A Contingent Claim Pricing Model for Valuing Non-Recourse Loan Programs and Target Prices *

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ABSTRACT

Options pricing theory is used to value the 1985 Farm Bill program provisions using loan rates and target prices as option exercise prices. Premiums on these options are high, indicating substantial value to farmers. This method of valuation should prove useful in evaluating whether or not farmers should participate in farm programs.

A Contingent Claim Pricing Model for Valuing Non-Recourse Loan Programs and Target Prices

Commodity programs provide economic value by protecting participating farmers against adverse price movements. In the absence of farm programs farmers would be willing to pay a premium for such protection in much the same way that hedgers use put options on futures contracts. This paper applies contingent claim - option pricing theory to the 1987 crop of corn, soybeans and wheat to determine the value to farmers of participating in government farm programs. By treating target price and non-recourse loan programs as contingent claims the value of the program to farmers is reflected in a single quantitative measure. This measure, referred to as the implied premium, is useable by policy makers, farmers and extension agents in determining the value of program benefits and in evaluating participation decisions.

Unlike options on futures contracts, farmers do not have to pay cash to participate in the farm program. However, there are acreage restrictions and cross-compliance measures which are economic opportunity costs. The value of the program is the value of the implied premium less the opportunity costs of set-aside acreage and cross compliance. If the opportunity costs exceed the implied premium then a participating farmer would, in essence, be paying too much for the program. Ray et al have shown that for some farmers revenues foregone through acreage set-aside and cross compliance are sometimes greater than program benefits.

The objectives of this paper are to describe a method for determining implied premiums on government programs and to illustrate the use of these premiums in evaluating the participation decision. Contingent claim pricing and option based theory has previously been applied to farm programs (Marcus and Modest; Witt and Reid; Irwin). This paper is like Witt and Reid in that it examines the participation choice for a single farmer holding the participation of all other farmers constant. In contrast, Irwin, and Marcus and Modest, investigated a different issue since they considered equilibrium effects of all farmers, participating or not participating. The current application extends previous analyses by incorporating the cost of post-harvest storage with the Commodity Credit Corporation (CCC). Furthermore, the premiums associated with particular provisions in the 1985 Farm Bill are incorporated into the empirical model.

A Contingent Claim Pricing Model for Agricultural Policies

Contingent claims are written by the government with strike prices equal to the basic loan rate, the minimum loan rate attainable under the Secretary of Agriculture's discretionary authority, and the target price. For each of these strike prices a claim is written with either a harvest or post-harvest expiration date. The claims are analogous to European options and therefore cannot be exercised early[1]. At the time of expiration any of three outcomes are possible. These terminal boundary conditions are: 1) if $P_\tau > P_T > P_L$ the farmer will sell the crop in the cash market and receive no deficiency payment; 2) if $P_T > P_\tau > P_L$ the farmer sells crop in the cash market and receives a deficiency payment, $P_T - P_\tau$; and 3) if $P_T > P_L > P_\tau$ the farmer release his crop to the commodity credit corporation for P_L and receives a deficiency payment, $P_T - P_L$. P_τ is the cash market price at expiration, P_T

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1 This is a simplification. In reality stocks held by the CCC can be sold at any time. Thus the storage decision should be modeled as an American type option. However, previous research on securities (Barone-Adesi and Whaley) and Futures (Hauser and Neff) have shown little, if any, difference between the premiums paid on American and European puts.

is the target price, and P_L is the loan rate. The intrinsic value of this contract at expiration time, τ, is zero if the harvest price is greater than the target price and is positive otherwise. The maximum value of this claim is,

(1) $G(P_\tau, P_T, P_L, \tau) = e^{-r\tau}[\max(0, P_T - P_\tau)]$, if target prices and deficiency payments are part of the program provisions (e.g. corn and wheat), or

(2) $G(P_\tau, P_L, \tau) = e^{-r\tau}[\max(0, P_L - P_\tau)]$, if program provisions do not include target prices or deficiency payments (e.g. soybeans). The term, r, is the riskless interest rate.

