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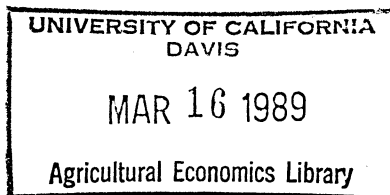
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THE DISTRIBUTION OF STANDARDIZED FUTURES PRICE CHANGES



by

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Selected Paper of the American Agricultural Economics Association Annual Meetings, July 31 - August 3, 1988, Knoxville, Tennessee.

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Future trading -- Mathematical models 1988

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Abstract

The observed non-normality of futures price changes has been attributed to non-constant variance. This paper tests whether the non-normality is due to changing variances or additional factors. The data are adjusted for heteroskedasticity and the stability-under-addition test of stable distributions performed on the original and the rescaled data sets. Rescaled data are less leptokurtic than the original data, but the rescaled data are still not normal. Thus, factors other than changing variance may also be responsible for the observed leptokurticity of daily futures returns.

THE DISTRIBUTION OF STANDARDIZED FUTURES PRICE CHANGES

I. Introduction:

Knowledge of the underlying distribution of futures price changes is required in several applications in the fields of finance and marketing, such as hypothesis testing, option pricing models and risk management studies. For instance, portfolio analysis is based on the assumption that the returns from the different assets under consideration are normally distributed with constant finite variance. However, if the variance is not constant and/or finite, this method may give misleading results. Research on the distribution of futures and stock price changes indicate that price changes are not distributed normally, but are leptokurtic i.e. characterized by a higher peak and thicker tails than the normal distribution (Fama (1963); Mandelbrot (1963); Officer; and Teichmoeller). The most common explanation for this occurrence is that futures and stock price distributions are mixture of normals distribution with changing variance. (Barnea and Downes; Doukas and Rahman; Hall et. al; and Richard and Sundaresan). According to the mixture of normals distribution, the observed distribution is a combination of several normal distributions with different variances. Support for the mixture of normals distribution also comes from studies that describe asset returns as a subordinated stochastic process and attribute the non-normality of futures and stock price changes to the non-normality of the random elements that constitute them (Clark; Epps and Epps; Mandelbrot and Taylor; and Upton and Shannon).

If the deviation from normality is due to changing variance, correcting for heteroskedasticity will result in a normal distribution and thus permit the use of statistical procedures that assume normality. Several models have been tried to correct for heteroskedasticity. For example, McCullough adjusted for heteroskedasticity in the monthly returns to financial assets using an adaptive conditional heteroskedastic (ACH) model and found that the disturbance terms were still non-normal. Bollerslev used extended autoregressive conditional heteroskedastic (ARCH) and

generalized autoregressive conditional heteroskedastic (GARCH) models to fit foreign exchange rates and stock price index data. Results indicate that neither model fully captured the observed leptokurtosis. Taylor (1985,1986) proposed that daily futures returns divided by their forecast conditional standard deviations would be better than original data, to study the distribution of daily returns, as the rescaled data had a reasonably homogenous variance and resulted in more accurate results. Such a rescaled data series was used by Taylor to test the random walk hypothesis for several U.S. and U.K. financial markets. The study results rejected the random walk hypothesis.

This paper corrects for heteroskedasticity following the procedure suggested by Taylor (1985,1986). The original data series is rescaled by dividing each observation by its forecast conditional standard deviation generated by the generalized autoregressive conditional heteroskedastic (GARCH) model. The data for the study consists of daily futures prices for 27 commodities including agricultural commodities, livestock, metals, and financial instruments. The nature of the distribution of the rescaled returns is to be determined and compared with the distribution of the original data series using the stability-under-addition test. This test is based on the fact that any linear combination of independently and identically distributed stable variables will also be stable. In each case, the test is performed on the entire data set and also on a subset (1981-1986) of the entire data and the results compared. If the departure of daily futures returns from normality is due only to changing variances, then the rescaled returns will be normally distributed. If the non-normality is due to factors other than changing variances, then the rescaled returns though not exactly normal, will be less leptokurtic than the original data.

