Dependence Analysis between Foreign Exchange Rates:
A Semi-Parametric Copula Approach

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Dependence Analysis between Foreign Exchange Rates: A Semi-Parametric Copula Approach

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Abstract

Not only currencies are assets in investor's portfolio, central banks use them for implementing economic policies. This implies existence of some type of dependence pattern among the currencies. We investigate such patterns among daily Deutsche Mark (DM) (Euro later), UK Sterling (GBP) and the Japanese Yen (JPY) exchange rate, all considered against the US Dollar during various economic conditions. To overcome the short-comings of mis-specification, normality and linear dependence for such time series, a flexible semi-parametric copula methodology is adopted where the marginals are non-parametric but the copula is parametrically specified. Dependence is estimated both as a constant and time-varying measure. During the Pre-Euro period, we find slightly more dependence when both DM (Euro)/USD and GBP/USD jointly appreciate as compared to joint depreciation, especially in the late 90s. Such results are reversed for GBP/USD and JPY/USD in the early 90s. Post-Euro, DM (Euro)/USD and GBP/USD exhibit stronger dependence when they jointly appreciate, which could indicate preference for price-stability in EU zone. Whereas the dependence of JPY/USD with both DM (Euro)/USD and GBP/USD is stronger when they jointly depreciate, this could imply preference for export competitiveness among the countries. In the beginning of Recent-Crisis period, DM (EURO)/USD and GBP/USD show more dependence when they jointly depreciate, but later we see the similar tendency for these currencies to be related more when they jointly appreciate. Such measures of asymmetric dependence among the currencies provide vital insight into Central banks preferences and investors portfolio balancing.

JEL Classification: C14, C58, F31.
Keywords: Copula, exchange rates, semi-parametric, time series.

1 Introduction

Exchange rates are a vital aspect of International Economics and are used to implement various economic policies. Along with GDP and interest rate, exchanges rates are an indicator of a country's economic outlook and are determined through various cross-country economic fundamentals. In this era of globalisation countries are not simply interested in closely monitoring their own currencies, but also the currency of other countries, which causes them to frequently intervene in the foreign exchange market. Such an intervention to guide their currency in a particular direction due to another currency, creates a dependence among the exchange rates. A synchronisation of business cycles, or difference in short-term interest rates across countries causing capital inflow/outflow, can also create comovement of exchange rates. Currencies are also held in investors portfolio along with other financial assets, and their preference over holding such currencies also creates a relationship between the exchange rates.

Not only are the exchange rates dependent upon each other, they could also exhibit non-linear dependence and non-constant dependence through time. Takagi (1999) states if there are two countries A and B, who export to foreign countries and in order to ensure their export prices are competitive to each other, then if country A's exchange rate depreciates, country B would ensure their exchange rate does too, which creates joint depreciation dependence. On the other hand if the countries prefer price stability among each other (maybe regional), then if country A's exchange rate appreciates, country B will intervene in the foreign exchange rate to ensure similar appreciation of their currency, and hence causing dependence due to joint appreciation. The variation in the preference of being competitive in terms of export, or ensuring price stability, creates an asymmetric dependence among the two currencies. Another contributing reason for such asymmetric dependence patterns could be associated to the common denominating currency in the two exchange rates. US Dollar has long been considered as a reserve currency, meaning investors prefer to hold it more in their portfolio as compared to other currencies. So when US Dollar appreciates, investors forgo their holdings of other currencies and shift their funds into the US Dollar. On the other hand, when US Dollar depreciates, they might not prefer to hold other currencies similarly. Such shifting of funds to and from the US Dollar could also create an asymmetric dependence. These asymmetries are not only found

in exchange rates, but also for other financial assets, Longin and Solnik (2001) show assets returns exhibit stronger dependence during market downturns as compared to market upturns.

Measures of dependence is mostly restricted to linear correlation, or based on methods assuming exchange rates (or other financial assets) to be normally distributed. Multivariate Normal and t-distribution have frequently been used to measure dependence between assets, Hansen (1994), Harvey and Siddique (1999) and Engle and Manganelli (2004) employ such models for applications to risk-management and portfolio allocation. Embrechts et al. (2002) points out the limitations for such methods, as linear correlation is only one measure of stochastic dependence. Other models such as Multivariate GARCH by Engle and Kroner (1995) and Dynamic Conditional Correlation (DCC) of Engle (2002) have also been applied, but they present various estimation problems, and are also bounded by distribution specifications, and in presence of asymmetric dependence do not provide adequate fit.

Recently Copula models have seen an increase in their use in finance and economics, though being around since Sklar (1959) theorem. They had success in applications, where dependence is non-linear and the random variables involved have different marginal distribution specifications. Copula methodology requires the decomposition of the marginal distributions of the random variables from the dependence among them. There are various copula families available to capture complex dependence patterns, such as non-elliptical forms and tail dependence. In finance, Embrechts et al. (2001) and Cherubini and Luciano (2001), use them for Value At Risk (VaR) and pricing analysis. Trivedi and Zimmer (2006) provide details of their use in health economics applications. Nelson (2006) and Joe (1997) cover various statistical and mathematical properties of copulas, including estimation techniques.

Within exchange rate dependence analysis, Patton (2006) and Dias and Embrechts (2010) adopt a fully parametric copula approach, where the marginals are chosen from a parametric family along with a parametric copula. Within a constant and time-varying dependence measure framework, they show that Deutsch Mark and Japanese Yen (both against US Dollar) tend to exhibit stronger dependence when they jointly depreciate, as compared to when they jointly appreciate. In both papers, exchange rate returns are assumed to be specified through t-distribution, with varying degrees-of-freedom. The copula estimation relies on no misspec-

ification of the marginals, and hence any parametric family chosen requires testing for the appropriateness of the chosen marginals specification. It is easy to misspecify the marginals, especially when the time series in question is of high frequency (daily exchange rates). Genest et al. (1995), show how non-parametric marginals produce consistent and asymptotically normal copula estimates, given the unspecified margins are of continuous type. Kim et al. (2007) also show such a specification to produce efficient results for sample size larger than 100. Boero et al. (2011) adopt such an approach, which is generally termed as semi-parametric copula estimation, where a parametric copula is specified. They show that for constant dependence measures, asymmetric patterns of strong depreciation or strong appreciation can vary for different currencies involved during various economic periods.

This paper extends the work of Boero et al. (2011) to a time-varying dependence measure within a semi-parametric copula approach. It gives a completely robust measure of dependence in a time-varying case. We also very specifically look at the dependence pattern between Deutsche Mark (DM, Euro later), UK sterling (GBP) and Japanese Yen (JPY) all against the US Dollar (USD) before and after the Euro's introduction and also during the recent financial crisis. Such measures can reveal valuable insight into the exchange rates comovement during various times for both policy makers and investment strategies purposes. We employ two copula families, the Gaussian and Symmetrized Joe-Clayton (SJC) copula of Patton (2006). The SJC copula is a two parameter based copula measuring the lower and upper joint tail dependence separately and the Gaussian copula acts as a benchmark specification. First, we filter the daily returns through a ARMA(p,q)-GARCH(1,1) model to obtain i.i.d observations required for the copula methodology, and then using the filtered returns we estimate both constant and time-varying measure of dependence through copula families between the 6-pairs for before (1990 - 1998) and after Euro (1999 - 2006), and for the recent crisis (2007 - 2009). The time-varying measure of dependence for both copula families evolve according to a ARMA type process, same as Patton (2006).

