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LAND RENTS AND PRICES : AN ECONOMETRIC TEST OF THE CAPITALIZATION FORMULA

by

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ABSTRACT

This paper uses cross-equation restrictions on the parameters of vector autoregressions of land prices and rents, implied by the capitalization formula, to test its validity in a mid-western state. The formula was found to be valid for the period 1921-1953 but was not supported by data for the subsequent period 1954-1986.

Introduction

The sharp decline in land prices during this decade has led to a critical re-examination of land pricing models. In the words of one author "....increased analysis of land prices apparently was precipitated by the perceived divergence between the time path of rents and land prices in recent years" (Oscar Burt, p 10). Such perceptions, being at odds with the well known capitalization formula, have motivated the search for alternative models[1]. However,there has been no direct test of the capitalization formula itself. The excercise is attempted in this paper.

The validity of the capitalization formula has been indirectly refuted by two recent papers. Using the general equilibrium asset pricing formula, Vasavada and White estimate the pricing equation of land under the assumption that agents have constant relative risk aversion utility functions[2]. They find the risk aversion parameter to be significant and reject the hypothesis of risk neutrality. Another paper, by Featherstone and Baker, finds a "...tendency towards bubbles..." as a possible cause for the presumed inconsistency between land rents and prices. Neither of these papers, however, directly test for consistency between land prices and rents as implied by the capitalization formula.

The capitalization formula used to evaluate land prices is among the simplest dynamic stochastic models of economics. While there have been many studies about the validity of the formula for other capital assets, like bonds and stocks we do not find any formal test of the formula applied to agricultural assets. This paper describes an econometric test of the capitalization formula and applies it to the agricultural land market in Minnesota for the period 1921-86. Implications of the present value model are tested using stationary time series analysis for the bivariate stochastic process of land rents and prices. The model imposes specific restrictions on the relation between the time series of rents and prices. The restrictions, which occur in non-linear form, are then tested using the maximum likelihood estimation procedure.

Methodology

Let A_t be the land price at time t. Then the capitalization formula or the present value model of land prices asserts that

$$A_t = \beta \sum_{i=0}^{\infty} \beta^i E(R_{t+i}|\Omega_t) \qquad (1)$$

where R_t is the cash rent paid during period t, β is the constant discount rate and $E(\cdot|\Omega_t)$ is the conditional expectations operator, conditional on information set Ω_t available in period t. A_t and R_t are expressed in real terms. The right hand side of equation (1) is the present value of expected rents and is often referred to as the market fundamental of the price of the underlying asset.

Several test procedures have been proposed in the literature to test the present value model of equation (1). These include the single-equation regression test, the test of cross-equation restrictions on a vector autoregression, and the variance bound test. This paper is based on the second method which assumes that land prices and cash rents can be described by a bivariate stochastic process.

The test procedure is briefly as follows. Equation (1) which states that land value is the discounted sum of expected future cash rents, implies, that the conditional expectations, at t-1, of A_t and R_{t+j} (j=0,1,2,.......) are related in a certain manner (equation (9) in the text below). The conditional expectations are obtained from a vector autoregression of land rents and prices. Substituting them in equation (9), we obtain the restrictions on the parameters of the vector autoregression. These restrictions, which are a direct consequence of equation (1), are then tested by the likelihood ratio test.

To derive the restrictions, the first step is to express the stochastic process of land rents and prices in vector autoregression form. But, to be represented by a vector autoregression, the stochastic process has to be stationary . Since A_t and R_t, in equation (1), are non- stationary, we take the first difference of the process (A_t, R_t) to obtain stationarity. By Wold's theorem, the vector autoregression of finite order for $(\Delta A_t, \Delta R_t)$ exists and is given by

$$\begin{aligned} \Delta R_t &= \sum_{i=1}^{m} a_{1i} \Delta R_{t-i} + \sum_{i=1}^{m} b_{1i} \Delta A_{t-i} + u_t \\ \Delta A_t &= \sum_{i=1}^{m} a_{2i} \Delta R_{t-i} + \sum_{i=1}^{m} b_{2i} \Delta A_{t-i} + v_t \end{aligned} \tag{2}$$

where u_t and v_t are innovations. Equation (2) can be written compactly as

$$x_t = \theta x_{t-1} + \epsilon_t, \tag{3}$$

where

$$x_t = \begin{vmatrix} \Delta R_t \\ \Delta R_{t-1} \\ \vdots \\ \Delta R_{t-m+1} \\ \Delta A_t \\ \Delta A_{t-1} \\ \vdots \\ \Delta A_{t-m+1} \end{vmatrix} \qquad \epsilon_t = \begin{vmatrix} u_t \\ 0 \\ \vdots \\ 0 \\ v_t \\ 0 \\ \vdots \\ 0 \end{vmatrix}$$