II. The Stable Paretian Distributions:

The family of probability distributions known as stable paretian are the most commonly used candidates for describing the distribution of stock returns. Stable paretian distributions are the only possible limiting

distributions for sums of independent and identically distributed random variables (Fama (1963)). The symmetric stable paretian distributions are determined by four parameters (α, β, γ and δ) and are described using a characteristic function instead of a probability density function (Fielitz and Rozelle, 1983). The characteristic function in the natural logarithmic form is given by:

$$\ln f(t) = \ln E(e^{iyt}) = i\delta t - \gamma |t|^\alpha \{1 + i\beta(t/|t|)\} \omega(t, \alpha) \quad (1)$$

where y is the random variable under consideration, namely, the actual or rescaled daily futures price changes in this study; $i = \sqrt{-1}$ and t is some real number. If $\alpha \neq 1$, then $\omega(t, \alpha) = \tan(\pi\alpha/2)$ and if $\alpha = 1$, then $\omega(t, \alpha) = (2/\pi)\tan(|t|)$. The ranges for the parameters are: $0 < \alpha \leq 2$, $\beta = 0$, $\gamma > 0$ and $-\infty < \delta < \infty$.

The total probability contained in the extreme tails of the distribution is given by α - the characteristic exponent of the stable paretian distribution. The nature of the distribution of rescaled futures price changes, regardless of whether it is distributed as normal, non-normal stable paretian or as a mixture of normals, can be studied by using the stability under addition property of stable distributions. According to this property the sum of independent stable variables with characteristic exponent α will be stable with the same characteristic exponent. Following the stability-under-addition test, α is estimated for the entire sample and also for non-overlapping sums of sample observations. Hence, if the underlying distribution is normal stable with constant finite variance, then $\hat{\alpha}$ will be equal to 2 and remain stable across non-overlapping sums of sample observations. If the distribution is non-normal stable with infinite variance, α will be less than 2 and remain stable across non-overlapping sums. If $\alpha < 2$ and approaches 2 as the sum size increases, it is a case of mixed normals distribution with finite but changing variance.

III. Data and Procedure:

The data used is the first differences of natural logarithms of the daily closing futures prices, since this can be interpreted as the returns

to futures contracts held for a day under continuous compounding. The data set includes 27 commodities including agricultural commodities, livestock, metals and financial instruments. The number of days and years of daily data available vary with the commodity, but information for each commodity is available through 1986 (Table 1). In order to maintain a continuity of data and minimize differences in the maturity of contracts, the data set consists of the changes in the log of daily closing prices of futures contracts until the third Tuesday of the month prior to delivery, after which the next nearest delivery month is used and this process is continued.

Rescaled data are more likely to be independently and identically distributed than the original daily returns (Taylor 1985, 1986). In this study, the procedure suggested by Taylor will be used to rescale the daily futures returns. The actual returns are divided by their forecast standard deviation to obtain the rescaled return series $\{U_t\}$ where each element of the series, U_t is given as:

$$U_t = (X_t - \mu) / V_t \quad (2)$$

where U_t is the actual rescaled return, X_t denotes the actual futures returns on day t , μ refers to the true population mean of the X_t series over the entire period and V_t is the conditional standard deviation of the t 'th day's returns. Since the actual conditional standard deviation V_t and population mean μ are not observable, an approximation to the actual rescaled return U_t is obtained by substituting estimated values for μ and V_t . Using the sample mean \bar{X} for μ and a forecast \hat{V}_t made at time $t-1$ gives the rescaled returns given by:

$$Y_t = (X_t - \bar{X}) / \hat{V}_t = (X_t - \mu) / V_t \quad (3)$$

where Y_t is an approximation to the actual rescaled return U_t . The approximated rescaled return will be identical to the unobservable true rescaled return when \hat{V}_t is a good forecast. Following Taylor, the daily returns of the first 20 days will be used to obtain a forecast of the conditional standard deviation for the twenty first day (\hat{V}_{21}) as follows:

$$\hat{V}_{21} = 1.253 \sum_{t=1}^{20} (|X_t - \bar{X}|) / 20 \quad (4)$$

The constant 1.253 is the standard deviation of a normal distribution divided by its mean absolute deviation. Subsequently the forecast of conditional standard deviation is given by the following generalized autoregressive conditional heteroskedastic (GARCH) model:

$$\hat{V}_t = (1 - \theta) \hat{V}_{t-1} + 1.253 \theta |X_{t-1} - \bar{X}| \quad (5)$$

Taylor suggested $\theta = 0.04$ for stock series and $\theta = 0.1$ for all other series. Hence in this study, θ will be taken as 0.1. Since \hat{V}_t is available only from the twenty first day onwards, the rescaled series will have twenty observations less than the original series. As the stability under addition test will be performed on the original returns and also on the rescaled returns, the first twenty observations will also be deleted from the original data series. The stability test will be performed on both the entire data and also a subset of the data (1981-1986).

The individual adjacent returns in each series will be summed into groups of 1,2,3,4,5,10,15,20 and 30 observations and the stability-under-addition test performed on the original and rescaled data series. Sum size one will be the initial rescaled data and represent one-day continuously compounded returns. Similarly the sum size two will represent two-day continuously compounded returns and so on (Fama and Roll (1971); Fielitz and Rozelle). The characteristic exponent $\hat{\alpha}$ will then be estimated for each sum size following the procedure developed by Fama and Roll(1971). This procedure involves the use of an estimator of the characteristic exponent of the symmetric stable distributions. This estimator is:

$$\hat{Z}_f = (\hat{x}_f - \hat{x}_{1-f})/2\hat{c} \quad (6)$$

where \hat{Z}_f is an estimate of the f'th fractile of a standardized, symmetric stable cumulative distribution function and \hat{x}_f is the $(N+1)(f)$ 'th order statistic where N refers to the sample size. Here \hat{c} is an estimate of the scale parameter and is calculated as:

$$\hat{c} = (\hat{x}_{0.72} - \hat{x}_{0.28})/2(0.827) \quad (7)$$

The value of f used in this study is 0.96 which gives $\hat{\alpha}$ values that are relatively efficient and have insensitive sampling dispersion (Fama and Roll (1971)). The estimated value of \hat{Z}_f is then compared to a table of

standardized symmetric stable cumulative distribution functions (Fama and Roll, 1968) to find the value of α whose fractile most closely matches the estimate \hat{Z}_f . For any value of \hat{Z}_f which indicates values of α greater than the values available in the table, the $\hat{\alpha}$'s will be treated as two, as α is restricted to the interval $[0,2]$ by definition.

IV. Empirical Results:

The estimates of characteristic exponents for the original and rescaled data for the entire data series are presented in Tables 2 and 3 respectively. Since stable distributions are by definition stable under addition, estimates of α will not change with sum size if the price changes conform to the stable distribution but if they are mixtures of normal distribution, estimates of α will increase with sum size. The results obtained therefore suggest a mixture of normals, since $\hat{\alpha}$ increases with the sum size in the original and rescaled data.

In the case of original data - entire series (Table 2), out of a total of 270 $\hat{\alpha}$'s, $\hat{\alpha} = 2.0$ in only 16 cases (6 %) and $\hat{\alpha} \geq 1.90$ in 25 cases (9 %). The $\hat{\alpha}$'s range from 1.15 to 2.0. The estimated $\hat{\alpha}$ increases as the sum size increases from 1 to 30. In general, the characteristic exponents in the rescaled data (Table 3) are much larger than the corresponding values in Table 2. For example, in the case of sum size one, corn has an $\hat{\alpha}$ value of 1.37 in the case of original data (Table 2). However, in the case of rescaled data (Table 3), the corresponding value has increased to 1.84. The estimated $\hat{\alpha}$'s in the case of rescaled data - entire series (Table 3) also exhibit a tendency to increase towards 2.0, as the sum size is increased. In Table 3, only 7 out of the 270 estimates or less than 3 % have an $\hat{\alpha}$ value below 1.6. However the corresponding number in Table 2 is 151, accounting for 56 per cent. The estimated $\hat{\alpha}$ reaches the value of 2.0 in 26 out of the 27 commodities (96 %) included in the study, as compared to just 7 commodities (26 %) under Table 2.

