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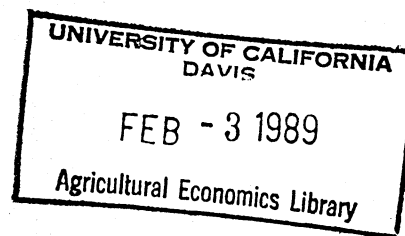
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APPROXIMATING A GSD-EFFICIENT SET OF
MIXTURES OF RISKY ALTERNATIVES FOR
RISK AVERSE DECISION MAKERS*

by

Francis McCamley**

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**The author is an associate professor of Agricultural Economics at the University of Missouri-Columbia.

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Risk-- Mathematical models

AN
ABSTRACT
OF
APPROXIMATING A GSD-EFFICIENT SET OF
MIXTURES OF RISKY ALTERNATIVES FOR
RISK AVERSE DECISION MAKERS

Conditions for a restricted version of GSD-efficiency of mixtures of risky alternatives are reviewed. These conditions and other characteristics of the (restricted) GSD-efficient set form the basis of a tentative procedure for approximating this efficient set. Hazell's data are used to illustrate a critical aspect of the procedure.

APPROXIMATING A GSD-EFFICIENT SET OF
MIXTURES OF RISKY ALTERNATIVES FOR
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Stochastic dominance criteria have largely replaced mean-variance and mean-absolute deviations criteria for ranking mutually exclusive alternatives. However, stochastic dominance criteria are not as commonly applied to problems involving mixtures of risky alternatives.

In those cases where stochastic dominance criteria have been applied to mixture problems, they have been applied to mixtures which have been randomly or systematically selected from the set of all feasible mixtures (Anderson). As Bawa et al. suggest, this approach can provide a reasonably good approximation to stochastic dominance efficient sets. Its major shortcoming is related to the fact that, typically, most of the feasible mixtures are not efficient. A sampling strategy which exploits the properties of stochastic dominance efficient sets would often be less costly.

McCarl et al. discuss conditions which can help determine whether there exists a mixture of two risky alternatives which dominates the "pure" strategy of specializing in one of the alternatives. It appears that their conditions could be extended to help guide the search for stochastic dominance-efficient sets.

An alternative approach is based on ideas presented by Dybvig and Ross. It has resulted in a method for identifying the second degree stochastic dominance (SSD) efficient set of mixtures of risky alternatives (McCamley and Kliebenstein, 1987). Conditions for Meyer's generalized stochastic dominance (GSD) efficiency of mixtures of risky alternatives have also been presented but only preliminary ideas about identifying GSD-efficient sets have been discussed (McCamley and Kliebenstein, 1986).

The purpose of this paper is to propose a method for approximating the efficient set of mixtures for a restricted version of the GSD criterion. To ensure that the reader understands the class of problems being considered and the conditions for GSD-efficiency, these matters are reviewed in the next two sections. Then two sections discuss characteristics of the GSD-efficient set. The final sections present a procedure for approximating the GSD-efficient set, illustrate one part of this procedure and offer some concluding remarks.

BASIC ASSUMPTIONS AND NOTATION

The class of problems considered here is similar to that associated with Tauer's version of the Target MOTAD model. The number of states of nature, s , is assumed to be finite. A row vector of probabilities, p , is associated with these states of nature. The elements of the column vector, y , are the (total) net returns associated with the various states of nature. This net returns vector is a linear homogeneous function of enterprise activity levels:

$$(1) \quad y - Cx = 0.$$

In (1), x is a column vector of n activity levels and C is a matrix of per unit net returns associated with the activities and the states of nature. Specifically, C_{ij} is the net return per unit of activity j when the i th state of nature occurs.

Activity levels are restricted by resource and/or technical constraints and nonnegativity constraints.

$$(2) \quad Ax \leq b$$

$$(3) \quad x \geq 0$$

In (2), A is a matrix of resource or technical requirements coefficients and b is a column of resource levels. To simplify the discussion in the balance of the paper, it is assumed that the set of feasible enterprise levels is bounded. An enterprise mixture (activity level vector), x^0 , will be regarded as being

GSD-efficient if, and only if, the associated net returns vector, y^0 , is GSD-efficient.