$$\theta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1m} & b_{11} & b_{12} & \cdots & b_{1m} \\ 1 & 0 & & 0 & 0 & 0 & & 0 \\ 0 & 1 & & 0 & 0 & 0 & & 0 \\ \vdots & & & & & & & \\ a_{21} & a_{22} & \cdots & a_{2m} & b_{21} & b_{22} & \cdots & b_{2m} \\ 0 & 0 & & 0 & 1 & 0 & & 0 \\ 0 & 0 & & 0 & 0 & 1 & & 0 \\ \vdots & & & & & & & \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots\ 1 & 0 \end{vmatrix}$$

The next step is to obtain the conditional expectations of ΔR_t and ΔA_t in terms of the parameters of the vector autoregression (3). Denote c as the (1x2m) row vector with one in first column, zeros elsewhere, and d as the (1x2m) row vector with one in the (m+1)st column, zeros elsewhere. Then $\Delta R_t = cx_t$, and $\Delta A_t = dx_t$. Using (3) we get

$$\Delta R_t = c\theta x_{t-1} + u_t$$

$$\Delta A_t = d\theta x_{t-1} + v_t \qquad (4)$$

Now using equation (3) x_{t+j} can be expressed recursively as

$$x_{t+j} = \theta^{j+1} x_{t-1} + \theta^{j} \epsilon_{t} + \theta^{j-1} \epsilon_{t+1} + \ . \ . \ + \epsilon_{t+j} \quad (5)$$

Then the conditional expectation of x_{t+j} based on the information at time t-1 is,

$$Ex_{t+j} | \theta_{t-1} = \theta^{j+1} x_{t-1} \qquad (6)$$

where $\theta_t = (R_t, R_{t-1}, \ldots R_{t-m+1}, A_t, A_{t-1}, \ldots A_{t-m+1}) \subset \Omega_t$.

Using the expression for $(\Delta R_t, \Delta A_t)$ in (4) and applying (5), we get

$$E\Delta A_t | \theta_{t-1} = d\theta x_{t-1} \text{ and } E\Delta R_{t+j} | \theta_{t-1} = c\theta^{j+1} x_{t-1} (j \geq 0). \quad (7)$$

Finally to obtain the restrictions express equation (1) in first difference form. This gives

$$(A_t - A_{t-1}) = \beta((R_t - R_{t-1}) + \beta(E_t R_{t+1} - E_{t-1} R_t)$$

$$+ \dots + \beta^i(E_tR_{t+i} - E_{t-1}R_{t+i-1}) + \;\; .. \} \quad (8)$$

where E_t denotes $E(\cdot|\Omega_t)$.

If we project both sides of (7) on θ_{t-1}, we get

$$E\Delta A_t|\theta_{t-1} = \beta(E\Delta R_t|\theta_{t-1} + \beta E\Delta R_{t+1}|\theta_{t-1} + \beta^2 E\Delta R_{t+2}|\theta_{t-1} + \,.. \\ + \beta^i E\Delta R_{t+i}|\theta_{t-1} + \dots \;). \quad (9)$$

Then, using equation (7), (9) can be rewritten as,

$$d\Theta x_{t-1} = \beta(c\Theta + \beta c\Theta^2 + \beta^2 c\Theta^3 + \,..\, + \beta^i c\Theta^{i+1} + \,..)x_{t-1} \quad (10).$$

Rearranging (9) gives,

$$d\Theta = \beta c\Theta \sum_{i=0}^{\infty} \beta^i \Theta^i \qquad (11)$$

or

$$d\Theta = \beta c\Theta(I - \beta\Theta)^{-1} \qquad (12)$$

Equation (12) is nothing more than the capitalization formula expressed in terms of the parameters of the bivariate autoregression of $(\Delta R_t, \Delta A_t)$. Therefore we can test the validity of the formula using the testable compact restrictions of (12). The maximum likelihood algorithm estimating the vector autoregression (2) under restriction (12) is given in Sargent (1979,b). The procedure is as follows. First, estimate by ordinary least squares the first row of Θ,i.e., estimate the first equation of (2). Then, the (m+1)st row of Θ, i.e, the second equation of (2), is calculated by an iterative procedure. Form a preliminary estimate of Θ ,call Θ_0, by setting the elements of row (m+1) to zero and all other rows to their known values. Then, at iteration i+1, calculate the (m+1)st row of Θ , as