The base model used is Merton's put option pricing model for continuous dividend yields. The model has been generalized in Barone-Adesi and Whaley, Asay, and Wolf to incorporate the cost of carrying inventories. In this formulation the cost-of-carry, b, is defined as the difference between the riskless interest rate, r, and the storage costs as a percentage of the loan rate or target price , g, charged by the CCC (i.e. $b = r - g$)(Barone-Adesi and Whaley). Post-harvest storage costs are annualized over the the pre and post-harvest time horizon in order to provide a cost-of-carry measure which is a constant over time and proportional to the loan rate or target price. When carrying costs are assumed to be zero, as is the case for the pre-harvest time horizon, the model reduces to Black's option pricing model. Four contingent claim formulas (put options) are described by;

(3) $G_{i,t} = e^{-r\tau}P_i\, N(d1_{i,t}) - e^{(b-r)\tau}P_o N(d2_{i,t})$,

where $i=L$ for the loan rate or $i=T$ for the target price, and $\tau=t_1$ for a harvest date expiration or $\tau=t_2$ for post-storage expiration ; P_o is the cash market price of the physical commodity at time of program sign up; N( ) is the cumulative standard normal density function;

$$d1_{i,\tau} = -[\ \ln(P_o/P_i) + (b + \sigma^2/2)\tau\ ]/\sigma(\tau^{.5})\ ;$$

$$d2_{i,\tau} = -[\ \ln(P_o/P_i) + (b - \sigma^2/2)\tau\ ]/\sigma(\tau^{.5})\ ;$$ and

σ is the standard deviation of the rate of change of cash prices.

Equation (3) can be used to determine the incremental premium on the deficiency payment over the loan rate (Witt and Reid). This measure is important if the loan rate acts as a price floor. Then the contingent claim is only an option on the amount of the deficiency payment, since a farmer could receive the benefit of the loan rate without participating or incurring any opportunity costs. The implied premium on the deficiency payment is:

(4) $D1 = G_{T,t_1} - G_{L,t_1}$, for harvest expiration put, and

(5) $D2 = G_{T,t_2} - G_{L,t_2}$, for post-harvest expiration put.

The implied premiums described by (3), (4) and (5) are estimated in this paper. Data for Cash prices, interest rates, expiration dates, target prices and loan rates were collected (Glaser; Jones and Martin; Wall Street Journal, April 15, 1987). These data are reported in Table 2. The cash price at the time of program sign-up was adjusted for seasonality by adding the spread between the futures contract nearest the harvest month and the May futures contract (Wall Street Journal, April 15, 1987). The only unobserved variable is the variance of the rate of change in spot prices, σ^2. This variance must be estimated and it is this estimation which makes the use of the above formulas difficult. The following section outlines the procedures used in estimating the implied premiums and volatilities.

Estimating the Standard Deviation of Expected Price Movements

The implied volatility of an option written on a futures contract is a proxy for the volatility observed in cash prices. Rather than using time series data of cash prices to calculate the variance term, the volatility of price movements implied by option premia was used. The implied volatility is the variance which equates the theoretical Black (or Black-Scholes) model to observed option premia. This assumes that all participants value options according to the assumptions of the Black model and obey the dominance restrictions outlined by Merton. The volatility implied by the option pricing model is attractive because it theoretically includes investor's probability assessments of future prices based on past, present and expected future outcomes (Schmalensee and Trippi). It will also reflect the impact of government programs on market prices.

When more than one option is written on a futures contract the implied volatilities of all options should be taken into account. Chiras and Manaster suggest that each option written on the futures contract should be weighted by the elasticity of the option with respect to its standard deviation. A computer program was used to estimate the implied volatilities using the Newton-Raphson iterative procedure and a polynomial approximation to the normal cumulative density function. These estimates were then used to derive the weights and implied premiums. Data used in estimation are presented in Tables 1 and 2 [1].

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1 Other measures of implied volatility can be found in Beckers, Trippi, Schmalensee and Trippi, and Latane' and Rendleman. Each of these measures were also computed but with very little difference found between them. The reported implied volatilities using the method of Chiras and Manaster is representative of these values.

RESULTS

Column 7 in Table 1 presents the implied standard deviations estimated from the Black model for each option. The weighted implied volatilities were .2242 for corn, .1496 for soybeans and .1658 for wheat. These values are used as data for the contingent claim models.

Estimated implied premiums are presented in Table 3. For a harvest expiration date the premium for an option on the basic loan rate is $.54/bu., $.23/bu., and $.27/bu. for corn, soybeans and wheat respectively. If the Secretary of Agriculture lowers loan rates to their minimum values, these values decrease to $.16/bu., $.12/bu. and $.006/bu. . The low implied premium for wheat reflects, and is consistent with, an option which is out-of-the-money.