CONDITIONS FOR GSD-EFFICIENCY

McCamley and Kliebenstein (1986) derived efficiency conditions for a restricted version of Meyer's GSD criterion. The general version of the GSD criterion assumes that the absolute risk aversion coefficient, $r(m) = -u''(m)/u'(m)$, is bounded by two functions, $g(m)$ and $h(m)$, of the income or wealth level, m . McCamley and Kliebenstein assumed that $g(m)$ is nonnegative; it will be assumed to be positive in this paper. To be consistent with most applications of the GSD criterion and to simplify the notation, it will also be assumed here that g and h are constants.¹

Any given enterprise mixture, x^0 , and the associated income distribution vector, $y^0 = Cx^0$, are GSD-efficient only if there exist vectors z^0 and w^0 such that

$$(4) \quad z^{0'} y^0 \geq z^{0'} y$$

for all y vectors which satisfy (1), (2) and (3),

$$(5) \quad z_i^0 = p_i w_i^0$$

$$(6) \quad w_i^0 \exp[-g(y_j^0 - y_i^0)] \geq w_j^0 \geq w_i^0 \exp[-h(y_j^0 - y_i^0)]$$

if $y_j^0 \geq y_i^0$ and

$$(7) \quad w^0, z^0 > 0$$

When g is positive the foregoing conditions are also sufficient for GSD-efficiency.

By solving either of two linear programming problems, it is possible to determine whether conditions (4) through (7) are met. Inasmuch as both of them are related to discussion later in this paper, they are stated below. In each formulation, it is assumed that the states of nature have been permuted so that the elements of the y^0 vector are in ascending order.

The dual is

$$(8) \quad \text{minimize } b'v - \sum_{j=1}^s w_j p_j y_j^0$$

subject to

$$(9) \quad A'v - C'z \geq 0$$

$$(10) \quad z_j - p_j w_j = 0 \quad \text{for } j=1, 2, \dots, s$$

$$(11) \quad w_s = 1$$

$$(12) \quad w_j \exp[-g(y_{j+1}^0 - y_j^0)] - w_{j+1} \geq 0 \quad \text{for } j=1, 2, \dots, s-1$$

$$(13) \quad -w_j \exp[-h(y_{j+1}^0 - y_j^0)] + w_{j+1} \geq 0 \quad \text{for } j=1, 2, \dots, s-1$$

$$(14) \quad v \geq 0$$

and (7).

The primal is

$$(15) \quad \text{maximize } f$$

$$(16) \quad t_1 \exp[-g(y_2^0 - y_1^0)] - q_1 \exp[-h(y_2^0 - y_1^0)] - p_1 y_1 \leq -p_1 y_1^0$$

$$(17) \quad t_j \exp[-g(y_{j+1}^0 - y_j^0)] - q_j \exp[-h(y_{j+1}^0 - y_j^0)] - t_{j-1} + q_{j-1} - p_j y_j \leq -p_j y_j^0 \quad \text{for } j=2, 3, \dots, s-1$$

$$(18) \quad f - p_s y_s - t_{s-1} + q_{s-1} \leq -p_s y_s^0$$

$$(19) \quad y - Cx = 0$$

$$(20) \quad Ax \leq b$$

$$(21) \quad x, t, q \geq 0$$

The vectors x^0 and y^0 are GSD-efficient if and only if the optimal value of f is zero.

GENERAL NATURE OF THE (RESTRICTED) GSD-EFFICIENT SET

When g is positive, the (restricted) GSD-efficient set is a subset of the SSD-efficient set. It is possible to show that the SSD-efficient set of mixtures is connected and is the union of a finite number of closed convex subsets. A simplified version of Dybvig and Ross's proof of their Theorem 3 (pp. 1538-9)

can be used to show that the restricted GSD-efficient set is connected. It appears that it is also the union of a finite number of closed convex subsets.

The most appropriate way to define GSD-efficient subsets is not yet known. In this paper, each of these subsets is defined as the set of mixtures for which a specific basis is optimal for the the primal linear programming problem reviewed in the previous section. One implication of this approach is that each GSD-efficient subset is a subset of an SSD-efficient subset.

A SIMPLE EXAMPLE

Some characteristics of the GSD-efficient set can be illustrated by considering a simple example. Let

$$C = \begin{pmatrix} 60 & 80 \\ 100 & 60 \end{pmatrix},$$

$p = (.5 \ .5)'$, $A = (1 \ 1)$, $b = 1$, $g = .02$, $h = .08$, $x^0 = (.75 \ .25)'$ and $y^0 = (65 \ 90)'$. It is possible to show that the primal linear programming problem is solved by $f = 0$, $x = x^0$, $y = y^0$, and $t_1, t_2, q_1 = 0$. The solution to the dual linear programming problem is $v = 110$, $z^0 = (1 \ .5)'$ and $w^0 = (2 \ 1)'$.