The rescaled returns therefore show a greater tendency towards a normal distribution (compared to the original returns) as evidenced by the larger

values of the characteristic exponents. In other words, the rescaled returns though not normal, are less leptokurtic than the original returns. In Table 3, the average $\hat{\alpha}$ for one-day rescaled returns was only 1.78, compared to 1.87 for the thirty-day rescaled returns. This implies that there could be some short-run changes in variance that the time series model failed to capture. Also, there may be factors in addition to the changing variance that contribute to the observed leptokurticity in the data. Or, the forecast standard deviation may be an imperfect rescaling factor. Other researchers also found similar results. For example, McCulloch also estimated values of $\hat{\alpha}$ between 1.614 and 1.714 for six selected financial assets using monthly data and a different model of adaptive conditional heteroskedasticity (ACH). This implies that the ACH model did not correct the observed leptokurticity completely. Similarly Bollerslev concluded that the ARCH or GARCH models with conditionally normal errors do not fully capture the leptokurtosis of the distribution of speculative price changes.

A comparison of Tables 4 and 5 reveal a similar relationship observed in Tables 2 and 3; i.e., the rescaled returns though not perfectly normal, are closer to the normal distribution than the original returns. The estimated values of $\hat{\alpha}$ in the 1981-86 original data (Table 4) have a greater tendency to approach 2.0 than the original data using the entire data set (Table 2). The values of $\hat{\alpha}$ in the subset are slightly more stable than the $\hat{\alpha}$'s obtained from the entire data set. This implies that the period prior to 1981 was characterized by a less constant variance compared to the later period. Surprisingly, quite the contrary was observed in the case of rescaled returns (Tables 3 and 5) i.e. in most of the cases, the values of estimated $\hat{\alpha}$'s were smaller in the subset compared to the entire data set.

Changing conditional variances do appear as an important factor responsible for the observed departure of futures returns from the normal distribution. However, results obtained in this study indicate that there are additional factors contributing to the observed leptokurticity of futures returns.

V. Conclusion:

This paper corrects for heteroskedasticity following the procedure suggested by Taylor (1985,1986). Daily futures returns data for 27 commodities are used in the study. The original data are corrected for heteroskedasticity by dividing each observation by its forecast conditional standard deviation generated by the generalized autoregressive conditional heteroskedastic (GARCH) model. The stability-under-addition test of stable distributions is then performed on the original data and also on the rescaled data using the entire data and a subset of the original data (1981-86).

The results indicate that the underlying distribution of the original and rescaled returns are both leptokurtic. The returns corrected for changing conditional variances are not normal but are less leptokurtic than the original. This indicates that the method of forecasting the standard deviation used here may not be an adequate rescaling factor. If this is true, then this procedure though not able to correct the observed leptokurticity, still provides a more appropriate data series to use and may yield more accurate results. This also implies that there may be factors in addition to the changing conditional variances that contribute to the observed leptokurticity in the data. An investigation into the other factors responsible for the observed leptokurticity is a possible avenue for further research. Another possible extension of this study is to try alternate rescaling procedures and observe its impact on the underlying distribution of futures returns. The time series model developed by Bollerslev which allows the error terms to follow a conditional t-distribution is one such alternative.

