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### A Simultaneous Econometric Model of World Vegetable Trade: Implications for Market Development

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Vegetables -- Marketing

International trade in fresh vegetables has become increasingly important to both developed and developing nations. United Nations data indicate that trade in fresh vegetables, SITC 054, increased about four fold between the years 1962-1982. The total quantities of imports and exports of fresh vegetables among the major participating regions of Latin America, the United States, Canada, the European Economic Community, the Middle East, the Far East, Africa, and the non-E.E.C. Western European nations in 1963 were 3.69 million and 3.59 million metric tons, respectively. By 1982 those totals had increased to 14.82 and 13.33 million metric tons. These data clearly demonstrate tremendous growth in the last two decades. Trading partners have changed, as well as the degree of regional competition. potential for market growth and, hence, changing market shares, is of paramount importance to those regions attempting to expand their export markets.

There are two major blocs of trade in fresh vegetables, within and among the Americas, and within and among Eurasia. The majority of trade in fresh vegetables occurs within these two blocs, however, there are economic forces at work changing these patterns. The fastest growing import markets, besides W. Europe, are the Far East, Africa, and the Middle East. While they are still relatively small markets, they do represent windows of opportunity for vegetable exporters. These markets are also becoming more accessible due to technological changes and improvements in infrastructure. However, trade between the

American regions and the E.E.C. is seriously hampered by the Common Agricultural Policy.

This article describes a study in which world trade in fresh vegetables from 1962 to 1982 was examined. Vegetable trade depends directly on the importing region's demand for the particular product; data indicate that supply levels have little impact on the quantities or patterns of vegetable trade. Demand may differ according to the source of supply. An empirical measurement of these demands is essential to evaluate pricing policies and to form long run trade projections. For the purposes of empirical implementation the SITC code of 054, fresh vegetables, was used. This is a high level of aggregation but it is justified because the purpose of the study was to examine world vegetable trade, its levels and patterns of demand and supply, not the characteristics of particular subgroups.

#### Methodology

A world trade model based on one outlined by Armington which distinguishes between products by place of production was constructed (Armington, 1969). This is a two stage allocation model where total demand for a good such as vegetables is first determined; this market demand is then independently allocated among competing sources of supply, or products. A constrained estimation procedure is used to guarantee that estimated world exports equals world imports. Weak separability and homotheticity are both assumed by this theoretical approach.

The structure of the international trade environment was

built into the trade model. Demand and export supply equations were specified for only the major trading regions listed earlier. Tariff barriers and preferential treatments were incorporated into the model, as were exchange rates, transportation and handling costs, regional incomes and demographic patterns.

An important issue in a model which distinguishes products by place of production is the degree of substitutability which exists between products. With eight differentiated products, all competing in eight different markets, there would be an inordinate number of parameters to be estimated if no limits were placed on their substitutability. To alleviate that problem, the CRES technical relationship was imposed on the trade system. This constrains the elasticities of substitution between any pair of competing products in a market to vary by a constant proportion, but the substitutability between every product in that market is not necessarily identical. The CRES technical relationship determines the functional nature of the product demands from competing supply regions. 1

#### Specific Functional Representation

The CRES technical relationship imposed on the market demands for vegetables for each region is represented as: $^2$ 

(1)  $X(i.) = (\sum_{j=1}^{\Sigma\beta} (ij) X(ij)^{\alpha\{ij\}})^{\{1/\alpha\{i.\}\}}$ 

Given (1), it can be shown that the product demand functions are

<sup>1</sup> The derivation is not included here, but is available from the author.

<sup>&</sup>lt;sup>2</sup> The i and j subscripts are used to represent regions of demand and supply, respectively.

represented as:

where

$$a(ij) = 1 / (1 - \alpha (ij))$$

and

 $\sigma$  (ij) = elasticity of substitution

 $\sigma(ij) = a(ii) a(ij) / \sum S(ij) a(ij)$ 

 $S(ij) = P(ij) X(ij) / \sum P(ij) X(ij)$ 

S(ij) = value share of the jth product in the ith vegetable market.

