Stochastic Technology in a Programming Framework: A Generalized E. V. Model

by

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Abstract

Production uncertainty is important in studying the behavior of risk averse firms and the making of successful agricultural policies. A model which extends the E-V method to incorporate stochastic technology in a programming framework is developed. Risk effects of factor inputs are measured for the irrigated multi-crop sector of Sudan. Supply responses of agricultural producers are more elastic when factor inputs are allowed to influence production risks.

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Uncertain decision environments have several implications on the behavior of the competitive firm and associated policy issues. Random prices (market risks) and stochastic production processes are commonly identified as the main sources of uncertainty for producers. Considerable space in the economics literature has been devoted to studying the impact of market risks on output supply and factor demands (Baron 1970, Sandmo 1971, Leland 1972, Batra and Ullah 1974). Various policies and institutions have been developed in the face of market risks, e.g. insurance, hedging, future contracts etc.

On the other hand, less attention was given to production uncertainty, in spite of its significant behavioral and policy implications (Just and Pope 1978, Pope and Kramer 1979). While market risks are exogenous, production risks are internal to the firms' decision problem. There is empirical evidence, particularly in agriculture that moments higher than the mean (variance and skewness) of the distribution of output are functions of input levels (Fuller 1965, de Janvry 1972a, Just and Pope 1979, Griffith and Anderson 1982, Antle and Goodger 1984). Consequently, the very instruments and production strategies used for risk management in agriculture such as fertilization, crop rotations, pest control etc. have uncertain effects on farm output. While diversification reduces market risks its impact on yield variability could be unfavorable. Moreover, most of the policies and innovations prescribed to developing countries to enhance productivity and promote agricultural development, such as the green revolution packages of high yielding varieties, fertilization and mechanization were based on their yield effects merits, whereas their risk effects were not considered. If farmers respond to risk (have a risk preference), then the rate of adoption and diffusion of such farming methods and technologies is dependent on their risk effects as well as their yield effects. It is therefore essential for a better understanding of producers' behavior and the making of sound and successful agricultural policies, to adequately investigate and quantify the relationship between factors of production and the upper moments of the distribution of crop yields.

The Mean-Variance (E-V) model is commonly used as a conditionally normative tool to study the behavior of farm firms under uncertainty. While the E-V method minimizes the variance of the firms' portfolio of enterprises, the individual variances of the production activities in the mix are constants (Markowitz 1959). Though the second moment of the distribution enters the objective function in the E-V formulation it does not account for the risk effects of factor inputs. To provide for the risk effects of factor inputs, Antle (1983) proposed a flexible moment-based econometric (FMB) approach to characterize the decision problem of risk averse producers under production uncertainty. Due to some limitations of econometric methods in handling multi-output supply structures, the FMB method has been applied in a single output framework (Just, Zilberman and Hochman 1983, Antle 1984, 1987).

This paper suggests an alternative method for analyzing the supply behavior of the multi-product firm under production uncertainty and risk aversion. The method extends the standard E-V programming model to incorporate stochastic technology in its structure. In this case the variances of the individual enterprises and hence the variance-covariance matrix of the portfolio of expected returns are functions of the levels of factor inputs and no longer constants. The model is used to study the supply behavior of farmers in the irrigated multi-crop sector of Sudan (Rahad Scheme) under production uncertainty. Iterative non-linear 3 stage least squares (IN3SLS) and multi-stage nonlinear generalized least squares (MNGLS) are used to estimate the parameters of the first two moments of the distribution of crop yields. Stochastic production functions are specified to allow for risk increasing as well as risk decreasing input effects. This production technology is then used in a conditional, normative analysis of the firms utility maximization process assuming mean-variance preferences.

The next section discusses the production technology. Section two develops the programming model using the stochastic technology. Section three discusses the area studied and crops considered. The empirical model and results are discussed in section four. Section five summarizes the findings and concludes the study.

Stochastic Production Function Estimation

Conditional probability distribution functions can be used to summarize the risk attributes of a random production technology (Day 1965, and Anderson 1973):

$$F(Y|X, \beta) \tag{1}$$

This formulation assumes that part of the variability in the random output (Y) can be explained by controllable factors, such as input levels (X).