Given the connectedness property of the GSD efficient set, one way of identifying it would be to "start" at the x^0 vector considered above and determine the range(s) in variation in x^0 for which the optimal value of f , f^* , remains equal to zero. For this simple example, it is possible to exploit the fact that f^* will be equal to zero for a set of x^0 vectors which is at least as large as the set of x^0 vectors for which the optimal basis (set of basis vectors) is the same as for the x^0 vector considered above.

As x^0 is varied, an alternative basis could become optimal when any of the following conditions is satisfied:²

1. x_1^0 or x_2^0 equals zero
2. the sum of x_1^0 and x_2^0 is less than 1.0
3. $y_1^0 = y_2^0$ (since the linear programming formulation assumes that $y_1^0 \leq y_2^0$)
4. the "reduced cost" for t_1 or q_1 becomes zero as reflected by the status of an inequality in (12) or (13), respectively, changing from a strict inequality to an equality.

In more general problems, an alternative basis could become optimal when:

5. the status of a resource constraint changes from strict inequality to equality
6. the reduced cost for a nonbasic x variable becomes zero as reflected by the status of the associated inequality in (9) changing from a strict inequality to an equality
7. a basic v variable becomes zero.

If condition 1, 2, 3 or 5 is satisfied while varying x^0 , it indicates that a boundary (or an additional boundary) of the SSD-efficient subset (as well as a boundary of the GSD-efficient subset) has been reached. Conditions 4, 6 and 7 are more relevant for the GSD criterion than for the SSD criterion. Of these, only condition 4 is unique to the GSD criterion; the others are shared with the third degree stochastic dominance criterion.

Condition 4 is easily illustrated with the problem described above. For that problem there are three SSD-efficient subsets. Two of them are the individual mixtures $(1 \ 0)'$ and $(1/3 \ 2/3)'$. The third subset consists of all (weakly) convex combinations of these mixtures and is the subset to which the x^0 vector presented above belongs.³ It is relatively easy to determine the GSD-efficient portion of this subset. Condition 4 is satisfied when $y_2^0 - y_1^0$, or equivalently, $40x_1 - 20x_2$ equals either 8.66 or 34.66. The intersection of the

third SSD-efficient subset of mixtures and the set for which $8.66 \leq 40x_1 - 20x_2 \leq 34.66$ is the set of all convex combinations of $(.48 \ .52)'$ and $(.91 \ .09)'$.

It was easy to identify the GSD-efficient set for this simple problem because there was only one GSD-efficient subset and the same w^0 vector was optimal for the whole subset. For most problems, there is more than one GSD-efficient subset and w^0 is a nonlinear function of x^0 . Moreover, it appears that the boundaries of some GSD-efficient subsets may be nonlinear and the subsets may "overlap".

In principle, it is possible to determine when condition 4, 6 or 7 is satisfied for a given basis. In practice, it may be easier to simply approximate the GSD-efficient portion of any given SSD-efficient subset by solving the primal (or dual) linear programming formulation for selected x^0 vectors.

A TENTATIVE PROCEDURE FOR APPROXIMATING THE GSD-EFFICIENT SET

One procedure for identifying the GSD-efficient set involves four steps:

1. Identify a mixture which maximizes expected utility for any utility function for which $g \leq r(m) \leq h$. Since g is greater than zero, the y^0 vector associated with this mixture is GSD-efficient and provides a starting point for identifying the GSD-efficient set.
2. Identify an SSD-efficient subset which includes this mixture.
3. Approximate the GSD-efficient portion of this SSD-efficient subset by solving the linear program discussed earlier in this paper for appropriately selected mixtures.
4. Identify another SSD-efficient subset which is adjacent ("connected") to the portion of GSD-efficient set which has thus far been identified. If none is found, stop; otherwise to go step 3.

The major difficulties in implementing this procedure are keeping track of the portion of the GSD-efficient set identified at any stage, determining which unexamined SSD-efficient subsets are adjacent to this set (step 4) and approximating the GSD-efficient portion of any given subset (step 3). Of these, the first two are essentially just messy "record keeping" problems. The next section of the paper illustrates one approach to step 3.

