An Economic Analysis of Sorghum as an Alternative Crop in the Mid-Atlantic Region

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Abstract

This study utilizes characteristics theory to answer the marketing question of how to price an alternative crop (sorghum). Portfolio analysis is also used to address both the marketing and production questions concerning potential supply, changes in relative crop distribution, and effectiveness as a risk management tool for producers.
An Economic Analysis of Sorghum as an Alternative Crop in the Mid-Atlantic Region

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INTRODUCTION

Farm production regions that depend substantially on the livestock industry need to continually evaluate their source of feed grains due to several important reasons. First, the cost of feed grains has a significant impact on the financial viability of the livestock sector. Second, the availability of feed supply is obviously critical to the efficient and productive performance of the livestock sector. In relation to the supply issue, weather problems such as drought conditions can alter the supply of the traditional feed grains (i.e. corn). Sorghum has been identified as a more drought resistant crop and is considered a suitable substitute for corn at various levels in feed mixtures.

Several studies have been done that evaluate the usefulness of sorghum as a substitute for corn in feed mixtures. (Grant and Krenz 1985, Jackson, Grant, and Shafer 1986) However none of these studies have focused on the Mid-Atlantic region, which is a poultry-intensive region. In addition, these studies have looked at regions where sorghum is already being produced in volume. Thus, sorghum production figures and prices are available for comparison with corn production and farm prices received within that area. It is, however, a problem to evaluate the appropriateness of using sorghum as a substitute crop if a particular region is not currently producing sorghum and historical farm prices are not available for comparison with other feed grain prices. Soil and climatic differences also exist that cause results of research from other regions to be not applicable to the Mid-Atlantic area.

In view of this dilemma, this study has two objectives. First, a methodology will be developed to provide a measure of regional sorghum price for non-sorghum producing areas. The unobserved price of sorghum will be estimated using its nutritive attributes and information available from other feed grain inputs. Two estimating techniques, characteristics regression and linear programming will be utilized for the first objective.
The second objective is to determine the economic feasibility of including sorghum in a set of alternative crops for the Mid-Atlantic region. Quadratic programming will be used in the second objective.

**METHODOLOGICAL FRAMEWORK**

**Linear Programming (LP):**

The technique for deriving the price of a certain product with specific qualities is based on the so-called "characteristics theory". The seminal work on characteristics theory was initiated by Waugh (1928, 1929) when he estimated the relationship between the wholesale price of fresh vegetables and the corresponding product characteristics. Following Waugh, the literature on characteristics theory remained mostly dormant until Cowling & Cubbin (1971) and Dhrymes (1971) published their work in the early seventies, where they explained the variation in automobiles prices as a function of their characteristics. Ladd and Martin (1976) used the characteristics theory in developing a linear programming model. They proposed that an input price can be described by its attributes and the utility derived from those attributes. Following Ladd and Martin's procedure and knowing the important physical characteristics of feed ingredients, the LP model used in this study takes the following form:

$$\text{Min } \sum_{i=1}^{m} p_i x_i ,$$

subject to $\sum_{j=1}^{n} a_{ij} x_i \geq a_{i0}$, and 

$$\sum_{i=1}^{m} x_i \leq 1.10 .$$

where $a_{i0}$ = some minimum level of the characteristics, 
$p_i$ = price of the $i$th input, 
$x_i$ = quantity of the $i$th input,
\[ a_{ij} \] = quantity of the jth characteristic in the ith input,
\[ n \] = the number of characteristics, and
\[ m \] = the number of input products.

The objective function represents a cost minimization problem subject to certain minimum levels of characteristic quantities and maximum allowable levels of other inputs. The constraints of this model are derived by maintaining that the sum of the characteristic levels (\( \sum x_{ia_{ij}} \)) must equal or exceed (\( \geq \)) the levels present in grain sorghum (\( a_{i0} \)). Two other constraints are that the levels of some inputs can not exceed certain levels dictated by the maximum allowable levels in a commercial broiler ration, and the sum of all ingredients can not total more than 1.10, which restricts the rations potential bulkiness.

**Characteristics Regression:**

The other procedure, characteristics regressions, will be used to derive hedonic or implicit prices for grain sorghum. The hedonic pricing model for sorghum was patterned after Ladd and Martin's procedure. Following the assumption that input characteristics are homogeneous across inputs and linearly related to product prices, the linear hedonic price model for grain sorghum takes the following matrix form:

\[
\Pi_i = X \beta + E_i
\]

where
- \( \Pi_i \) = i by 1 vector of prices of ingredients at time T,
- \( X = i \) by \( x \) matrix of the \( x \) characteristic levels in the i inputs,
- \( \beta = x \) by 1 vector of characteristic coefficient estimates, and
- \( E = i \) by 1 vector of error terms that meet the Gauss-Markov criteria for BLUE estimates.