Alternatively, a single function of output can be used to represent stochastic technology. In this form, a random disturbance term is appended to a deterministic neo-classical production function:

$$Y = F(X, \beta, \epsilon) \tag{2}$$

Alternative specifications of the stochastic component (ϵ) lead to different production function representations and behavioral implications.

Endogeneity of factor inputs and the simultaneous equations bias problem encountered when working with survey data have been discussed using log-linear and multiplicative disturbance forms of model 2 (Marschak and Andrews 1944, Mundlak and Hock 1965, Zellner, Kmenta and Dreze 1966).

Concerned with the risk effects of factor inputs, Just and Pope (1978) found that several popular formulations of stochastic production functions are very restrictive. They show that the main deficiency of the common forms is the implication that all factors are risk increasing and thus all are used less under risk aversion. A more flexible form is proposed in Just and Pope (1978):

$$Y = F(X, \beta) + H(X, \alpha)\epsilon \tag{3}$$

The stochastic specification in (3) defines separate effects of the decision variables (X) on the deterministic (F) and random (H) components of production. This heteroscedatic, additive-error form, allows for risk increasing and risk reducing as well as zero risk effects of factor inputs $(\frac{\partial H}{\partial x_i} \gtrless 0)$. Given that (ϵ) is an independently distributed random variable with zero mean and variance σ, model (3) has the following moments:

$$E[Y] = F(X, \beta) \tag{4}$$

$$V(Y) = H' \sigma H = H_i \sigma_{ij} H_j \quad \forall\ i, j = 1,\dots,n \tag{5}$$

where V(Y) is the (nxn) variance-covariance matrix of crop yields.

For the multi-product firm, jointness in production is usually modelled with the implicit production frontier:

$$G(Y, X) = 0 \tag{6}$$

where Y is a (kx1) vector of outputs and X is a (KxJ) matrix of input allocations (X_{kj}). Jointness holds when there are technical interdependencies among outputs or in presence of allocable fixed resources (Pfouts, 1961). It is assumed that technological externalities do not exist within the farming system under study, e.g. the production of any crop k is not directly dependent on other products or inputs used in them (output determination). Crops grown, on the other hand, compete for a fixed amount of some elements in X, and hence jointness still holds.

Just, Zilberman and Hochman (1983) point out that under the assumptions of allocated inputs, physical constraints and output determination, there is a function G_k that uniquely determines output Y_k from inputs X_{kj} when the technology is nonjoint in inputs. Using the implicit function theorem, model 6 can be written, under these assumptions, as:

$$Y_k = G_k (X_k) \qquad k = 1,\ldots,K. \tag{7}$$

where outputs are linked only by the fixed input constraints given in:

$$\sum_k X_{kj} = X_j \qquad j = 1,\ldots,D \qquad \text{where D indexes a subset of X.} \tag{8}$$

The fact that the three assumptions of allocated inputs, physical constraints and output determination hold for many decision problems including the case in this study, plus availability of input allocations by crop (X_k), made model 3 appropriate for estimating the production technology of the Rahad farmers, where G_k is specified to follow the Just-Pope technology:

$$G_k = F_k(X_k, \beta_k) + H_k (X_k, \alpha_k) \epsilon_k \tag{9}$$

Just, Zilberman and Hochman propose estimation of production function and supply response parameters by econometric estimation of the first order conditions for profit maximization using price and quantity data. The approach taken here is to directly estimate the production function and then use optimization techniques in a normative approach to supply analysis. Jointness due to allocable fixed resources is provided for by assuming cross-equation correlations in the econometric estimation of parameters of model 9. The programming model imposes the physical constraints and the estimated covariance structure of residuals on the model's solutions to account for jointness in production.

A third approach to analyzing producers risk behavior under production uncertainty is the FMB method of Antle (1983). The FMB approach postulates distribution functions to represent random production, revenue or profits. An expected utility of profits model is then approximated by a Taylor's series expansion, where expected utility is expressed as a function of moments of the distribution of profits and risk preferences. Econometric methods are then utilized to estimate the parameters of the model from a system of first order condition equations for the expected utility maximization problem.

