proposition implies that the elasticity of debt with respect to equity is less than 1, that the derivative of debt with respect to age is negative, and that the derivative with respect to opportunity cost is positive. Debt was regressed against age, equity, and opportunity cost; we used Tobit since debt is constrained to be nonnegative. The results, shown in table 2, are consistent with the theory. The derivatives of debt with respect to age and opportunity cost have the expected sign and are significant. The elasticity of debt with respect to equity was calculated using Thraen et al. The elasticity at the sample mean was .0526 and was less than .2 at all data points.

Table 2. Dependent Variable: Debt

Independent variable	Estimated coefficient	T-ratio
Constant	12.5256	2.570
Opportunity cost ($1,000)	3.12345	5.263
Age (years)	-.384612	-4.427
Equity ($10,000)	.048492	10.974

Conclusion

This paper has provided a model to explain the evolution of the capital structure of a noncorporate firm over the lifetime of the proprietor. Even if the expected rate of return and probability of failure are stationary, optimal capital structure is likely to change as retirement approaches. The cost of failure is likely to be greater for older farmers, because their opportunities for alternative employment are less attractive; to the extent that they have more equity, they also have more to loose.

The theory is consistent with cross sectional data of Arkansas farmers. A more comprehensive test will require time series data.

References

Bulow, Jeremy I. and Shoven, John B., "The Bankruptcy Decision," The Bell Journal of Economics, Vol. 9, 1978.

Collins, Robert, "The Financial Condition of Arkansas Farm Survey Respondents by Farm Type and Location: 1986," Arkansas Agriculture Experiment Station Bulletin, No. 910, 1987.

Census of Agriculture, "Geographic Area Series Part 4," Arkansas State and County Data, United States Department of Commerce, Bureau of Consensus, Vol. 1, 1982.

Innes, R., "Agency Cost, Farm Debt and Foreclosure: Positive and Policy Issues," unpublished Working Paper, Department of Agricultural Economics, University of California, Davis, 1987.

Modigliani, F. and Miller, M., "The Cost of Capital, Corporation Finance, and the Theory of Investment," American Economic Review, Vol. 48, 1958.

Myers, Stewart C., "The Capital Structure Puzzle," The Journal of Finance, Vol. 39, 1984.

Thraen, C. S.; Hammond, J. W.; and Buxton, B. M., "Estimating Components of Demand Elasticities from Cross-Sectional Data," American Journal of Agricultural Economics, 1978.