We see that SJC copula provides a better fit in terms of the likelihood of the data. Before the introduction of the Euro, DM/USD and GBP/USD do not show any asymmetric dependence, but still there is presence of tail dependence. There seems to be some asymmetric depen-

dence between GBP/USD and JPY/USD. After the introduction of the Euro, the SJC copula reports higher probability of joint appreciation between DM (EURO)/USD and GBP/USD, which could be associated with higher co-operation within the EU for price stability in the region. Both DM (Euro)/USD and GBP/USD when paired with JPY/USD, show higher joint tail depreciation probability compared to joint tail appreciation probability. However the time-varying SJC reveals that there are points in time where the difference in tail dependence could be quite large, but in same direction. In the recent-crisis period, for DM (Euro)/USD and GBP/USD, the constant SJC copula measure could be misleading as both tend to jointly depreciate with greater probability, then compared to joint appreciation in the beginning of 2007. Although later they revert back to appreciating jointly with greater probability, which could be due the uncertainty in the beginning of the crisis. Whereas the dependence between JPY/USD and the other two exchange rate seems weaker. Generally our results indicate greater preference for price stability between DM(Euro)/USD and GBP/USD. Whereas, when paired with JPY/USD, more export competitive behaviour is suggested. Overall we see strong indication of asymmetric tail dependence over various periods, which could be explained through economic events. Also the time-varying measure is necessary to fully reveal the dependence pattern.

We start our analysis by first explaining the copula methodology in Section 2, where details over the constant and time-varying dependence measure is provided, along with non-parametric margins. Section 3 sets out the data, and describes some vital summary statistics. In Section 4, we present the results for both the constant and time-varying copula measures and present some economic intuition for the result. Finally concluding in Section 5.

2   Semi-Parametric Copula Framework

2.1   Formal Copula Definition

According to Sklar (1959)'s theorem, any p-dimensional joint distribution H for some random variables $Y_1, \ldots . Y_p$ can be decomposed to copula C measuring their dependence, and their margins $F_1, \ldots, F_p$, specifying their individual characteristics (fat tails, skewness etc.). Formally given as

$$H(y_1, \ldots, y_p) = C(F_1(y_1), \ldots F_p(y_p)).$$

Where $C : [0, 1]^p \mapsto [0, 1]$. The copula C is unique, if all the margins $F_1, \ldots, F_p$ are continuous. Distribution given above could also be stated as,

$$C(u_1, \ldots, u_p) = \text{pr}\ (U_1 \le u_1, \ldots, U_p \le u_p).$$

u_j is the uniform variable computed through the marginal distribution $u_j = F(y_j)$, where $j = 1, \ldots, p$, also referred to as Probability Integral Transformation (PIT) (see Diebold et al. (1998)). If the joint distribution F is p-times differentiable, then by taking its pth cross-partial derivatives we get,

$$f(y_1, \ldots, y_p) = \prod_{j=1}^{p} f_j(y_j) \cdot c(F_1(y_1), \ldots, F_p(y_p)).$$

Such a decomposition provides a very flexible framework, where each F_j could belong to a different parametric families, and the dependence among the random variables is not confined to elliptical distributions (Gaussian or t-distribution). Joe (1997) and Nelson (2006) provide a detailed coverage of theoretical and modelling aspects of copulas.

We presented the copula definition for a multivariate case of dimension p. In our empirical analysis. We are interested in capturing dependence among two random variables at a time, hence from now we will present the specifications for $p = 2$. Instead of denoting the random variables as Y_1 and Y_2, we denote them as X and Y and their respective uniforms through the marginal distributions as u and v.

There exist a wide array of copulas families to choose from, depending upon the type of dependence a practitioner is interested in capturing, Nelson (2006) describes most of the commonly used copulas. Given our interest is in identifying any asymmetric dependence, we will confine ourselves to the SJC copula, which has two parameters capturing the joint lower and upper tail dependence. We will also use a Gaussian copula as a benchmark.

2.1.1 Gaussian Copula

A Gaussian copula is the most used copula, along with a t-copula. It is analogous to a multivariate Normal distribution when the margins are assumed/chosen to be normally distributed.

It is based on an elliptical distribution, which implies symmetric and zero tail dependence. For a bivariate case, it is given as

$$
\begin{aligned}
C_g(u,v|\rho) &= \Phi_g(\Phi^{-1}(u), \Phi^{-1}(v); \rho) \\
&= \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}} \\
&\times \left\{ \frac{-(s^2 - 2\rho st + t^2)}{2(1-\rho^2)} \right\} dsdt,
\end{aligned}
$$

where Φ is the Cumulative Distribution Function (CDF) of a standard normal distribution, $\Phi_g(u,v)$ is a standard bivariate normal distribution and ρ the correlation parameter defined over $[-1,1]$.

2.1.2 SJC Copula

In order to capture asymmetric dependence in the tails, we have to employ a copula which separately parameterizes the left and the right tail. Joe (1997) proposes a copula termed as "BB7", also referred as Joe-Clayton copula. It is a two parameter based copula, given as

$$
C_{JC}(u,v; \tau^U, \tau^L) = 1 - (1 - \{[1-(1-u)^\kappa]^{-\gamma} + [(1-v)^\kappa]^{-\gamma} - 1\}^{-\frac{1}{\gamma}})^{\frac{1}{\kappa}},
$$

where $\tau^U \in (0,1)$ and $\tau^L \in (0,1)$ are the upper and lower tail dependence measure respectively, and

$$
\begin{aligned}
\kappa &= \frac{1}{\log_2(2-\tau^U)}, \\
\gamma &= -\frac{1}{\log_2(\tau^L)}.
\end{aligned}
$$

When $\kappa = 1$, Joe-Clayton copula reduces to Clayton Copula, and when $\gamma \to 0$ it reduces to Joe Copula. Patton (2006) points out that Joe-Clayton copula tends to report asymmetric dependence, even if the dependence in both tails is perfectly symmetric. He proposes a slight modification to the Joe-Clayton copula and terms it as "Symmetric Joe-Clayton" (SJC) copula. It is computed as

$$
C_{SJC}(u,v; \tau^U, \tau^L) = 0.5(C_{JC}(u,v; \tau^U, \tau^L) + (C_{JC}(1-u, 1-v; \tau^U, \tau^L) + u + v - 1).
$$

It treats symmetry as a special case and is consistent in reporting any asymmetry.

2.2 Time-Varying dependence

There is evidence that dependence among financial assets does not stay constant over time (see Bouyé et al. (2008) and Longin and Solnik (2001)). Such dynamics have huge implications from portfolio diversification perspective and can identify how two assets behave jointly in various economic conditions. Given that we are dealing with exchange rates which tend to be highly volatile, we should account for changes in the contemporaneous dependence.