The difficulties associated with econometric specification of joint production structures precluded the use of the FMB approach in multiple-output framework analyzed here. Restrictive assumptions such as stochastic nonjointness must be invoked in order to validate the use of single output data to estimate the parameters of the moment based model (Antle 1984, 1987). The method proposed in this paper to study the supply behavior of multi-product firms under production uncertainty utilizes the advantages of normative methods over econometric techniques in handling jointness in inputs and outputs (Shumway et al. 1984) and is developed in the next section.

A Mean-variance Model with Stochastic Technology

An average farm programming model is developed in this section to characterize the decision problem of agricultural producers under production uncertainty. Crop yields are stochastic and assumed to be the only source of risk in the model. Prices are non-random and independent of the individual farmers' actions. This is consistent with the environment in many developing countries when prices are set by the government and is certainly the case for the study at hand. The model could be generalized to handle price uncertainty in a straightforward manner. The average farm model assumes identical preferences and risk aversion as opposed to a distribution of risk preferences (Antle 1987). The farmers' objective criterion is to maximize the expected utility of net returns to the fixed resources of the farm firm. The two moment (E-V) model postulates the following objective function:

$$U(N) = E[N] - \frac{1}{2} \lambda V(N) \tag{10}$$

where expected utility U is a weighted function of the mean--E[N] and variance--V(N) of the random returns (N). The weight is the coefficient of absolute risk aversion--(ARA). Constant ARA and normality of (N) are maintained by the structural nature of the model (Markowitz 1959, Freund 1965). The E-V model has been extensively used to study supply behavioral under risk aversion (Hazell 1971, Wiens 1976, Kramer et al. 1983, Chen 1973). Net farm returns (N) is defined to be the sum of the net returns to individual enterprises:

$$N = A'R \tag{11}$$

where A and R are respectively (nxl) vectors of crop areas and net returns per unit area. And:

$$R = PY \tag{12}$$

where P is an (nxn) diagonal matrix of net returns per unit of output and Y is an (nxl) vector of output per unit area. Y represents the random yield functions discussed in section one and is given by:

$$Y = F(X, \beta) + H(x, \alpha)\epsilon \quad (13)$$

$$\text{with } E[\epsilon] = 0 \text{ and } V(\epsilon) = \sigma$$

where 0 is an (nx1) null vector and σ is an (nxn) variance-covariance matrix of the yield functions residuals e.g. $\sigma_{ij}\ \forall\ i, j = 1,\ldots,n$.

Combining equation 13 with equation 12 gives:

$$E[R] = P\ F(x, \beta) = R(x, \beta, P) \quad (14)$$

$$V(R) = P\ V(Y)\ P = V(x, \alpha, P) \quad (15)$$

Then according to (7) expected returns are given by:

$$E[N] = A'R(x, \beta, P) \quad (16)$$

while return variance is given by:

$$V(N) = A'\ V(x, \alpha, P)\ A \quad (17)$$

The farmers' decision problem can thus be defined as the maximization of the expected utility function in (10) subject to the stochastic technology constraints, in 16 and 17, plus the other institutional and resource constraints of the system:

$$\Sigma_i\ G_{ij} \leq G_j \quad i = 1,\ldots,n\ ;\ j = 1,\ldots,m. \quad (18)$$

where G_{ij} is the amount of resource j used in the production of crop i, and G_j represents the upper limit of the total amount of resource j available. In this formulation of the E-V model it is clear that the mean and variance of net farm returns are functions of factor inputs as shown in 16 and 17, e.g. the variance-covariance matrix of net returns is no longer constant. In a compact form the decision problem is written as:

$$\text{(19)} \qquad \max_{x,\,A} U = A'R - \frac{1}{2}\lambda A' V A$$

$$\text{subject to:} \quad R = P\,F(x, \beta)$$

$$V = P\,V(Y)\,P$$

$$V(Y) = H'(x, \alpha)\,\sigma\,H(x, \alpha)$$

$$\Sigma_i\, G_{ij} \le G_j$$

$$x, A \ge 0 \qquad \forall\ i = 1,\ldots,n\ ;\ j = 1,\ldots,m \text{ and}$$

variables are defined as in (10-18) above.