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TABLE 1

COMMODITIES INCLUDED, PERIOD COVERED AND THE NUMBER OF OBSERVATIONS

Commodities	Period	No. of observations (All data)	No. of observations (1981-86 series)
<u>I. Agrl. Commodities:</u>			
1. Oats	1960-86	6772	1514
2. Soybeans	1960-86	6769	1516
3. Soybean Meal	1960-86	6772	1515
4. Soybean Oil	1960-86	6772	1515
5. Wheat-Chi	1960-86	6775	1516
6. Wheat-Kc	1979-86	1992	1511
7. Coffee	1979-86	1987	1505
8. Cocoa	1960-86	6704	1506
9. Corn	1960-86	6774	1516
10. Cotton	1961-86	6481	1509
11. Orange Juice	1968-86	4728	1508
12. Lumber	1974-86	3256	1516
<u>II. Livestock:</u>			
13. Live Cattle	1966-86	5292	1517
14. Feeder Cattle	1974-86	3256	1516
15. Pork Bellies	1965-86	5514	1515
16. Live Hogs	1970-86	4260	1516
<u>III. Metals:</u>			
17. Gold	1976-86	2748	1512
18. Silver	1964-86	5716	1507
19. Copper	1960-86	6720	1510
<u>IV. Finl. Instruments:</u>			
20. Japanese Yen	1977-86	2500	1515
21. Swiss Franc	1977-86	2502	1517
22. U.S. T-Bills	1976-86	2749	1516
23. British Pound	1977-86	2499	1516
24. Canadian Dollar	1977-86	2501	1516
25. Deutsche Mark	1977-86	2502	1517
26. T-Bonds	1978-86	2251	1516
<u>V. Miscellaneous:</u>			
27. Heating Oil	1980-86	1745	1501

TABLE 2: ESTIMATES OF CHARACTERISTIC EXPONENTS FOR THE ENTIRE DATA SERIES-ORIGINAL DATA

COMMODITIES	Sum Size									
	1	2	3	4	5	10	15	20	30	
<u>I. Agrl. Commodities:</u>										
1. Oats	1.43	1.57	1.58	1.55	1.57	1.55	1.53	1.59	1.60	
2. Soybeans	1.37	1.29	1.31	1.32	1.26	1.26	1.20	1.30	1.22	
3. Soybean Meal	1.36	1.42	1.45	1.37	1.43	1.41	1.42	1.55	1.46	
4. Soybean Oil	1.47	1.49	1.51	1.49	1.52	1.55	1.66	1.64	1.55	
5. Wheat-Chi	1.40	1.45	1.39	1.48	1.48	1.43	1.44	1.41	1.43	
6. Wheat-Kc	1.56	1.65	1.72	1.51	1.51	1.53	1.75	1.64	1.74	
7. Coffee	1.48	1.49	1.47	1.50	1.44	1.40	1.51	1.44	1.37	
8. Cocoa	1.73	1.63	1.62	1.61	1.69	1.78	1.67	1.77	1.91	
9. Corn	1.37	1.43	1.44	1.39	1.46	1.44	1.51	1.45	1.53	
10. Cotton	1.20	1.29	1.28	1.29	1.28	1.28	1.41	1.37	1.28	
11. Orange Juice	1.48	1.47	1.45	1.51	1.48	1.43	1.69	1.60	1.54	
12. Lumber	2.00	1.83	2.00	1.96	1.95	2.00	2.00	2.00	1.68	
<u>II. Livestock:</u>										
13. Live Cattle	1.54	1.53	1.61	1.62	1.59	1.73	1.70	1.62	1.51	
14. Feeder Cattle	1.81	1.68	1.83	1.69	1.69	1.70	1.65	1.49	1.52	
15. Pork Bellies	2.00	1.71	1.78	1.82	1.81	1.81	1.93	1.86	1.74	
16. Live Hogs	1.66	1.64	1.69	1.78	1.71	2.00	1.70	1.90	2.00	
<u>III. Metals:</u>										
17. Gold	1.40	1.45	1.55	1.46	1.48	1.52	1.65	1.38	1.58	
18. Silver	1.34	1.30	1.29	1.34	1.29	1.40	1.46	1.46	1.29	
19. Copper	1.49	1.45	1.48	1.55	1.54	1.64	1.71	1.67	1.74	
<u>IV. Finl. Instruments:</u>										
20. Japanese Yen	1.57	1.56	1.61	1.76	1.63	1.65	1.62	1.93	1.73	
21. Swiss Franc	1.66	1.63	1.63	1.74	1.66	1.97	1.69	2.00	2.00	
22. U.S. T-Bills	1.20	1.27	1.30	1.30	1.28	1.44	1.27	1.35	1.18	
23. British Pound	1.51	1.61	1.54	1.66	1.79	1.65	2.00	1.78	1.78	
24. Canadian Dollar	1.54	1.55	1.54	1.59	1.53	1.47	1.67	1.56	1.36	
25. Deutsche Mark	1.53	1.69	1.63	1.68	1.68	2.00	1.67	1.70	2.00	
26. T-Bonds	1.51	1.52	1.60	1.60	1.86	1.63	1.76	1.63	1.75	
<u>V. Miscellaneous:</u>										
27. Heating Oil	1.42	1.48	1.46	1.39	1.40	1.57	1.47	1.82	1.38	