Similar to Patton (2006), we let the dependence parameter evolve according to an ARMA process, both for the Gaussian and the SJC copula dependence parameters. We assume the functional form of the copula remains constant over time, but the copula parameters can evolve with time. As Patton mentions, the problem lies in defining the "Forcing Variable" for the evolution equation, as there is uncertainty to what causes the variation in the parameters. First we define the evolution of the upper and lower tail dependence parameter in the SJC copula. We adopt the same specification as Patton (2006) for both tails, given as,

$$\tau_t^U = \Lambda\left(\omega_U + \beta_U \tau_{t-1}^U + \alpha_U \cdot \frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|\right), \tag{2.1}$$

$$\tau_t^L = \Lambda\left(\omega_L + \beta_L \tau_{t-1}^L + \alpha_L \cdot \frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|\right), \tag{2.2}$$

where $\tilde{\Lambda}(x) \equiv (1 - e^{-x})^{-1}$ is the logistic transformation, which keeps τ^U and τ^L bounded to $(0,1)$. Both (2.1) and (2.2) are similar to an ARMA(1,10) process, where both the upper tail τ_t^U and the lower tail τ_t^L at period t depend upon their respective 1-period lag and a forcing variable for the time-varying limit probability, which is the mean absolute difference between u_t and v_t over the last 10 observations. Different specifications were also tried, but yielded no major improvements, so we adopted the dynamics specified as of Patton (2006). The mean value in both (2.1) and (2.2) is inversely related to the concordance ordering of the copulas, value of zero corresponds to perfect positive dependence, 1/3 corresponds to independence and 1/2 implies perfect negative dependence.

As we saw earlier the Gaussian copula only has one dependence parameter ρ, and we specify

its evolution as

$$\rho_t = \tilde{\Lambda}\Big(\omega_\rho + \beta_\rho \cdot \rho_{t-1} + \alpha \frac{1}{10}\sum_{j=1}^{10}\Phi^{-1}(u_{t-j})\cdot\Phi^{-1}(v_{t-j})\Big), \qquad (2.3)$$

where $\tilde{\Lambda}(x) \equiv (1-e^{-x})(1+e^{-x})^{-1} = \tanh(x/2)$ is the modified logistic transformation required to keep ρ_t in [-1,1] at all instances. The evolution of ρ is similar to the one of SJC copula parameters, where in (2.3) the lag ρ_{t-1} captures any persistency in the dependence parameter. To be able to compare the SJC copula dynamics with the Gaussian copula dynamics a similar MA term is included, which is the mean of the product of the last 10 standard normals obtained through $\Phi^{-1}(u_{t-j})$ and $\Phi^{-1}(v_{t-j})$.

From the non-structural equations (2.1) - (2.3) we can easily compute the 1-step ahead forecast for the dependence measure, as for 1-step forecast we would have the observed x and y (returns in our case) and hence the corresponding u and v. This implies the forcing terms in all the equations can be computed. For the given set-up, we are unable to compute dynamic forecasts, for which we would require to change the forcing terms or be able to first forecast x and y (which would give us predicted u and v), maybe either through a parametric ARMA(p,q) - GARCH(1,1) set-up, or even a multivariate equation which would account for the covariance matrix.

2.3 Marginal Specification

We just stated the specifications related to the copula families to be used in this paper. Both the Gaussian and the SJC copula are parametrically specified. Before a copula is estimated, we need to compute u and v. Let n be the total number of observations, $i = 1,\ldots,n$, and F and G be the marginal distribution function for x and y respectively.

Copula modelling relies upon the assumption that we have i.i.d observations, but daily exchange rate returns tend to exhibit some serial correlation and high volatility. Following previous literature (see Patton (2006), Dias and Embrechts (2010) and Boero et al. (2011)), we first apply an ARMA(p,q)-GARCH(1,1) with normally distributed error term, on each exchange rate return series, through which we obtain the filtered residuals. We consider these filtered residuals to be our filtered returns, where each series now has i.i.d observations. Such filtration

still preserves the contemporaneous dependence among the returns. The order of p and q for the series along with the results are provided in Table 3.

After obtaining the filtered returns, we have to decide the functional form of F and G. In practice, the true marginal distribution function are not completely known, and if F and G are misspecified the employed copula will also be misspecified. An assumed parametric family for each margin requires careful testing for any misspecification. Assuming that all the margins are continuous, we can adopt the approach of Genest et al. (1995), where the margins are left unspecified and computed non-parametrically based on the observed ranks. Boero et al. (2011) adopt the same approach for the filtered returns. So u and v are computed as

$$u = \widetilde{F}(x) = \frac{1}{n+1} \sum_{i=1}^{n} \mathbb{1}(X_i \leq x),$$

$$v = \widetilde{G}(y) = \frac{1}{n+1} \sum_{i=1}^{n} \mathbb{1}(Y_i \leq y),$$

where $\mathbb{1}(.)$ is an indicator function and we divide the summation by $n + 1$ to avoid CDF boundaries. $\widetilde{F}$ and $\widetilde{G}$ are employed for all $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ respectively.

Now we can proceed with the estimation of the copula parameters.

2.4   Estimation

Given that we have specified the copula parametrically and the margins non-parametrically, a semi-parametric estimation technique follows. Let Θ denote the parameter vector associated to a copula C, required to be estimated. The estimation is performed in two steps, first through ARMA(p,q)-GARCH(1,1) filtering and the ranks of the filtered returns, the pseudo observations $u_1, \ldots, u_n$ and $v_1, \ldots, v_n$ are obtained. Then the second step involves maximum likelihood estimation of the pseudo log-likelihood function

$$\widehat{\Theta} = \underset{\Theta}{\operatorname{argmax}} \sum_{i=1}^{n} \log c(\widetilde{F}(x_i), \widetilde{G}(y_i); \Theta),$$

Where c denotes the copula density function. Genest et al. (1995) states the semi-parametric estimator $\widehat{\Theta}$ is consistent and asymptotically normal under suitable regularity conditions. Kim

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et al. (2007) show such an estimator to be robust when the margins are misspecified. Alternatives to the above estimator would be Inference Function for Margins (IFM) and Maximum Likelihood (ML) in fully parametric setting (see Joe (1997)). However in a fully parametric setting, the assumptions on the margins would have to be tested, for example Patton (2006) provides a goodness-of-fit test for the margins. Our approach avoids such issues related to the assumed margins, but it relies on the assumption that the random variables are continues. Kim et al. (2007) show the semi-parametric estimator to be as efficient as ML with sample size larger than 100.

$\widehat{\Theta}$ is the estimated constant copula parameters. We are also interested in capturing any possible dynamics in the dependence parameters through an ARMA process, as described above. The parameters for both the Gaussian copula evolution $(\omega_\rho, \alpha_\rho, \beta_\rho)$ and for the SJC copula evolution $(\omega_U, \alpha_U, \beta_U, \omega_L, \alpha_L, \beta_L)$ are estimated through maximum likelihood. The estimated constant copula parameters act as the starting observations for the time series of dependence.

3   Data

The data consists of daily exchange rates for Deutsch Mark (DM) (later converted at the conversion rate of Euro), UK Sterling (GBP) and Japanese Yen (JPY). All these currencies are denoted against the U.S. Dollar (USD). The full sample is over the period of 1st January 1990 up to 31st December 2009 and collected from Bank of England database[1]. We converted all the series to obtain log-differenced returns.

Three sub-samples are considered from the full sample. First, the Pre-Euro period from 1st January 1990 to 31st December 1998 (2276 observations). Second, the Post-Euro period from 1st January 1999 to 31st December 2006 (2020 observations) and finally the Recent-Crisis period from 1st January 2007 to 31st December 2009 (760 observations). We present time series plots for the three exchange rates in Figure 4 - 6, and summary statistics in Table 2 for the returns series. Time plots for DM(EURO)/USD and GBP/USD, show similar trends throughout the sample, especially in the Post-Euro period, when both currencies heavily appreciate together.