The Farming System of the Rahad Scheme

The Rahad Scheme is the second largest irrigated scheme in Sudan. Three hundred thousand feddans(1) of cotton, groundnuts and sorghum are grown under regular irrigation and mechanical power. A fixed cropping pattern is imposed on the 22 feddan tenancies allotted to farmers. Levels of most of the production inputs are determined by the Rahad Corporation (land, water, seeding rates, chemical and mechanical inputs). The farming families on the other hand allocate their labor, entrepreneurial and working capital resources among the various production activities of the three crops. Labor hiring is an important activity controlled by tenant farmers. Crop yields are responsive to the quantity and quality of hired and family labor and managerial resources under the farmers control.

The stochastic specification in (9) is employed to represent the multicrop production technology of the Rahad tenants. Three yield equations are specified for the three crops grown. Five factors are considered variable, namely hours of family (FL) and hired (HL) labor, weeding and harvesting and sowing dates (SD). Years in farming the crop (FR) is treated as a fixed

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(1) One feddan is approximately one acre.

input. Other inputs are considered fixed across farmers. Farm firms are assumed to use the same production technology across the scheme.

Estimation of the Model's Parameters

Production Technology. The three equation model in (9) is fit to data collected from a sample of 54 farmers collected during the 1984/85 season. Flexible functional forms are utilized in estimating relation (9).

Several forms of the generalized power production function -- GPPF (de Janvry, 1972b) are estimated for the three crops. The general form of the GPPF is:

$$Y_k = a_k \prod_{i=1}^{n} x_{ik}^{f_{ik}(x)} \quad g_k(x) \qquad (20)$$

where: x is a vector of factor inputs

Y_k is the kth output

x_{ik} is the ith factor in the production of crop k.

Admissible forms that gave the best statistical fit are presented in Table (1). Specification tests were not used but determination was based on simple tests on individual coefficients. The Cobb-Douglas structure was accepted for groundnuts, where $g(x) = 1$ and $f_i(x_i) = \alpha_i$. The transcendental form ($g(x) = \Sigma_i \beta_i x_i$ and $f_i(x_i) = \alpha_i$) was chosen for cotton and the Cobb-Douglas with variable elasticities of substitution ($g(x) = 1$) form was the best for sorghum. An iterative non-linear least squares procedure was employed to estimate the parameters of the mean and variance of the yield functions of the system (9). This corrects for heteroscedasticity, possible endogeneity of factor inputs and cross-equations correlations from jointness in production (Hassan, Hallam and D'Silva 1987b).

While the endogeneity problem associated with production function estimation does not occur if producers maximize expected profit (Zellner, Kmenta and Dreze) or expected utility of profit (Blair and Lusky), both IN3SLS and MNGLS were used to estimate the parameters to check for possible endogeneity problems. As would be expected estimates differed little between methods implying either expected profit or expected utility maximization.

The estimated equations for the mean crop yields are given in Table (1). While weeding and harvesting labor could be aggregated in cotton and sorghum, separability tests indicated that it should not be aggregated in groundnuts. The residual covariances of the econometric estimation of model (4) are used as consistent-estimates of the yield covariances (V_{ij}). The covariance estimates are given in part B of Table (1).

The covariances of the yield functions (the off-diagonals of V(Y) in (5) e.g. $V_{ij} = H_i \sigma_{ij} H_j \; \forall \; i \neq j$) are non-zero but assumed to be constants and independent of the X's. However, the diagonal elements of V(Y) e.g. $V_{ii} = H_i^2 \sigma_{ii}$, are functions of input levels (X) as measured by H in (4). Hired labor is found to increase production risks in cotton and sorghum--where cash wages are paid, whereas it is risk decreasing in groundnuts production--where share cropping prevails. The reverse is true for family labor.