A MORE COMPLEX EXAMPLE

Data from Hazell are used. For purposes of the illustration, each state of nature is assumed to be equally likely. It is also assumed that $g = .000025$ and $h = .000065$. To simplify the discussion, only the approximation of the GSD-efficient portion of one SSD-efficient subset will be considered in detail.

Assume that the mixture, $x^0 = (40 \ 40 \ 60 \ 60)'$, is known to be GSD-efficient (i.e., maximizes expected utility for some utility function in the relevant class of functions) when $r(m)$ belongs to the risk aversion coefficient interval $(.000025, .000065)$. This x^0 vector belongs to a two-dimensional SSD-efficient subset which lies on the face of the set of feasible mixtures associated with the land and rotation constraints. The line segment ad is one edge of the face. The line segment which begins at a and extends upward along the vertical axis is a second edge. Another edge is a line segment which begins at d and passes through e and f . The fact that $x_1 + x_3 = 100$ and $x_2 + x_4 = 100$ on the face being considered means that it is only necessary to plot x_1 and x_2 coordinates in figure 1. The SSD-efficient set to which the initial x^0 vector belongs is $bcefg$.

There are several ways of approximating the GSD-efficient portion of $bcefg$. A grid approach is adopted here but it differs from that proposed by Bawa et al. in several ways. The least significant difference is that a very coarse grid is adopted first and then finer grids are used to further refine the