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[1]http://www.bankofengland.co.uk

Table 1: Pair-Wise Linear Correlation

	Pre-Euro			Pre-Euro			Recent-Crisis		
	EURO	GBP	JPY	EURO	GBP	JPY	EURO	GBP	JPY
EURO	1			1			1		
GBP	0.719	1		0.633	1		0.651	1	
JPY	0.500	0.352	1	0.343	0.355	1	0.124	-0.120	1

This is also confirmed by the linear correlation values in Table 1, which shows strong correlation in both exchange rates, even in different sub-samples. JPY/USD on the other hand does not seem to follow any particular trends with DM(Euro)/USD or GBP/USD, and the correlation seems to be much weaker, becoming negative with GBP/USD in the Recent-Crisis period.

Table 2, shows all of the series have skewness and excess kurtosis. DM/USD in the Pre-Euro period has almost zero skewness and GBP/USD in the Post-Euro period has kurtosis of almost 3, but apart from these two cases, none of the other series can be described through a normal distribution. The Jarque-Bera Statistic rejects normality with very large values. The ARCH-LM test, suggests the presence of heteroscedasticity for most of the series, hence it is appropriate for us to employ an ARMA(p,q)-GARCH(1,1) type filtering for the returns series.

4 Copula Results & Economic Interpretation

4.1 Pre-Euro

The constant Gaussian copula reports correlation of 0.71 between DM(Euro)/USD and GBP/USD in Table 4, which is of course similar to the pair-wise linear correlation in Table 1. Such strong correlation is not surprising, as the Pound shadowed closely the Deutsch Mark since 1988 to tackle inflation. The time-varying Gaussian copula in Figure 7 suggests the dependence stayed quite constant among the pair, except by the end of 1996, where there is a slight decline to about 0.4. Such a decline could be due to the interest rate lowering announcement in August 1996 by Bank of England to tackle inflation. Lower interest rate causes investors to shift their funds from GBP (causing depreciation), but DM (Euro) did not get effected by it. The
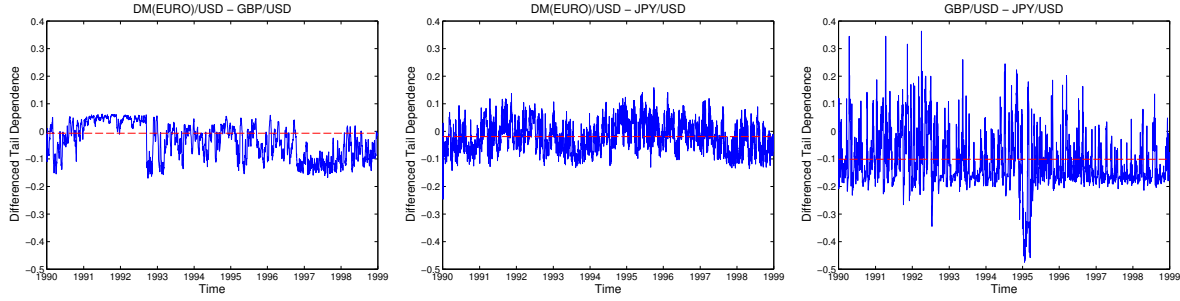
Figure 1: Pre-Euro TV-SJC $(\tau_t^U - \tau_t^L)$ tail differences

constant SJC copula results in Table 4 suggest no asymmetric dependence, as the differenced tail dependence measure $(\tau^U - \tau^L)$ is insignificant at 5%. Although the time-varying tailed differenced series in Figure 1 shows after 1993 the difference in upper tail and lower tail to be negative, this could correspond to greater preference for price stability in the region.

The relationship between DM(Euro)/USD and JPY/USD seems very stable through this period. The constant Gaussian copula reports correlation of 0.52 in Table 4 and the time-varying Gaussian shows no deviation from this level in Figure 7. Such patterns, might be suggestive of the fact that these two countries shared similar economic conditions and foreign trade patterns, which created a unique and constant tie between them. The constant SJC copula measure reports no asymmetric dependence in Table 4, but from Figure 1 we see the difference in the tails of about 0.1. The results indicate the correlation patterns through a Gaussian copula can be appropriately described by a constant measure (similar likelihood in Table 4 and Table 5), the time-varying SJC copula also does not reveal more information about the dependence through this period, than what the constant SJC copula reports.

GBP/USD and JPY/USD are among the most volatile currencies, and due to this volatility investors seek to gain profits from short buying and selling. The correlation is relatively lower compared to the previous pairs above, of 0.37. The constant Gaussian copula correlation does fairly well until 1996, where the correlation drops and reaches the minimum of 0.1. The constant measure of SJC copula in Table 4 suggests the difference between joint upper and lower tails to be -0.1 and significant at 5%, but from Figure 1, we see the difference in the tails is very volatile and changes sign frequently. To associate such changes due to some form of economic policy of one or both of the central banks, would not be suitable. Investors hold

various currencies in their portfolios and take positions which could imply they shift out (joint depreciation) of the two currencies in a similar manner. GBP and JPY are not considered as candidates for being a reserve currency, and investors frequently buy and sell them. Therefore a time-varying copula should be employed in order to provide a more adequate representation of the dependence between these pair of currencies.

Unlike Patton (2006), we report the dependence between DM (Euro)/USD and JPY/USD to be symmetric, but similar to Patton, the time-varying measure of differenced tail dependence is not zero. To remind again we follow a semi-parametric copula estimation, whereas Patton (2006) sets out a fully parametric copula approach and have a slightly larger backdating period.

4.2   Post-Euro

Around the time the Euro was introduced, economic policies among the european countries including Great Britain were strengthened, especially trade policies. The constant Gaussian copula again shows strong correlation of 0.64 between DM (Euro)/USD and GBP/USD, and the time-varying measure in Figure 8 suggests also such correlation to have stayed constant, except a dramatic fall in early 2001, which could be due to pessimism about the newly formed currency causing smaller proportion of DM (Euro) to be held in investors portfolio. The constant SJC copula measure reports a stronger tendency ($\tau_U = 0.36$) towards large joint appreciations ($\tau_L = 0.53$) with respect to USD, than towards joint depreciation. Such a result could be due to the strong bounds created by the EU and the preference for price stability through the EU, rather than export competitiveness. Although from the differenced time-varying SJC copula in Figure 2, we see that at the beginning of the period the difference sometimes in the tails is reversed, later on the lower tail dependence measure exceeds that off the upper tail. Even though the constant SJC copula accurately predicts the directions of the asymmetry, it under predicts the magnitude which at points reaches up to -0.4. This asserts the point even strongly that among a unified EU pricing stability is more preferred as compared to having a preference for being competitive in exports. Also Euro is the currency for most of the European countries, and hence for UK to be competing the rest of Europe is very unlikely. The constant SJC copula does report the right sign on the tail difference in the later half of this period, but the
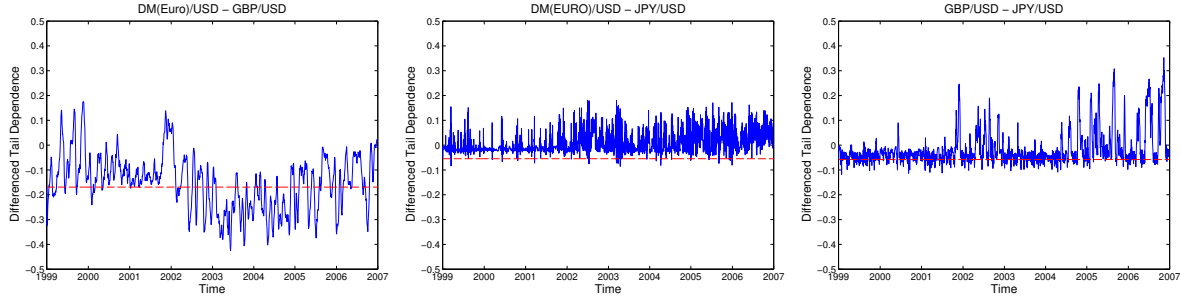
Figure 2: Post-Euro TV-SJC $(\tau_t^U - \tau_t^L)$ tail differences

magnitude is surely not appropriate to represent the period.