A land constraint is specified such that the total area under cotton (C), groundnuts (G) and sorghum (D) must not exceed the tenancy size (22 feddans). The family (F) and hired (H) labor constraints are similarly constructed as shown in Table 1. Total number of family man hours available are calculated on the basis of the average effective family size (250 hrs./feddan). The highest level of hired hours observed is considered the average upper limit of hired labor per feddan (460 hrs./feddan). Average net returns per unit of output (P_i) are calculated by deducting the cost of operations and factors other than land and family labor from the ruling market price. Labor hiring is allowed at the average wage rates calculated in Table 1 part E.

Programming Model. The General Interactive Optimizer (GINO) was used to solve the programming model. Different solutions to the model were generated by varying the coefficient of ARA (λ). Simulated solutions were compared to the actual farm plans. The value of (λ) that best simulated observed choices of the farmers was found to be 0.001 (Hazell 1971). A similar coefficient was estimated by Hassan, D'Silva, and Hallam (1987a) for dry land traditional farmers in the Sudan. This implies that risk preferences of agricultural producers in the irrigated commercial agriculture and dry land subsistence farming are not significantly different.

To exemplify the importance of risk effects in supply response, discrete supply responses were generated by varying the output and input price vectors. Continuous partial supply and demand equations were then obtained from the step response functions by regression analysis. The following functional form was estimated to study own and cross-price effects.

$$Q = aP^{b}W^{d} \tag{21}$$

where Q refers to outputs and inputs quantities, P represents output prices, W is a wage rate and a, b and d are parameters of the response functions. Equation (21) was estimated for output supply and labor input demands (Table 2). Output responses were generated by varying crop prices simultaneously while maintaining the wage rate fixed. Input demands, on the other hand, are conversely derived.

While this is not the long run equilibrium supply equation (as some prices are fixed), it measures partial supply adjustments when risk effects of factor inputs are taken into consideration (Pope, Chavas and Just 1983). The estimated elasticities are not directly interpretable, due to the variance component of the risk averse firm supply response structure, but represent elasticities along a risk adjusted supply curve.

The above response functions were estimated for different behavioral and structural specifications to analyze the risk effects of factor inputs or the changes in the supply function when risk behavior is included. Three scenarios were employed to represent different specifications of the model.

1. Risk neutrality (RN): $\lambda = 0$ and thus effects of factor inputs on the second moment of the distribution of returns do not affect farmers decisions.
2. Risk aversion and zero risk effects (RA1): λ is positive but farmers' actions cannot influence production risks. In other words the diagonal elements as well as the off-diagonals in the covariance matrix (V) are constants.
3. Risk aversion and non-zero risk effects (RA2): λ is positive and factor inputs have non-zero risk effects. In this scenario farmers are allowed to alter production risks by optimally choosing input levels, e.g. the diagonal elements of the (V) matrix are functions of input levels (X).

Optimal production plans obtained by solving the model under the mentioned scenarios, are compared. As expected, the risk averse firm (RA2) is found to use more (less) of marginally risk reducing (increasing) factor than does the risk neutral firm (RN). Risk neutral firms on the other hand produce more and use more of all inputs than risk averters when zero risk effects are assumed (RA1). The estimated partial demand and supply elasticities are summarized in Table 2. Farmers supply responses are more elastic when factor inputs are allowed to influence production risks (RA2).

Summary and Conclusions

Production uncertainty is important in studying the behavior of risk averse producers and essential to the making of successful agricultural policies. A normative supply response model is developed to analyze the supply behavior of farmers in the irrigated multi-crop sector of Sudan. The model extends the two-moment (E-V) method to incorporate stochastic technology in the programming framework. Stochastic production functions are specified for the 3 crops grown. Risk increasing as well as risk decreasing input-effects are allowed. The IN3SLS procedure is used to correct for possible endogeneity of factor inputs, heteroscedasticity and cross-equations correlations in estimating the parameters of the mean and variance of yield functions.

Farmers supply responses are simulated under different behavioral and structural specifications of the model. The results show that supply responses are more elastic when the risk effects of factor inputs are taken into consideration. Less (more) of the marginally risk increasing (decreasing) factors are demanded under risk aversion. It follows that if farmers have risk preferences, it is then very important to adequately investigate the risk effects of the policy instruments to be employed to influence change, control farmers responses and promote the desired adjustments in agricultural supply. And thus the significance of studying the risk attributes of the technological opportunities and production choices open to agricultural producers.