The CRES property is clear in that all  $\sigma(ij)$  vary proportionately with the common factor of proportion being  $1/\frac{\Sigma}{2}S(ij)$  a(ij). The market demand, export supply and CIF product price

relationships were each estimated in double log form.

Prices are the crucial linking and determining mechanism of the model, serving to allocate products so that supplies equal demand on both the product and good level. The average FOB export price of the product, F(.j), is determined by the interaction of region j's export supply of the product and world demand less domestic demand for X(.j). It is represented as:

(3) 
$$F(.j) = (\sum_{i} F(ij) X(ij) / X(.j)) = F(jj)$$
 where

F(jj) = the domestic price of product j. A product produced and consumed domestically does not incur costs associated with shipping and barriers to entry. It is therefore assumed to be equal to the average of all FOB export prices for that producing region.

F(ij) = the FOB export price of vegetables produced in region j and demanded in i.

(4)  $C(ij) = {}^{\Phi}(0ij) F(ij)$  Year where

- $^{\Phi}(\text{Oij})$  captures the costs associated with the distance between regions i and j, as well as the costs associated with non-tariff barriers and the market organization of the importing countries.
- $^{\Phi}$ (lij) measures directly the strength of the relationship between the CIF import price and the FOB export price.
- $\Phi(2ij)$  allows the costs of insurance and freight to vary on a yearly basis.

The CIF import prices were adjusted for the costs of tariffs and preferential treatments with:

(5) P(ij) = ((1 + T(ij)) C(ij))where

P(ij) = the market price in region i for region j's product

T(ij) = the costs of tariffs and preferential treatments

between importing region i and exporting region

j, expressed in percentage terms.

The average price paid for vegetables in region i was defined as:

(6) 
$$P(i.) = (\sum_{j} P(ij) X(ij) / X(i.))$$

In order to assure that the system is in equilibrium, demand and supply restrictions were placed on the model. Equation (7) restricts market demand in region i be equal to the sum of all its product demands while (8) restricts the total amount supplied by region j to be equal to the sum of all the regional demands of product j.

(7) 
$$X(i.) = \sum_{j} X(ij)$$

(8) 
$$X(.j) = \sum_{i} X(ij)$$

Exchange rates were taken into account by conversion of all price and income data to one currency, the U.S. dollar.

The system for trade in fresh vegetables as specified in this study is nonlinear. For estimation purposes it was advantageous to convert the functional relationships to a linear form. The identities were left in their nonlinear forms. The log linear functional relationships are specified below.

Market Demands

(9) 
$$\log X(i.) = \log \delta(i0) + \delta(i1) \log P(i.) + \delta(i2) \log GDP(i) + \delta (i3) \log Pop(i)$$

Product Demands

(10) 
$$\log X(ij) = \log^{\phi}(0ij) + {\phi(1ij)} (\log P(ij) - \log P(i.))$$
  
+  ${\phi(2ij)} \log X(i.)$ 

where

$$\phi \text{ (0ij)} = ((\alpha \text{ (i.)} / \alpha \text{ (ij)} \beta \text{ (ij)})^{\{1/\{\alpha\{ij\}-1\}\}}$$

$$\phi \text{ (1ij)} = (1 / (\alpha \text{ (ij)} -1))$$

$$\phi \text{ (2ij)} = (\alpha \text{ (i.)} -1 / \alpha \text{ (ij)} -1)$$

Export Supply

(11) 
$$\log (X(.j) - X(jj)) = \log^{\rho}(0j) + {\rho(1j)} \log F(.j) + {\rho(2j)} \log X(.j)$$

CIF Import Price

(12) 
$$\log C(ij) = \log^{\Phi}(0ij) + {\Phi}(1ij) \log F(ij) + {\Phi}(2ij) \log$$
  
Year

The model was estimated with nonlinear two stage least squares using principal components as instruments. The parameters, while biased, are consistent. A simultaneous estimation technique was used as it is likely, when estimating demand and supply equations in international trade research, that the use of OLS would yield biased and inconsistent results (Goldstein and Khan).