$$d\Theta_{i+1} = \beta c\Theta_i(I - \beta\Theta_i)^{-1}$$

where Θ_i is the estimate of Θ on the i-th iteration. At each step in forming

θ_i, leave the other rows of θ at their initial values and recalculate θ again and iterate until matrix θ converges. The condition for convergence is that roots of $\beta\theta$ be less than one in modulus. This two step procedure computes the a_{2i}'s and b_{2i}'s of (2) that satisfy (12) as a function of the a_{1i}'s and b_{1i}'s. Denote the solution to the iteration as the set function

$$(a_2,b_2) = \phi(a_1,b_1) \qquad (13)$$

ϕ maps the a_{1i}'s and b_{1i}'s into a set of a_{2i}'s and b_{2i}'s that satisfy restriction (12).

If we assume that (u_t,v_t) is bivariate normal, the likelihood function of a sample of (u_t,v_t) for $i=1,\ldots,T$ is

$$L(a_1,b_1,a_2,b_2,V|\{\Delta R_t\},\{\Delta A_t\}) = (2\pi)^{-T}|V|^{-T/2}\exp(-\frac{1}{2}\sum_{t=1}^{T} e_t V^{-1} e_t'), \qquad (14)$$

where $e_t = (u_t,v_t)'$, $V = E\, e_t e_t'$. Maximizing (14) without any restriction on the parameters of the vector autoregression gives least square estimates of (13).

Under restriction (12), or equivalently (13), the likelihood function (14) becomes a function of (a_1,b_1). We can use the convenient result (Wilson,1973) that maximum likelihood estimates with an unknown V are obtained by minimizing the determinant of the estimated V (denoted by $\hat{V}$) with respect to the a_{1i}'s and b_{1i}'s.

$$|\hat{V}| = |\frac{1}{T}\sum_{t=1}^{T} e_t(a_1,b_1)e_t(a_1,b_1)'|, \qquad (15)$$

where the e_t's are functions of the a_1's and b_1's by virtue of them being calculated from (2) with (12) being imposed. Broyden- Fletcher- Goldfarb-Shanno(BFGS) nonlinear minimization algorithm is used to minimize (15) numerically. The least squares estimates of a_1's and b_1's are used as the starting values.

Let $|V_u|$ be the determinant of the estimated variance-covariance matrix of the residuals of the unrestricted maximum likelihood estimate of (14). Also let $|V_r|$ be the value of (15) under the restriction (12). Then under the null hypothesis that the capitalization formula holds, the likelihood ratio statistic $T\{\log_e|V_r|-\log_e|V_u|\}$ is asymptotically distributed as $\chi^2(2m)$ (See Wilson(1973), p.80) . High values of the likelihood ratio lead to the rejection of the restriction (12) that are implied from the present value formula.

Empirical results

data

Data on land values and cash rents were obtained from the USDA's land value surveys. The data set contains observations from 1921 to 1986. The original survey is conducted per crop reporting district and the data are compiled as state averages[3]. The sample is divided into two sub-periods of 1921-1953 and 1954 -1986 in order to get two test periods with sufficient number of observations. In this study land values and cash rents have been deflated by the implicit price deflator with 1953 as base year.

estimation results

The χ^2 test suggested by Sims was used to determine the lag length of the autoregression. Lag lengths of 6 and 4 were found to be appropriate for the first and second period respectively. The discount rate β is chosen to be .97 (we tried the alternative values of .96, and .95 but the results were not affected) . Table 1 and 2 report estimates of equation (2) under restriction (12) for the two subperiods. The table contains the unconstrained estimates of