Both corn and wheat programs have provisions which include a target price. Options on the target price are currently in-the-money by a substantial amount. For a harvest date expiration, the implied premiums are respectively $1.25/bu. for corn and $1.76/bu. for wheat. The value to farmers of the wheat program is greater than the corn program because the intrinsic value (target price less cash price) is greater ($1.78/bu. vs. $1.31/bu.).

Including storage costs in the options formula decreases the implied premiums for all commodities. For example storing soybeans at the basic loan rate for 100 days and charging a storage cost of $.35/bu./year decreases the implied premium by $.0676/bu. ($.2296/bu - $.1620/bu. (Table 3)) . For the minimum loan rate the maximum decrease was about $.0306/bu. for soybeans. The maximum decrease in implied premiums for the storage target price option was $.1146/bu. for wheat. Dominance restrictions applied to conventional Black or Black-Scholes option formula, without storage costs, imply that as the time to maturity increases the value of the option also increases since some of the uncertainty about future price movements is diminished (Merton; Wolf). However, when storage costs are included in the model, the value of the option may decrease if carrying costs off-set the benefits from an extended time horizon. These results imply that the degree by which farmers have to pay storage costs affects the value of the program. Policies which require full payment of storage costs may induce farmers to release crops immediately to the CCC at harvest rather than incurring the costs of storage. Alternatively, policies which eliminate storage costs or require only partial payment of storage may encourage storage under the non-recourse loan provisions.

The value of the deficiency payment is given by the difference between the target price and loan rate premiums (Table 4). The implied premium on the deficiency payment for the basic loan rate with harvest expiration date is about $.71/bu. for corn and $1.48/bu. for wheat. These values reflect the value of program benefits if the government completely stored supplies below the loan rate so that the loan rate becomes an effective price floor. If the Secretary of Agriculture uses his discretionary authority and decreases the loan rate to the allowable minimum the value of the deficiency payment increases substantially to about $1.09/bu. for corn and $1.75/bu. for wheat. Including post-harvest storage decreases these values somewhat.

The implied premiums reflect the amount that farmers would be willing to pay for the price protection that the farm program offers. Gardner (1977,1981) suggests that the government is offering the same service that options markets offer and that these programs provide disincentives to using options and futures markets to hedge. The results neither corroborate or refute this claim. However, the results do indicate that the value to farmers of the program provisions may be greater than the price protection that the options market offers. From Table 1 the market premium of a put option on corn futures with a $1.90 strike price is $.15/bu.. This contrasts with the implied premium of the farm program ,$.16/bu., using the loan rate of $1.82/bu. and the April 14, 1987 adjusted cash price. Similar conclusions can be reached by comparing the premiums in Tables 1 and 3 for soybeans and wheat.

The implied premiums can be used by farmers to determine whether or not to participate in farm programs. In most cases participation requires acreage set-aside and cross compliance with other program crops. These restrictions on production incur opportunity costs to participating farmers. An important policy and management issue to be resolved is to determine the amount of gross revenue that farmers could give up before program restriction costs exceed program benefits. Implied premiums can be used to determine the break-even point. The proposed measure adjusts the loan rate (target price) by the cost-benefit ratio of the implied premium to the loan rate (target price). For example, corn farmers would pay $1.2478/bu. to receive a target price of $3.03/bu.. The cost-benefit ratio of .412 (1.2478/3.03) implies that gross revenues cannot be reduced by more than 41.2% of gross revenues attainable without acreage restrictions. In other words, a farmer would be indifferent to receiving a market price of $1.78/bu. ($3.03/bu.* (1-.412)) on 100 acres of unrestricted corn acreage or $3.03/bu. on 58.8 (100*(1-.412)) acres of restricted corn acreage. If acreage reduction and cross-compliance reduced the average price received below $1.78/bu. the opportunity cost of the program would exceed the implied value of the program. Thus participation would not be profitable. The cost-benefit ratios and break-even prices are respectively 40% and $2.63/bu. for the wheat target price and 4.8% and $4.54/bu. for the soybean loan rate. Although it is unlikely that average production revenues will be reduced enough to provide disincentives for corn and wheat producers to participate in farm programs, soybean farmers may be giving up more than the value of the premium if set-aside restrictions of more than 4.8% were imposed on soybean acreage.