TABLE 3: ESTIMATES OF CHARACTERISTIC EXPONENTS FOR THE ENTIRE DATA SERIES-RESCALED DATA

COMMODITIES	1	2	3	4	Sum Size 5	10	15	20	30
<u>I. Agrl. Commodities:</u>									
1. Oats	1.79	1.89	1.85	1.77	1.95	1.77	2.00	2.00	1.89
2. Soybeans	1.89	1.84	1.84	1.84	1.92	1.86	1.80	2.00	1.79
3. Soybean Meal	1.76	1.88	1.88	1.91	1.80	1.81	1.86	2.00	2.00
4. Soybean Oil	1.89	1.93	2.00	1.94	1.88	1.94	2.00	2.00	2.00
5. Wheat-Chi	1.82	1.85	1.73	1.80	1.89	1.87	2.00	1.88	2.00
6. Wheat-Kc	1.65	1.84	1.83	1.65	1.81	1.64	1.77	1.51	1.54
7. Coffee	1.73	1.84	1.92	1.95	1.79	2.00	2.00	1.69	1.65
8. Cocoa	1.70	1.64	1.61	1.63	1.61	1.77	1.62	1.50	1.65
9. Corn	1.84	1.74	1.93	1.77	1.80	1.83	1.86	1.72	1.83
10. Cotton	1.61	1.64	1.69	1.71	1.77	1.74	2.00	2.00	2.00
11. Orange Juice	1.64	1.64	1.67	1.69	1.71	1.72	1.99	1.80	1.89
12. Lumber	2.00	2.00	2.00	2.00	2.00	2.00	2.00	1.96	1.88
<u>II. Livestock:</u>									
13. Live Cattle	2.00	1.94	2.00	2.00	2.00	2.00	2.00	2.00	1.80
14. Feeder Cattle	2.00	2.00	2.00	2.00	2.00	1.99	2.00	1.91	2.00
15. Pork Bellies	2.00	2.00	2.00	2.00	1.96	2.00	2.00	2.00	1.64
16. Live Hogs	1.95	1.86	1.84	2.00	1.97	2.00	1.90	1.96	2.00
<u>III. Metals:</u>									
17. Gold	1.59	1.57	1.64	1.61	1.65	1.75	2.00	1.67	1.88
18. Silver	1.77	1.71	1.69	1.76	1.72	1.89	1.91	1.78	1.92
19. Copper	1.83	1.68	1.81	1.82	1.84	1.94	1.91	2.00	2.00
<u>IV. Finl. Instruments:</u>									
20. Japanese Yen	1.65	1.74	1.79	1.83	1.71	1.70	1.70	1.75	2.00
21. Swiss Franc	1.79	1.90	1.86	1.89	1.95	2.00	1.79	2.00	2.00
22. U.S. T-Bills	1.72	1.86	1.86	2.00	2.00	2.00	2.00	2.00	1.77
23. British Pound	1.62	1.69	1.83	1.85	2.00	2.00	2.00	1.87	2.00
24. Canadian Dollar	1.66	1.72	1.75	1.63	1.84	1.97	1.98	1.78	1.49
25. Deutsche Mark	1.65	1.84	1.84	1.77	1.87	2.00	1.78	2.00	2.00
26. T-Bonds	1.78	1.89	1.89	2.00	2.00	2.00	2.00	2.00	1.76
<u>V. Miscellaneous:</u>									
27. Heating Oil	1.79	1.85	1.78	1.66	1.92	1.84	2.00	2.00	2.00