Using a linear model, the beta (\( \beta \)) values can be interpreted as the marginal expenditure for a characteristic. The nutrient quantities for each input are assumed to be constant, consequently the hedonic price for sorghum can be specified as:

\[
Psorghum = \beta_0 + \beta_1 \text{nutrient}_1 + \ldots + \beta_i \text{nutrient}_i.
\]
The usual method of estimation would be ordinary least squares (OLS) regression on the pooled time series and cross sectional data, but data on composition (characteristics) of feed inputs are unavailable to match local prices. Instead, the quantities of the characteristics in the inputs are drawn from Nutrius Inc. Since the values are constant through time with relatively few exceptions, this restricts the methodology to a cross-sectional data analysis.

When dealing with several cross-sectional data, the problem of noncomparability arises because parameter estimates derived by regressing the raw price on the corresponding attributes would only be a valid indicator of the marginal expenditures given the particular time period the estimates were made. This problem was corrected by expressing the endogenous variable as a price ratio using corn price as the base. Another reason for using the price ratio was to make all prices relative to a common factor that is of a known value. Corn was chosen as the numeraire because it is universally accepted as a standard for feed comparability. The model utilizing the index was:

\[
P_1/P_{\text{corn}} = XB + E
\]

where

- \( P_1/P_{\text{corn}} \) = x by 1 vector of price ratios at time T,
- \( X \) = i by x matrix of the x characteristic levels in the i inputs,
- \( B \) = x by 1 vector of characteristic coefficient estimates, and
- \( E \) = i by 1 vector of error terms that meet the Gauss-Markov criteria for BLUE estimates.

The use of OLS (ordinary least squares) regression yields estimates of B that are best linear unbiased estimates (BLUE) only if the underlying assumptions hold true. To address the heteroscedasticity problem, two tests, the Glejser test and Goldfeld & Quandt test, were utilized and the residuals were plotted against any variables thought to be associated with the increase or decrease in error variance. In the event of heteroscedasticity, weighted least squares was utilized for deriving the coefficient estimates. The Durbin-Watson statistic calculated on the ordered residuals was used to test for positive and negative autocorrelation of the first degree. The nature of cross-sectional
data would lead one to assume that serial correlation (autocorrelation) should not be a problem. In many aspects of science most factors interact with others and there are rarely exogenous (X) variables that are truly orthogonal. Given this problem, variance inflation factors (VIF) were also used to assess the effects of multicollinearity among the regressors.

**Quadratic Programming:**

The first step in solving for a QP risk optimal solution is to estimate the variance-covariance (W) matrix of net returns which requires the calculation of net return data. Net returns, for the purpose of this study, do not include payments for land or returns to management and was estimated as gross revenue less variable cost. The variable costs represented costs for nitrogen, mixed fertilizer, lime, seed, chemicals, labor, custom operations, machinery cost, machinery repair, fuel & lubrication, interest, and drying. The different crop activities included in the analysis included full season corn, soybeans, sorghum, wheat, and barley as well as double crop wheat and soybeans, wheat and sorghum, barley and soybeans, and barley and sorghum. These crops were chosen because they have very similar resource requirements. The model can be completely specified by the Lagrangean risk minimization function, subject to the set of relevant income and resource constraints. The quadratic programming model was specified as:

\[
\begin{align*}
\text{Minimize } Z &= U - \mathbf{\xi}(x_6 + x_7 + x_8 + x_9) - \pi(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9) - \zeta(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9) \\
\text{subject to: } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 &= 1
\end{align*}
\]

where

- \( Z \) = the total variance of the crop mix,
- \( U \) = a variance-covariance matrix of net returns,
- \( \mathbf{\xi} \) = a lagrangean multiplier restricting the sum of the double crops to be less than or equal to 50% of the land resource,
- \( \pi \) = a lagrangean multiplier restricting the expected returns to be greater than or equal to some level m,
- \( \zeta \) = a lagrangean multiplier constraining the resources to sum 1,
- \( x \) = the portions of each crop, and
- \( r \) = the mean net return level.
The first derivatives with respect to the variables included in the model yield a set of linear constraints that forms the convex polyhedral which bounds the objective function. The solution to the QP model gives a value for the variance of the whole crop combination and the portion of each crop to be planted.