The constant Gaussian copula no longer adequately captures the correlation pattern among the DM(Euro)/USD and JPY/USD. Figure 8, shows the correlation goes to negative values in the infancy of Euro. This again could be due to the uncertainty over the newly created currency, and investors regarding Yen as a more secure holding in their portfolios, as compared to Euro. Constant SJC copula measure indicates the tails to be symmetric, but the time-varying SJC copula in Figure 2 shows instances of upper tail dependence being greater than lower tail dependence, which is understandable as Japan is not really part of EU trade treaties and now an export competitive position is preferred. This results is opposite to that off Patton (2006), who report stronger joint appreciation after the Euro's introduction, but their post-euro sample size is only of 2 years. It could be argued that given it is a new currency which needs more time to become stable and hence also the stable correlation among them.

The correlation between GBP/USD and JPY/USD seems more stable and constant, also the time-varying Figure 8 does not show much deviation from the constant level. The constant SJC reports no asymmetry, but this is true for the beginning of the period, but later in the period as we see from Figure 2, there is a greater probability of joint depreciation as compared joint appreciation. The linear correlation for this pair of currencies can be specified through a constant Gaussian copula, but for asymmetric dependence the constant SJC copula fails to capture the variation in the joint tails.

We cannot compare the results here with previous literature, as our sample for post Euro is much longer and unlike other works the correlation/dependence attains stable values after the uncertainty due to the new currency reduces.
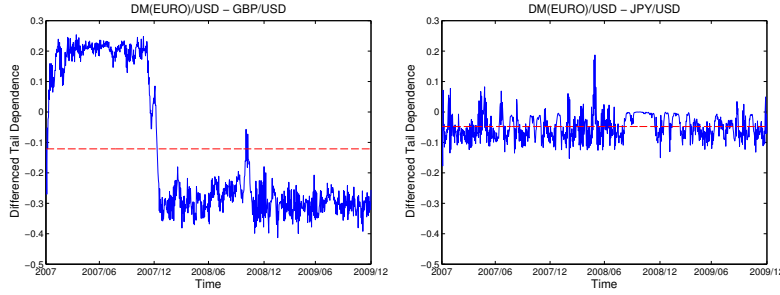
15

Figure 3: Recent-Crisis TV-SJC $(\tau_t^U - \tau_t^L)$ tail differences

4.3   Recent-Crisis

This period represents turmoil and uncertainty from many aspects. Investor do not know what currencies to hold. The crisis originated from the U.S. soon had spillover effects into other major currencies. From the constant Gaussian copula results, we see the correlation between DM (Euro)/USD and GBP was almost the same as in previous periods. The time-varying measure reveals similar constant correlation until the end of 2008 when correlation dropped significantly, this could be associated to bail-outs of the UK banks. The SJC constant copula reveals again a significant (at 5%) asymmetry in the tail, where there is higher probability for these currencies to depreciate together. The time-varying SJC copula shows in Figure 3 that at the beginning of the crisis there is higher probability to depreciate together. The US Dollar appreciated in the beginning of the crisis, which is quite unusual given the crisis originated from there. This was possibly due to short-term interest rate differentials, which investors tried to take advantage of and hence moved away from Euro and Pound. But such directions were reversed as soon as the risk aversion abated. Through such times price stability in the EU was strongly among the agenda, and therefor we see a much stronger probability of joint appreciation between DM (Euro)/USD and GBP/USD.

Between DM (Euro)/USD and JPY/USD, the correlation fell to 0.12. The time-varying Gaussian copula confirms this in Figure 9. By the mid 2008, the correlation becomes very volatile, which could be due to investors trying to seek safe portfolio holdings. The constant and time-varying SJC copula indicates no asymmetries in the tails.

The correlation between GBP/USD and JPY/USD became negative, −0.12. This is also confirmed in the time-varying Gaussian copula case, the correlation patterns in late 2008 is

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similar to the correlation between DM (Euro)/USD and JPY/USD, indicating similar positions for Euro and Pound as compared to Yen. The constant and time-varying SJC copula are not reported, due to zero tail dependence found. Constant copula fails to address the extent of negative correlation in late 2008.

In terms of the best copula specification, we see the time-varying SJC copula has the highest log-likelihood value in all the sub-samples. As all sub-samples are large, the Akaike Information Criteria reports the same best fitting copula.

Overall, we discussed reasons for observing dependence patterns for the currencies considered, though there could be many more reasons for observing asymmetric dependence patterns. We have not discussed the role of US Dollar, which through out the years has served investors as a reserve currency and movements to/from US Dollar to other currencies might not be the same. Exchange rate not only serves as an economic tool for policy implementation, they are also considered an asset along with other stock assets. But unlike other financial assets, investors hold projections over economic conditions which lead them to hold specific holdings on currencies, and this could create complex dependence patterns. We need to use the time-varying measure to have a full understanding of the dependence structure.

5 Conclusion

Various currencies are related to each other due to economic interaction among countries and how they are held in investor's portfolio. Their relationship in various economic conditions not only can reveal vital information to policy makers, but can also provide insight to investors for diversification purposes.

Given non-normality of daily exchange rates and joint non-linear dependence among exchange rate returns, we adopt a semi-parametric copula approach which overcomes the short-comings of multivariate Normal and t-distribution. Our approach is similar to Patton (2006) and Dias and Embrechts (2010), but unlike them we do not assume any parametric distribution for the marginals. Along with a parametric copula we specify the marginals to be non-parametric. Such a specification is robust to any misspecification of the marginals. Genest

et al. (1995) show an estimator based on the ranks of the observed data is efficient and asymptotically normal for continuous data. Kim et al. (2007) report such a specification is robust to any misspecification of the marginals. Boero et al. (2011) employ a similar technique, but to estimate constant dependence only. We extended their approach to study dependence in a time-varying case.

We examine the dependence pattern between DM (Euro)/USD, GBP/USD and JPY/USD in different economic conditions. From using the Gaussian copula and the SJC copula, we see varying patterns of dependence in period before introduction of Euro, after and the most recent financial crisis.