TABLE 4: ESTIMATES OF CHARACTERISTIC EXPONENTS FOR THE 1981-86 SERIES-ORIGINAL DATA

COMMODITIES	Sum Size									
	1	2	3	4	5	10	15	20	30	
<u>I. Agrl. Commodities:</u>										
1. Oats	1.69	1.65	1.66	1.77	1.78	1.61	1.93	1.41	1.88	
2. Soybeans	1.59	1.65	1.74	1.68	1.68	1.66	1.63	1.55	1.68	
3. Soybean Meal	1.59	1.66	1.66	1.72	1.68	1.60	2.00	1.87	2.00	
4. Soybean Oil	1.62	1.60	1.59	1.67	1.83	1.89	1.64	1.75	1.71	
5. Wheat-Chi	1.79	1.84	1.65	2.00	1.84	1.73	2.00	1.85	2.00	
6. Wheat-Kc	1.63	1.77	1.60	1.89	1.74	1.64	1.57	1.65	1.59	
7. Coffee	1.47	1.47	1.45	1.46	1.44	1.49	1.20	1.57	1.38	
8. Cocoa	1.68	1.77	1.77	1.76	2.00	1.84	1.95	1.79	1.46	
9. Corn	1.67	1.52	1.66	1.69	1.76	1.62	1.62	1.46	1.76	
10. Cotton	1.57	1.63	1.61	1.65	1.56	1.72	1.79	2.00	2.00	
11. Orange Juice	1.41	1.44	1.44	1.44	1.43	1.43	1.51	1.58	1.33	
12. Lumber	2.00	1.93	2.00	1.96	1.74	2.00	2.00	2.00	1.66	
<u>II. Livestock:</u>										
13. Live Cattle	1.62	1.73	1.87	1.97	1.87	2.00	2.00	2.00	1.95	
14. Feeder Cattle	1.59	1.66	1.60	1.71	1.71	2.00	1.96	2.00	1.77	
15. Pork Bellies	2.00	1.93	1.95	2.00	2.00	1.88	1.75	1.84	1.59	
16. Live Hogs	1.70	1.79	1.82	1.96	2.00	2.00	2.00	1.79	1.75	
<u>III. Metals:</u>										
17. Gold	1.40	1.48	1.55	1.54	1.58	1.56	2.00	1.69	1.58	
18. Silver	1.51	1.54	1.61	1.48	1.40	1.60	1.98	1.63	1.39	
19. Copper	1.72	1.72	1.66	1.58	1.67	1.70	2.00	2.00	2.00	
<u>IV. Finl. Instruments:</u>										
20. Japanese Yen	1.59	1.70	1.72	1.63	1.65	1.60	1.82	1.62	2.00	
21. Swiss Franc	1.72	1.81	2.00	1.69	1.80	1.85	2.00	1.83	1.45	
22. U.S. T-Bills	1.28	1.29	1.38	1.35	1.54	1.45	1.42	1.37	1.32	
23. British Pound	1.61	1.65	1.50	1.59	1.79	1.74	1.87	1.59	1.77	
24. Canadian Dollar	1.48	1.42	1.48	1.47	1.52	1.43	1.95	1.75	1.26	
25. Deutsche Mark	1.68	1.91	1.88	1.71	1.83	2.00	2.00	2.00	1.64	
26. T-Bonds	1.68	1.82	1.87	1.80	2.00	2.00	2.00	1.90	1.83	
<u>V. Miscellaneous:</u>										
27. Heating Oil	1.44	1.50	1.43	1.32	1.33	1.39	1.61	1.72	1.20	