**DATA REQUIREMENTS**

There are two sets of data in this study. The first data set included prices for ingredients purchased to be used in a broiler ration, since broilers substantially contribute to the farm income in the Mid-Atlantic area. The inputs considered for this part of the analysis were barley, corn gluten, corn, crab meal, fat, fish meal, meat meal, soybeans hulls, soybean meal 48%, and wheat. The above cross-section of ingredients represent the largest portion of the inputs included in a broiler ration. The nutritional attributes that were considered for the initial analysis were percent fat, fiber, calcium, phosphorus, protein, arginine, lysine, methionine, and energy (calories per pound). The time period for which data were collected and analyzed was July 1983 to December 1985.

The second set of data, used in the risk analysis portion, was based on a study of yields conducted by the Delaware Cooperative Extension Service. The data generated included distinctions between yields for full season soybeans, double crop soybeans after barley, double crop soybeans after wheat, and corn. The sorghum yields used to match this set of actual yield data were generated using a combination of survey information from sorghum producers and the variety yield trials conducted by the Extension Service. The yield for full season grain sorghum was derived as a logarithmic function of corn. The price of sorghum was calculated as a direct ratio of the price of corn. Three price ratios were considered, i.e. 85, 90, and 95 percent of the price of corn. The figures for variable cost were obtained from the crop production budgets provided by the Extension Service.
LINEAR PROGRAMMING

The first method used to derive the implicit price for sorghum was linear programming. Similar to many least cost feed formulation programs, the linear programming model has a minimization objective function. The objective function can be represented by the minimization of price (Psorghum). In this case price is equal to the sum of the unit prices of other feed inputs multiplied by the quantity needed to equal or exceed the nutritive composition of sorghum. The final model took the following form:

\[
\text{Minimize} \quad Psorghum = p_1 x_1 + p_2 x_2 + p_3 x_3 + \ldots + p_i x_i ,
\]

where \( p_i \) = unit price of ingredient \( i \), and \( x_i \) = quantity of ingredient \( i \).

The set of constraints comprising the convex polyhedral boundary represented the minimum nutritive levels of the important attributes found in sorghum. This means that the proportions of those characteristics (attributes) described by the solution must be at least equal to those found in grain sorghum (milo). These constraints take the form shown below:

\[
\text{Energy}_1 x_1 + \text{Energy}_2 x_2 + \ldots + \text{Energy}_i x_i \geq \text{Energy sorghum},
\]
\[
\text{Protein}_1 x_1 + \text{Protein}_2 x_2 + \ldots + \text{Protein}_i x_i \geq \text{Protein sorghum},
\]

\( x_i \leq w_i \), and

\( x_1 + x_2 + \ldots + x_i \leq 1.10. \)

Where \( w_i \) is the maximum level of the \( i \)th ingredient allowed in a ration.

A second set of physical constraints were imposed on the model to place limits on the amount of certain inputs that can be included in the objective function. This set of
constraints was constructed using the same values as the weights used in the regression analysis. In the linear programming model, the weights were converted to form boundaries of the objective function. Many feed inputs, although still having beneficial characteristics, have detrimental aspects that may limit their effectiveness. These detrimental aspects include properties like palatability problems that can cause reduced feed consumption and problems with feed storage and handling in conventional facilities. Additionally, in poultry diets, as well as other intensified animal production diets, animal feed intake is a limiting factor in what and how much of certain inputs can be included in a ration.

Using the linear programming model the implicit sorghum price was derived for each set of cross-sectional data. The price ratios generated as a result of the linear programming model had a mean value of .9539 and a standard deviation of .0492. The linear programming ratios are shown in Figure 1.

**Characteristics Regression:**

The second method used was characteristics regression model using weighted least squares to estimate the coefficients for energy and protein of the given poultry feed inputs. Although several other attributes were proposed for the model, problems such as multicollinearity, lack of significance, and need for increased degrees of freedom limited the model to include only protein and energy as exogenous variables. The model in its final form is shown below:

\[
\text{Psorghum/Pcorn} = b_0 + b_1 \text{ protein in sorghum} + b_2 \text{ energy in sorghum}
\]

The average portion of the variation (R^2) in the price ratio explained by the exogenous variables was above .90 or 90 percent for the entire time span of the data set. The estimated value of the price ratio of sorghum was very stable with a mean of .9685 and
a standard deviation of .0106. The range of the predicted values was .9506 to .9891. A plot of the regression fitted price ratio values against time are shown in Figure 1.