Table 1: Parameters of the Programming Model(1, 2)

A) Mean yield functions (equation 4)

Cotton: $Y_C = 1.23 * CF^{.22} * CH^{.185} * \exp\ (.01 * (CF/CH))$

Sorghum: $Y_D = 1.74 * DF^{(.24 - .0003 * DH)} * DH^{.14}$

Groundnuts: $Y_G = 1.94 * GWF^{.11} * GWH^{.21} * GHF^{.12} * GHH^{.38}$ (3)

B) Variance of yield functions (equation 5)

Cotton: $v_{cc} = 210 - .68 * CF + .23 * CH - .0003 * CF * CH$

Sorghum: $v_{dd} = 129 - .43 * DF + .15 * DH - .001 * DF * DH$

Groundnuts: $v_{gg} = 388 + .26 * GWF + .31 * GHF - .23 * GWH - .29 * GHH$

C) Covariances (v_{ij} of equation 5):

Cotton (C)	Sorghum (D)	Groundnuts (G)	
v_{cc}	158	330.5	C
v_{dd}	215		D
		v_{gg}	G

D) Net returns per unit of output:

	Cotton	Sorghum	Groundnuts
in Sudanese Pounds	30/kanter	15/sack	12/sack

E) Resources:

	Area	Family labor	Hired Labor
i. constraints levels	22 fed.	250 hr/fed.	460 hr/fed.

ii. input prices

PH	28	piasters/hr. (4)
PGWH	30	piasters/hr.
PGHH	35	piasters/hr.

Table 1, continued

(1) The estimated mean and variance of yield functions are evaluated at average sowing dates and farming years to focus on allocation of the labor inputs.

(2) Y_C, Y_D, Y_G represent yield of cotton, sorghum and groundnuts respectively. CF, CH, DF and DH measure hours of family (F) and hired (H) labor per unit area of cotton (C) and sorghum (D) respectively.

(3) Separability in weeding and harvesting family (WF, HF) and hired labor (WH, HH) was not rejected in groundnut production (GN).

(4) The wage rate for hired labor in cotton and sorghum production is given by PH, whereas PGWH and PGHH give the wage rate for weeding and harvesting hired labor in groundnuts.

Table 2: Supply Response Elasticities

	Cotton			Groundnuts			Sorghum		
	C(1)	FL(2)	HL(2)	G	FL	HL	D	FL	HL
P_C									
RA1	.19			-.14			-.11		
RA2	.24			-.16			-.16		
RN	.18			-.14			-.10		
P_G									
RA1	-.17			.16			-.07		
RA2	-.20			.19			-.11		
RN	-.15			.16			-.04		
P_D									
RA1	-.25			-.06			.45		
RA2	-.31			-.07			.48		
RN	-.25			-.06			.44		
WH(3)									
RA1		.12	-.15		.15	-.20		.07	-.10
RA2		.14	-.20		.17	-.23		.09	-.12
RN		.11	-.13		.15	-.18		.07	-.09

(1) Yield function

(2) Hours per feddan

(3) Wage rate (Ls/Hour)

Bibliography

Anderson, J. R. "Sparse Data, Climatic Variability and Yield Uncertainty in Response Analysis." Amer. J. of Agr. Econ. 55 (1973):77-82.

Antle, J. M. "Econometric Estimation of Producers' Risk Attitudes." Amer. J. Agr. Econ. 69, August (1987):509-522.

__________. "Testing the Stochastic Structure of Production: A Flexible Moment Based Approach." J. of Bus. and Econ. Statist. 1 (1983):192-201.

Antle, J. M., and W. J. Goodger. "Measuring Stochastic Technology: The Case of Tulane Milk Production." Amer. J. Agr. Econ., August (1984):342-350.

Baron, D. P. "Price Uncertainty, Utility and Industrial Equilibrium in Pure Competition." International Econ. Rev. October (1970):463-480.

Batra, R. N. and A. Ullah. "Competitive Firm and the Theory of Input Demand Under Price Uncertainty." J. Polit. Econ. May/June 1974.