Statistics regarding the fit and performance of the model indicate that, in general, the model does a good job in capturing the economic forces involved in international fresh vegetable trade. The Durbin Watson statistics are usually close to 2, indicating little serial correlation and a well specified model.

Due to the highly restrictive, nonlinear nature of the model, R square statistics are not valid for statistical tests. For this reason, root mean square percent errors and Theil inequality coefficients were also used to evaluate the performance of the fresh vegetable trade model. They indicate that, in general, the

model predicts the observed data points well and captures turning points in the data.

Implications of Empirical Results for World Vegetable Trade, by

#### Region

The EEC is the largest importer and a major exporter of fresh vegetables. Estimated product demand parameters indicate that market shares are determined by price in the EEC. This is evidence that the CAP has a major impact on trading patterns, reducing the ability of the U.S., Latin America, or Canada to be price competitive in this region. The EEC is not participating in the growth of the Far Eastern, Middle Eastern, or African import markets and its export markets are small, not rapidly growing, or both.

The growth of U.S. imports and exports of fresh vegetables from 1962-1982 was below the world average. CCIF product price parameters indicate that transportation costs are a major factor hindering U.S. trade, particularly with the growing markets of the Middle East, Far East, and Africa. Technological advances will, in the long run, reduce the costs of transporting fresh vegetables, allowing the U.S. to be more price competitive in these growing markets, and allowing their products to be more competitive in the U.S. market. (Empirical results indicate that inegotiations reducing the CAP levies on fresh vegetable imports would facilitate increased penetration of the EEC by U.S.

Latin America's pattern of exports of fresh vegetables is

Changing. It is breaking out of its established markets of the U.S. and Canada and building strength in other import markets such as the EEC, Middle East, and Africa. As these markets grow or Latin America increases the competitiveness of its product on the world market, product demand parameters indicate that Latin American exports of fresh vegetables should increase. On the import side, some change is also occurring in Latin America's pattern of participation, but it is not a growth market.

Canada is a growth region both in terms of its exports and imports, although this is primarily occurring within previously established patterns of trade. The regions receiving virtually all of its exports are the U.S., Latin America, the EEC, and the non-EEC W. European region. However, Canada did increase its exports to the Middle East, the Far East, and Africa, thus participating in the growth of these regions. Virtually all of Canada's imports are from the U.S. and Latin America.

While still a small supplier, Middle Eastern exports more than doubled from 1962 to 1982. Its exports to the other relatively small markets of Latin America, Africa, and the Far East increased; it did not make inroads into the established markets of the U.S., Canada, the EEC, or the non-EEC W. European region. Middle Eastern imports of fresh vegetables increased by more than three fold; it is a growth region of some importance for exporters.

The Far East increased the level of its exports more than 70 times and has become a major supplier of fresh vegetables. Its

largest customer was the EEC. If the trends observed from 1962 to 1982 continue, Far Eastern exports of fresh vegetables to the EEC will continue to grow, as will its share of that market.

The Far East is the fastest growing market for fresh vegetables. Its market demand shows inelastic responses to changes in the average price of vegetables in the Far East, income, or population levels. The relationships point to a stable, long term expansion of the Far Eastern market. However, product demand functions indicate that the demands for the imports from the different regions are highly volatile with regards to changes in price or market size. Market size parameters indicate that as this market grows, most regions will be able to increase their exports to it.

Africa lost most of its market shares of its important export markets from 1962 to 1982. The parameters on its export supply equation indicate vegetable production increases are being absorbed by the domestic market. Clearly Africa is becoming a net importer even with the growth in domestic production.

Africa was the second fastest growing import market from 1962 to 1982. Empirical results indicate that all regions can expand their exports to Africa as this market grows. African price and market size responses are such that the Middle East and the EEC will probably be able to increase their market shares easier than will the other regions. The Far East and Latin America also have advantages in this market, more so than do the U.S. and Canada.

The primary market for the mon-EEC W. European region's exports is the EEC. Also, the EEC is its primary supplier, although it does receive some imports from Latin America, Canada, and the U.S. There is nothing in the empirical evidence to findicate a change in these patterns.