**Figure 1:** Milo / Corn Price Ratio Derived by Linear Programming & Regression

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**Quadratic Programming (Risk Analysis):**

The quadratic programming model was utilized to derive risk efficient (EV) sets of crops for a representative county in the Mid-Atlantic Region. The risk efficient sets or EV frontiers represent the set of returns with minimum variance given different minimum desired levels of expected returns. This study evaluated two types of sensitivity analysis. First, the model analyzed the sensitivity of the results with or without sorghum in the optimum solution. The nature of the model dictates that the addition of sorghum or any other crop cannot raise the variance of the optimal solution but could lower it. Conversely, the removal of a crop from the possible crop mix cannot lower the total variance but could raise it. The magnitude by which the objective function value changes is greatly affected by the correlation among alternative crops. Ideally, if crops can be found that have high negative correlations and therefore have negative covariances, the objective function value will be greatly reduced. The second type of sensitivity analysis looked at the effect of different sorghum prices, relative to corn, on the presence of sorghum in the optimal solution. The price of sorghum relative to corn directly affects net returns. The three price
ratios used, 85%, 90%, and 95% of the price of corn, verified sorghum's presence in the model and how volatile its proportions are to relative changes in price.

The model also evaluated the prospective mix of crops that represented the least risk combination for certain set levels of returns. The return levels were set at $10.00 per acre and incremented by $10.00 per acre to the maximum attainable based on the mean returns per acre for the available crops. The changes in relative crop proportions that occurred as a result of adding sorghum are presented in Table 1. Following the addition of sorghum to the model, corn and soybeans acreages were shifted to sorghum in the quadratic programming optimal solutions. Raising the relative price of sorghum caused it to enter the model at upper income levels. The addition of sorghum at the 90 and 95 percent levels caused the maximum attainable income to rise from 60 dollars per acre to 80 and 90 dollars per acre, respectively. The presence of sorghum at 85 percent of the price of corn in the optimal solution caused a 20% reduction in the minimum coefficient of variation. At a return level of 60 dollars per acre, the 85 percent solution had a standard deviation in returns of 39.70 with a C.V. level of .66. Sorghum at 90 and 95 percent caused even further reductions in coefficient of variation to .61 and .57 at increased levels of mean income of 70 and 80 dollars per acre.

The shape of the expected value variance (EV) frontiers in Figure 2 illustrate the increasing presence of risk in returns for crop production in the Mid-Atlantic region. The relative rate at which the lines become parallel with the horizontal axis is an indication of how the risk per unit of returns is rising. The fact that the addition of sorghum causes an increase in the farmer's income potential, while at the same time reducing risk, allows farmers to bolster farm income with no sacrifice of income security.
Summary

The price ratio of grain sorghum, net of any value attributed to xanthophyll, was found to be between 95 and 97 percent of the price of corn for poultry production. The results from the two characteristics models were very similar, although the LP results were more volatile than those from the regression analysis. Both models derived the sorghum to corn price ratio as a linear function of protein and energy content as well as a set of relevant constraints.

The effect of including sorghum to the crop choices for the Mid-Atlantic Region was found to be beneficial to farmers. Sorghum came into many of the optimal (minimum risk) solution sets. Sorghum's effect and presence was quantified by using sensitivity analysis and was found to reduce the risk associated with crop production by as much as 20 percent.
Table 1: Risk Optimal Crop Proportions for Mid-Atlantic Representative Farm County

<table>
<thead>
<tr>
<th>Mid-Atlantic Representative County</th>
<th>Return Constraint</th>
<th>Mean Return</th>
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<tr>
<td>&quot;No Milo&quot;</td>
<td>10 20 30 40 50 60 70 80 90 100 110 120 130 140 150</td>
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</tr>
<tr>
<td>soybeans</td>
<td>.000 .103 .220 .337 .471 .590 .000</td>
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<tr>
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<td>69.75</td>
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<tr>
<td>wheat beans</td>
<td>.000 .000 .000 .000 .000 .000 .000</td>
<td>41.52</td>
</tr>
<tr>
<td>barley beans</td>
<td>.000 .000 .000 .000 .009 .037 .000</td>
<td>65.63</td>
</tr>
<tr>
<td>Return</td>
<td>11 20 30 40 50 60 70 80 90 100 110 120 130 140 150</td>
<td></td>
</tr>
<tr>
<td>With Milo</td>
<td>85%</td>
<td>10 20 30 40 50 60 70 80 90 100 110 120 130 140 150</td>
</tr>
<tr>
<td>soybeans</td>
<td>.000 .029 .096 .163 .235 .313</td>
<td>62.49</td>
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<tr>
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<tr>
<td>corn</td>
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REFERENCES