Figure 1

Value of Land

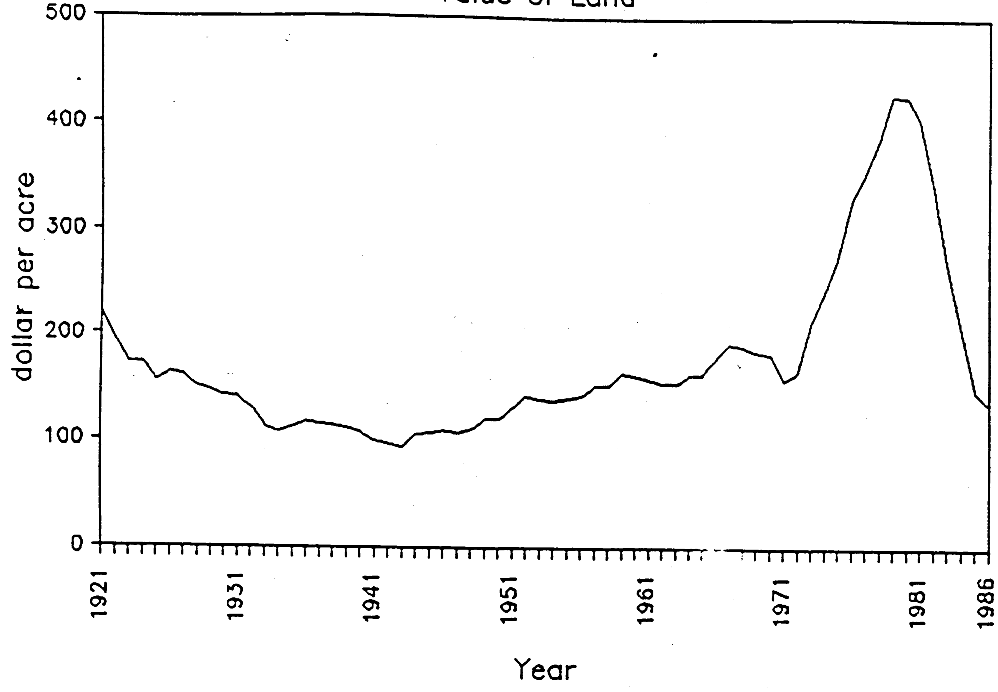
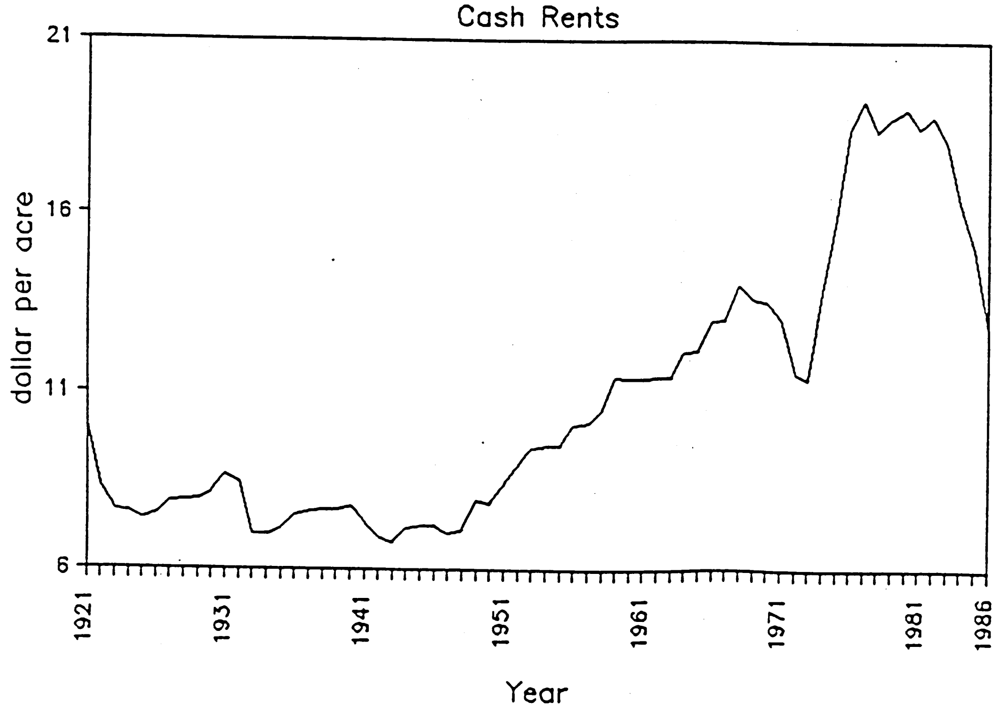


Figure 2

Cash Rents



the bivariate autoregression (2) and the row sums of a_{1i}'s, b_{1i}'s, a_{2i}'s and b_{2i}'s. The likelihood ratio statistic for testing the restrictions is distributed χ^2 with 12 degrees of freedom for 1921-1953 and 8 degrees of freedom for 1954-1986 and the marginal significance level is .103 and 0.0 approximately for the two periods respectively.

Clearly then, the null hypothesis is accepted in the first subperiod and rejected in the second subperiod at the 5% level of significance. This suggests that the capitalization formula worked well to describe land prices in the period 1921-1953. But the data does not support a similar characterization for the subsequent period.