We show linear correlation measures do not reveal dependence completely and to capture any possible asymmetric tail dependence we should adopt a two parameter copula like the SJC copula. In the Pre-Euro period DM (Euro)/USD and GBP/USD are highly correlated and such correlation persists through the other sub-samples. A time-varying analysis however shows that there are periods when the correlation weakens. From measuring asymmetric tail dependence, we find that the constant SJC copula fails to capture the variation in the joint tails, as there seems to be some pairs which have a higher probability to jointly appreciate as compared to probability of joint depreciation during different sub-samples. For DM (Euro)/USD and JPY/USD, correlation is quite constant as confirmed by the time-varying measure. There does not seem to be any particular preference from central banks to create export competitive environment or create price stability. The relationship between GBP/USD and JPY/USD seems very volatile through all the samples, and there is asymmetric tail dependence which the constant SJC copula does not completely capture. After Euro's introduction, DM (Euro)/USD and GBP/USD become more dependent when they jointly appreciate, reflecting the preference for price stability of both central banks, this is understandable as EU has trade policies in place, which are very co-operative and protect EU countries. Although there are certain periods (early Recent-Crisis period), where the probability to jointly depreciate is higher than probability to jointly appreciate, this could be due to shifting of funds into USD from both currencies. Both DM (Euro)/USD and GBP/USD have a similar stance towards JPY/USD, and hence the correlation between DM (Euro)/USD and JPY/USD and GBP/USD and JPY/USD show similar

patterns.

We show how dependence evolves over time and assuming a constant dependence measure fails to capture the variations. The whole analysis is performed in a setting which ensure no misspecification of the marginal behaviours (distributions). We also show how dependence patterns change with different economic conditions.

Table 2: Sample Statistics

	Pre-Euro			Post-Euro			Recent-Crisis		
	DM	GBP	JPY	EURO	GBP	JPY	EURO	GBP	JPY
Mean	−0.001	−0.001	−0.011	−0.005	−0.008	0.003	−0.011	0.025	−0.033
Median	0.0006	−0.013	0.019	0.000	−0.014	0.017	−0.029	0.020	0.010
Std.Deviation	0.666	0.607	0.736	0.620	0.510	0.622	0.727	0.814	0.811
Skewness	0.055	0.239	−0.787	−0.126	−0.083	−0.253	−0.257	0.073	−0.730
Kurtosis	5.244	6.328	9.147	4.391	3.748	4.546	8.253	7.330	6.811
Jarque-Bera Statistic	478.6	1007	3819	168.3	49.34	222.6	882.3	594.3	527.4
Arch-LM Statistic	77.29	114.8	110.4	4.572	7.377	7.706	74.00	43.60	34.04
Number of Observations	2276	2276	2276	2020	2020	2020	760	760	760

Note: ARCH-LM Test employed for presence of heteroscedasticity at 5 lags.

Table 3: ARMA(p,q)-GARCH(1,1)

	Pre-Euro			Post-Euro			Recent-Crisis		
	DM	GBP	JPY	EURO	GBP	JPY	EURO	GBP	JPY
C	−0.002	−0.004	−0.012	−0.012	0.004	−0.012	−0.033*	−0.024	−0.004
	(0.013)	(0.014)	(0.011)	(0.013)	(0.013)	(0.011)	(0.019)	(0.027)	(0.020)
AR(1)	—	—	0.074***	—	—	—	—	—	—
			(0.021)						
AR(3)	—	−0.040*	—	—	—	—	—	—	—
		(0.022)							
GARCH Constant	0.007***	0.011***	0.002**	0.003	0.009***	0.005**	0.001**	0.006**	0.002**
	(0.002)	(0.002)	(0.001)	(0.002)	(0.003)	(0.002)	(0.000)	(0.003)	(0.001)
ARCH Term	0.044***	0.067***	0.040***	0.020***	0.026***	0.032***	0.053***	0.069***	0.062***
	(0.006)	(0.007)	(0.004)	(0.005)	(0.006)	(0.008)	(0.009)	(0.013)	(0.012)
GARCH Term	0.948***	0.914***	0.954***	0.972***	0.950***	0.947***	0.947***	0.927***	0.937***
	(0.008)	(0.008)	(0.005)	(0.008)	(0.012)	(0.013)	(0.006)	(0.011)	(0.011)

*** - 1%, ** - 5% and * - 10% significance level. GBP/USD and JPY/USD for the Pre-Euro period required the 1st and the 3rd lag, respectively.

Table 4: Constant Copula Results for DM-GBP, EURO-JPY and GBP-JPY

		Pre-Euro			Post-Euro			Recent-Crisis		
		DM-GBP	DM-JPY	GBP-JPY	EURO-GBP	EURO-JPY	GBP-JPY	EURO-GBP	EURO-JPY	GBP-JPY
Gaussian Copula	(ρ)	0.717***	0.516***	0.377***	0.635***	0.360***	0.355***	0.635***	0.116***	−0.103***
		(0.007)	(0.013)	(0.017)	(0.011)	(0.018)	(0.018)	(0.017)	(0.036)	(0.036)
	LL	−817.0	−349.5	−172.3	−520.4	−140.0	−142.31	−196.5	−5.168	−4.013
SJC	(τ^U)	0.547***	0.296***	0.133***	0.360***	0.161***	0.160***	0.404***	–	–
		(0.016)	(0.025)	(0.030)	(0.028)	(0.032)	(0.031)	(0.039)		
	(τ^L)	0.554***	0.315***	0.231***	0.530***	0.215***	0.218***	0.526***	0.064	–
		(0.015)	(0.025)	(0.026)	(0.016)	(0.029)	(0.029)	(0.026)	(0.045)	
	$(\tau_t^U - \tau_t^L)$	−0.007	−0.019	−0.101***	−0.169***	−0.005	−0.058*	−0.121***	−0.064	–
		(0.019)	(0.029)	(0.032)	(0.027)	(0.034)	(0.034)	(0.041)	(0.045)	
	LL	889.3	338.3	167.4	560.3	147.2	152.3	221.8	11.73	–

*** - 1%, ** - 5% and * - 10% significance level.

Table 5: Time-Varying Copula Results

	Pre-Euro			Post-Euro			Recent-Crisis		
	DM-GBP	DM-JPY	GBP-JPY	EURO-GBP	EURO-JPY	GBP-JPY	EURO-GBP	EURO-JPY	GBP-JPY
Normal									
Constant	-0.342^{***}	0.645	-0.020^{***}	0.020^{***}	0.015^{*}	0.001	0.335^{*}	0.396	-0.225
	(0.000)	(0.800)	(0.001)	(0.007)	(0.008)	(0.006)	(0.194)	(0.136)	(0.154)
α	0.107^{***}	-0.054	0.018^{***}	0.282^{***}	0.163^{***}	0.081^{***}	-0.158^{***}	0.762	0.557^{***}
	(0.000)	(0.066)	(0.001)	(0.031)	(0.028)	(0.020)	(0.052)	(0.153)	(0.169)
β	2.857^{***}	1.030	2.123^{***}	2.114^{***}	1.962^{***}	2.052^{***}	2.095^{***}	-1.763	-1.875^{***}
	(0.012)	(1.487)	(0.008)	(0.025)	(0.041)	(0.033)	(0.239)	(0.186)	(0.167)
LL	827.0	350.1	179.0	583.4	183.0	108.5	213.4	17.94	9.470
SJC									
Constant(U)	1.130^{***}	1.505	3.650^{***}	-1.526^{***}	4.658^{***}	-0.253	-1.943^{***}	3.956	—
	(0.145)	(1.275)	(0.905)	(0.421)	(0.703)	(0.497)	(0.041)	(2.601)	—
$\alpha(U)$	0.911^{***}	-1.255	-3.014^{***}	3.684^{***}	-3.149^{***}	2.615^{***}	4.365^{***}	2.249	—
	(0.150)	(2.022)	(0.709)	(0.511)	(0.933)	(0.931)	(0.082)	(3.750)	—
$\beta(U)$	-9.471^{***}	-9.158^{***}	-22.65^{***}	-2.284^{*}	22.62^{***}	-7.637^{***}	-1.563^{***}	-23.13	—
	(0.127)	(3.667)	(4.659)	(1.303)	(2.762)	(2.029)	(0.372)	(9.380)	—
Constant(L)	1.984^{***}	2.059^{***}	-2.009^{***}	-1.993^{***}	2.355^{***}	1.256	2.842^{***}	3.919	—
	(0.062)	(0.496)	(0.032)	(0.012)	(1.167)	(1.289)	(0.629)	(0.730)	—
$\alpha(L)$	-0.636^{***}	-4.192^{***}	4.300^{***}	4.091^{***}	-1.307	-1.310	-3.451^{***}	-3.802	—
	(0.139)	(0.278)	(0.052)	(0.076)	(1.549)	(2.265)	(0.952)	(0.709)	—
$\beta(L)$	-8.582^{***}	-6.673^{***}	-0.761	-0.283^{***}	-14.52^{***}	-9.561^{***}	-5.061^{**}	-18.28^{***}	—
	(0.675)	(2.344)	(0.192)	(0.017)	(4.022)	(3.642)	(2.534)	(3.024)	—
LL	1024	355.60	189.4	668.3	224.3	188.1	251.67	34.00	—