#### Summary and Conclusions

This paper has discussed a study of international trade in fresh vegetables. The study contributes to agricultural economics literature in several ways. First, the construction of the model is such that vegetables produced in different regions are not considered perfect substitutes. Typically in international trade research, products of the same type of good produced in different regions or nations are considered homogeneous and therefore perfect substitutes. This implies that the elasticities of substitution between the products are infinite. The use of an Armington-type model, with the constant ratio of elasticities of substitution property imposed, allows more realistic substitution characteristics between products to be reflected in the empirical results.

The second contribution this study makes is that the trade system is estimated simultaneously. Typically international trade models are estimated with single equation techniques. This approach, where OLS is used to estimate the equations of a trade system, implicitly assumes that the export and import supply price elasticities facing any individual country or region are infinite. This may be reasonable in the case of the supply of

imports to a single country or region, but probably is not for the case of the supply of exports from an individual country or region. It is not likely that an increase in world demand for a country or region's exports can be satisfied without an increase in the price of its exports, unless idle capacity exists or production is subject to constant or increasing returns to scale. The quantity supplied will be a function of price. Because demand is also a function of price, simultaneity exists. The use of OLS will yield biased and inconsistent estimators (Goldstein and Khan). To the researcher's knowledge, this is only the second study to estimate a model based on Armington's theory of demand as distinguished by place of production in a simultaneous system (Winters). Thus, in this study Armington's original model is extended in that it is rendered more flexible by use of the CRES function and it is estimated in a theoretically consistent manner.

The third major contribution is its broad analysis of international fresh vegetable trade. The trade of this good grew tremendously from 1962 to 1982. The economic forces driving this trade have been virtually unstudied; there has been little economic analysis of fresh vegetable trade. This study, then, fills a gap in agricultural economics literature with respect to internationally traded commodities.

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### Market Demand Parameters and t-Statistics

		δ(io)	δ(il)	δ <u>(i2)</u>	δ(i3)
	Latin America	17.019 (9.68)	-0.455 (-1.58)	0.430 (2.15)	-0.234 (65)
i = 2	U.S.	15.144 (1.76)	-0.431 (-1.04)	0.642 (.85)	-0.528 (20)
i = 3	Canada	3.179 (1.15)	0.117 (.71)	-1.115 (-2.08)	5.552 (3.27)
i = 4	E.E.C.	14.205 (5.16)	-0.722 (-1.64)	0.153	0.272 (.86)
i = 5	Middle East	48.222 (.55)	-1.781 (66)	3.119 (.55)	-9.48 (41)
i = 6	Far East	14.506 (5.28)	0.204	-0.023 (21)	0.800 (1.68)
i = 7	Africa	12.475 (22.15)	0.011 (.43)	-0.033 (39)	1.009 (6.87)
i = 8	Non-E.E.C. W. Europe	11.417 (4.55)	-0.113 (-1.06)	0.045	1.027 (2.45)

Log X(i.) =  $\log \delta(io) + \delta(il) \log P(i.) + \delta(i2) \log GDP(i)$ 

+ δ(i3) log POP(i.)

Degrees of freedom for each of these equations is 17.