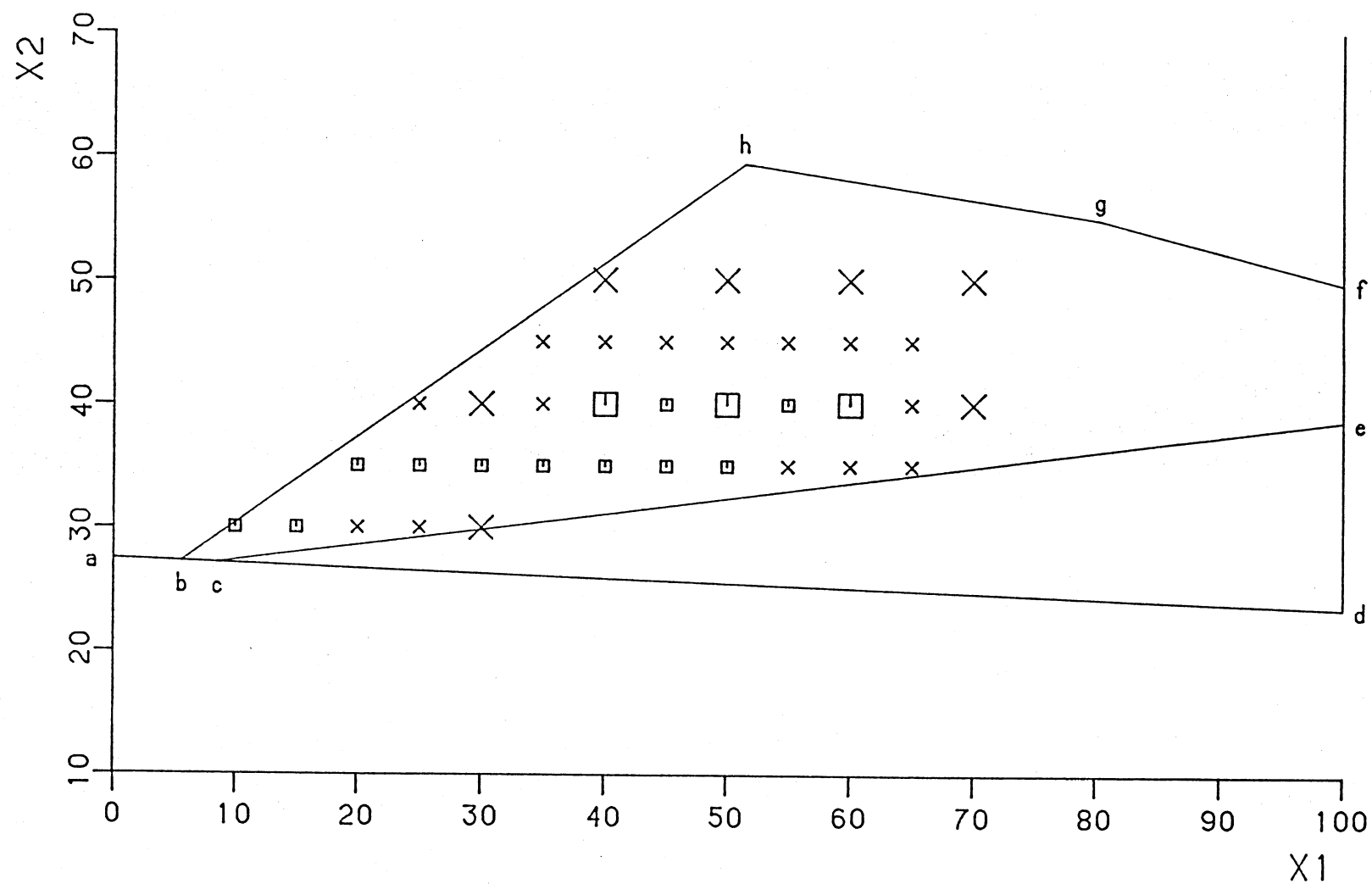


Figure 1. GSD-Efficient Mixtures on Coarse Grids in Subset bcefg

approximation. Of greater significance is that for any given grid, only those grid points (mixtures) which are "adjacent" to mixtures known to be GSD-efficient and in the same SSD-efficient subset are considered at any given time.⁴ A third difference is due to the stochastic dominance test procedure rather than the grid itself. The linear programming formulations which are used here effectively compare a given mixture (x^0 vector) with all other feasible mixtures. Once a mixture has been tested its efficiency status is known. Changing the grid size may refine the approximation of the efficient set but it will not provide any additional information about the efficiency status of a mixture which has already been tested. By contrast, when the pairwise tests assumed (at least implicitly) by Bawa et al. are employed, each grid point (mixture) can essentially only be classified as dominated or "not yet" dominated.⁵

The initial grid involves ten acre increments. Those mixtures which are both on grid points adjacent to the initial mixture and in the same SSD-efficient set have x_1, x_2 coordinates of (30,30) (30,40), (40,50), (50,50), and (50,40). Of these, only (50,40) is GSD-efficient. The untested grid point mixtures which are adjacent to (50,40) are (60,40) and (60,50). The mixture (60,40) is GSD-efficient but the mixture (60,50) is not. Likewise, the only untested grid points adjacent to (60,40) are (70,40) and (70,50); neither of these is GSD-efficient. The tested (as well as the initial) grid points are represented in figure 1 by large X or square symbols. The squares denote GSD-efficient mixtures; X's represent inefficient mixtures.

The approximation of the GSD-efficient portion of bcefg h can be improved as much as desired by employing successively finer grids. Consider next a grid with increments of five acres. The relevant mixtures on this new grid are

represented by smaller symbols. As before, the squares represent GSD-efficient mixtures while the x's represent inefficient mixtures.

Even with the relatively coarse five acre grid, a reasonably accurate "picture" of the GSD-efficient portion of bcefgH emerges. By reducing the grid increment to one acre, a better approximation is obtained. It is shown in figure 2. In this figure, only the grid points associated with GSD-efficient mixtures are represented by symbols.

The complete GSD-efficient set of mixtures almost certainly includes mixtures in other SSD-efficient subsets. The SSD-efficient subset which shares "boundary" bh with bcefgH is an obvious candidate. A less obvious but very relevant candidate is the three dimensional SSD-efficient subset of which bcefgH is one face.

CONCLUDING REMARKS

The partial discussion of the Hazell example suggests that it would not be too difficult to approximate the GSD-efficient set of mixtures for problems of that size. The Hazell example involves only four enterprises, three resource constraints and six states of nature. The number of states of nature for this example is not much smaller than the numbers used in some application of MOTAD and Target MOTAD. However, most practical problems have larger numbers of enterprises and constraints.

The cost of identifying the GSD-efficient set of mixtures depends, in part, upon the size of the risk aversion coefficient interval chosen. With a relatively short risk aversion coefficient interval, the restricted GSD-efficient set of mixtures may be considerably smaller than the SSD-efficient set and relatively easy to approximate.