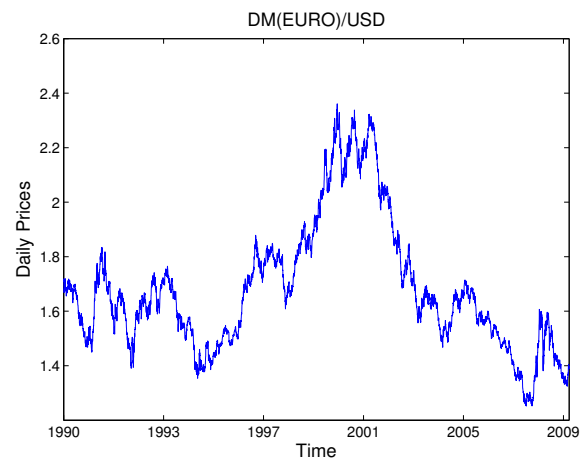*** - 1%, ** - 5% and * - 10% significance level.

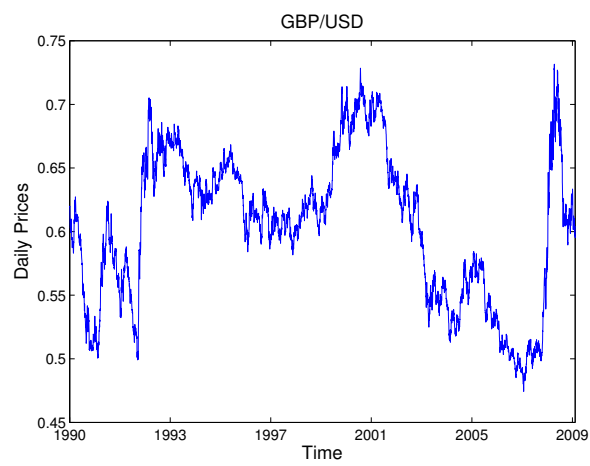Figure 4: Daily DM/USD Exchange Rate

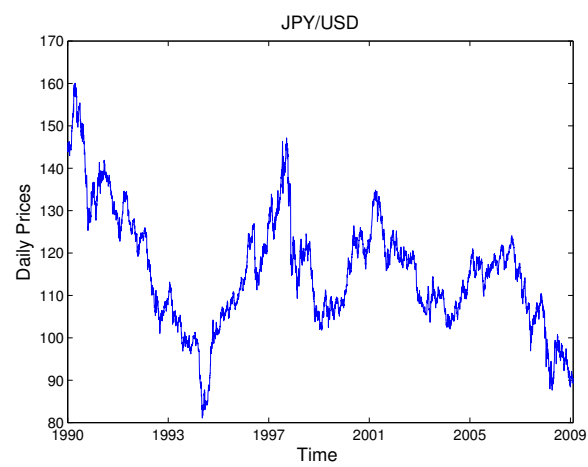
Figure 5: Daily GBP/USD Exchange Rate

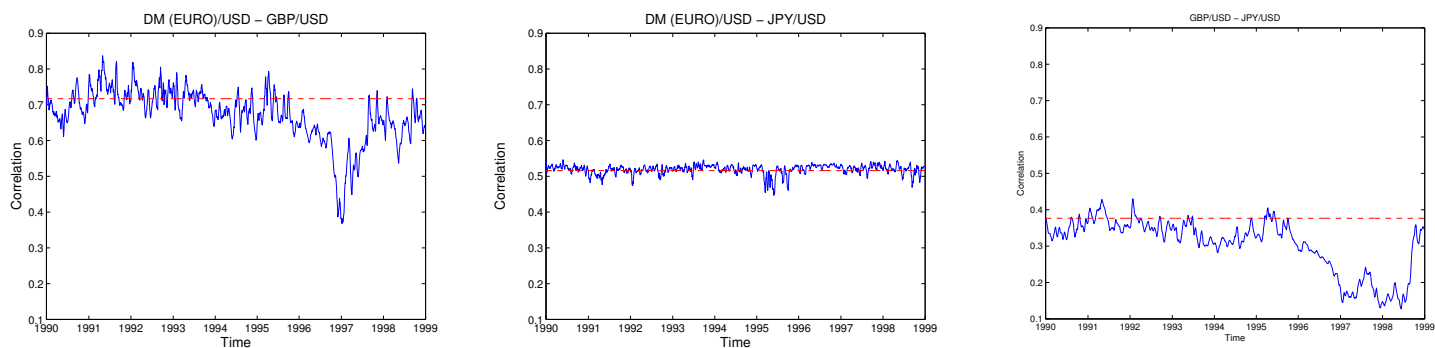Figure 6: Daily JPY/USD Exchange Rate

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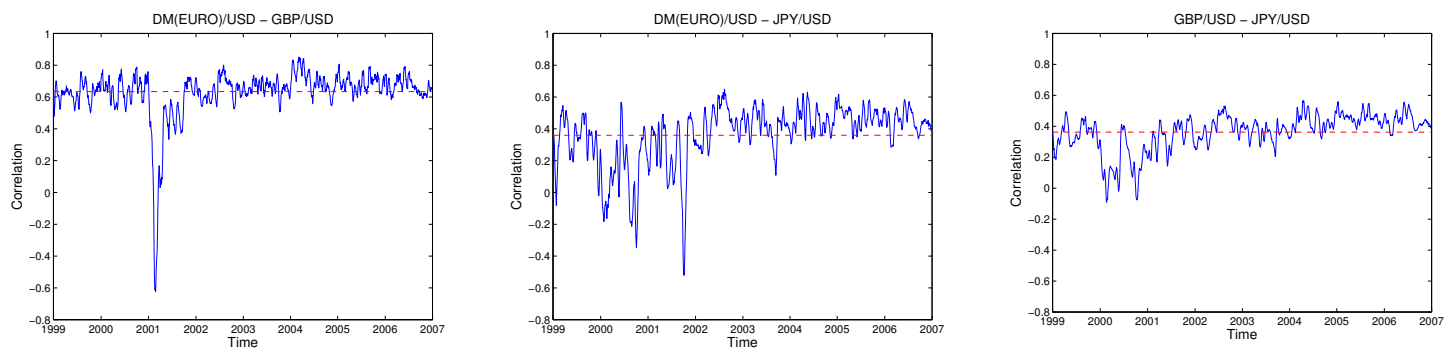Figure 7: Pre-Euro TV Gaussian Copula