TABLE 5: ESTIMATES OF CHARACTERISTIC EXPONENTS FOR THE 1981-86 SERIES-RESCALED DATA

COMMODITIES	1	2	3	4	Sum Size 5	10	15	20	30
<u>I. Agrl. Commodities:</u>									
1. Oats	1.81	1.83	1.80	1.71	1.80	1.62	2.00	1.50	1.48
2. Soybeans	1.88	1.84	2.00	1.78	1.85	1.86	1.71	1.62	1.85
3. Soybean Meal	1.68	1.70	1.64	1.51	1.78	1.58	2.00	1.95	2.00
4. Soybean Oil	1.76	1.71	1.78	2.00	2.00	2.00	2.00	2.00	1.69
5. Wheat-Chi	1.83	1.58	1.71	1.54	1.57	1.72	2.00	1.74	1.70
6. Wheat-Kc	1.62	1.89	1.76	2.00	1.79	1.75	1.57	1.53	1.61
7. Coffee	1.75	1.88	1.99	2.00	1.86	2.00	2.00	1.93	2.00
8. Cocoa	1.72	1.97	1.71	1.62	2.00	1.67	1.90	1.93	2.00
9. Corn	1.79	1.68	1.78	1.81	1.79	1.71	1.73	1.79	1.88
10. Cotton	1.68	1.75	1.82	1.60	1.63	1.94	1.98	2.00	2.00
11. Orange Juice	1.63	1.60	1.75	1.60	1.68	1.63	1.70	1.69	1.52
12. Lumber	2.00	2.00	2.00	1.98	2.00	2.00	2.00	2.00	2.00
<u>II. Livestock:</u>									
13. Live Cattle	1.84	1.96	2.00	2.00	2.00	2.00	2.00	1.86	2.00
14. Feeder Cattle	1.78	1.80	1.99	2.00	2.00	2.00	2.00	2.00	2.00
15. Pork Bellies	2.00	2.00	2.00	2.00	2.00	1.91	2.00	1.73	1.47
16. Live Hogs	1.79	1.79	1.93	2.00	2.00	2.00	1.80	2.00	1.80
<u>III. Metals:</u>									
17. Gold	1.55	1.51	1.62	1.58	1.41	1.74	2.00	1.94	1.89
18. Silver	1.59	1.58	1.71	1.54	1.57	1.72	2.00	2.00	2.00
19. Copper	1.76	1.78	1.71	1.83	1.79	2.00	2.00	2.00	2.00
<u>IV. Finl. Instruments:</u>									
20. Japanese Yen	1.67	1.93	1.77	1.82	1.90	1.66	1.79	1.59	2.00
21. Swiss Franc	1.79	1.97	1.96	1.99	2.00	2.00	2.00	2.00	1.58
22. U.S. T-Bills	1.67	1.79	1.89	2.00	2.00	2.00	2.00	2.00	1.69
23. British Pound	1.68	1.76	1.65	1.82	1.75	2.00	2.00	1.93	2.00
24. Canadian Dollar	1.65	1.66	1.58	1.66	1.88	1.74	1.95	2.00	1.55
25. Deutsche Mark	1.66	2.00	1.97	1.87	1.98	1.80	2.00	1.91	1.64
26. T-Bonds	1.77	1.94	1.99	2.00	2.00	2.00	2.00	2.00	1.91
<u>V. Miscellaneous:</u>									
27. Heating Oil	1.78	1.78	1.79	1.64	1.74	1.79	2.00	2.00	1.95