### **Product Demand Parameters and t-Statistics**

Regions									
		Latin America	u.s.	Canada	E.E.C.	Middle East	Far East	Africa	Non-E.E.C. W. Europe
Latin America	A	( <b>.</b> )	(.)	1.810	0.481	0.067	-2.242 (-1.27)	-0.989 (65)	-1.173 (-1.37)
	8 <sup>b</sup>	(.)	(.)	1.029	2.577 (1.68)	7.127 (1.20)	0 .848 (.38)	1.431 (.75)	2.691 (4.28)
U.S.	<b>A</b>	-0.620 (-1.98)	(:)	(.)	-0.086 (55)	-1.654 (-2.06)	-1.285 (-2.90)	-0.481 (81)	-2.158 (-1.44)
	В	4.131 (8.01)	· · · ·	(·)	-0.098 (16)	9.571 (4.83)	3.271 (2.63)	2.279 (2.37)	-0.991 (62)
Canada	<b>A</b>	-0.537 (-3.69)	-0.523 (-5.58)	(.)	(.)	-0.761 (51)	-0.508 (56)	1.058	-0.573 (-1.10)
	B	0.416 (1.17)	1.162 (9.72)	· · · · · ·	<b>(.)</b>	7.746 (3.35)	2.079 (2.40)	-3.239 (-1.14)	-1.627 (-1.35)
E.E.C.	λ	-0.280 (36)	-4.079 (-2.28)	3.823 (1.83)	· · ·	(;)	-3.921 (-14.98)	-0.400 (49)	1.851 (2.53)
	В	-3.240 (-1.57)	0.393	.811	(.)	<i>(</i> .)	-0.838 (63)	-0.005 (01)	-0.140 (11)

### Product Demand Parameters and t-Statistics--continued

Regions		Partner Regions									
		Latin America	u.s.	Canada	E.E.C.	Hiddle East	Far East	Africa	Non-E.E.C. W. Europe		
Middle East	<b>A</b>	1.517	-1.841 (-1.84)	-0.849 (69)	-2.131 (-6.41)	· (.)	(.)	-2.476 (-1.90)	-2.156 (-3.38)		
	В	0.858	1.077 (2.75)	1.786 (3.84)	1.808 (8.95)	(:)	· (.)	0.860 (2.43)	0.739 (4.49)		
Far East	A	-0.798 (53)	0.955		-1.265 (-2.29)	-2.888 (-3.18)	(.)	(:)	-0.005 (1.24)		
	В	2.135 (.37)	4.716 (3.77)	11.952 (2.42)	3.285 (3.77)	-2.136 (58)	(·)	<b>(.)</b>	4.742 (1.29)		
Africa	λ	-1.264 (78)	-2.567 (-3.71)	-1.928 (-2.57)	-1.550 (-1.06)	0.472	-1.277 (-1.11)	(.)	(.)		
	В	15.830 (7.43)	3.683 (2.53)	4.961 (2.18)	2.391 (3.61)	7.234 (8.20)	3.142 (1.70)	(:)	(:)		
Non-E.E.C.	λ	( <b>.</b> )	-5.348 (-1.57)	3.706 (5.23)	2.704 (3.08)	-3.484 (-7.55)	-0.070 (16)	-4.773 (-4.12)	<b>(:)</b>		
	В	· (.)	1.529	0.353	-0.106 (16)	3.729 (1.68)	-7.623 (-1.61)	6.293 (1.31)	(.)		

A - Relative price parameter

 $\log X(ij) = \log \phi(0ij) + \phi(1ij) \log P(ij) - \phi(1ij) \log P(i.) + \phi(2ij) \log X(i.)$ 

Degrees of freedom for each of these equations is 18.