From a policy perspective the implied premiums can also be used to determine the maximum cost of participation under alternative policy regimes. For example, if target prices and deficiency payments were eliminated on corn and wheat there would have to be substantial changes in acreage restrictions in order to induce farmers to participate. The cost benefit ratio for corn would be .236 (.5384/2.28) and for wheat .114 (.2737/2.40). Under the 1985 Farm Bill acreage reduction is set at 12.5%-20.0% for corn and 20%-27.5% for wheat. Thus, it is unlikely that wheat farmers would participate in the program if deficiency payments were eliminated without a corresponding adjustment in acreage restrictions.

CONCLUSIONS

This paper reported a method by which contingent claim analysis can be used to provide a single quantitative dollar value of farm programs to farmers. The method incorporates loan rates, target prices, deficiency payments and storage costs. Furthermore, it was shown how premiums implied by these claims could be used to determine the set-aside requirements at which program participation becomes break-even for farmers. The method, and results have several uses in farm management and policy analysis. For example, in the past, extension programs and policy analysis have had to rely on "what if" or sensitivity analyses to evaluate government programs and participation. The method proposed in this study allows program valuation and participation to be evaluated with a single quantitative measure.

Table 1: Option Data Requirements and Estimated Implied Volatilities for Black Model (as of April, 14, 1987)

Contract	Strike Price ($/bu.)	Current Future Price ($/bu.)	Annual Interest rate (%)	CCC Storage Costs ($/bu)	Days To Expir. (days)	Current Put Premium ($/bu)	Estimated Implied Volatility (std. dev.)
CORN	1.60	1.855	9.35	.34	250	.0375	.2278
(Dec.)	1.70	1.855	9.35	.34	250	.06325	.2229
	1.80	1.855	9.35	.34	250	.1025	.2241
	1.90	1.855	9.35	.34	250	.15	.2202
	2.00	1.855	9.35	.34	250	.21	.2198
SOYBEANS	4.75	5.17	9.35	.35	160	.05	.1482
(Sept.)	5.00	5.17	9.35	.35	160	.125	.1521
WHEAT	2.60	2.655	9.35	.34	100	.06375	.1646
(July)	2.70	2.655	9.35	.34	100	.115	.1678

Source: Wall Street Journal, April 15, 1987.

Table 2: Data Used in Estimating Implied Premiums on Program Participation

Input Variable	Corn	Soybeans	Wheat
Basic Loan Rate ($/bu.)	2.28	5.02	2.40
Minimum Loan Rate ($/bu.)	1.82	4.77	2.28
Target Price ($/bu.)	3.03	n.a[a]	4.38
Spot Price ($/bu.)	1.575	4.97	2.745
Days Until Harvest	185	185	130
Storage Days	100	100	100

[a]There is no target price or deficiency payment for soybeans
Source: Jones and Martin; Glaser.

Table 3: Implied Premiums ($/bu.) on Program Participation by Commodity

Commodity	Weighted Implied Standard Deviation	Harvest Loan Premium	Harvest Minimum Loan Premium	Harvest Target Premium	Storage Loan Premium	Storage Minimum Loan Premium	Storage Target Premium
Corn	.2242	.5384	.1629	1.2478	.4806	.1588	1.1421
Soybeans	.1496	.2296	.1172	n.a.[a]	.1620	.0866	n.a.[a]
Wheat	.1658	.2737	.0055	1.7565	.2583	.0183	1.6419

[a]not applicable, there is no target price for soybeans

Table 4: Implied Premiums ($/bu.) on Deficiency Payments for Corn and Wheat

Commodity	Weighted Implied Standard Deviation	Harvest[a] Date, Basic Loan	Harvest[b] Date, Minimum Loan	Storage[c] Date, Basic Loan	Storage[d] Date, Minimum Loan
Corn	.2241	.7094	1.0849	.6616	.9833
Wheat	.1658	1.4828	1.7510	1.3838	1.6236

[a] harvest target premium minus harvest basic loan premium

[b] harvest target premium minus harvest minimum loan premium

[c] storage target premium minus storage basic loan premium

[d] storage target premium minus storage minimum loan premium

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