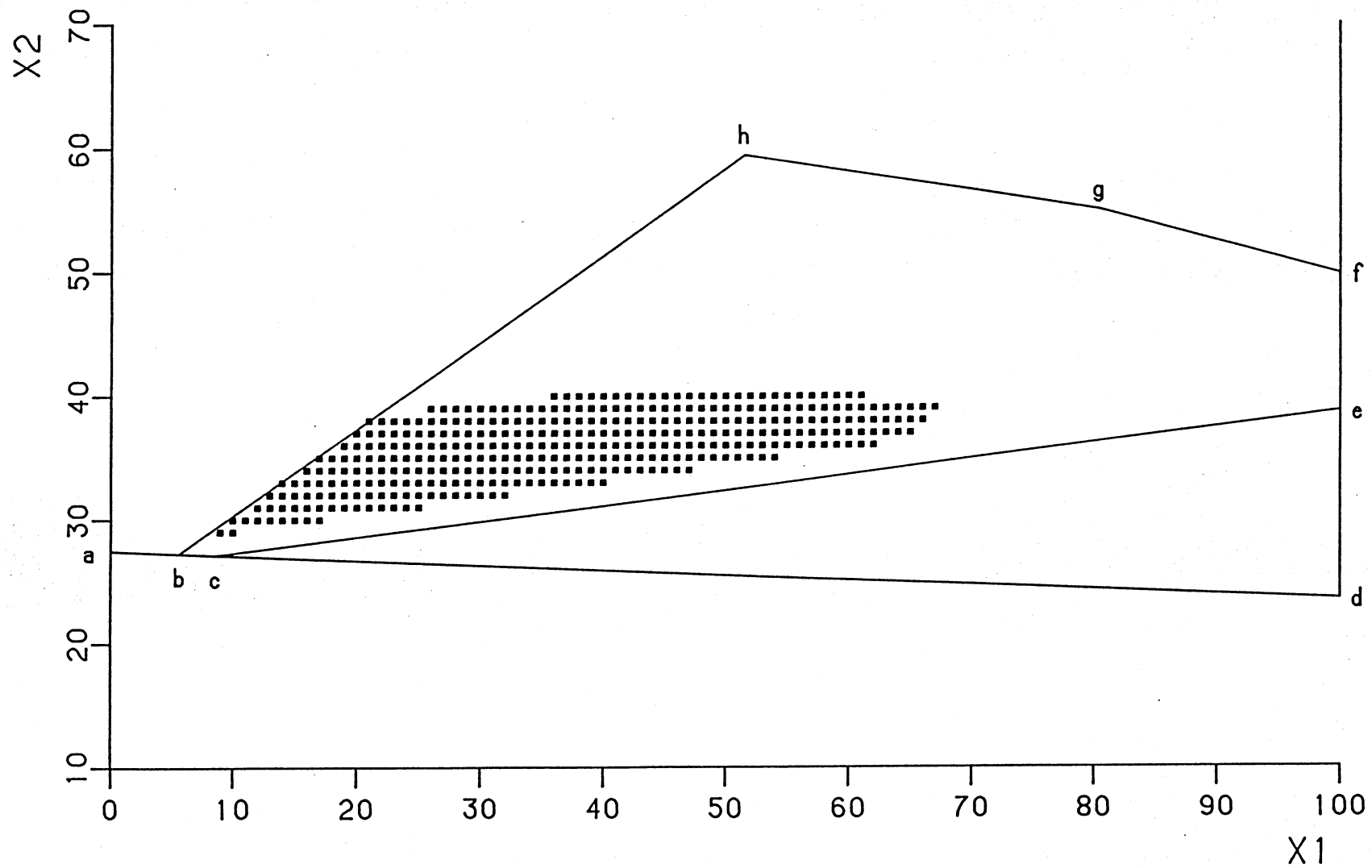


Figure 2. GSD-Efficient Portion of One SSD-Efficient Subset

FOOTNOTES

¹In the balance of the paper, expressions such as $g(y_j^0 - y_i^0)$ (or $h(y_j^0 - y_i^0)$) mean the product of g (or h) and $y_j^0 - y_i^0$.

²Note that when any of these conditions occur both the "old" and "new" bases are optimal.

³Since the first two subsets are subsets of the third one, it would be appropriate, for the purpose of describing the SSD-efficient set, to say that there is one SSD-efficient subset. However, for the purpose of approximating the efficient sets for the GSD criterion, complete enumeration of the collection of SSD-efficient subsets can be helpful.

⁴The connectedness property of the GSD-efficient motivates this strategy.

⁵Even with pairwise tests, the degree of uncertainty about a grid point's efficiency status decreases as the grid becomes finer. Another limitation of pairwise tests is that the efficient set may include alternatives (mixtures in this paper) which no decision maker in the relevant class would choose. That is, they maximize expected utility for no utility function in the relevant class. It can be shown that this limitation also becomes less serious as finer grids are employed.

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