Table 1
Estimates of Bivariate Autoregression
Unrestricted and Restricted (1921-1953)

i	1	2	3	4	5	6	Row Sum
			Unrestricted Estimates				
a_{1i}	-.263	.282	.131	-.405	-1.078	-.345	-1.678
b_{1i}	.039	-.052	-.030	.004	.061	-.001	.021
a_{2i}	-7.025	-4.015	-4.343	3.648	-4.985	3.237	-13.483
b_{2i}	.840	.001	.240	-.049	.292	-.226	1.098
			$\lvert\hat{V}\rvert = 9.87$				
			Maximum Likelihood Estimates				
a_{1i}	.128	.489	.339	-.685	-.826	-.554	-1.109
b_{1i}	-.012	-.053	-.044	.008	.045	.013	-.043
a_{2i}	-.420	-.518	-.832	-1.077	-.741	-.304	-3.892
b_{2i}	-.027	-.029	-.009	.035	.032	.007	.009
			$\lvert\hat{V}\rvert = 20.66$				

Likelihood ratio statistics = 18.45
Marginal significance level = .1027

Table 2

Estimates of Bivariate Autoregression
Unrestricted and Restricted (1954-1986)

i	1	2	3	4	Row Sum
			Unrestricted Estimates		
a_{1i}	.061	.012	-.255	-.481	-.663
b_{1i}	.025	.003	-.003	.012	.037
a_{2i}	-3.605	7.915	.442	4.994	9.746
b_{2i}	.928	-.079	.177	-.809	.217
			$\|\hat{V}\| = 1535.57$		
			Maximum Likelihood Estimates		
a_{1i}	.044	-.167	-.274	-.589	-.986
b_{1i}	.018	.010	-.001	.018	.045
a_{2i}	-.489	-.516	-.435	-.298	-.76
b_{2i}	.023	.014	.009	.009	.055
			$\|\hat{V}\| = 6620.50$		

Likelihood ratio statistics = 43.8
Marginal significance level = .0000

Summary and Concluding Remarks

The objective of our study was to test the validity of the widely used capitalization formula of land prices. The formula imposes cross-equation restrictions on the parameters of vector autoregression of land rents and prices on their past values. The null hypothesis, that the restrictions are satisfied, was tested for the adjacent periods 1921-1953 and 1954-86 using data on farmland prices and rents for Minnesota.

Our results reveal that the land price deviated from its market fundamental in the post-war period (1953-1986). For the earlier period, the null hypothesis is accepted. Comparison of this result with previously published research is difficult because, to the best of our knowledge, no other study has directly tested for the validity of the present value formula in the land market. Other studies[4] which document a strong relationship between land prices and rents are not necessarily inconsistent with our result. But the assumption that land prices are determined only by expectations of future rents is not supported by the data.

As the figures on page 8 show, the second period is marked by increased volatility in land prices and rents after about 1971. It is, of course, well known that agricultural activity expanded in the 1970's on the strength of foreign markets. The expansion was largely financed by farmers taking on new debt which was secured by high and rising land values. Between 1976 and 1980, land values in Minnesota shot up by an astonishing 49%. It is tempting to conclude, on the basis of the results in this paper, that the rising land values were spurred on by the prospect of capital gains.

The rejection of the capitalization formula is, however, consistent with at least, three possible interpretations .

1. The test used in the paper assumes a constant discount rate. It is possible that a present value model with time varying discount rates is consistent with data.

2. The land price may not be equal to its fundamental value due to the existence of a speculative bubble. This refers to a situation where self-fulfilling expectation of price changes result in actual price changes independent of market fundamentals. It should be noted that the no-arbitrage condition of efficient markets is consistent with the existence of a rational speculative bubble[5]. For this reason rejection of the capitalization formula should not be interpreted as a rejection of the market efficiency hypothesis.

3. It is also possible that agents are not rational. Shiller for instance believes that the excess volatility of stock prices relative to dividends is best explained by 'fads' or changes in mass psychology in the market.

Since each of these hypotheses have different implications for the behaviour of the land market, the results reported in this paper should be considered preliminary to a larger investigation capable of sorting out the issues.

Footnotes

1. For a recent review of research, see Featherstone and Baker.
2. The general equilibrium pricing equation reduces to the capitalization formula if agents are risk neutral.
3. It should be noted that the weighting scheme for the computation of state averages was changed from 1972 in order to correspond to the 1974 census for agricultural land in farms. A minor change also occured in 1984.
4. See Robison, Lins and VenkataRaman and the papers cited therein, pp 795.
5. See Blanchard and Watson.

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