b<sub>B</sub> - Market size parameter

## CIF Product Price Parameters and t-Statistics

Regions					Partner R	legions			
		Latin America	u.s.	Canada	E.E.C.	Hiddle East	Par East	Africa	Non-E.E.C. W. Europe
Latin America	λ <sup>a</sup>	(.)	0.690	0.523 (4.07)	0.995 (8.42)	1.099 (2.07)	0.040	0.699 (3.51)	0.655
ABELICA	вр	(.)	13.625	117.200 (5.08)	-1.074 (05)	-159.760 (-1.25)	155.130 (6.24)	82.507 (2.64)	70.573
U.S.	<b>A</b>	-0.348 (-1.49)	(.)	0.842	0.921 (13.68)	1.156 (2.02)	2.885 (1.57)	0.597 (3.08)	0.754
•	В	166.790 (4.05)	· (.)	18.762 (.80)	13.416 (1.64)	82.401 (1.97)	-42.448 (38)	23.924 (.96)	12.988
Canada	<b>A</b>	-0.372 (-2.30)	0.781 (5.27)	<b>(:)</b>	0.921 (2.62)	0.381 (1.39)	0.874	1.028 (2.38)	1.432 (2.95)
•	В	196.890 (5.52)	60.883	<b>(.)</b>	-45.135 (-1.11)	98.922 (1.82)	52.920 (.47)	15.875	-180.140 (-1.63)
E.E.C.	<b>A</b>	1.164 (4.45)	0.558 (2.57)	0.993 (3.26)	(.)	-0.530 (85)	1.241 (9.28)	1.202	1.217
	В	-4.825 (12)	65.081 (2.20)	-18.750 (24)	(:)	183.340 (2.56)	-83.852 (-4.49)	12.933	-61.637 (89)
Hiddle Bast	<b>λ</b>	1.696 (1.59)	1.775 (1.15)	0.997 (2.72)	1.017 (3.59)	<b>(.)</b>	-0.363 (30)	1.047 (2.34)	0.958 (5.12)
<u> </u>	В	105.730	-50.389 (23)	3.795 (.06)	-13.740 (22)	(:)	216.620 (1.46)	47.512 (.81)	134.240 (3.49)

### CIF Product Price Parameters and t-Statistics -- continued

Regions		 	Partner	Regions					
		Latin America	<u>u.s.</u> ,	Canada	E.E.C.	Middle East	Par East	Africa	Non-E.E.C. W. Europe
Par East	<b>A</b>	0.567 (2.05)	0.752	0.572	· 1.318 (1.56)	-0.150 (33)	(.)	0.694 (3.58)	0.603 (2.98)
•	В	154.520 (2.16)	61.850 (3.79)	13.103 (.09)	-41.259 (29)	103.360 (1.95)	(:)	96.066 (2.12)	17.633 (1.01)
Africa	<b>A</b>	1.360 (1.59)	-0.016 (03)	0.145	0.737 (6.17)	0.748 (1.52)	0.916 (1.12)	(.)	1.519 (6.36)
	В	-21.480 (22)	97.282 (1.37)	45.411 (1.01)	25.249 (1.14)	-21.911 (24)	-10.692 (09)	(.)	-26.356 (69)
Non-E.E.C. W. Europe	<b>A</b>	0.237	1.280 (4.99)	0.564 (2.02)	0.990 (35.46)	0.622	1.139	0.006	<b>(.)</b>
	В	163.520 (6.38)	-18.640 (47)	54.142 (2.01)	-7.421 (-1.65)	121.890 (2.72)	72.529 (2.22)	66.791 (3.11)	(·)

A - FOB price parameter

 $\log C(ij) = \log \Phi(0ij) + \Phi(1ij) \log P(ij) + \Phi(2ij) \log Year$ 

Degrees of freedom for each of these equations is 18.

b<sub>B</sub> = Parameter on year

### Export Supply Parameters and t-Statistics

	<u>ρ(ΰj)</u>	<u>P(lj)</u>	<u>ρ(2j)</u>
i = 1	Latin -30.476	0.427	2.478
	America (-3.05)	(3.11)	(4.50)
i = 2.	31.663	1.504	-0.901
	U.S. (1.03)	(1.63)	(53)
i = 3	0.185	0.192	0.835
	Canada (.04)	(2.18)	(2.93)
i = 4	5.407	0.156	0.472
	E.E.C. (.62)	(2.11)	(.97)
i = 5	Middle 3.159	0.015	0.567
	East (1.15)	(.12)	(3.71)
i = 6	Far -590.36	-5.420	30.474
	East (-1.69)	(-1.18)	(1.74)
i = 7	52.710	0.243	-2.136
	Africa (5.20)	(1.52)	(-3.93)
i = 8	Non-E.E.C. 11.417 W. Europe (.43)	-0.123 (8.88)	0.045 (1.28)

log  $[X(.j) - X(jj)] = log \rho(0j) + \rho(lj) log F(.j)$ 

 $\log [X(.]) - X(]]) = \log \rho(0]) + \rho(1]) \log r$ +  $\rho(2j) \log X(ij)$ 

Degrees of freedom for each of these equations is 18.