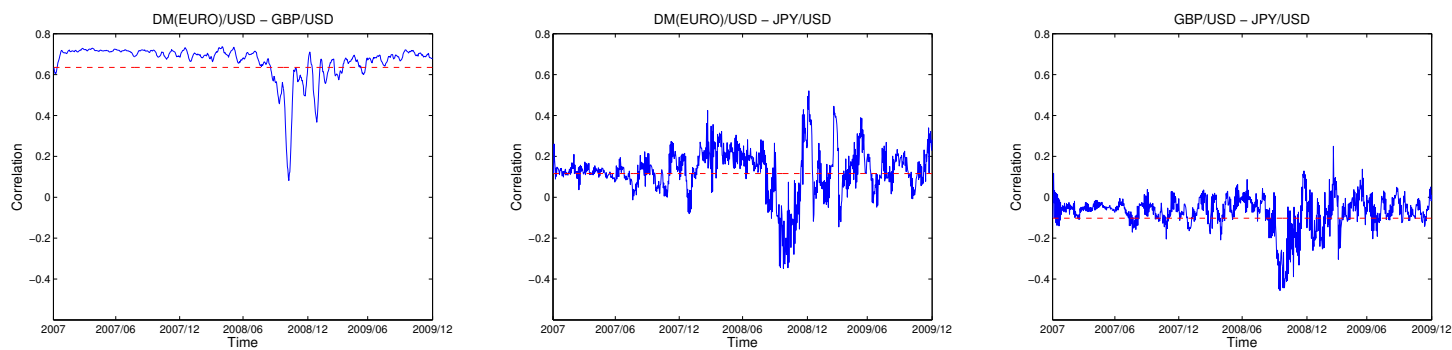

Figure 8: Post-Euro TV Gaussian Copula

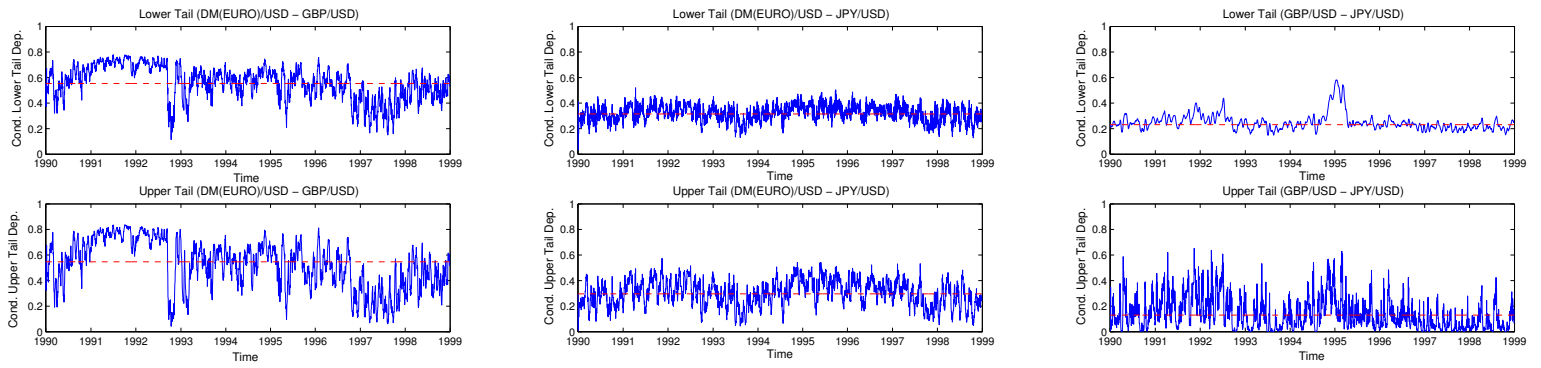Figure 9: Recent-Crisis TV Gaussian Copula

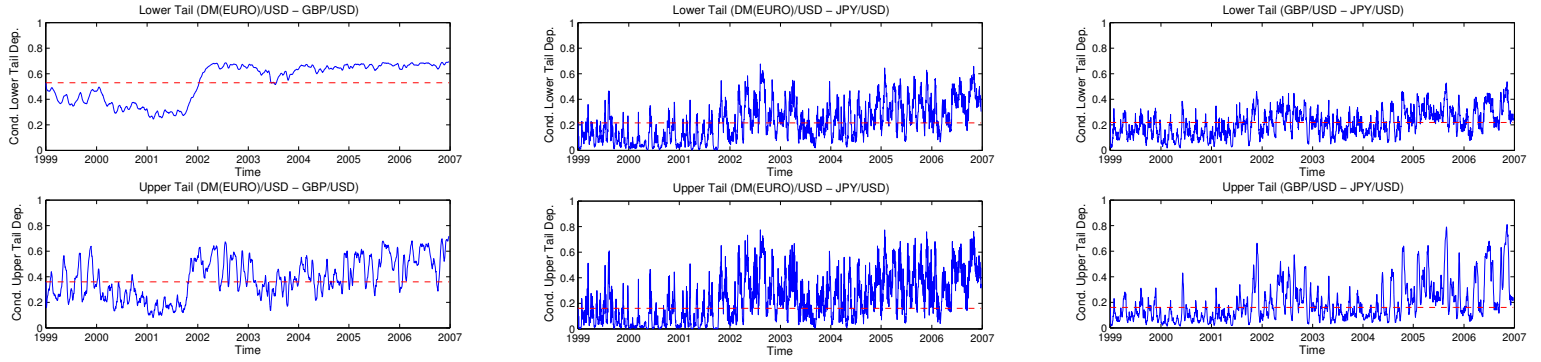Figure 10: Pre-Euro TV SJC Copula

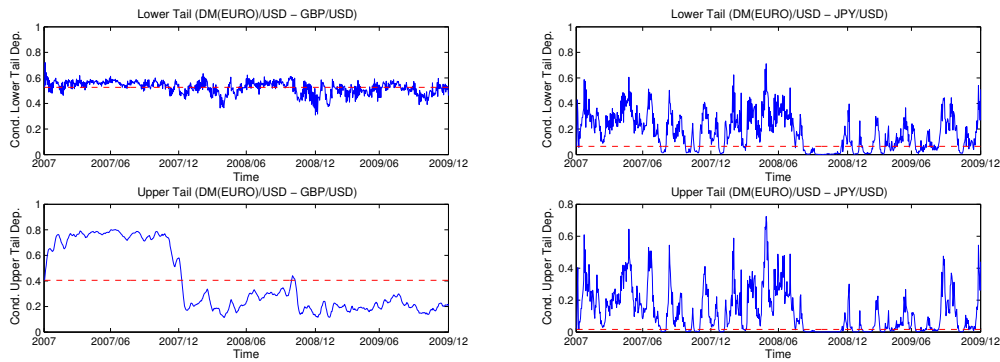Figure 11: Post-Euro TV SJC Copula

Figure 12: Recent-Crisis TV SJC Copula

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