Blair, R. D., and R. Lusky. "A Note on the Influence of Uncertainty on Estimation of Production Function Models," Journal of Econometrics 3 (1975):391-394.

Chen, Joyce T. "Quadratic Programming for Least Cost Feed Formulations under Probabilistic Protein Constraints." Amer. J. Agr. Econ. 55 (1973):175-83.

Day, R. H. "Probability Distribution of Field Crops." Jour. of Farm Economics 47 (1965):713-41.

de Janvry, A. "Optimal Levels of Fertilization Under Risk: The Potential for Corn and Wheat Fertilization Under Alternative Price Policies in Argentina." Amer. J. Agr. Econ. 54 (1972):1-10.

__________. "The Generalized Power Production Function." Amer. J. Agr. Econ. (1972):234-237.

Freund, R. J. "The Introduction of Risk into a Programming Model." Econometrica 24 (1965):253-63.

Fuller, W. A. "Stochastic Fertilizer Production Functions for Continuous Corn." J. Farm Econ. 47 (1965):105-119.

Griffith, W. E. and J. Anderson. "Using Time Series and Cross-Section Data to Estimate a Function with Positive and Negative Marginal Risks." J. Amer. Stat. Assoc. 77 (1982):529-536.

Hassan, R. M., B. D'Silva and A. Hallam. "Testing for Decreasing Risk Aversion in Traditional Farming: The Case of Sudan." (under review Cambridge Journal of Economics), 1987a.

__________, A. Hallam and B. D'Silva. "Measuring Stochastic Technology for the Multi-product Firm: The Irrigated Farms of Sudan." (under review Southern Economic Journal), 1987b.

Hazell, P. "A Linear Alternative to Quadratic and Semi-variance Programming for Farm Planning under Uncertainty." Amer. J. Agr. Econ. 53 (1971):53-62.

Just, R. E. and R. Pope. "Stochastic Specification of Production Functions and Econometric Implications." Journal of Econometrics 7 (1978):67-86.

__________. "Production Function Estimation and Related Risk Considerations." Amer. J. Agr. Econ. 61 (1979):276-84.

Just, R. E., D. Zilberman, and E. Hochman. "Estimation of Multicrop Production Functions." AJAE (November 1983):770-780.

Kramer, R., W. McSweeny, and R. Starvos. "Soil Conservation with Uncertain Revenues and Input Supplies." Amer. J. Agr. Econ. 65 (1983):694-702.

Leland, H. E. "The Theory of the Firm Facing Uncertain Demand." Amer. Econ. Rev. LXII (1972):278-91.

Marschak, J., and W. H. Andrews. "Random Simultaneous Equations and the Theory of Production." Econometrica (Oct. 1965):814-501.

Markowitz, H. M. "Portfolio Selection: Efficient Diversification of Investments." Cowels-Wiley, New York, 1959.

Mundlak, Y., and I. Hock. "Consequences of Alternative Specifications in Estimation of Cobb-Douglas Production Functions." Econometrica (1955):143-205.

Pfouts, R. W. "The Theory of Cost and Production in the Multi-product Firm." Econometrica (1961):650-58.

Pope, R. and R. Kramer. "Production Uncertainty and Factor Demands for the Competitive Firm." South Econ. J. 46 (1979):489-501.

Pope, R., Chavas, J. P. and Just, R. E., "Economic Welfare Evaluation for Producers Under Uncertainty," AJAE, 65 (1983), pp. 98-107.

Sandmo, A. "On the Theory of the Competitive Firm Under Price Uncertainty." Amer. Econ. Rev. LXI (1971).

Shumway, C. R., R. D. Pope, and E. K. Nash. "Allocatable Fixed Inputs and Jointness in Agricultural Production: Implications for Economic Modelling." AJAE (February 1984):72-78.

Wiens, T. B. "Peasant Risk Aversion and Allocative Behavior: A Quadratic Programming Experiment." Amer. J. Agr. Econ. 58 (1976):629-35.

Zellner, A., J. Kmenta, and J. Dreze. "Specification and Estimation of Cobb-Douglas Production Function Models." Econometrica 34 (1966):784-95.